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Research Article

Applying and comparing finite difference, differential quadrature, and radial basis function-based differential quadrature numerical methods in confined aquifers

Atena Naghipour Karder [1], Ali Khoshfetrat [2*]

1 Department of Civil Engineering, Shahid Ashrafi Esfahani University, Isfahan, Iran

2 Department of Civil Engineering, Isfahan (Khorasgan) Branch, Islamic Azad University, Isfahan, Iran

Corresponding Author email: khoshfetrat@khuisf.ac.ir

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Extended Abstract

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Introduction

Numerical approaches have become indispensable tools for simulating groundwater issues. Traditional numerical methods such as Finite Difference Method (FDM) and Finite Element Method (FEM) have been predominantly employed for this purpose. These methods typically utilize low-order polynomials to estimate derivatives, which limits their accuracy and flexibility. Recent advancements in high-order numerical methods, particularly the Differential Quadrature (DQ) method, offer a promising alternative. DQ is a high-order numerical method characterized by its high accuracy and low computational cost, but it lacks geometric flexibility for modeling complex domains. To address this limitation, the Radial Basis Function-based Differential Quadrature (RBF-DQ) method has emerged, combining the advantages of DQ with the flexibility of meshless numerical methods. This study explores the application of both DQ and RBF-DQ methods for solving the governing equations of groundwater flow in confined aquifers under steady and unsteady conditions. The efficiency of these methods is evaluated by comparing their results with those obtained from traditional finite difference solutions.

Materials and Method

The DQ method approximates the n-th derivative of a function defined on a rectangular domain using a grid of nodes. For a function (f(x,y)), the n-th derivative at point ((x_i, y_j)) in the x-direction is approximated by a weighted sum of function values at surrounding nodes. The weights are determined using explicit formulas. The RBF-DQ method extends this by estimating the derivative at a point using a weighted combination of function values at all nearby points, allowing for greater flexibility in handling irregular domains. In this study, two numerical problems are investigated: the first focuses on steady-state flow in a confined aquifer, while the second addresses unsteady flow conditions. For both problems, boundary conditions are derived from analytical solutions, allowing for error assessment of the numerical

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methods. The average error is computed using a defined formula, comparing numerical results against analytical solutions obtained via traditional methods.

Results and Discussion

Results from the numerical modeling of both problems using DQ, RBF-DQ, and FDM are presented in several tables. For the first problem, average error values indicate that the DQ method consistently yields lower errors compared to both RBF-DQ and FDM methods. Specifically, as the number of nodes increases, the error decreases significantly for the DQ method, demonstrating its superior accuracy. In contrast, RBF-DQ shows a dependency on the chosen shape parameter, which affects its accuracy. The second problem's results similarly indicate that DQ outperforms RBF-DQ in terms of error reduction, reinforcing the findings from the first problem. Additionally, computational times reveal that while DQ and FDM methods have comparable computational efficiency, RBF-DQ requires significantly more time due to matrix inversions necessary for weight coefficient determination.

Conclusion

This study marks the first application of DQ and RBF-DQ methods in modeling groundwater flow in confined aquifers. The results demonstrate that while DQ provides superior accuracy and computational efficiency compared to traditional FDM, RBF-DQ, despite its flexibility for irregular domains, exhibits lower accuracy and higher computational demands. The dependency of RBF-DQ's performance on the shape parameter necessitates careful selection to minimize error, which can be time-consuming. Overall, the DQ method is recommended for regular domains, while RBF-DQ may be advantageous in complex geometries where traditional methods struggle. Future research could explore optimizing the shape parameter for RBF-DQ to enhance its applicability in groundwater modeling. The findings contribute valuable insights for numerical groundwater modeling, emphasizing the potential of high-order methods in improving simulation accuracy and efficiency.

Keywords: DQ Method, RBF-DQ Method, Unsteady Flow, Steady Flow, Groundwater Equations, Confined aquifers

Extended Abstract