FUZZY OPTIMIZATION AND MODELLING 2(2) (2021) 12-21
E-ISNN: 2676-7007

Contents lists available at FOMJ

Fuzzy Optimization and Modelling

Journal homepage: http://fomj.qaemiau.ac.ir/

Paper Type: Research Paper

Optimistic and Pessimistic Fuzzy Data Envelopment Analysis: Empirical Evidence from Tehran Stock Market

Pejman Peykani[a,*], Mohammad Namakshenas[a], Nasim Arabjazi[b], Fatemeh Shirazi[c], Neda Kavand[b]

[a] *School of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran*
[b] *Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran*
[c] *Department of Industrial Engineering, Amirkabir University of Technology, Tehran, Iran*

A R T I C L E I N F O

Article history:
Received 23 May 2021
Revised 26 June 2021
Accepted 3 July 2021
Available online 3 July 2021

Keywords:
Fuzzy DEA
Possibility Theory
Stock Market
Data Ambiguity

A B S T R A C T

In this paper, the fuzzy chance-constrained data envelopment analysis (FCCDEA) approach is presented for stock evaluation and portfolio selection under data ambiguity. To propose FCCDEA method, data envelopment analysis (DEA), possibilistic programming (PP), and chance-constrained programming (CCP) approaches are applied. It should be noted that FCCDEA models can be used by decision makers (DMs) under optimistic and pessimistic viewpoints. To show the applicability of the proposed fuzzy chance-constrained DEA approach, FCCDEA models based on possibility and necessity measures are implemented in a real-life case study from Tehran stock market. The results show the efficacy of the proposed FCCDEA approach for stock assessment in the presence of fuzzy data.

1. Introduction

Portfolio selection is one of the important and practical areas in financial market and investment issues. So far, numerous researchers and investors have attempted to present different models and approaches in this field [14, 15, 33, 44, 59, 66, 69]. One of the most important points in proposing models and approaches for stock portfolio selection is to pay attention to the multi-criteria feature of this issue. In other words, make a decision to identify and purchase good stocks can be difficult since there are many attributes from different financial perspectives including liquidity, asset utilization, valuation, leverage, profitability and growth must be considered simultaneously [20, 22, 28, 32, 36, 37, 39]. In recent years, multi-criteria decision making (MCDM) approaches are applied for portfolio selection problem by decision-makers (DMs) and investors. Data envelopment analysis (DEA) is one of the popular and applicable MCDM methods that is widely used in financial market and investment problems. DEA approach measures the relative efficiency of peer decision-making units (DMUs) considering the multiple inputs and multiple outputs [1, 2, 3, 6, 25, 24, 40, 43, 53, 70].

[*] Correspondig author
E-mail address: pejman.peykani@yahoo.com (Pejman Peykani)

DOI: 10.30495/fomj.2021.1931398.1028

As a result, DEA models can be employed in portfolio selection problem by calculating efficiency of stocks to identify good (desirable) stocks and filter bad (undesirable) stocks. Another important point that should be considered in the proposed approach for portfolio selection is the uncertainty of financial data. Because, financial data such as rate of return, rate of liquidity, and risk measures, most of the times are tainted by uncertainty and ambiguity [21, 27, 31, 35, 38, 42, 60]. Accordingly, it is essential to implement an extended DEA approach that can assess stocks in the presence of imprecise and vague data. Fuzzy chance-constrained data envelopment analysis (FCCDEA) is one of the applicable and popular approaches that is capable to be employed for performance measurement of DMUs under ambiguity [5, 8, 9, 19, 26, 41, 54, 55, 65, 67, 68].

Therefore, in this paper, two FCCDEA models based on optimistic and pessimistic viewpoints are implemented in a real-world case study from Tehran stock exchange for stock assessment in the presence of fuzzy data. It should be noted that possibilistic programming and chance-constrained programming approaches are used to deal with data ambiguity. Also, the experimental results are evaluated to show the efficacy and applicability of fuzzy DEA approach in stock market. The remainder of this study is organized as follows. The modeling of fuzzy data envelopment analysis approach based on possibility and necessity measures will be proposed in Section 2. Then, the implementation of fuzzy DEA models in real-life case study will be discussed in Section 3. Finally, conclusions as well as some directions for future research are introduced in Section 4.

2. Optimistic and Pessimistic Fuzzy DEA Models

Suppose that there are n homogenous decision-making units $\mathrm{DMU}_k\,(k=1,\dots,n)$ that convert m inputs $x_{ik}\,(i=1,\dots,m)$ into s outputs $y_{rk}\,(r=1,\dots,s)$, and DMU_p is the DMU under evaluation. Also, the non-negative weights $v_i\,(i=1,\dots,m)$ and $u_r\,(r=1,\dots,s)$ are assigned to inputs and outputs, respectively. Model (1) presents classic DEA approach under constant return to scale assumption that inputs and outputs are tainted by ambiguity:

$$\begin{aligned}
\text{Max} \quad & \sum_{r=1}^{s} \tilde{y}_{rp} u_r && (1)\\
\text{S.t.} \quad & \sum_{r=1}^{s} \tilde{y}_{rk} u_r - \sum_{i=1}^{m} \tilde{x}_{ik} v_i \le 0, \quad \forall k = 1,\dots,n \\
& \sum_{i=1}^{m} \tilde{x}_{ip} v_i = 1 \\
& v_i, u_r \ge 0, \quad \forall i = 1,\dots,m, \quad \forall r = 1,\dots,s
\end{aligned}$$

Now suppose that inputs and outputs have a trapezoidal fuzzy distribution, and δ is a confidence level for satisfying the fuzzy chance-constraints. Accordingly, fuzzy DEA models based on possibility and necessity approaches are presented as Models (2) and (3), respectively:

$$\begin{aligned}
\text{Max} \quad & F && (2)\\
\text{S.t.} \quad & \sum_{r=1}^{s} \left((\delta) y_{rp}^{3} + (1-\delta) y_{rp}^{4} \right) u_r \ge F \\
& \sum_{r=1}^{s} \left((1-\delta) y_{rk}^{1} + (\delta) y_{rk}^{2} \right) u_r - \sum_{i=1}^{m} \left((\delta) x_{ik}^{3} + (1-\delta) x_{ik}^{4} \right) v_i \le 0, \quad \forall k = 1,\dots,n \\
& \sum_{i=1}^{m} \left((1-\delta) x_{ip}^{1} + (\delta) x_{ip}^{2} \right) v_i \le 1 \\
& v_i, u_r \ge 0, \quad \forall i = 1,\dots,m, \quad \forall r = 1,\dots,s
\end{aligned}$$

$$
\begin{aligned}
&\text{Max} \quad G \qquad (3)\\
&\text{S.t.} \quad \sum_{r=1}^{s}\left(\left(\delta\right)y_{rp}^{1}+\left(1-\delta\right)y_{rp}^{2}\right)u_{r}\geq G\\
&\sum_{r=1}^{s}\left(\left(1-\delta\right)y_{rk}^{3}+\left(\delta\right)y_{rk}^{4}\right)u_{r}-\sum_{i=1}^{m}\left(\left(\delta\right)x_{ik}^{1}+\left(1-\delta\right)x_{ik}^{2}\right)v_{i}\leq 0, \quad \forall k=1,\ldots,n\\
&\sum_{i=1}^{m}\left(\left(1-\delta\right)x_{ip}^{3}+\left(\delta\right)x_{ip}^{4}\right)v_{i}\leq 1\\
&v_{i},u_{r}\geq 0, \qquad \forall i=1,\ldots,m, \quad \forall r=1,\ldots,s
\end{aligned}
$$

It is worth noting that Models (2) and (3) can be applied by decision makers (DMs) under optimistic and pessimistic viewpoints, respectively [4, 18, 29, 34, 58]. Also, above FDEA models can be presented to deal with triangular fuzzy distribution in a similar manner [57, 63, 64].

3. A Real-World Case Study

In this section, the real data set for 20 stocks are extracted from Tehran stock exchange (TSE). Accordingly, two inputs including quick ratio (QR) and leverage ratio (LR) as well as three outputs including rate of return (RoR), rate of liquidity (RoL), and earning per share (EPS) are considered for implementation of FDEA approach. Trapezoidal fuzzy data set for QR, LR, RoR, RoL, and EPS are presented in Tables 1 to 5, respectively:

Table 1. Data Set for Quick Ratio

	Stocks	x^1	x^2	x^3	x^4
First Input (I1)	Stock 01	0.39	0.46	0.51	0.59
	Stock 02	0.47	0.56	0.62	0.71
	Stock 03	0.68	0.81	0.90	1.03
	Stock 04	0.85	1.01	1.12	1.28
	Stock 05	0.78	0.92	1.02	1.17
	Stock 06	1.49	1.77	1.96	2.24
	Stock 07	0.85	1.01	1.12	1.28
	Stock 08	0.72	0.85	0.94	1.08
	Stock 09	0.58	0.68	0.75	0.86
	Stock 10	1.02	1.22	1.35	1.54
	Stock 11	0.63	0.75	0.83	0.95
	Stock 12	0.97	1.15	1.28	1.46
	Stock 13	0.58	0.69	0.76	0.87
	Stock 14	0.66	0.78	0.87	0.99
	Stock 15	0.97	1.15	1.27	1.45
	Stock 16	0.81	0.96	1.06	1.21
	Stock 17	0.78	0.93	1.02	1.17
	Stock 18	0.48	0.57	0.63	0.72
	Stock 19	0.80	0.95	1.05	1.20
	Stock 20	0.85	1.00	1.11	1.27

Table 2. Data Set for Leverage Ratio

	Stocks	x^1	x^2	x^3	x^4
Second Input (I2)	Stock 01	3.09	3.67	4.06	4.64
	Stock 02	2.28	2.71	3.00	3.42
	Stock 03	1.82	2.16	2.39	2.73
	Stock 04	1.47	1.75	1.93	2.21
	Stock 05	1.09	1.29	1.43	1.63
	Stock 06	0.21	0.25	0.27	0.31
	Stock 07	0.98	1.17	1.29	1.47
	Stock 08	4.76	5.65	6.24	7.14
	Stock 09	2.40	2.85	3.15	3.60
	Stock 10	0.97	1.15	1.27	1.45
	Stock 11	3.54	4.21	4.65	5.32
	Stock 12	0.75	0.89	0.99	1.13
	Stock 13	1.82	2.16	2.39	2.73
	Stock 14	4.12	4.90	5.41	6.19
	Stock 15	0.67	0.79	0.88	1.00
	Stock 16	0.76	0.90	1.00	1.14
	Stock 17	1.16	1.38	1.52	1.74
	Stock 18	3.85	4.57	5.05	5.77
	Stock 19	1.55	1.83	2.03	2.32
	Stock 20	1.25	1.49	1.65	1.88

Table 3. Data Set for Rate of Return

	Stocks	y^1	y^2	y^3	y^4
First Output (O1)	Stock 01	164.60	195.46	216.03	246.89
	Stock 02	54.87	65.16	72.02	82.31
	Stock 03	218.11	259.01	286.27	327.17
	Stock 04	162.98	193.54	213.91	244.47
	Stock 05	211.30	250.92	277.33	316.95
	Stock 06	13.79	16.37	18.09	20.68
	Stock 07	153.97	182.84	202.09	230.96
	Stock 08	58.79	69.82	77.17	88.19
	Stock 09	126.93	150.73	166.59	190.39
	Stock 10	199.12	236.45	261.34	298.67
	Stock 11	201.73	239.55	264.76	302.59
	Stock 12	250.84	297.87	329.23	376.26
	Stock 13	459.50	545.66	603.10	689.26
	Stock 14	251.42	298.56	329.98	377.12
	Stock 15	144.00	171.01	189.01	216.01
	Stock 16	116.85	138.76	153.36	175.27
	Stock 17	332.15	394.43	435.95	498.22
	Stock 18	176.92	210.09	232.21	265.38
	Stock 19	119.35	141.72	156.64	179.02
	Stock 20	212.11	251.88	278.39	318.16

Table 4. Data Set for Rate of Liquidity

	Stocks	y^1	y^2	y^3	y^4
Second Output (O2)	Stock 01	146.79	174.31	192.66	220.18
	Stock 02	88.22	104.76	115.79	132.33
	Stock 03	98.21	116.62	128.90	147.31
	Stock 04	131.25	155.86	172.27	196.88
	Stock 05	183.10	217.44	240.32	274.66
	Stock 06	33.48	39.76	43.95	50.23
	Stock 07	135.76	161.21	178.18	203.63
	Stock 08	182.29	216.47	239.25	273.43
	Stock 09	150.39	178.59	197.39	225.58
	Stock 10	164.17	194.96	215.48	246.26
	Stock 11	141.66	168.22	185.93	212.49
	Stock 12	175.49	208.40	230.33	263.24
	Stock 13	117.81	139.90	154.63	176.72
	Stock 14	158.69	188.44	208.28	238.03
	Stock 15	139.52	165.67	183.11	209.27
	Stock 16	21.09	25.05	27.69	31.64
	Stock 17	130.97	155.52	171.89	196.45
	Stock 18	145.12	172.33	190.48	217.69
	Stock 19	182.74	217.00	239.85	274.11
	Stock 20	177.38	210.64	232.81	266.07

Table 5. Data Set for Earning per Share

	Stocks	y^1	y^2	y^3	y^4
Third Output (O3)	Stock 01	170.40	202.35	223.65	255.60
	Stock 02	639.20	759.05	838.95	958.80
	Stock 03	554.40	658.35	727.65	831.60
	Stock 04	1021.60	1213.15	1340.85	1532.40
	Stock 05	96.80	114.95	127.05	145.20
	Stock 06	413.60	491.15	542.85	620.40
	Stock 07	797.60	947.15	1046.85	1196.40
	Stock 08	422.40	501.60	554.40	633.60
	Stock 09	244.80	290.70	321.30	367.20
	Stock 10	764.80	908.20	1003.80	1147.20
	Stock 11	532.80	632.70	699.30	799.20
	Stock 12	527.20	626.05	691.95	790.80
	Stock 13	181.60	215.65	238.35	272.40
	Stock 14	1026.40	1218.85	1347.15	1539.60
	Stock 15	977.60	1160.90	1283.10	1466.40
	Stock 16	104.80	124.45	137.55	157.20
	Stock 17	556.80	661.20	730.80	835.20
	Stock 18	323.20	383.80	424.20	484.80
	Stock 19	334.40	397.10	438.90	501.60
	Stock 20	524.00	622.25	687.75	786.00

The results of stocks evaluation based on possibility and necessity approaches are presented in Tables 6 and 7, respectively:

Table 6. The Results of FDEA Model under Possibility Approach

Stocks	$\delta = 0\%$	$\delta = 25\%$	$\delta = 50\%$	$\delta = 75\%$	$\delta = 100\%$
Stock 01	2.269	1.942	1.664	1.427	1.225
Stock 02	2.119	1.814	1.556	1.336	1.149
Stock 03	1.863	1.594	1.367	1.174	1.009
Stock 04	2.258	1.933	1.658	1.424	1.225
Stock 05	2.249	1.928	1.656	1.424	1.225
Stock 06	2.228	1.905	1.633	1.401	1.204
Stock 07	2.201	1.885	1.617	1.389	1.194
Stock 08	1.790	1.533	1.315	1.130	0.971
Stock 09	2.067	1.773	1.523	1.310	1.128
Stock 10	2.120	1.813	1.554	1.333	1.146
Stock 11	1.881	1.609	1.380	1.185	1.018
Stock 12	2.260	1.936	1.662	1.429	1.230
Stock 13	2.250	1.927	1.654	1.421	1.222
Stock 14	2.252	1.931	1.661	1.430	1.233
Stock 15	2.242	1.923	1.656	1.427	1.231
Stock 16	1.095	0.938	0.806	0.693	0.596
Stock 17	2.250	1.925	1.650	1.416	1.217
Stock 18	2.028	1.733	1.484	1.273	1.093
Stock 19	2.222	1.902	1.632	1.402	1.205
Stock 20	2.245	1.926	1.655	1.424	1.226

Table 7. The Results of FDEA Model under Necessity Approach

Stocks	$\delta = 0\%$	$\delta = 25\%$	$\delta = 50\%$	$\delta = 75\%$	$\delta = 100\%$
Stock 01	0.817	0.702	0.603	0.517	0.442
Stock 02	0.765	0.658	0.566	0.485	0.416
Stock 03	0.672	0.578	0.497	0.426	0.365
Stock 04	0.817	0.703	0.603	0.517	0.443
Stock 05	0.816	0.703	0.604	0.519	0.445
Stock 06	0.838	0.719	0.617	0.528	0.452
Stock 07	0.797	0.686	0.590	0.506	0.434
Stock 08	0.649	0.558	0.479	0.411	0.352
Stock 09	0.755	0.649	0.558	0.479	0.411
Stock 10	0.765	0.659	0.567	0.487	0.417
Stock 11	0.679	0.584	0.502	0.431	0.370
Stock 12	0.813	0.700	0.602	0.517	0.443
Stock 13	0.821	0.706	0.606	0.519	0.444
Stock 14	0.815	0.702	0.603	0.518	0.444
Stock 15	0.819	0.705	0.606	0.520	0.447
Stock 16	0.397	0.342	0.294	0.252	0.216
Stock 17	0.824	0.708	0.607	0.520	0.444
Stock 18	0.729	0.628	0.540	0.464	0.398
Stock 19	0.804	0.693	0.596	0.512	0.440
Stock 20	0.816	0.702	0.604	0.520	0.446

As it can be seen from Tables 6 and 7, by increasing the confidence level from 0 to 1, the results of FDEA approach are decreased. Also, as expected, the optimistic and pessimistic results for stock evaluation are obtained from possibility and necessity approaches, respectively. The results indicate on the applicability and efficacy of fuzzy DEA method in stock market.

4. Conclusions and Future Research Directions

In the current study, stocks are evaluated using possibilistic data envelopment analysis approach. It should be explained that possibilistic programming and chance-constrained programming approaches are applied to handle epistemic uncertainty. Finally, fuzzy data envelopment analysis models based on possibility and necessity measures are implemented in a real-world case study from Tehran stock exchange. For the future studies, robust optimization approach can be applied in order to deal with uncertainty of data [7, 10, 16, 17, 23, 30, 51, 52, 56, 61]. Additionally, machine learning models can be used for forecasting and prediction of financial data in portfolio selection problem [11, 12, 13, 45, 46, 47, 48, 49, 50, 62].

Conflict of interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

1. Azadeh, A., & Kokabi, R. (2016). Z-number DEA: A new possibilistic DEA in the context of Z-numbers. *Advanced Engineering Informatics, 30*(3), 604-617.
2. Banker, R.D., Charnes, A., & Cooper, W.W. (1984). Some models for estimating technical and scale inefficiencies in data envelopment analysis. *Management Science, 30*(9), 1078-1092.
3. Charnes, A., Cooper, W.W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. *European Journal of Operational Research, 2*(6), 429-444.
4. Dubois, D., & Prade, H. (1978). Operations on fuzzy numbers. *International Journal of Systems Science, 9*(6), 613-626.
5. Ebrahimnejad, A., & Amani, N. (2021). Fuzzy data envelopment analysis in the presence of undesirable outputs with ideal points. *Complex & Intelligent Systems, 7*(1), 379-400.
6. Emrouznejad, A., & Yang, G.L. (2018). A survey and analysis of the first 40 years of scholarly literature in DEA: 1978–2016. *Socio-Economic Planning Sciences, 61*, 4-8.
7. Ghassemi, A. (2019). *System of systems approach to develop an energy-water nexus model under uncertainty*. Doctoral Dissertation, University of Illinois at Chicago.
8. Ghassemi, A., & Scott, M.J. (2021). A mathematical approach to improve energy-water nexus reliability using a novel multi-stage adjustable fuzzy robust approach. *Progress in Intelligent Decision Science. IDS 2020. Advances in Intelligent Systems and Computing*. Springer, Cham.
9. Ghassemi, A., & Scott, M.J. (2021). Investigating the role of renewable energies in integrated energy-water nexus planning under uncertainty using fuzzy logic. *Progress in Intelligent Decision Science. IDS 2020. Advances in Intelligent Systems and Computing*. Springer, Cham.
10. Ghassemi, A., Hu, M., & Zhou, Z. (2017). Robust planning decision model for an integrated water system. *Journal of Water Resources Planning and Management, 143*(5), 05017002.
11. Kumar, G., Jain, S., & Singh, U.P. (2021). Stock market forecasting using computational intelligence: a survey. *Archives of Computational Methods in Engineering, 28*, 1069–1101.
12. Li, A.W., & Bastos, G.S. (2020). Stock market forecasting using deep learning and technical analysis: a systematic review. *IEEE Access, 8*, 185232-185242.
13. Li, Z., Crook, J., & Andreeva, G. (2017). Dynamic prediction of financial distress using Malmquist DEA. *Expert Systems with Applications, 80*, 94-106.
14. Markowitz, H. (1952). Portfolio selection. *The Journal of Finance, 7*(1), 77-91.
15. Markowitz, H. (1959). *Portfolio Selection: Efficient Diversification of Investments*. New York: John Wiley.

16. Namakshenas, M., & Pishvaee, M.S. (2019). Data-driven Robust Optimization. *Robust and Constrained Optimization: Methods and Applications*, 1-40, Nova Science Publishers, Inc.
17. Namakshenas, M., Pishvaee, M.S., & Mahdavi Mazdeh, M. (2017). Event-driven and Attribute-driven Robustness. *Iranian Journal of Operations Research, 8*(1), 78-90.
18. Peykani, P., & Gheidar-Kheljani, J. (2020). Performance appraisal of research and development projects value-chain for complex products and systems: the fuzzy three-stage DEA approach. *Journal of New Researches in Mathematics, 6*(25), 41-58.
19. Peykani, P., & Mohammadi, E. (2018). Fuzzy network data envelopment analysis: a possibility approach. *The 3th International Conference on Intelligent Decision Science*, Iran.
20. Peykani, P., & Mohammadi, E. (2018). Portfolio selection problem under uncertainty: a robust optimization approach. *The 3th International Conference on Intelligent Decision Science*, Iran.
21. Peykani, P., & Mohammadi, E. (2018). Robust data envelopment analysis with hybrid uncertainty approaches and its applications in stock performance measurement. *The 14th International Conference on Industrial Engineering*, Iran.
22. Peykani, P., & Roghanian, E. (2015). The application of data envelopment analysis and robust optimization in portfolio selection problem. *Journal of Operational Research in Its Applications, 12*(44), 61-78.
23. Peykani, P., Edalatpanah, S.A., Najafi, S.E., Amirteimoori, A., & Ebrahimnejad, A. (2021). Uncertain range directional measure model under deep uncertainty: a robust convex programming approach. *The 2nd International Conference on Challenges and New Solutions in Industrial Engineering and Management and Accounting*, Iran.
24. Peykani, P., Farzipoor Saen, R., Seyed Esmaeili, F.S., & Gheidar-Kheljani, J. (2021). Window data envelopment analysis approach: a review and bibliometric analysis. *Expert Systems,* e12721.
25. Peykani, P., Hosseinzadeh Lotfi, F., Mohammadi, E., & Tehrani, R. (2021). Performance assessment of investment firms under uncertainty. *Financial Knowledge of Securities Analysis, 13*(48), 35-46.
26. Peykani, P., Hosseinzadeh Lotfi, F., Sadjadi, S.J., Ebrahimnejad, A., & Mohammadi, E. (2021). Fuzzy chance-constrained data envelopment analysis: a structured literature review, current trends, and future directions. *Fuzzy Optimization and Decision Making*.
27. Peykani, P., Mohammadi, E., & Emrouznejad, A. (2021). An adjustable fuzzy chance-constrained network DEA approach with application to ranking investment firms. *Expert Systems with Applications*, *166*, 113938.
28. Peykani, P., Mohammadi, E., & Seyed Esmaeili, F.S. (2019). Stock evaluation under mixed uncertainties using robust DEA model. *Journal of Quality Engineering and Production Optimization, 4*(1), 73-84.
29. Peykani, P., Mohammadi, E., Emrouznejad, A., Pishvaee, M.S., & Rostamy-Malkhalifeh, M. (2019). Fuzzy data envelopment analysis: an adjustable approach. *Expert Systems with Applications, 136*, 439-452.
30. Peykani, P., Mohammadi, E., Farzipoor Saen, R., Sadjadi, S.J., & Rostamy-Malkhalifeh, M. (2020). Data envelopment analysis and robust optimization: a review. *Expert Systems, 37*(4), e12534.
31. Peykani, P., Mohammadi, E., Hosseinzadeh Lotfi, F., Tehrani, R., & Rostamy-Malkhalifeh, M. (2019). Performance evaluation of stocks in different time periods under uncertainty: fuzzy window data envelopment analysis approach. *Financial Engineering and Securities Management, 10*(40), 304-324.
32. Peykani, P., Mohammadi, E., Jabbarzadeh, A., & Jandaghian, A. (2016). Utilizing robust data envelopment analysis model for measuring efficiency of stock, a case study: Tehran stock exchange. *Journal of New Research in Mathematics, 1*(4), 15-24.
33. Peykani, P., Mohammadi, E., Jabbarzadeh, A., Rostamy-Malkhalifeh, M., & Pishvaee, M.S. (2020). A novel two-phase robust portfolio selection and optimization approach under uncertainty: a case study of Tehran stock exchange. *Plos One, 15*(10), e0239810.
34. Peykani, P., Mohammadi, E., Pishvaee, M.S., Rostamy-Malkhalifeh, M., & Jabbarzadeh, A. (2018). A novel fuzzy data envelopment analysis based on robust possibilistic programming: possibility, necessity and credibility-based approaches. *RAIRO-Operations Research, 52*(4-5), 1445-1463.
35. Peykani, P., Mohammadi, E., Rostamy-Malkhalifeh, M., & Hosseinzadeh Lotfi, F. (2019). Fuzzy data envelopment analysis approach for ranking of stocks with an application to Tehran stock exchange. *Advances in Mathematical Finance and Applications, 4*(1), 31-43.
36. Peykani, P., Mohammadi, E., Sadjadi, S.J., & Rostamy-Malkhalifeh, M. (2018). A robust variant of radial measure for performance assessment of stock. *The 3th International Conference on Intelligent Decision Science*, Iran.

37. Peykani, P., Namakshenas, M., Kavand, N., Nouri, M., & Rostamy-Malkhalifeh, M. (2021). Mean-absolute deviation-beta portfolio optimization under ambiguity: a real-world case study. *The 2nd International Conference on Challenges and New Solutions in Industrial Engineering and Management and Accounting*, Iran.
38. Peykani, P., Namakshenas, M., Shirazi, F., Jabbarzadeh, A., & Kavand, N. (2021). Possibilistic data envelopment analysis approach for stock evaluation. *The 2nd International Conference on Challenges and New Solutions in Industrial Engineering and Management and Accounting*, Iran.
39. Peykani, P., Nouri, M., & Farrokhi-Asl, H. (2021). Uncertain multi-period portfolio management under fuzzy environment: an alpha-cut method. *The 2nd International Conference on Challenges and New Solutions in Industrial Engineering and Management and Accounting*, Iran.
40. Peykani, P., Rahmani, D., Gheidar-Kheljani, J., Jabbarzadeh, A., & Karimi Gavareshki, M.H. (2021). A novel ranking method based on uncertain DEA model. *The 2nd International Conference on Challenges and New Solutions in Industrial Engineering and Management and Accounting*, Iran.
41. Peykani, P., Seyed Esmaeili, F.S., Rostamy-Malkhalifeh, M., & Hosseinzadeh Lotfi, F. (2018). Measuring productivity changes of hospitals in Tehran: the fuzzy Malmquist productivity index. *International Journal of Hospital Research, 7*(3), 1-17.
42. Peykani, P., Seyed Esmaeili, F.S., Rostamy-Malkhalifeh, M., Hosseinzadeh Lotfi, F., & Tehrani, R. (2019). Fuzzy range directional measure: the pessimistic approach. *The 11th National Conference on Data Envelopment Analysis*, Iran.
43. Ren, T., Zhou, Z., & Xiao, H. (2021). Estimation of portfolio efficiency considering social responsibility: evidence from the multi-horizon diversification DEA. *RAIRO-Operations Research, 55*(2), 611-637.
44. Sehgal, R., & Mehra, A. (2021). Robust reward–risk ratio portfolio optimization. *International Transactions in Operational Research, 28*(4), 2169-2190.
45. Shafiee, M., Hosseinzadeh Lotfi, F., & Saleh, H. (2021). Benchmark forecasting in data envelopment analysis for decision making units. *International Journal of Industrial Mathematics, 13*(1), 29-42.
46. Shahhosseini, M., Hu, G., & Archontoulis, S.V. (2020). Forecasting corn yield with machine learning ensembles. *Frontiers in Plant Science, 11*, 1120.
47. Shahhosseini, M., Hu, G., & Pham, H. (2019). Optimizing Ensemble Weights for Machine Learning Models: A Case Study for Housing Price Prediction. *INFORMS International Conference on Service Science*, 87-97. Springer, Cham.
48. Shahhosseini, M., Hu, G., Huber, I., & Archontoulis, S.V. (2021). Coupling machine learning and crop modeling improves crop yield prediction in the US Corn Belt. *Scientific Reports, 11*(1), 1-15.
49. Shahhosseini, M., Martinez-Feria, R.A., Hu, G., & Archontoulis, S.V. (2019). Maize yield and nitrate loss prediction with machine learning algorithms. *Environmental Research Letters, 14*(12), 124026.
50. Shaverdi, M., & Yaghoubi, S. (2021). A technology portfolio optimization model considering staged financing and moratorium period under uncertainty. *RAIRO-Operations Research, 55*, S1487-S1513.
51. Su, X., Bai, M., & Han, Y. (2021). Robust portfolio selection with regime switching and asymmetric dependence. *Economic Modelling, 99*, 105492.
52. Toloo, M., & Mensah, E.K. (2019). Robust optimization with nonnegative decision variables: a DEA approach. *Computers & Industrial Engineering, 127*, 313-325.
53. Veiga, G.L., de Lima, E.P., Frega, J.R., & Da Costa, S.E.G. (2021). A DEA-based approach to assess manufacturing performance through operations strategy lenses. *International Journal of Production Economics, 235*, 108072.
54. Wu, D.D., Yang, Z., & Liang, L. (2006). Efficiency analysis of cross-region bank branches using fuzzy data envelopment analysis. *Applied Mathematics and Computation, 181*(1), 271-281.
55. Wu, S.C., Lu, T., & Liu, S.T. (2021). A fuzzy approach to support evaluation of fuzzy cross efficiency. *Symmetry, 13*(5), 882.
56. Xidonas, P., Steuer, R., & Hassapis, C. (2020). Robust portfolio optimization: a categorized bibliographic review. *Annals of Operations Research, 292*(1), 533-552.
57. Xu, J., & Zhou, X. (2011). *Fuzzy-Like Multiple Objective Decision Making*. Berlin: Springer.
58. Xu, J., & Zhou, X. (2013). Approximation based fuzzy multi-objective models with expected objectives and chance constraints: application to Earth-Rock work allocation. *Information Sciences, 238*, 75-95.
59. Yaman, I., & Dalkılıç, T.E. (2021). A hybrid approach to cardinality constraint portfolio selection problem based on nonlinear neural network and genetic algorithm. *Expert Systems with Applications, 169*, 114517.

60. Yang, X., Liu, W., Chen, S., & Zhang, Y. (2021). A multi-period fuzzy mean-minimax risk portfolio model with investor's risk attitude. *Soft Computing, 25*(4), 2949-2963.
61. Yanıkoğlu, İ., Gorissen, B.L., & Den Hertog, D. (2019). A survey of adjustable robust optimization. *European Journal of Operational Research, 277*(3), 799-813.
62. Yeh, C.C., Chi, D.J., & Hsu, M.F. (2010). A hybrid approach of DEA, rough set and support vector machines for business failure prediction. *Expert Systems with Applications, 37*(2), 1535-1541.
63. Zadeh, L.A. (1978). Fuzzy sets as a basis for a theory of possibility. *Fuzzy Sets and Systems, 1*(1), 3-28.
64. Zadeh, L.A. (2011). A note on Z-numbers. *Information Sciences, 181*(14), 2923-2932.
65. Zerafat Angiz, M., Mustafa, A., Ghadiri, M., & Tajaddini, A. (2015). Relationship between efficiency in the traditional data envelopment analysis and possibility sets. *Computers & Industrial Engineering, 81*, 140-146.
66. Zhang, Y., Li, X., & Guo, S. (2018). Portfolio selection problems with Markowitz's mean–variance framework: a review of literature. *Fuzzy Optimization and Decision Making, 17*(2), 125-158.
67. Zhou, W., & Xu, Z. (2020). An overview of the fuzzy data envelopment analysis research and its successful applications. *International Journal of Fuzzy Systems, 22*(4), 1037-1055.
68. Zhou, X., Pedrycz, W., Kuang, Y., & Zhang, Z. (2016). Type-2 fuzzy multi-objective DEA model: an application to sustainable supplier evaluation. *Applied Soft Computing, 46*, 424-440.
69. Zhou, Z., Gao, M., Xiao, H., Wang, R., & Liu, W. (2021). Big data and portfolio optimization: a novel approach integrating DEA with multiple data sources. *Omega*, 102479.
70. Zhu, Q., Li, F., Wu, J., & Sun, J. (2021). Cross-efficiency evaluation in data envelopment analysis based on the perspective of fairness utility. *Computers & Industrial Engineering, 151*, 106926.

Peykani, P., Namakshenas, M., Arabjazi, N., Shirazi, F., & Kavand, N. (2021). Optimistic and Pessimistic Fuzzy Data Envelopment Analysis: Empirical Evidence from Tehran Stock Market. *Fuzzy Optimization and Modelling Journal*, 2 (2), 12-21.

https://doi.org/10.30495/fomj.2021.1931398.1028

Received: 23 May 2021 Revised: 26 June 2021 Accepted: 3 July 2021