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Congestion Status Identification Using Slack Based Models in Data Envelopment Analysis

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Abstract

Congestion is a wasteful stage of the production process where outputs are reduced due to excessive amount of inputs. There are several approaches in data envelopment analysis (DEA) literatures to treat congestion. The concept of strong and weak congestion first developed by Tone and Sahoo [Tone, K., Sahoo, B.K., 2004. Degree of scale economies and congestion: A unified DEA approach. European Journal of Operational Research 158, 755–772]. Evidence of strong congestion occurs whenever reducing proportionally all inputs can increase all outputs. Tone and Sahoo extended the relationship between congestion and other economic concept such as marginal productivity. They proved that existence of strong congestion is hinged to negative marginal productivity. Nonetheless the definition of strong congestion is too restrictive, since it has severe condition of "proportionate'' reduction in all inputs causes an augmentation in all outputs. So the number of strongly congested DMUs appeared to be limited. In this paper we define a new less restrictive definition of congestion namely " strict congestion'' which is identified if and only if a reduction in all inputs causes an increase in all outputs. Therewith this study first proposes a new approach for the congestion recognition. Then another scheme is presented to determine the status of congestion (weak or strict). The validity of the proposed approach is demonstrated using numerical examples.

Keywords: Data envelopment analysis; Weak congestion; Strict congestion; Strong congestion.

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1. Introduction

Data envelopment analysis(DEA) which first proposed by Charnes, Cooper and Rhodes [3] is a non-parametric and mathematical programming based approach for evaluating performance of a set of homogeneous decision making units(DMUs) using multiple inputs to produce multiple outputs. In performance analysis, in particular in DEA, the concept of congestion plays a seminal role in theory and application. Congestion is a special phenomenon in the production process which is defined in economics where outputs are reduced along of excessive amount of inputs or an increase in one or more outputs is along of a reduction in one or more inputs. For an actual example of congestion in a coal mine where the miners are working in an narrow underground, when a lot of workers were crowded, the amount of minerals excavated will be reduced [7].

Heretofore, various approaches have been presented in DEA for the treatment of congestion. First, Far and Svensson [11] discussed congestion in the theoretical aspect. Far and Grosskopf [9], then considered the concepts of congestion from a operational aspect in the content of DEA. Subsequently, it was given an operationally implementable model via DEA to evaluate congestion by Far, Grosskopf and Lovell [10]. Cooper, Thompson and Thrall (CTT, [4]) proposed a slack- based approach which not only can successfully identify the sources of congestion but also can evaluate the amount of congestion in each inputs as the difference between the observed amounts and the expected amounts. CTT model has been extended by Brocket et al. [2] in their study of congestion in chinese industrials. Cooper et al. [5] extended a unified additive model for determining congestion. Next Cooper et al. [6] proposed a one-model approach that can be used instead of such two-model approaches. Jahanshahloo and Khodabakhshi [12] provided an approach of input congestion based on the relaxed combinations of inputs. Also Khodabakhshi [13] proposed a method to detect the input congestion in the stochastic DEA . Wei and Yan [16] used DEA output additive models and proposed a necessary and sufficient condition for existence of congestion. Tone and Saho [15] developed a new slack-based approach to evaluate the scale elasticity in the presence of congestion with a unified framework. They developed new concepts of strong and weak congestion and proved that existence of strong congestion is hinged to negative marginal productivity. Nonetheless the definition of strong congestion is too restrictive , since it has severe condition of "proportionate'' reduction in all inputs causes an augmentation in all outputs. So the number of strongly congested DMUs appeared to be limited .

In this paper we define a new less restrictive definition of congestion as "strict congestion'' which is identified if and only if a reduction in all inputs causes an increase in all outputs. Therewith this study first proposes a new approach for the congestion recognition. Then another scheme is presented to determine the status of congestion (weak or strict).

The rest of this paper unfolds as follow: In the next section some basic definitions and related DEA models will be presented to facilitate later discussions. The proposed congestion approach will be explained in section 3. In section 4, two numerical examples are discussed. Finally, Section 5 gives the conclusion of this paper.

2. Some basic concepts

In this section we first briefly describe some fundamental concepts of DEA. DEA successfully divided DMUs into two categories: efficient DMUs and inefficient DMUs. Efficient and inefficient DMU in DEA was defined as follows [6]:

Efficiency (Technical efficiency): *Efficiency is achieved by* DMU_0 *if and only if it is not possible to improve some of its inputs or outputs without worsening some of its other inputs or outputs.*

Inefficiency (Technical inefficiency): *Technical inefficiency is said to be present in the performance of* DMU_0 *when the evidence shows that it is possible to improve some input or output without worsening some other input or output.*

One research field which has received widespread attention in DEA is identification and evaluation congestion. Congestion is a special phenomenon in the production process which was defined the following form [7] :

Congestion: *Congestion is said to occur when the output that is maximally possible can be increased by reducing one or more inputs without worsening any other input or output. Conversely, congestion is said to occur when some of the outputs that are maximally possible are reduced by increasing one or more inputs without improving any other input or output.*

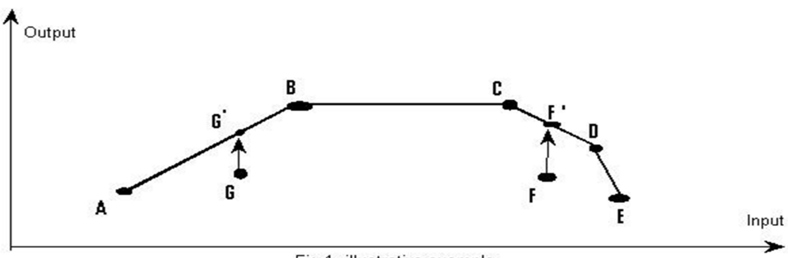

Fig.1. illustrative example

Fig.1 helps us to better distinguish these concepts. The points A and B and the segments connecting them represent the efficiency frontier. The segments connecting BC and the segments connecting CDE represent the inefficiency part of the frontier and the congested part of the frontier respectively. To help facilities better understand the difference between the definitions of technical inefficiency and congestion, consider two points F and G that both lie below the frontier. As can be seen, first we obtain maximal possible output of these two points without changing their input, this stage includes concept of technical inefficiency for both point, but in point F', further increase in output can be obtained by decreasing the input amount whereas this matter dose not happen in point G'. Therefore F is congested but D is technical inefficient.

Thus, congestion may be regarded as a particularly severe form of technical inefficiency.

Suppose there are n DMUs with m inputs and s outputs. The input and output vectors of $DMU_j(j=1,\dots,n)$ are $x_j=(x_{1j},\dots,x_{mj})^t, y_j=(y_{1j},\dots,y_{sj})^t$, respectively, where $x_j \ge 0, x_j \ne 0, y_j \ge 0, y_j \ne 0$.

The production possibility set T is represented as

$$T=\{(x,y)\in R_+^{m+s} \mid y \text{ can be produced from } x\} \quad (1)$$

Banker et al. [1] deduced the following production possibility set. This set is denoted by T_{BCC}, regarding the prevalence of variable returns to scale assumption of the production technology.

$$T_{BCC}=\{(x,y)\in R_+^{m+s} \mid x \ge \sum_{j=1}^{n}\lambda_j x_j, \; y \le \sum_{j=1}^{n}\lambda_j y_j, \sum_{j=1}^{n}\lambda_j=1, \lambda_j \ge 0 \; j=1,\dots,n\} \quad (2)$$

In T_{BCC} it is possible to increase inputs $(x \geq \sum_{j=1}^{n} \lambda_j x_j)$ without changing outputs $(y = \sum_{j=1}^{n} \lambda_j y_j)$. However in the case of congestion, outputs may not stay unchanged when inputs increase. Therefore input disposability postulates violates in the presence of congestion. In order to potentially deal with such situation, Tone and Sahoo [15] modified the production possibility set of T_{BCC} as follows:

$$T_{Convex} = \{(x, y) \in R_{+}^{m+s} \mid x = \sum_{j=1}^{n} \lambda_j x_j,\; y \leq \sum_{j=1}^{n} \lambda_j y_j, \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0 \; j = 1,\ldots,n\} \tag{3}$$

Definition 2.1 *Suppose that $DMU_p = (x_p, y_p)$ is under evaluation. The output-oriented form of BCC model, introduced by Banker et al.[1] to evaluate the efficiency of DMU_p is as follows :*

$$\begin{aligned} Max \;\; z = \phi + \varepsilon\Big(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+\Big) & \\ s.t. \;\; \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{ip} \qquad & i = 1,\ldots,m \\ \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = \phi y_{rp} \qquad & r = 1,\ldots,s \\ \sum_{j=1}^{n} \lambda_j = 1 & \\ \lambda_j \geq 0, \;\; j = 1,\ldots,n & \\ s_i^- \geq 0, \;\; i = 1,\ldots,m \;\; s_r^+ \geq 0, \;\; r = 1,\ldots,s & \end{aligned} \tag{4}$$

where ε is a non-Archimedean infinitesimal. DMU_p is called "BCC-strong efficient'' if and only if $\phi^* = 1$ and all optimal slacks are zero. Through this paper "*'' denotes optimality.

Notice that model (4) is solved in two stages. First ϕ is maximized and in the second stage $\sum_{i=1}^{m} S_i^- + \sum_{r=1}^{s} S_r^+$ will be maximized while ϕ is kept at the optimal value.

Definition 2.2 *Consider the following output-oriented model to evaluate the efficiency of DMU_p under set T_{Convex} :*

$$\begin{aligned} Max \;\; z = \beta + \varepsilon\Big(\sum_{r=1}^{s} t_r^+\Big) & \\ s.t. \;\; \sum_{j=1}^{n} \lambda_j x_{ij} = x_{ip} \qquad & i = 1,\ldots,m \\ \sum_{j=1}^{n} \lambda_j y_{rj} - t_r^+ = \beta y_{rp} \qquad & r = 1,\ldots,s \\ \sum_{j=1}^{n} \lambda_j = 1 & \\ \lambda_j \geq 0, \;\; j = 1,\ldots,n \;\; t_r^+ \geq 0, \;\; r = 1,\ldots,s & \end{aligned} \tag{5}$$

where ε is a non-Archimedean infinitesimal. DMU_p is called "strongly efficient" with respect to T_{Convex} if and only if $\beta^* = 1$ and all optimal slacks are zero.

Notice that model (5) is solved in two stages. First β is maximized and in the second stage $\sum_{r=1}^{s} t_r^+$ will be maximized while β is kept at the optimal value.

Here , we redefine the concept of "weak congestion'' and "strong congestion" from Tone and Sahoo [15][Tone and Sahoo =TS hereafter].

Definition 2.3 *(weak congestion)*

$DMU_p = (x_p, y_p)$ is weakly congested if it is strongly efficient with respect to T_{Convex} and there exists $(\hat{x}, \hat{y}) \in T_{Convex}$, such that $\hat{x} \le x_p, \hat{x} \ne x_p$ and $\hat{y} \ge y_p, \hat{y} \ne y_p$.

Definition 2.4 *(strong congestion)*

$DMU_p = (x_p, y_p)$ is strongly congested if it is strongly efficient with respect to T_{Convex} and there exists an activity $(\bar{x}, \bar{y}) \in T_{Convex}$ such that $\bar{x} = \alpha x_p$ (with $0 < \alpha < 1$) and $\bar{y} \ge \beta y_p$ (with $\beta > 1$), that is, if a proportionate reduction in all inputs of a DMU, warrantees an increase in all outputs, then strong congestion is occurred.

It is worthy of note that one of considerable previous research on congestion is TS approach [15]. In the following we briefly describe TS congestion approach.

2.1. TS congestion approach

Definitions 2.3 and 2.4 assume that a DMU is strongly efficient with respect to T_{Convex}. Hence the approach proposed by TS projects all DMUs onto the efficiency frontier of T_{Convex} and then recognizes whether each DMU is congested (weak or strong) or not. So, their approach needs to solve model (5). Therefore the projected point for DMU_p will be obtain as follows :

$$\begin{aligned} \hat{x}_p &= x_p \\ \hat{y}_p &= \beta^* y_p + t^{+^*} \end{aligned} \tag{6}$$

Trivially the projected point $(\hat{x}_p, \hat{y}_p)$ is strongly efficient with respect to T_{Convex} .

TS have proposed the following procedure to find out the congestion (weak or stong) for DMU_p on the projected point $(\hat{x}_p, \hat{y}_p)$

Step1. Let $(\phi^*, s^{-^*}, s^{+^*})$ be the optimal value of model (4) .

(a) If $\phi^* = 1, s^{-^*} = 0, s^{+^*} = 0$ then $(\hat{x}_p, \hat{y}_p)$ is BCC-efficient and not congested under variable return to scale and stop.

(b) If $\phi^* = 1, s^{-^*} \ne 0 \; and \; s^{+^*} = 0$ then $(\hat{x}_p, \hat{y}_p)$ is BCC-inefficient and stop.

(c) If $(\phi^* = 1 \; and \; s^{-^*} \ne 0) \; or \; \phi^* > 1$, then $(\hat{x}_p, \hat{y}_p)$ is congested and go to step 2.

Step2. Let $\bar{\rho}$ be the optimal value of the following model :

$$\begin{aligned} &\boldsymbol{Max}\ \ \rho = v\hat{x}_p \\ &s.t.\ -vx_j + uy_j + w \le 0,\quad j = 1,...,n,\ j \neq p \\ &\qquad -v\hat{x}_p + u\hat{y}_p + w = 0 \qquad\qquad (7) \\ &\qquad u\hat{y}_p = 1 \\ &\qquad u \ge 0, v \ge 0 \end{aligned}$$

if $\bar{\rho} < 0$, $(\hat{x}_p, \hat{y}_p)$ is strongly congested. Otherwise, $\bar{\rho} \ge 0$ then $(\hat{x}_p, \hat{y}_p)$ is weakly, but not strongly congested.

It is noticeable that the $\bar{\rho}$ in above procedure is the upper bound of scale elasticity (ρ) or DSE[1]. TS [15] discussed the scale elasticity issue with respect to the new production possibility set T_{Convex} and proved that "strong'' congestion is recognized when $\bar{\rho} < 0$.(see TS [15] for more details).

3. Proposed approach to recognize weak and strict DEA congestion

In this section we introduce a new status of congestion, namely strict congestion. Our motivation of this definition is that in TS congestion research [15] the term of "proportionally'' decrease in all inputs in definition of strong congestion has been specified but in some preambles in the same paper, this subject has been neglected and the term of decrease in all inputs has been equated, whiles they are differs actually and they have different results. This issue intended us to introduce "strict congestion''" as a distinguished definition of congestion as follows:

Definition 3.1 *(strict congestion)*

$DMU_p = (x_p, y_p)$ is strictly congested if it is strongly efficient with respect to T_{Convex} and there exists an activity $(\bar{x}, \bar{y}) \in T_{Convex}$ such that $\bar{x} < x_p$ and $\bar{y} > y_p$. This means that $DMU_p = (x_p, y_p)$ lies in the area of congestion where there exists an activity $(\bar{x}, \bar{y}) \in T_{Convex}$ such that uses less resources in all inputs to produce more products in all outputs.

Note that strong congestion implies strict congestion and strict congestion implies weak congestion but not vice versa and that in the case of single input and output, there is no distinction between strong and strict and weak congestions.

In following we first propose a new approach based on definition 2.4 to identify congestion. Then another scheme is presented to determine the status of congestion (weak or strict). For this end it is assumed that DMU_p $, (p \in \{1,...,n\})$ is strongly efficient with respect to T_{Convex}. If

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not, we solve model (5). Suppose that $(t^{+^*}, \beta^*, \lambda^*)$ be the optimal solution of (5). we project DMU_p on the strongly efficient frontier of T_{Convex} as following:

$$\begin{aligned} \hat{x}_p &= x_p \\ \hat{y}_p &= \beta^* y_p + t^{+^*} \end{aligned} \tag{8}$$

Now, we solve the following model:

$$\begin{aligned} & Max\ Min\ \{ \sum_{i=1}^{m} s_i^- , \sum_{r=1}^{s} s_r^+ \} \\ & s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \hat{x}_{ip} \qquad i = 1,\dots,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = \hat{y}_{rp} \qquad r = 1,\dots,s \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \geq 0, \ j = 1,\dots,n \\ & s_r^+ \geq 0, \ r = 1,\dots,s \quad s_i^- \geq 0, \quad i = 1,\dots,m \end{aligned} \tag{9}$$

Remark 1 *Problem (9) is non-linear, but it can easily be converted to a linear programming problem by replacement* $q = Min\ \{\sum_{i=1}^{m} s_i^-, \sum_{r=1}^{s} s_r^+\}$ *as follows :*

$$\begin{aligned} & q^* = Max\ \ q \\ & s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \hat{x}_{ip} \qquad i = 1,\dots,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = \hat{y}_{rp} \qquad r = 1,\dots,s \\ & \sum_{j=1}^{n} \lambda_j = 1, \ \ \sum_{i=1}^{m} s_i^- \geq q, \ \sum_{r=1}^{s} s_r^+ \geq q \\ & \lambda_j \geq 0, \quad j = 1,\dots,n \\ & s_r^+ \geq 0, \ r = 1,\dots,s \quad s_i^- \geq 0, \quad i = 1,\dots,m \end{aligned} \tag{10}$$

Theorem 3.1 *Congestion is appeared in performance of* DMU_p *if and only if the optimal objective value of model (6) is positive.*

If DMU_p has been identified congested then we solve the following model to detect wether DMU_p is strictly or weakly congested.

$$\begin{aligned} & Max\ \ Min\ \{ \ s_1^- ,\dots, s_m^- , s_1^+ ,\dots, s_s^+ \ \} \\ & s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \hat{x}_{ip} \qquad i = 1,\dots, m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = \hat{y}_{rp} \qquad r = 1,\dots, s \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \geq 0, \quad j = 1,\dots, n \\ & s_r^+ \geq 0, \quad r = 1,\dots, s \quad s_i^- \geq 0, \ i = 1,\dots, m \end{aligned} \tag{11}$$

Remark 2 *Problem (11) is non-linear, but it can easily be converted to a linear programming problem by replacement* $p = Min\ \{s_1^-, s_2^-, s_m^-, s_1^+, s_2^+, s_s^+\}$ *as follows*

$$\begin{aligned} & p^* = Max \quad p \\ & s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = \hat{x}_{ip} \qquad i = 1,\dots,m \\ & \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = \hat{y}_{rp} \qquad r = 1,\dots,s \\ & \sum_{j=1}^{n} \lambda_j = 1 \\ & \lambda_j \geq 0, \ j = 1,\dots,n \quad s_i^- \geq p, \quad i = 1,\dots,m \\ & s_r^+ \geq p, \ r = 1,\dots,s \quad p \geq 0, \end{aligned} \tag{12}$$

Theorem 3.2 *DMU_p has evidence strict congestion if and only if the optimal objective value of model (12) is positive.*

4. Numerical examples

In this section, we apply our proposed procedure and TS approach[15] to detect congestion and determine the status of congestion in order that discriminate between weak and strict and strong congestion in some simple numerical examples .

Example 1

We compare the proposed approach with TS approach [15] with an example of TS ([15], p.756) which is listed in Table 1 of our study . This example consists of four DMUs A, B, C and D ,with two inputs and two outputs each. The congestion results of TS [15] and the proposed approach are summarized in Table 2. As can be seen, both TS [15] and the proposed approach indicate that DMUs A and B have no congestion and DMU C has weak congestion. DMU D is strongly congested because $\bar{\rho}_D = -3 < 0$ and by proposed approach is strictly congested since $p_D^* = 1 > 0$.

Notice that the column of $\bar{\rho}$ is only calculated for congested DMUs to determine whether are strongly congested or not and for the ones that are not congested thre is no need to calculate that.

DMUs	**Input1**	**Input2**	**Output1**	**Output2**
A	1	1	1	1
B	2	2	2	2
C	2	3	2	1
D	3	3	1	1

Tabel 1: Data set in Example 1

DMUs	TS **Approach**							**Proposed Approach**		
	φ^*	s_1^{-*}	s_2^{-*}	s_1^{+*}	s_2^{+*}	$\bar{\rho}$	Status	p^*	q^*	status
A	1	0	0	0	0		No congestion	0	0	No congestion
B	1	0	0	0	0		No congestion	0	0	No congestion
C	1	0	1	0	1	∞	weak congestion	1	0	weak congestion
D	2	1	1	0	0	-3	strong congestion	2	1	strictly congestion

Table 2: Results of TS and proposed congestion approach for Example 1

Example 2

In this example we consider a group of 15 DMUs using three inputs and producing three outputs. The data set of DMUs are given in Table 3 and the congestion results of TS [15] and the proposed approach are summarized in Table 4. As shown in Table 4 , both TS approach [15] and the proposed approach provide same results in recognition congested units , that is DMUs 4, 5, 8, 11, 12 and 15 are recognized congested but for status of congestion we have : TS approach indicates that none of congested units have evidence strong congestion and using our proposed approach all of congested units except unit 12 are strictly congested. Actually DMU 12 among all congested DMUs is only weakly but not strictly congested , since $p_{12}^{*}=0$ and based upon proposed model this means that it is not possible to decrease all inputs and increase all outputs of the its projection and hence regarding definition 3.1 DMU 12 is not strictly congested. This example indicates that the occurence of strong congestion unlike strictly congestion is very slim, because the definition of strong congestion is too restrictive in that a “proportionat’’ reduction in all inputs warrants an increase in all outputs. Notice that the column of $\overline{\rho}$ is only calculated for congested DMUs to determine whether are strongly congested or not and for the ones that are not congested, there is no need to calculate that.

DMUs	Input1	Input2	Input3	Output1	Output2	Output3
1	225935	405	1000	178285	4967	3388
2	102200	179	575	75526	3808	2083
3	94900	135	388	63868	5435	1246
4	51100	95	263	31835	1222	559
5	43800	43	186	19360	1112	373
6	31755	36	116	23372	2095	275
7	31390	28	99	17798	462	277
8	29930	31	159	14067	794	184
9	27740	41	226	15895	1130	386
10	23360	39	153	18127	1143	269
11	20440	31	94	14860	949	157
12	10585	15	67	4498	148	29
13	8030	15	62	8311	335	70
14	11680	26	110	9449	435	161
15	11315	20	92	4797	71	53

Tabel 3: Data set in Example 2

DMUs	Proposed approach			TS approach	
	p^*	q^*	status	$\bar{\rho}$	Status
1	0	0	no congestion		Efficient
2	0	0	no congestion		Efficient
3	0	0	no congestion		Efficient
4	5586.47	34.80	strict congestion	1.20	weak congestion
5	7245.76	4.28	strict congestion	1.45	weak congestion
6	0	0	no congestion		Efficient
7	0	0	no congestion		Efficient
8	5584.80	6.5	strict congestion		weak congestion
9	0	0	no congestion		Efficient
10	414.58	68	strict congestion	15	weak congestion
11	1145.72	6.89	strict congestion	1.57	weak congestion
12	2560	0	weak congestion	1.10	weak congestion
13	0	0	no congestion		Efficient
14	0	0	no congestion		Efficient
15	3320	5	strict congestion		weak congestion

Tabel 4: Results of TS and proposed congestion approach for Example 2

5. conclusion

The main focus of this research lies in developing a new status of congestion defined as “strict congestion’’ and offering a simplified implementation for congestion identification. Indeed, by relaxing the sever condition of “proportionate’’ reduction in all inputs within the definition of strong congestion, just to the condition “reduction in all inputs’’, we define a new less restrictive definition of congestion that we call it “strict congestion’’. In following Theorems 3.1 and 3.2 provide directly procedures to assess existence of congestion and distinguish the status of congestion (weak or strong). Illustrative examples implied the truth that weakly congested DMUs often are strictly congested but the probability of occurrence strong congestion for congested DMUs is very slim. This is pertain to the definitions of such status of congestion.

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