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Productivity changes of units: A directional measure of cost Malmquist index

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Abstract

This study examines the productivity changes of decision making units in situation where input price vectors are varying between them and inputs are heterogeneous; that is a non-competitive market. We present a directional measure of cost Malmquist productivity index where incorporates the decision maker's preference over productivity change over time. A simple numerical example is designed to illustrate the new measure of the cost Malmquist index.

***Keywords**: Data envelopment analysis; directional measure; profit efficiency; productivity change; cost Malmquist index*

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1. Introduction

The productivity change over time is an important subject for decision making units (DMUs). Data envelopment analysis (DEA) can be applied as a non-parametric approach in studying the productivity change and its decomposition. DEA for the first time was developed by Charnes et al. (1978) and has been used so far in many literatures that focus on the efficiency and productivity of DMUs. For example, Berger (2007), Sathye (2003), Coli et al. (2011), Bruni et al. (2011). The method is also applied to compute the Malmquist index that is used for evaluating the productivity changes of DMUs over time.

Caves et al. (1982) defined the Malmquist index based on efficiency score for the first time and then Färe et al. (1989) decomposed the index into efficiency and technical change. Malmquist index is extended in different ways. For instance, Chen (2003) presented the non-radial Malmquist index and developed a non-radial DEA model for computing it. Arabi et al. (2015) as well as used a slacks-based measure (SBM) model to compute the productivity change of DMUs in the presence of undesirable outputs. Maniadakis and Thanassoulis (2004) extended the index to the cost Malmquist (CM) index for the case where the prices of the inputs are known. They used the Farrell cost efficiency in definition of the index. See also Portela and Thanassoulis (2010), Tohidi et al. (2010) for other applications of Malmquist index.

The cost efficiency (CE) measure used by Maniadakis and Thanassoulis (2004) in definition of CM index can be applied when inputs are homogenous and the prices are exogenously fixed. Thus, the CE measure only incorporates the input inefficiency and the contribution of inefficiency that is made by market prices (market inefficiency) is not considered. To solve these problems the alternate CE model was presented in Tone (2002) by considering the production technology in a cost/input space. Following the presented CE measure, Fukuyama and Weber (2004) and Färe and Grosskopf (2006) developed a directional input-cost distance function and therefore a directional measure of value-based technical inefficiency. This measure was extended in Sahoo et al. (2014). They developed a directional value-based measure of technical efficiency and also a directional cost-based measure of efficiency which satisfied the properties: translation invariant, unit invariant and strong monotonocity.

In this paper we estimate a directional measure of cost Malmquist index (DCM) based on the cost and technical efficiency measure presented in Sahoo et al. (2014), in order to examine the productivity changes of units in situation where DMUs act in a non-competitive market that inputs are heterogeneous and DMUs can control to some extent the market prices. In fact, when the input price vectors of DMUs are different because of the different competitive environments, comparing the productivity changes of DMUs can not be right. Using the proposed index the environmental factors can be incorporated in the comparison between DMUs. In addition, DMUs can control their productivity changes by considering the suitable direction vector.

The rest of paper proceeds as follows. Section 2 expresses the previous studies on efficiency and productivity change when input prices are known. Section 3 develops a cost Malmquist index for evaluating DMUs in a non-competitive market. In section 4 we design a simple numerical example to show the application of the proposed approach and Section 5 concludes.

2. Cost Malmquist productivity index

Suppose that there are n DMUs, observed in time period t $(t=1,\ldots,T)$, each DMU_j $(j=1,\ldots,n)$ consumes a non-negative input vector $x_j^t=(x_{1j}^t,\ldots,x_{mj}^t)$ with the price vector $c_j^t=(c_{1j}^t,\ldots,c_{mj}^t)$ to produce a non-negative output vector $y_j^t=(y_{1j}^t,\ldots,y_{sj}^t)$. Farrell (1957) defined the cost efficiency of a DMU as the ratio of minimum cost of production to the observed cost. This definition of cost efficiency requires that the input prices be fixed and the exact information of them is at hand. By using the concept of Farrell's cost efficiency, Maniadakis and Thanassoulis (2004) presented the cost Malmquist productivity index, which evaluates the productivity change of DMUs between time periods t and $t+1$ in the case where the input price vector is known. They assumed that all DMUs face the same input price vectors. Consider $c^t=(c_1^t,\ldots,c_m^t)$ as the input price vector of period t $(c_j^t=c^t,\forall j)$, and define the production technology of period t as

$$T^t=\{(x^t,y^t)\in R_{\geq 0}^{m+s}: x^t\in R_{\geq 0}^{m} \ canproduce \ y^t\in R_{\geq 0}^{s}\} \quad (1)$$

The Farrell cost efficiency measure for (x_o^t,y_o^t), observed in period t, under the input price vector c^t is defined as

$$\text{CE}_o^t(y_o^t,x_o^t,c^t)=\frac{C_o^t(y_o^t,c^t)}{c^t x_o^t}, \quad (2)$$

Where $C_o^t(y_o^t,c^t)$ is the minimum production cost and $c^t x_o^t$ is the observed cost of producing y_o^t. $C_o^t(y_o^t,c^t)$ can be obtained by solving the following linear programming model:

$$\begin{aligned} C_o^t(y_o^t,c^t) = \min\ & \sum_{i=1}^{m} c_i^t x_i, \\ \text{s.t.}\quad & \sum_{j=1}^{J} \lambda_j x_{ij}^t \le x_i, \quad i=1,\ldots,m, \\ & \sum_{j=1}^{J} \lambda_j y_{rj}^t \ge y_{ro}^t, \quad r=1,\ldots,s, \\ & \lambda_j \ge 0, \qquad j=1,\ldots,J, \\ & x_i \ge 0, \qquad i=1,\ldots,m. \end{aligned} \tag{3}$$

Based on the cost efficiency defined in (2), and by considering time period t as the reference period, the cost Malmquist productivity index (CM^t) is (Maniadakis and Thanassoulis, 2004):

$$\mathrm{CM}^t = \frac{c^t x_o^{t+1} \big/ C_o^t(y_o^{t+1},c^t)}{c^t x_o^t \big/ C_o^t(y_o^t,c^t)}. \tag{4}$$

CM^t compares the cost efficiency of (x_o^{t+1},y_o^{t+1}), under evaluation DMU observed in time period $t+1$, to that of (x_o^t,y_o^t) by measuring their distances from the cost boundary of period t as a benchmark, where the cost boundary is defined as

$$\mathrm{Iso}\,\bar{\mathrm{C}}_o^t(y_o^t,c^t) = \left\{x^t : c^t x^t = \mathrm{C}_o^t(y_o^t,c^t)\right\}. \tag{5}$$

Similarly, CM^{t+1} index can be defined based on the cost boundary of period $t+1$ as a benchmark,

$$\mathrm{CM}^{t+1} = \frac{w^{t+1} x^{t+1} \big/ C^{t+1}(y^{t+1},w^{t+1})}{w^{t+1} x^t \big/ C^{t+1}(y^t,w^{t+1})}. \tag{6}$$

CM^{t+1} compares the cost efficiency of (x_o^{t+1},y_o^{t+1}) to that of (x_o^t,y_o^t) by measuring their distances from the cost boundary of period $t+1$.

Because two indexes CM^t and CM^{t+1} may provide different measures of productivity change (the distances are computed based on different benchmarks), Maniadakis and Thanassoulis (2004) defined the CM index as the geometric mean of these two indexes as follows:

$$CM^{t,t+1} = \left[\frac{\frac{w^t x^{t+1}}{C^t(y^{t+1},w^t)}}{w^t x^t / C^t(y^t,w^t} \times \frac{w^{t+1} x^{t+1} / C^{t+1}(y^{t+1},w^{t+1})}{w^{t+1} x^t / C^{t+1}(y^t,w^{t+1})}\right]^{1/2} \tag{7}$$

If the $\mathrm{CM}^{t,t+1}$ index has a value less than 1, productivity progress, a value greater than 1 means that productivity regress and a value of 1 means that productivity remains unchanged.

Using the CM index to examine the productivity change of DMUs, they could incorporate allocative efficiency in the measurement of productivity change. They decomposed the presented index into cost efficiency and cost technical change. In addition, they decomposed each of these components into two components. Cost efficiency was decomposed into technical and allocative efficiency change, and cost technical change into a part capturing shifts of input quantities and shifts of input prices.

Farrell's CE model used for computing the CM index requires that the input price vector of DMUs are fixed and exogenously given. In fact, this model is applied for evaluating the cost efficiency of DMUs in a competitive market. In most of the real application, we deal with the cases where the market is not fully competitive and input prices may vary between DMUs. In such situation Farrell CE measure can not reflect the market inefficiency. In the next section we suggest a directional cost Malmquist index for use in situations where DMUs operates in a non-competitive market characterized by heterogeneous inputs.

3. A directional measure of cost Malmquist index

Cost efficiency measure and cost Malmquist index discussed in the former section can be applied for the case where DMUs are homogenous and input prices are exogenously fixed (that is DMUs are price taker). To estimate the cost efficiency of DMUs in non-competitive market, Sahoo et al. (2014) developed a directional cost based measure of efficiency (DCE) and also directional value based measure of technical efficiency (TE) which satisfy three important properties, translation invariance, strong monotonicity and unit invariance if the units of measurement for each component of the selected direction vector $g^t = (g_x^t, g_y^t)$, $g_x^t = (g_1^t, g_2^t, \ldots, g_m^t)$, $g_y^t = 0 \in R^s$, be the same as that of ith input-cost, $\bar{x}_i^t$. They assumed that inputs are heterogeneous and their prices vary across DMUs. In order to incorporate these assumptions in the model, they defined the production technology as

$$T^t = \{(x^{-t}, y^t) \in R_{\geq 0}^{m+s} : x^{-t} \in R_{\geq 0}^m \; can\, produce \; y^t \in R_{\geq 0}^s\} \tag{8}$$

where $\bar{x}^t = c^t * x^t$.

Their presented model to evaluate the DCE measure of DMU_o observed in period t based on the production technology of period t is as follows:

$$DCE_0^t(y_0^t, \bar{x}_0^t) - \min 1 - \sum_{i=1}^{m} \frac{g_{ix}^t}{G^t} \beta_i^{t,t}$$

$$\sum_{j=1}^{n} \lambda_j \bar{x}_{ij}^{t} \leq \bar{x}_{io}^{t} - \beta_i^{t,t} g_{ix}^{t}, i = 1, \dots, m$$
$$\sum_{j=1}^{n} \lambda_j y_{rj}^{t} \geq y_{ro}^{t}, \ r = 1, \dots, s \quad (9)$$
$$\sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, \ j = 1, \dots, n$$

Where $G^t = \sum_{i=1}^{m} g_{ix}^{t}$ and to guarantee that $\mathrm{DCE}_o^t(y_o^t, \bar{x}_o^t) \leq 1$, the direction vector g must satisfy the following condition:

$$\max_{i=1,\dots m} \left\{ \frac{\bar{x}_{io}^{t} - \min_{j=1,\dots n} \left\{ \bar{x}_{ij}^{t} \right\}}{g_{ix}^{t}} \right\} \leq 1. \quad (7)$$

Similarly, the DCE measure of DMU_o observed in time period $t+1$ with respect to the technology of period $t+1$ is,

$$\mathrm{DCE}_0^{t+1}(y_0^{t+1}, \bar{x}_0^{t+1}) - \min 1 - \sum_{i=1}^{m} \frac{g_{ix}^{t+1}}{G^{t+1}} \beta_i^{t+1,t+1}$$
$$\sum_{j=1}^{n} \lambda_j \bar{x}_{ij}^{t+1} \leq \bar{x}_{io}^{t+1} - \beta_i^{t+1,t+1} g_{ix}^{t+1}, i = 1, \dots, m$$
$$\sum_{j=1}^{n} \lambda_j y_{rj}^{t+1} \geq y_{ro}^{t+1}, \ r = 1, \dots, s \quad (11)$$
$$\sum_{j=1}^{n} \lambda_j = 1, \quad \lambda_j \geq 0, \quad j = 1, \dots, n$$

To measure $\mathrm{DCE}^{t,t+1}$ and $\mathrm{DCE}^{t+1,t}$, we modify

Error! Reference source not found. into the following models, respectively:

$$DCE_0^{t+1}(y_0^{t+1}, \bar{x}_0^{t+1}) - \min 1 - \sum_{i=1}^{m} \frac{g_{ix}^{t}}{G^{t}} \beta_i^{t,t+1}$$
$$\sum_{j=1}^{n} \lambda_j \bar{x}_{ij}^{t} \leq \bar{x}_{io}^{t+1} - \beta_i^{t,t+1} g_{ix}^{t}, i = 1, \dots, m$$
$$\sum_{j=1}^{n} \lambda_j y_{rj}^{t} \geq y_{ro}^{t+1}, \ r = 1, \dots, s \quad (12)$$
$$\sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, \ j = 1, \dots, n$$

$$DCE_0^{t+1}(y_0^{t}, \bar{x}_0^{t}) - \min 1 - \sum_{i=1}^{m} \frac{g_{ix}^{t+1}}{G^{t+1}} \beta_i^{t+1,t}$$
$$\sum_{j=1}^{n} \lambda_j \bar{x}_{ij}^{t+1} \leq \bar{x}_{io}^{t} - \beta_i^{t+1,t} g_{ix}^{t+1}, i = 1, \dots, m$$
$$\sum_{j=1}^{n} \lambda_j y_{rj}^{t+1} \geq y_{ro}^{t}, \ r = 1, \dots, s \quad (13)$$
$$\sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, \ j = 1, \dots, n$$

Now, we define the directional cost Malmquist (DCM) productivity index of t, $t+1$ and their geometric mean respectively as follows:

$$\mathrm{DCM}^t = \frac{\mathrm{DCE}_o^t(y_o^{t+1}, \bar{x}_o^{t+1})}{\mathrm{DCE}_o^t(y_o^t, \bar{x}_o^t)}, \quad (8)$$

$$\mathrm{DCM}^{t+1} = \frac{\mathrm{DCE}_o^{t+1}(y_o^{t+1}, \bar{x}_o^{t+1})}{\mathrm{DCE}_o^{t+1}(y_o^t, \bar{x}_o^t)}, \quad (9)$$

$$DCM^{t,t+1} = \left[\frac{DCE_0^t(y_0^{t+1}, \bar{x}_0^{t+1})}{DCE_0^t(y_0^t, \bar{x}_0^t)} \times \frac{DCE_0^{t+1}(y_0^{t+1}, \bar{x}_0^{t+1})}{DCE_0^{t+1}(y_0^t, \bar{x}_0^t)} \right]^{1/2} \quad (16)$$

If the $\mathrm{DCM}^{t,t+1}$ index has a value less than 1, productivity regress, a value greater than 1 means that productivity progress and a value of 1 means that productivity remains unchanged.

The proposed productivity index **Error! Reference source not found.** is incorporated with the decision maker's preferences. Therefore, to improve the productivity of DMUs decision makers can select a specific direction vector. In fact, the productivity change of DMUs can be measured based on their specific direction vector. In addition, the measure of productivity change obtained from **Error! Reference source not found.** is translation invariant and can be used in situations dealing with negative data.

3.1. Decomposition of the proposed index

Now, we decompose the DCM index to illustrate how the index includes directional technical and allocative efficiency changes, shift of the production boundary between periods t and $t+1$, and also the effect of input price change on the productivity change of DMU_o between time periods t and $t+1$. The decomposition is similar to the decomposition of CM index presented in Maniadakis and Thanassoulis (2004).

In the first stage, DCM index can be decomposed into overall (cost) efficiency change (OEC) and cost technical change (CTC) as follows:

$$DCM^{t,t+1} = \frac{DCE_0^{t+1}(y_0^{t+1},\bar{x}_0^{t+1})}{DCE_0^{t}(y_0^{t},\bar{x}_0^{t})}\left[\frac{DCE_0^{t}(y_0^{t+1},\bar{x}_0^{t+1})}{DCE_0^{t+1}(y_0^{t+1},\bar{x}_0^{t+1})} \times \frac{DCE_0^{t}(y_0^{t},\bar{x}_0^{t})}{DCE_0^{t+1}(y_0^{t},\bar{x}_0^{t})}\right]^{1/2} \tag{17}$$

OEC component examine whether the observed cost of producing the given output vector, $\sum_{i=1}^{m}\bar{x}_{io}^{t}$, catches up the minimum cost of producing it from period t to period $t+1$. Using the optimal solution of model **Error! Reference source not found.**, the minimum cost of producing y^t can be calculated as,

$$\sum_{i=1}^{m}\bar{x}_{io}^{t*} = \sum_{i=1}^{m}(\bar{x}_{io}^{t} - \beta_i^{t,t*} g_{ix}^{t}). \tag{10}$$

Similarly, the minimum cost of producing y^{t+1} can be calculated by the optimal solution of model **Error! Reference source not found.**.

CTC component compares the minimum cost of producing the given output vector observed in period t with that of period $t+1$.

In the second stage of the decomposition of DCM index, each component obtained in stage 1 can be further decomposed into two components. OEC component is decomposed as

$$OEC = \frac{1-\beta^{t+1,t+1}}{1-\beta^{t,t*}}\left[\frac{DCE_0^{t+1}(y_0^{t+1},\bar{x}_0^{t+1})/(1-\beta^{t+1,t+1*})}{DCE_0^{t+1}(y_0^{t+1},\bar{x}_0^{t})/(1-\beta^{t,t*})}\right] = TEC \times AEC \tag{19}$$

that $\beta^{t,t*}$ is the possible technical improvement of the components of input-spending $\bar{x}_o^t$ (directional value-based measure of technical inefficiency). The value of $\beta^{t,t*}$ can be computed by the optimal solution of model **Error! Reference source not found.** as follows:

$$\beta^{t,t*} = 1 - \min_{i=1,\ldots,m} \beta_i^{t,t*} \sum_{i=1}^{m} \frac{g_{ix}^t}{G^t} = 1 - \min_{i=1,\ldots,m} \beta_i^{t,t*}. \quad (11)$$

$\beta^{t,t*}$ can be calculated also by solving the following model directly:

$$\beta^{t,t*} = max_{\lambda,\beta} \ \beta^{t,t}$$

$$\sum_{j=1}^{n} \lambda_j \bar{x}_{ij}^t \le \bar{x}_{io}^t - \beta^{t,t} g_{ix}^t, \quad i = 1, \ldots, m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj}^t \ge y_{ro}^t, \quad r = 1, \ldots, s \quad (21)$$

$$\sum_{j=1}^{n} \lambda_j = 1, \lambda_j \ge 0, j = 1, \ldots, n$$

Similarly, $\beta^{t+1,t+1*}$ can be calculated by the optimal solution of model **Error! Reference source not found.**, or directly by solving model **Error! Reference source not found.** after replacing time t with time $t+1$.

The component CTC obtained in the first stage of decomposition can be decomposed into TC and PE components as follows:

$$CTC = \left[\frac{1-\beta^{t,t+1*}}{1-\beta^{t+1,t+1*}} \times \frac{1-\beta^{t,t*}}{1-\beta^{t+1,t*}}\right]^{1/2} \times \left[\frac{DCE_0^t(y_0^{t+1}, \bar{x}_0^{t+1})/(1-\beta^{t,t+1*})}{DCE_0^{t+1}(y_0^{t+1}, \bar{x}_0^{t+1})/(1-\beta^{t+1,t+1*})} \times \frac{DCE_0^t(y_0^t, \bar{x}_0^t)/(1-\beta^{t,t*})}{DCE_0^{t+1}(y_0^t, \bar{x}_0^t)/(1-\beta^{t+1,t*})}\right]^{1/2} = TC \times PE \quad (12)$$

The value of $\beta^{t,t+1*}$ can be calculated by the optimal solution of model **Error! Reference source not found.** as

$$\beta^{t,t+1*} = 1 - \min_{i=1,\ldots,m} \beta_i^{t,t+1*} \sum_{i=1}^{m} \frac{g_{ix}^t}{G^t} = 1 - \min_{i=1,\ldots,m} \beta_i^{t,t+1*}$$

or solving the following model directly:

$$\beta^{t,t+1*} = max_{\lambda,\beta} \ \beta^{t,t+1}$$

$$\sum_{j=1}^{n} \lambda_j \bar{x}_{ij}^t \le \bar{x}_{io}^{t+1} - \beta^{t,t+1} g_{ix}^t, \ i = 1, \ldots, m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj}^t \ge y_{ro}^{t+1}, \quad r = 1, \ldots, s \quad (23)$$

$$\sum_{j=1}^{n} \lambda_j = 1, \lambda_j \ge 0, j = 1, \ldots, n$$

The value of $\beta^{t+1,t*}$ can be estimated similar to $\beta^{t,t+1*}$ after changing round the periods t and $t+1$.

TC component estimates the shift of the production boundary from period t to period $t+1$. PE component estimates the effect of input price changes on changes of the minimum cost of producing the given output vector. For each of the components of the DCM index, a value less than 1 indicates regress, a value greater 1 indicates progress and a value of 1 express the performance remains unchanged.

4. Numerical example

In order to illustrate the ability of the proposed approach we have analyzed 5 DMUs with two inputs and two outputs. Table 1 shows the input/output data and the input price vectors for 5 DMUs observed in two time periods 0 and 1. We apply the index **Error! Reference source not found.** and also the index **Error! Reference source not found.** to evaluate the productivity changes of DMUs between periods 0 and 1. We compute the index **Error! Reference source not found.** by considering two direction vectors as

$$g^t = \left(g_x^t, g_y^t\right), g_{ix}^t = \bar{x}_{io}^t, g_{ry}^t = 0, \quad i = 1, \dots, m, \ r = 1, \dots, s \tag{24}$$

$$\mathrm{g}^\mathrm{t} = \left(\mathrm{g}_\mathrm{x}^\mathrm{t}, \mathrm{g}_\mathrm{y}^\mathrm{t}\right), \mathrm{g}_\mathrm{ix}^\mathrm{t} = \max_{\mathrm{j}=1,\dots,\mathrm{n}}\left\{\bar{\mathrm{x}}_\mathrm{ij}^\mathrm{t}\right\}, \mathrm{g}_\mathrm{ry}^\mathrm{t} = 0, \quad \mathrm{i} = 1, \dots, \mathrm{m}, \ \mathrm{r} = 1, \dots, \mathrm{s} \tag{25}$$

Note that in period 2 DMU1 increases its input quantities and simultaneously decreases its output quantities while the input prices vector remains unchanged from time 0 to time1. Therefore we expect that productivity of DMU1 regress from time 0 to time1. It can also be derived from Tables 6 and 7. These Tables respectively show the results obtained from the indexes **Error! Reference source not found.** and **Error! Reference source not found.**. Two indexes report a regress in productivity for DMU1. From the results of Table 3 it can be seen that the amount of regress in productivity obtained from selecting the direction vector **Error! Reference source not found.** is more than that of the direction **Error! Reference source not found.**. It means that, $\mathrm{DCM}^{t,t+1}$ is incorporated with the decision maker's preferences.

Now consider DMU5 as DMU under evaluation. This DMU improves its outputs without any changes in its inputs quantities and prices. Thus we expect that its productivity improves from time 0 to time 1.

Table 1. Numerical example data

	t=0						t=1					
MU	**I1**	**I2**	**C1**	**C2**	**O1**	**O2**	**I1**	**I2**	**C1**	**C2**	**O1**	**O2**
DMU1	5	3	3	1	2	3	15	6	3	1	1	1.5
DMU2	9	5	3	1	5	4	4.5	2.5	1.5	0.5	15	12
DMU3	13	6	4	2	3	6	13	6	4	2	3	6
DMU4	15	14	2	3	7	9	15	14	10	15	7	9
DMU5	7	11	5	1	5	9	7	11	5	1	20	36

It can be seen that from Tables 2 and 3, the indexes $CM^{t,t+1}$ and $DCM^{t,t+1}$ provide the different results for DMU5. The $CM^{t,t+1}$ index shows a regress in the productivity while the $DCM^{t,t+1}$ index reports an improvement for the productivity change between two time periods. In addition, the value of $DCM^{t,t+1}$ obtained based on direction vector **Error! Reference source not found.** indicates higher productivity growth than the $DCM^{t,t+1}$ index based on direction vector **Error! Reference source not found.**. Therefore, it seems that the results obtained from the $DCM^{t,t+1}$ index are more reasonable than that of the $CM^{t,t+1}$ index. The results for the other DMUs and also the value of the $DCM^{t,t+1}$ index components can be interpreted similarly.

In a non-competitive market characterized by heterogeneous inputs where DMUs have the ability to influence somewhat the market prices, the environmental factors may affect on decisions of DMUS in specifying their input price vectors. In such situations the obtained results of comparing the productivity changes of DMUs using the cost Malmquist index presented in Maniadakis and Thanassoulis (2004) can not be right. In The current study we assumed that the input prices are varying between DMUs and estimated a directional measure of cost Malmquist index by considering the affects of these environmental factors on the productivity changes over time. Also, using the new cost malmquist index, decision maker's preference can be incorporated in the productivity changes of units by selecting the suitable direction vector.

5. Conclusion

Table 2. The results of CM index

DMU	DMU1	DMU2	DMU3	DMU4	DMU5
$CM^{0,1}$	2.83	1.87	1.00	1.00	2.36

Table 3. The results of DCM index

	Direction vector 1					Direction vector 2				
	Decomposition of DCM index					Decomposition of DCM index				
DMU	$DCM^{0,1}$	TEC	AEC	TC	PE	$DCM^{0,1}$	TEC	AEC	TC	PE
DMU1	0.35	0.21	0.75	2.31	0.97	0.95	0.98	0.9	1.03	1.05
DMU 2	6	1	1	11.59	0.52	1.26	1	1	1.15	1.1
DMU 3	1	0.22	1.12	4.49	0.89	1.7	1.08	1.19	1.1	1.2
DMU 4	0.2	0.05	0.49	4.44	2.03	0.12	0.05	0.49	2.84	1.87
DMU 5	3	1	1	4.45	0.67	1.34	1	1	1.22	1.1

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