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Revisit ordering on L-R fuzzy numbers and its application in fuzzy mathematical programming

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Abstract

Ordering on fuzzy quantities have been attracted a wide domains of studies in fuzzy sets theory in the two last decades. In many practical situations as well as fuzzy mathematical programming, it is necessary to the decision makers consider *L-R* fuzzy numbers according to their aims. But in the most of methods which are presented to order fuzzy numbers, the authors have been considered a special kind of fuzzy numbers such as triangular fuzzy numbers, trapezoidal fuzzy numbers and etc. But as we know the *L-R* fuzzy numbers as a general kind of these numbers have not been discussed. Hence in this paper, we focus on a general *L-R* fuzzy number and propose a new approach to order them as an extension of the method which is given by Nasseri in [14]. For validity of the proposed method, we will illustrate this method based on a convenient examples which is appeared in the literature of fuzzy ordering. Furthermore, we emphasize that the proposed method will be useful for evaluating the optimality conditions in the fuzzy primal simplex algorithms and the other related algorithms such as the fuzzy dual simplex algorithm and the fuzzy two phase simplex algorithm, fuzzy transportation models, fuzzy interval linear programming and etc.

Keywords: Fuzzy number, *L-R* fuzzy number, Fuzzy ordering, Fuzzy arithmetic, Fuzzy mathematical programming.

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1. Introduction

Ranking fuzzy numbers plays a very important role in linguistic decision making and some other fuzzy application system. *L-R* fuzzy number as the most general form of number has been used extensively as in [17] pointed out. A key issue in operation analyzing fuzzy set theory is how to compare fuzzy numbers. A significant number of ranking approaches have been suggested in the literature [1-9], [11], [14-17]. Some of them have been reviewed and compared by Bortolan and Degani [4], Chen and Hwang [6], and Wang and Kerre [16]. Almost each approach, however, has pitfalls in some aspect, such as inconsistency with human intuition, in discrimination, and difficulty of interpretation. But in this paper, we propose a new approach for ranking of *L-R* fuzzy numbers based on a fuzzy relation to make pair wise comparisons. In 2010, an issue of ordering triangular fuzzy numbers is encountered by Nasseri and Mizuno [14]. Similar to their approach here, we define relations $\overline{R}$ and $\underline{R}$ on $\mathbb{R}$ for *L-R* fuzzy numbers. First, we are going to introduce a modified defuzzification of a fuzzy quantity and then the objective is applying the proposed method to rank fuzzy numbers.

This paper is organized in 5 Sections. In Section 2, some fundamental results on fuzzy numbers are recalled. In Section 3, we introduce a new ranking technique for *L-R* fuzzy number. Also, the proposed method for ordering *L-R* fuzzy numbers is in this section. Discussion and an illustrative example is carried out in Section 4. The paper ends with conclusions in Section 5.

2. Background and preliminaries

In this section, we first introduce the basic concepts of fuzzy sets theory and *L-R* fuzzy numbers and then present briefly the arithmetic on *L-R* fuzzy numbers.

We review some basic notions of fuzzy sets [10, 13, 14] in this section. These notions are expressed as follows:

Definition 2.1: Fuzzy set and membership function. If X is a collection of objects denoted generically by x, then a fuzzy set A in X is defined to be a set of ordered pairs $A = \{(x, \mu_A(x)) \mid x \in X\}$, where $\mu_A(x)$ is called membership function for the fuzzy set. The membership function maps each element of X to a membership value between 0 and 1.

Remark 2.1: In throughout the paper, we assume that $X = \mathbb{R}$.

Definition 2.2: A fuzzy subset A of universe set X is normal if and only if $sup_{x\in X}\ \mu_A(x) = 1$, where X is the universe set.

Definition2.3: A fuzzy subset A of universe set X is convex if and only if

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq min\{\mu_A(x_1), \mu_A(x_2)\}$$

$x_1, x_2 \in X, \lambda \in [0,1]$

Definition 2.4: A fuzzy subset A is a fuzzy number if and only if A is normal and convex on X.

Definition 2.5: A trapezoidal fuzzy number A is a fuzzy number with a membership function μ_A defined by:

$$\mu_A(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & a_1 \leq x < a_2, \\ 1, & a_2 \leq x < a_3, \\ \frac{a_4-x}{a_4-a_3}, & a_3 \leq x < a_4, \\ 0, & otherwise, \end{cases} \quad (2-1)$$

which can be denoted as a quartet (a_1, a_2, a_3, a_4).

In these above situations a_1, a_2, a_3 and a_4, if $a_2 = a_3$, A becomes a triangular fuzzy number.

Definition 2.6: An extended fuzzy number A is described as any fuzzy subset of the universe set X with membership function μ_A defined as follows:

(a) $\mu_A(x)$ is a continuous mapping from X to the closed interval [0,1].

(b) $\mu_A(x) = 0$, for all $x \in (-\infty, a_1]$.

(c) μ_A is strictly increasing on $[a_1, a_2]$.

(d) $\mu_A(x) = 1$, for all $x \in [a_2, a_3]$.

(e) μ_A is strictly decreasing on $[a_3, a_4]$.

(f) $\mu_A(x) = 0$,for all $x \in [a_4, +\infty)$.

In these above situations a_1, a_2, a_3 and $a_4$are real numbers. If $a_1 = a_2 = a_3 = a_4, A$ becomes a crisp real number.

Definition 2.7: The $\alpha -$cut of a fuzzy number A, where $0 < \alpha \leq 1$ is a set defined as

$$A_\alpha = \{x \in \mathbb{R} | \ \mu_A(x) \geq \alpha\}.$$

According to the definition of a fuzzy number it is seen once that every $\alpha -$cut of a fuzzy number is a closed interval. Hence, we have

$$A_\alpha = [{A_\alpha}^-, {A_\alpha}^+],$$

where

$${A_\alpha}^- = inf\{x \in \mathbb{R} | \ \mu_A(x) \geq \alpha\}, (2-2)$$

$${A_\alpha}^+ = sup\{x \in \mathbb{R} | \ \mu_A(x) \geq \alpha\}. \quad (2-3)$$

A set of all fuzzy numbers on real line will be denoted by $\mathbb{F}(\mathbb{R})$.

Remark 2.2: A fuzzy set is convex if and only if all its $\alpha -$cuts are convex.

Here we are going to introduce a special kind of fuzzy number which is named as $L-R$ fuzzy numbers and will be use full in our discussion.

The fundamental idea of the $L-R$ representation of fuzzy numbers is to split the membership function $\mu_A(x)$ of a fuzzy number A into two curves $\mu_{L_A}(x)$ and $\mu_{R_A}(x)$, left and right of the modals values m and n. The membership function

$\mu_A(x)$ can then be expressed through parameterized reference function or shape and

$$R(1) = 0, \quad \text{if } min_x\, R(x) = 0,$$

$$lim_{x\to\infty} R(x) = 0, \quad \text{if } R(x) > 0,\ \forall x.$$

Now we give the definition of fuzzy arithmetic on $L-R$ fuzzy numbers as follows:

Let $A_i = \langle m_i, n_i, \alpha_i, \beta_i\rangle_{LR}$ and $A_j = \langle m_j, n_j, \alpha_j, \beta_j\rangle_{LR}$ be two $L-R$ fuzzy numbers and $x \in \mathbb{R}$. Define:

$$x \geq 0, xA_i = \langle xm_i, xn_i, x\alpha_i, x\beta_i\rangle_{LR},$$

$$x < 0, xA_i = \langle xn_i, xm_i, -x\beta_i, -x\alpha_i\rangle_{RL},$$

and

$$A_i + A_j = \langle m_i + m_j, n_i + n_j, \alpha_i + \alpha_j, \beta_i + \beta_j\rangle_{LR}.$$

3. Construction of a new method for ordering of L-R fuzzy number

Here, we establish a new ordering approach for $L-R$ fuzzy numbers which is very realistic and efficient and then introduce a new algorithm for ordering $L-R$ fuzzy numbers.

For any $L-R$ fuzzy number $A = \langle m, n, \alpha, \beta\rangle_{LR}$, define

$$\underline{A} = \frac{(m+n)}{2} - \frac{1}{(\alpha'+\beta'+n-m)}\left|\int_\lambda^1 L^{-1}(\tau)d\tau\right|, \quad \forall\lambda \in [0,1], \qquad (3-1)$$

and

$$\overline{A} = \frac{(m+n)}{2} + \frac{1}{(\alpha'+\beta'+n-m)}\left|\int_\lambda^1 R^{-1}(\tau)d\tau\right| \quad \forall\lambda \in [0,1], \qquad (3-2)$$

where

$$m - \alpha' = inf\{x \in \mathbb{R} |\, \mu_{A_L}(x) \geq \lambda\},$$

and

$$n + \beta' = sup\{x \in \mathbb{R} |\, \mu_{A_R}(x) \geq \lambda\}.$$

Now assume that $A_i = \langle m_i, n_i, \alpha_i, \beta_i\rangle_{LR}$ and $A_j = \langle m_j, n_j, \alpha_j, \beta_j\rangle_{LR}$ be two $L-R$ fuzzy numbers. Let

$$\overline{R}(A_i, A_j) = \overline{A}_i - \overline{A}_j, \qquad (3-3)$$

$$\underline{R}(A_i, A_j) = \underline{A}_i - \underline{A}_j, \qquad (3-4)$$

where $\overline{A}_i$, $\underline{A}_i$, $\overline{A}_j$ and $\underline{A}_j$ defined in (3-1) and (3-2).

Remark 3.1: Trapezoidal fuzzy numbers are special cases of E.Q. (2-4) with $L(x) = R(x) = 1 - x$.

Remark 3.2: Triangular fuzzy numbers are also special cases of E.Q. (2-4) with $L(x) = R(x) = 1 - x$ and $m = n$.

Remark 3.3: For triangular and trapezoidal fuzzy numbers the method is exactly match with the method which is given in [11, 14].

Lemma 3.1: Assume that A_i and A_j be two $L-R$ fuzzy numbers, which are $A_i = \langle m_i, n_i, \alpha_i, \beta_i\rangle_{LR}$ and $A_j = \langle m_j, n_j, \alpha_j, \beta_j\rangle_{LR}$. Then, we have

$$\overline{R}(A_i, A_j) = -\overline{R}(A_j, A_i)$$
$$= \overline{R}(-A_j, -A_i), \qquad (3-5)$$

and

$$\underline{R}(A_i, A_j) = -\underline{R}(A_j, A_i)$$
$$= \underline{R}(-A_j, -A_i). \qquad (3-6)$$

Proof: The result straightforward from (3-3), (3 - 4).

Definition 3.1: Assume that A_i and A_j be two $L-R$ fuzzy numbers, which are $A_i = \langle m_i, n_i, \alpha_i, \beta_i \rangle_{LR}$ and $A_j = \langle m_j, n_j, \alpha_j, \beta_j \rangle_{LR}$ such that $\underline{R}(A_j, A_i) \geq 0$. Define the relation $\approx$ and $\prec$ on $\mathbb{F}(\mathbb{R})$ as given below:

1) $A_i \approx A_j$ if and only if

$$\underline{R}(A_j, A_i) = \overline{R}(A_i, A_j). \qquad (3-7)$$

2) $A_i \prec A_j$ if and only if

$$\underline{R}(A_j, A_i) > \overline{R}(A_i, A_j). \qquad (3-8)$$

Remark 3.4: We denote $A_i \preccurlyeq A_j$, if and only if $A_i \approx A_j$ or $A_i \prec A_j$. Then $A_i \preccurlyeq A_j$ if and only if $\underline{R}(A_j, A_i) \geq \overline{R}(A_i, A_j)$. Also $A_i \prec A_j$ if and only if $A_j \succ A_i$.

4. Numerical example and motivation

In this section, we illustrate the mentioned method in the last section to emphasize the conformability of the approach.

Example 4.1: Consider two $L-R$ fuzzy numbers which is taken from [9], i.e., $A_1 = \langle 2,2,1,3 \rangle_{LR}$ and $A_2 = \langle 2,2,1,2 \rangle_{LR}$, as shown in Fig.1. The membership function of A_1 is as follows:

$$\mu_{A_1}(x) = \begin{cases} x-1, & 1 \leq x \leq 2, \\ \dfrac{(5-x)}{3}, & 2 \leq x < 5, \\ 0, & otherwise. \end{cases}$$

The membership function of A_2 is as follows:

$$\mu_{A_2}(x) = \begin{cases} \sqrt{1-(x-2)^2}, & 1 \leq x \leq 2, \\ \sqrt{1-0.25(x-2)^2}, & 2 \leq x < 4, \\ 0, & otherwise. \end{cases}$$

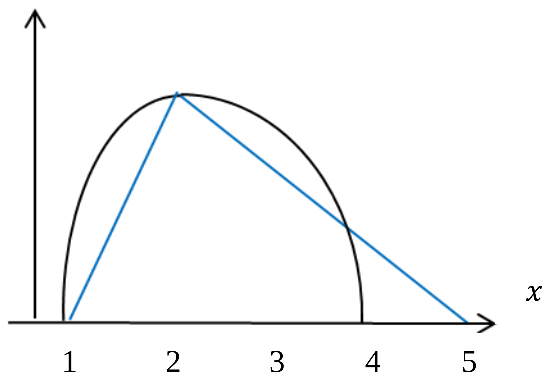

***Fig.* 1.** *Fuzzy numbers in Example* 4.1

According to E.q.s (3-1) and (3-2), for $\lambda = 0$ we can, respectively, obtain $\overline{A}_1$, $\underline{A}_1$, $\overline{A}_2$ and $\underline{A}_2$ as follows:

$$\underline{A}_1 = 0.875, \quad \overline{A}_1 = 2.375,$$
$$\underline{A}_2 = 1.738, \quad \overline{A}_2 = 2.523,$$

then by E.q.s (3-3) and (3-4), we get $\overline{R}(A_1, A_2)$ and $\underline{R}(A_1, A_2)$ as follows

$$\overline{R}(A_1, A_2) = -\overline{R}(A_2, A_1) = -0.148,$$

$\underline{R}(A_1, A_2) = -\underline{R}(A_2, A_1) = -0.863.$

Finally, by E.q (3-8), we conclude that $A_1 \prec A_2$.

As a motivation of the discussed study, we frankly emphasize that this approach can be extended for solving fuzzy and grey mathematical programming which is appeared in the literature such as [10], [12] and [13]. Furthermore, we emphasize that the proposed method will be useful for evaluating the optimality conditions in the fuzzy primal simplex algorithms and the other related algorithms such as the fuzzy dual simplex algorithm and the fuzzy two phase simplex algorithm, fuzzy transportation models, fuzzy interval linear programming and etc.

5. Conclusions

This paper presents a new approach for ordering $L-R$ fuzzy numbers, the researchers proposed a modified defuzzification using a fuzzy relation between two fuzzy numbers and by using that, they have proposed a method for ranking of $L-R$fuzzy numbers. Roughly, there is not much difference in the researcher method and theirs. The method can effectively rank various fuzzy numbers and their images. We also used comparative examples to illustrate the advantage of the proposed method. The calculation of the proposed method is far simpler than the other approaches.

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