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Extending Allocation Stages of Fixed Costs Between Decision Making Units Using DEA

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ABSTRACT

A good idea for a decision maker to protect and increase the efficiency of the decision-making units (DMUs) in an organization is to allocate the fixed costs between them based on their efficiencies. Since, data envelopment analysis (DEA) is a suitable method to calculate the efficiency, allocating fixed costs to DMUs based on the two-stage network DEA (NDEA) approach is done by researchers. But due to some limitations (like producing a product in several steps, receiving incomes in several stages and etc.) and some organization necessities, it is impossible to do this allocation just in two-stages. In this paper, we suggest a model for allocating the fixed cost among DMUs in more than two-stage network DEA approach, so that more than allocating the fixed cost, increasing in efficiency is also considered. Also, a benchmark example in reality is presented to illustrate the model and its applications.

1. Introduction

Evaluation and efficiency of decision-making units (DMUs) were strengthened by presenting data envelopment analysis models. Charnes et al. [6] were the first researchers that proposed a model for calculating the efficiency scores of DMUs and after that, a plenty of models have been presented in this issue (for instance Banker and Charnes [4], Cooper et al. [9], Gonzalez-Padron et al. [12]).

In the other hand one of the applications of DEA is to allocate costs among DMUs in an optimized way. Successful managers always believe that allocating budgets, bonuses and fixed costs to the units of the organization under their control, should be optimized. For this reason, the issue of allocation costs is very important. At first Cook and Kress [7] solved the allocation problem using DEA. They considered the fixed cost as an extra input for each DMU. Beasley [3] via DEA and base on efficiency of DMUs presented a nonlinear model for allocating the fixed costs. Amirteimoori and Kordrostami [1] proposed a model of combining the efficiency in variance and Beasly's additional constraints. Cook and Zhu [8] extend Cook and Kress method and

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introduced a new approach to the cost allocation problem. Lin [15] proposed a method to allocate the fixed cost, he shows that Cook and Zhu method has not a practical solution for some of additional specific constrains. Li et al. [14] presented a model based on satisfaction score of DMUs to allocate them the fixed cost. Lin et al. [17] under two assumptions efficiency invariance and zero slack, proposed a new approach for allocating the fixed costs between DMUs. Jahanshahloo et al. [16] presented two methods for fixed cost allocation using DEA; in the first method, the costs are allocated to DMUs in such a way that their efficiency score does not change. In the second method, the costs are allocated in such a way that input and output of all units have a common set of weights; this allocation has the minimum difference with the allocation that has been obtained in first method. Li, Zhu and Liang [13] allocated the fixed costs between DMUs, using DEA game cross efficiency approach. Ghasemi et al. [11] present the fixed resource allocation with the help of DEA. He proposed a new model by determining a common set of weights (CSW). The minimum resources allocated to each DMU were commensurate to the efficiency of that DMU and the share of DMU in the input resources and the output productions. Feng et al. [10] solved the fixed cost allocation problem using DEA approach and based on input and output scales of DMUs, such that in this manner, considered both the input consumption and output production scales .

Another important issue in DEA which has many applications is its applicability for network structures. Most organizations, like banks and commercial complexes companies have a network structure. Regarding this fact that the operational process of most organizations consists of several stags, therefore, the cost allocation using DEA Network (NDEA) is more suitable than other methods. Indeed, use of NDEA and considering the internal structure of DMUs will give better results that are more adapted with reality. In this approach the fixed cost based on the efficiency score, in each stage is allocated to DMUs. Yu et al. [18] in 2016 used NDEA and allocate the fixed cost to DMUs in two-stage. Zhu et al. [19] proposed three procedures with different objectives for allocating the fixed costs in among a set of DMUs based on two-stage NDEA, in these suggestions the fixed costs use as an additional input factor in two stage. An et al. [2] with idea of satisfaction degree and noncooperative game theory, introduced a model for solving the fixed cost allocation problem in two-stage NDEA. Since most organizations and companies are composed of several units and the process of their activities is more than two-stages (like car production companies), therefore, it is necessary to study how the fixed costs is allocated in more than two-stage. In this regard, here, we extend Yu et al. [18] method and allocate the fixed cost to DMUs in more than two-stages .

In this paper in Section 2 we introduce some preliminaries notions, definitions and properties which are required in the main discussion. Section 3 is devoted to the main discussions and the proposed models are presented. A real benchmarking as example is demonstrated in Section 4 to illustrate our approaches, and the final section is devoted to some conclusion remarks.

2. Some preliminaries

For obtaining the efficiency score a decision-making unit such as DMU_O, $o \in \{1,2,...,J\}$, Charnes et al. [6] presented the following model:

$$Max z_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}}$$

$$S.to: \frac{\sum_{r=1}^{s} u_r\, y_{rj}}{\sum_{i=1}^{m} v_i\, x_{ij}} \leqslant 1, j = 1,2,\dots,J;$$

$$u_i \geqslant 0, i = 1,2,\dots,m;$$

$$v_r \geqslant 0, r = 1,2,\dots,s \qquad (1)$$

where x_{ij} is the amount of i_{th} input consumed by DMU_j, y_{rj} is the amount of r_{th} output produced by DMU_j, u_r is the given weight to the r_{th} output and v_i is the weight given to the r_{th} input of DMU_j for $j=1,2,...,J$, $r=1,2,...,s$ and $i=1,2,...,m$. By considering the names of its creation persons, this model is called CCR.

To solve the linear fractional programming problem (1), usually, the method of Charnes and Cooper [5] is used. Assuming $t=\frac{1}{\sum_{i=1}^{m} v_i x_{io}}$, problem (1) can be converted into the following linear programming one which is much easier to be solved:

$$\begin{aligned} & Max z_o=\sum_{r=1}^{s} U_r y_{ro} \\ & S.to: \sum_{i=1}^{m} V_i x_{io}=1; \\ & \sum_{r=1}^{s} U_r y_{rj}-\sum_{i=1}^{m} V_i x_{ij} \leqslant 0,\, j=1,2,...,J; \\ & U_i \geqslant 0, i=1,2,...,m; \\ & V_r \geqslant 0, r=1,2,...,s \end{aligned} \tag{2}$$

where $U=tu$ and $V=tv$. By solving (2), the most performance of DMU_o is obtained; also, the optimal output price ($u^*=U^*/t^*$) and maximum input cost ($v^*=V^*/t^*$) of DMU_o are given. If the optimal objective value of (2) equals one, then the efficiency score is one and DMU_O is defined as efficient; otherwise, it is inefficient.

Definition 1: The value of C is called a fixed cost that is allocated to $DMUs$, so that the value of $c_j \leqslant C$ is allocated to each unit such that $\sum_{j=1}^{J} c_j = C$.

Definition 2: Some decision-making units consist of several sections or stages (or produce their products in several stages). These stages car produce their products in several stages. These stages indeed make up a network of sub-processes which are usually classified as series, parallel or dynamic. Figure 1 shows a DMU of the four-stage dynamic network.

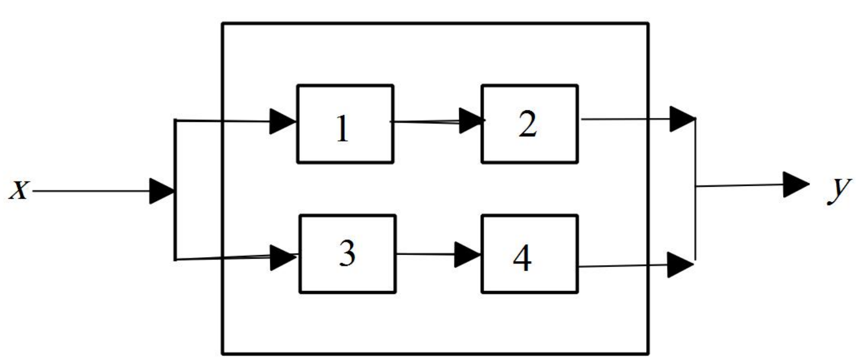

Figure 1. A DMU with a four-stage dynamic network

3. The proposed approaches

Yu et al. [18] allocated the fixed cost to all DMUs based on the two-stage network DEA approach. The proposed model by Ming, was taken a kind of CCR model with constant returns to scale. Here, we are going to generalize Ming method and allocate the fixed cost to $DMUs$ of an organization in more stages. To do this end, first, we consider that each $DMU_j\left(j=1,2,...,J\right)$ in the first stage has I_1 inputs $x_{ij}^1\left(i=1,2,...,I_1\right)$, and M_1 outputs $y_{mj}^1\left(m=1,2,...,M_1\right)$. Also in the second stage it has I_2 inputs $y_{ij}^1\left(i=1,2,...,I_2\right)$, M_2 outputs $y_{mj}^2\left(m=1,2,...,M_2\right)$. This procedure is assumed till stage n, when it has I_n inputs and M_n outputs and the output of each stage is used as the input of the next stage. This indicates that the number of outputs l -th stage is equal to the number of inputs $\left(l+1\right)$-th stage, i.e. $I_2=M_1, I_3=M_2,...,I_n=M_{n-1}$. We also must allocate fixed cost c_j to each DMU_j, such that $\sum_{j=1}^{J}c_j=C$ in n stages.

Because, we can freely allocate the fixed cost c_j between the first, second till n-th stages, therefore, we allocate part of the fixed cost c_j as input in each stage. We do this operation convexly. In the first stage α_j^1, in the second stage α_j^2 and in the n-th stage α_j^n part of the fixed cost c_j is allocated to DMU_j such that $\alpha_j^1+\alpha_j^2+...+\alpha_j^n=1$.

Now, according to Yu et al. [18] method, we introduce the n-stages allocation model for the series network of DMUs as following:

$$
\begin{aligned}
E_k = Max(&w_1\frac{\sum_{m=1}^{M_1} v_m^1\, y_{mk}^1}{\sum_{i=1}^{I_1} u_i^1\, x_{ik}^1+\varpi\alpha_k^1 c_k}+w_2\frac{\sum_{m=1}^{M_2} v_m^2\, y_{mk}^2}{\sum_{i=1}^{I_2} v_i^1\, y_{ik}^1+\varpi\alpha_k^2 c_k}+\ldots+w_n\frac{\sum_{m=1}^{M_n} v_m^n\, y_{mk}^n}{\sum_{i=1}^{I_n} v_i^{(n-1)}\, y_{ik}^{(n-1)}+\varpi\alpha_k^n c_k})\\
S.to{:}\;& E_j^1\leqslant\frac{\sum_{m=1}^{M_1} v_m^1 y_{mj}^1}{\sum_{i=1}^{I_1} u_i^1 x_{ij}^1+\varpi\alpha_j^1 c_j}\leqslant 1,\quad j=1,2,\ldots,J;\\
& E_j^2\leqslant\frac{\sum_{m=1}^{M_2} v_m^2 y_{mj}^2}{\sum_{i=1}^{I_2} v_i^1 y_{ij}^1+\varpi\alpha_j^2 c_j}\leqslant 1,\quad j=1,2,\ldots,J;\\
&\quad\vdots\\
& E_j^n\leqslant\frac{\sum_{m=1}^{M_n} v_m^n y_{mj}^n}{\sum_{i=1}^{I_n} v_r^{(n-1)} y_{ij}^{(n-1)}+\varpi\alpha_j^n c_j}\leqslant 1,\quad j=1,2,\ldots,J;\\
& \textstyle\sum_{j=1}^{J} c_j=C, L_j^1\leqslant\alpha_j^1\leqslant U_j^1,\quad j=1,2,\ldots,J;\\
& L_j^2\leqslant\alpha_j^2\leqslant U_j^2,\quad j=1,2,\ldots,J;\\
&\quad\vdots\\
& L_j^n\leqslant\alpha_j^n\leqslant U_j^n,\quad j=1,2,\ldots,J;\\
& \alpha_j^1+\alpha_j^2+\ldots+\alpha_j^n=1,\quad j=1,2,\ldots,J;\\
& c_j\geqslant 0,\quad j=1,2,\ldots,J;\\
& u^1, v^l\geqslant 1\varepsilon,\quad \varpi\geqslant\varepsilon,\quad l=1,2,\ldots,n.
\end{aligned}
\tag{3}
$$

In problem (3), we obtain the efficiency of DMU_k ($k \in \{1,2,...,J\}$) with I_l input and M_l output in l-th stage, in an n stage fixed cost allocation problem $(l = 1,2,...,n)$; note that the input of the l-th stage is the same as the output of the $(l-1)$-th stage for $l = 2,3,...,n$. Here, x^1 is the input vector of DMU_k and u^1 is the weight vector, y^l is the output vector of the l-th stage (which is also the input vector of the $(l+1)$-th stage) of DMU_k and v^l is the weight vector assigned to them for $l = 1,...,n$. Also, $\varpi\alpha_k^l c_k$ is the fixed cost allocated to l-th stage, which ϖ is the weight assigned to it. Also, in each stage, according to the importance of that stage, the weight w_j for $j = 1,2,...,n$ is assigned to it, such that $\sum_{j=1}^{n} w_j = 1$.

Regardless the fixed costs at each stage, the efficiency scores DMU_j ($j \in 1,2,...,J$) E_j^1, E_j^2, ..., E_j^n, in the sequel stages can be obtained by using CCR model. Because for DMU_j the number of inputs are increased when adding the fixed cost, the efficiency score of DMU_j increases at each stage. Therefore, the efficiency value DMU_j at each stage is greater than E_j and less than1; this fact is considered in the first set of in the constraints in problem (3).

But problem (3) is a non-linear fractional programming; to solve this problem, we first prefer to convert the problem to a linear programming problem, as an accepted instruction in DEA. For this purpose, like [18], we introduce the weights related to each stage (w_1, w_2, ... w_n) as follows:

$$w_1 = \frac{\sum_{i=1}^{I_1} u_i^1 x_{ik}^1 + \varpi\alpha_k^1 c_k}{\sum_{i=1}^{I_1} u_i^1 x_{ik}^1 + \varpi\alpha_k^1 c_k + \sum_{i=1}^{I_2} v_i^1 y_{ik}^1 + \varpi\alpha_k^2 c_k + \ldots + \sum_{i=1}^{I_n} v_r^{(n-1)} y_{ik}^{(n-1)} + \varpi\alpha_k^n c_k},$$

$$w_2 = \frac{\sum_{i=1}^{I_2} v_i^1 y_{ik}^1 + \varpi\alpha_k^2 c_k}{\sum_{i=1}^{I_1} u_i^1 x_{ik}^1 + \varpi\alpha_k^1 c_k + \sum_{i=1}^{I_2} v_i^1 y_{ik}^1 + \varpi\alpha_k^2 c_k + \ldots + \sum_{i=1}^{I_n} v_r^{(n-1)} y_{ik}^{(n-1)} + \varpi\alpha_k^n c_k},$$

$$\vdots$$

$$w_n = \frac{\sum_{i=1}^{I_n} v_r^{(n-1)} y_{ik}^{(n-1)} + \varpi\alpha_k^n c_k}{\sum_{i=1}^{I_1} u_i^1 x_{ik}^1 + \varpi\alpha_k^1 c_k + \sum_{i=1}^{I_2} v_i^1 y_{ik}^1 + \varpi\alpha_k^2 c_k + \ldots + \sum_{i=1}^{I_n} v_r^{(n-1)} y_{ik}^{(n-1)} + \varpi\alpha_k^n c_k}. \tag{4}$$

It is clear that $\sum_{j=1}^{n} w_j = 1$. In this regard, problem (3) can be converted into the following problem:

$$E_k = Max\Bigg(\frac{\sum_{m=1}^{M_1} v_m^1 y_{mk}^1 + \sum_{m=1}^{M_2} v_m^2 y_{mk}^2 + \cdots}{\sum_{i=1}^{I_1} u_i^1 x_{ik}^1 + \varpi\alpha_k^1 c_k + \sum_{i=1}^{I_2} v_i^1 y_{ik}^1 + \varpi\alpha_k^2 c_k + \cdots}$$
$$\frac{\ldots + \sum_{m=1}^{M_n} v_m^n y_{mk}^n}{\ldots + \sum_{i=1}^{I_n} v_i^{(n-1)} y_{ik}^{(n-1)} + \varpi\alpha_k^n c_k}\Bigg)$$

$$S.to: E_j^1 \leqslant \frac{\sum_{m=1}^{M_1} v_m^1 y_{mj}^1}{\sum_{i=1}^{I_1} u_i^1 x_{ij}^1 + \varpi\alpha_j^1 c_j} \leqslant 1, \quad j = 1,2,\ldots,J;$$

$$E_j^2 \leqslant \frac{\sum_{m=1}^{M_2} v_m^2\, y_{mj}^2}{\sum_{i=1}^{I_2} v_i^1\, y_{ij}^1 + \varpi \alpha_j^2 c_j} \leqslant 1, \quad j = 1,2,\ldots,J;$$

$$\vdots$$

$$E_j^n \leqslant \frac{\sum_{m=1}^{M_n} v_m^n\, y_{mj}^n}{\sum_{i=1}^{I_n} v_i^{(n-1)}\, y_{ij}^{(n-1)} + \varpi \alpha_j^n c_j} \leqslant 1, \quad j = 1,2,\ldots,J;$$

$$\sum_{j=1}^{J} c_j = C; L_j^1 \leqslant \alpha_j^1 \leqslant U_j^1, \quad j = 1,2,\ldots,J;$$

$$L_j^2 \leqslant \alpha_j^2 \leqslant U_j^2, \quad j = 1,2,\ldots,J;$$

$$\vdots$$

$$L_j^n \leqslant \alpha_j^n \leqslant U_j^n, \quad j = 1,2,\ldots,J;$$

$$\alpha_j^1 + \alpha_j^2 + \ldots + \alpha_j^n = 1, \quad j = 1,2,\ldots,J;$$

$$c_j \geqslant 0, \quad j = 1,2,\ldots,J; \quad u^1, v^l \geqslant 1\varepsilon, \quad \varpi \geqslant \varepsilon, \quad l = 1,2,\ldots,n. \tag{5}$$

Now, we transfer the above nonlinear fractional programming problem (3) to a linear programming one using transformation method Charnes and Cooper [5] as explained briefly in previous section. Let

$$\tau^k = \frac{1}{\sum_{i=1}^{I_1} u_i^1 x_{ik}^1 + \varpi \alpha_k^1 c_k + \sum_{i=1}^{I_2} v_i^1 y_{ik}^1 + \varpi \alpha_k^2 c_k + \ldots + \sum_{i=1}^{I_n} v_r^{(n-1)} y_{ik}^{(n-1)} + \varpi \alpha_k^n c_k},$$

$$\tau^k v^l = \mu^l, \tau^k u^1 = \psi^1, \tau^k \varpi = \sigma, \sigma c_j = \bar{c}_j, l = 1,2,\ldots,n, \sum_{j=1}^{J} \sigma\, c_j = \sigma C. \tag{6}$$

Then, the problem (5) is changed to the following problem by using (6):

$$E_k(k) = Max\left(\sum_{m=1}^{M_1} \mu_m^1\, y_{mk}^1 + \sum_{m=1}^{M_2} \mu_m^2\, y_{mk}^2 + \ldots + \sum_{m=1}^{M_n} \mu_m^n\, y_{mk}^n\right)$$

$$S.to\colon \sum_{i=1}^{I_1} \psi_i^1\, x_{ik}^1 + \alpha_k^1 \bar{c}_k + \sum_{i=1}^{I_2} \mu_i^1\, y_{ik}^1 + \alpha_k^2 \bar{c}_k + \ldots + \sum_{i=1}^{I_n} v_r^{(n-1)}\, y_{ik}^{(n-1)} + \alpha_k^n \bar{c}_k = 1;$$

$$E_j^1 \leqslant \frac{\sum_{m=1}^{M_1} \mu_m^1\, y_{mj}^1}{\sum_{i=1}^{I_1} \psi_i^1\, x_{ij}^1 + \alpha_j^1 \bar{c}_j} \leqslant 1, \quad j = 1,2,\ldots,J;$$

$$E_j^2 \leqslant \frac{\sum_{m=1}^{M_2} \mu_m^2\, y_{mj}^2}{\sum_{i=1}^{I_2} \mu_i^2\, y_{ij}^1 + \alpha_j^2 \bar{c}_j} \leqslant 1, \quad j = 1,2,\ldots,J;$$

$$\vdots$$

$$E_j^n \leqslant \frac{\sum_{m=1}^{M_n} \mu_m^n\, y_{mj}^n}{\sum_{i=1}^{I_n} \mu_i^{(n-1)}\, y_{ij}^{(n-1)} + \alpha_j^n \bar{c}_j} \leqslant 1, \quad j = 1,2,\ldots,J;$$

$$\sum_{j=1}^{J} \bar{c}_j = \sigma C;$$

$$L_j^1 \leqslant \alpha_j^1 \leqslant U_j^1, \quad j = 1,2,\dots,J;$$

$$L_j^2 \leqslant \alpha_j^2 \leqslant U_j^2, \quad j = 1,2,\dots,J;$$

$$\vdots\; L_j^n \leqslant \alpha_j^n \leqslant U_j^n, \quad j = 1,2,\dots,J;$$

$$\alpha_j^1 + \alpha_j^2 + \dots + \alpha_j^n = 1, \quad j = 1,2,\dots,J;$$

$$\bar{c}_j \geqslant 0, \quad j = 1,2,\dots,J;$$

$$\psi^1, \mu^l \geqslant 1\varepsilon, \quad l = 1,2,\dots,n. \tag{7}$$

Note that condition $a_j^1 + a_j^2 + \ldots + a_j^n = 1 \quad \forall j$, guarantees that the fixed costs are allocated convexly. Because $\alpha_j^1 \bar{c}_j, \alpha_j^2 \bar{c}_j, \ldots, \alpha_j^n \bar{c}_j$ are nonlinear terms in some constraints, problem (7) is still a nonlinear programming one. By using the transformations $\alpha_j^1 \bar{c}_j = \gamma_j^1, \alpha_j^2 \bar{c}_j = \gamma_j^2, \ldots, \alpha_j^n \bar{c}_j = \gamma_j^n$, the problem (7) can be converted to the following linear one:

$$E_k(k) = Max(\sum_{m=1}^{M_1} \mu_m^1\, y_{mk}^1 + \sum_{m=1}^{M_2} \mu_m^2\, y_{mk}^2 + \dots + \sum_{m=1}^{M_n} \mu_m^n\, y_{mk}^n)$$

$$S.to\colon \sum_{i=1}^{I_1} \psi_i^1\, x_{ik}^1 + \alpha_k^1 \bar{c}_k + \sum_{i=1}^{I_2} \mu_i^1\, y_{ik}^1 + \alpha_k^2 \bar{c}_k + \dots + \sum_{i=1}^{I_n} v_r^{(n-1)}\, y_{ik}^{(n-1)} + \alpha_k^n \bar{c}_k = 1;$$

$$\sum_{m=1}^{M_1} \mu_m^1\, y_{mj}^1 - (\sum_{i=1}^{I_1} \psi_i^1\, x_{ij}^1 + \gamma_j^1) \leqslant 0, j = 1,2,\dots,J;$$

$$\sum_{m=1}^{M_1} \mu_m^1\, y_{mj}^1 - E_j^1(\sum_{i=1}^{I_1} \psi_i^1\, y_{ij}^1 + \gamma_j^1) \geqslant 0, j = 1,2,\dots,J;$$

$$\sum_{m=1}^{M_2} \mu_m^2\, y_{mj}^2 - (\sum_{i=1}^{I_2} \mu_i^2\, y_{ij}^1 + \gamma_j^2) \leqslant 0, j = 1,2,\dots,J;$$

$$\sum_{m=1}^{M_2} \mu_m^2\, y_{mj}^2 - E_j^2(\sum_{i=1}^{I_2} \mu_i^2\, y_{ij}^1 + \gamma_j^2) \geqslant 0, j = 1,2,\dots,J;$$

$$\vdots$$

$$\sum_{m=1}^{M_n} \mu_m^n\, y_{mj}^n - (\sum_{i=1}^{I_n} \mu_i^{(n-1)}\, y_{ij}^{(n-1)} + \gamma_j^n) \leqslant 0, j = 1,2,\dots,J;$$

$$\sum_{m=1}^{M_n} \mu_m^n\, y_{mj}^n - E_j^n(\sum_{i=1}^{I_n} \mu_i^{(n-1)}\, y_{ij}^{(n-1)} + \gamma_j^n) \geqslant 0, \quad j = 1,2,\dots,J;$$

$$\sum_{j=1}^{J} \bar{c}_j = \sigma C; L_j^1 \bar{c}_j \leqslant \gamma_j^1 \leqslant U_j^1 \bar{c}_j, j = 1,2,\ldots,J;$$
$$L_j^2 \bar{c}_j \leqslant \gamma_j^2 \leqslant U_j^2 \bar{c}_j, j = 1,2,\ldots,J;$$
$$\vdots$$
$$L_j^n \bar{c}_j \leqslant \gamma_j^n \leqslant U_j^n \bar{c}_j, j = 1,2,\ldots,J;$$
$$\gamma_j^1 + \gamma_j^2 + \ldots + \gamma_j^n = \bar{c}_j, j = 1,2,\ldots,J;$$
$$\bar{c}_j \geqslant 0, j = 1,2,\ldots,J;$$
$$\mu^l, \psi^1 \geqslant 1\varepsilon, \quad \gamma_j^l \geqslant \varepsilon \quad l = 1,2,\ldots,n, \quad j = 1,2,\ldots,J. \tag{8}$$

In this manner, by solving (8), the optimal solution can be obtained as follows:

$$\mu^{l*}, \ \ \psi^{1*}, \ \sigma^*, \gamma_j^{l*}, \ c_j^* = \frac{\bar{c}_j^*}{\sigma^*}, \ \alpha_j^{l*} = \frac{\gamma_j^{l*}}{\bar{c}_j^*}, \quad j = 1, 2, \ldots, J, \quad l = 1, 2, \ldots, n. \tag{9}$$

In this section, how to allocate the fixed cost to DMUs was done in two stages and more than two stages, and this action is done using model (8), while the model provided by Yu et al. [18] allocates the fixed cost only in two stages. The fixed cost is allocated to DMUs in several stages, so that the DMUs obtain the highest efficient. In the next section, with providing a real example, how the fixed cost allocation is explained in three stages.

4. A benchmark example

Shiraz, as one of the most important tourism cities in Iran is always attractive for national and foreign tourists. For this reason, some large cultural institute, welfare and commercial complex centres have been built in this city. The income of these complex centres usually is obtained monthly through the commercial units and other sources of income such as parking; therefore, decisions to allocate fixed costs is ordinary designed monthly.

Suppose that the manager system of the Bin-ol-Harmain commercial complex centre in Shiraz, inclines to allocate the fixed cost to three of its sub-units in 12 month of year. These sub-units are coffee shop, food court and a sub-unit development plans, in which are connected to each other as a network structure, so that the coffee shop, food court and development plans sub-units are considered as the first, second and third stages, respectively (see Figure 2). As seen in the Figure 2, the first stage (coffee shop) includes two inputs (primitive material and number of customers) and two outputs (profit and the number of customers). The second stage (food court) is included two inputs (profit and the number of customers) and one output (benefit). The third stage (development plans) is included one input (food court benefit) and the two outputs (added value and quality). The decision maker intends to distribute the monthly fixed cost between these three stages according to the efficiency of the sub-units; so that, the efficiency score of these sub-units are maximized in each month of the year. Therefore, if we consider each month of the year as a DMU in DEA concepts, then in this example we have 12 DMUs and three sub-units. We did the allocation project for the commercial centre during the year 2016. The related given data are listed in the normalized form in Table 1. To solve this fixed cost allocation problem, at first, we obtain the efficiency of all the sub-units in each month, using CCR model by DEA solver software (2019). Then by substituting these results in (8), the new problem is solved and the obtained results are listed in Table 2.

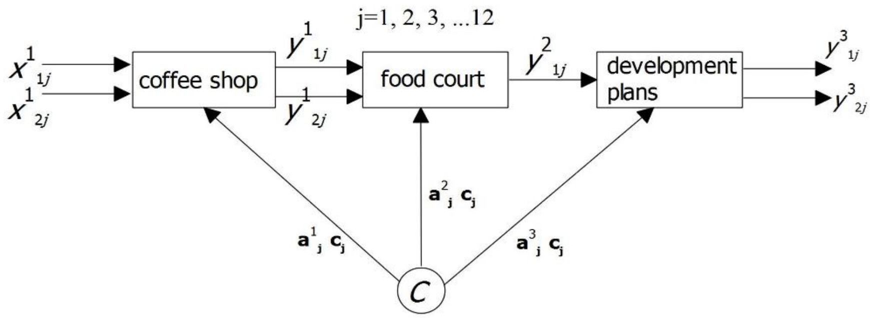

Figure 2: Allocation three-stages network diagram of the example

Table 1. The inputs and outputs data for each stage of the commercial complex

DMUs	c_j	x^1_{1j}	x^1_{2j}	y^1_{1j}	y^1_{2j}	y^2_{1j}	y^3_{1j}	y^3_{2j}
1	0.1246	.01452	0.1404	0.2182	0.1540	0.2468	0.1452	0.1452
2	0.0934	0.1034	0.1011	0.1091	0.1026	0.1070	0.1034	0.1034
3	0.0810	0.0817	0.0842	0.04	0.0855	0.0362	0.0817	0.0817
4	0.0910	0.0835	0.0870	0.0509	0.0907	0.0461	0.0835	0.0835
5	0.1121	0.0871	0.0932	0.0873	0.0949	0.0856	0.0871	0.0871
6	0.1121	0.0907	0.0926	0.1454	0.0941	0.1426	0.0907	0.0907
7	0.0623	0.0544	0.0533	0.0372	0.0428	0.0296	0.0544	0.0544
8	0.0561	0.0581	0.0556	0.0327	0.0470	0.0296	0.0581	0.0581
9	0.0498	0.0526	0.0477	0.02545	0.0393	0.0192	0.0526	0.0526
10	0.0467	0.0563	0.0533	0.0364	0.0428	0.0329	0.0563	0.0563
11	0.0561	0.0780	0.08591	0.0654	0.0898	0.0592	0.0780	0.0780
12	0.1059	0.1089	0.1055	0.1564	0.1163	0.1651	0.1089	0.1089

Table 2. The optimal results of the fixed cost allocation problem.

DMUs	E_j	ψ^1_{1j}	ψ^1_{2j}	μ^1_{1j}	μ^1_{2j}	μ^2_{1j}	μ^3_{1j}	μ^3_{2j}
1	0.8421	0.4662	0.6383	1.6205	1.6673	0.0001	0.0001	0.0001
2	0.9975	0.0001	1.9495	3.7377	3.8445	0.0001	0.0001	0.0001
3	1	3.4866	1.6609	8.4667	0.0001	5.0431	0.0001	2.3866
4	1	4.2779	0.8882	6.9904	4.5441	0.0001	2.6007	0.0001
5	1	2.3139	0.0001	4.6606	4.5813	0.0001	0.0001	0.5332
6	0.8617	0.5964	1.2041	2.8046	0.0001	2.8859	0.0001	0.0001
7	0.9999	0.0001	6.2591	11.8288	7.0209	0.0001	0.0001	4.0798
8	1	9.0204	0.0001	11.168	0.0001	6.4449	3.7089	0.0001
9	1	7.8427	0.0113	13.8883	0.0001	10.1387	0.0001	0.0001
10	1	7.1875	0.0001	10.5185	6.9681	0.0001	0.37229	0.0001
11	1	3.4605	0.0001	6.5335	0.0001	4.96	0.8431	0.0001
12	0.8714	0.5007	1.0466	2.4224	2.4925	0.0001	0.0001	0.0001

The obtained optimal results show that after allocating the fixed cost to the three sub-units of the complex centre in each month, the best efficiency score can be calculated. The mentioned results in Table 2, shows that in month 1, 2, 6, 7 and 12 the complex centre is inefficient. For example, in month 1, the efficiency score of the complex centre is 0.8421. Therefore, in the next years, managers must have a special pattern for this month to have efficiency. For instance, they can improve the efficiency of the sub-units with increasing the outputs or decreasing the inputs of them. This definitely causes increasing in efficiency of the complex centre. The optimal weights of the inputs and outputs of the sub-units are written in the Table 2 as well; as seen, the optimal weight of the second output of the coffee shop is more than the all outputs; so, increasing this output is better of increasing the other outputs. The same issue, for decreasing input 2 of coffee shop is also true. Of course, the manager can allocate more fixed costs distributing between sub-units as well. Also, in month 2, the complex centre is inefficient after allocating the fixed cost, and according to the coefficients obtained in table 2, with the increase of the second output of sub-unit 1, the efficiency score of the complex can improve better than the increase of the outputs of other sub-units. In month 3, 4, 5, 8, 9, 10, 11, the complex centre with the mentioned allocating the fixed cost to three sub-units has been the best efficiency.

5. Conclusion

In this paper, we generalized the presented model by Yu et al. [18] and introduced an n-stages model for allocating the fixed cost between DMUs in an organization. The purpose of presenting these models is to determine how the fixed cost is allocating in several stages to a DMU so that the highest efficiency score is obtained. The structure of the presented models is based on the DEA network model and the efficiency scores are calculated in the CCR sense. By calculating the presented model for allocating the fixed cost to the units in several stages, the maximum efficiency of the units is determined after allocating the fixed cost, and in case of inefficiency of the units, the managers can improve the efficiency of the units by using the information obtained from the model. In this regard a benchmark real example for allocating the fixed cost to commercial complex centre in Shiraz city of Iran is present and the obtained benefits are analysed. The numerical results show that the proposed method is completely successful to allocate the fixed costs for increasing the efficiency. For further study, one may consider the other suitable DEA models or other kind of networks.

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