Available online at http://jnrm.srbiau.ac.ir
Vol.1, No.4, Winter 2016
ISSN:1682-0169

Journal of New Researches in Mathematics   Science and Research Branch (IAU)

Utilizing Robust Data Envelopment Analysis Model for Measuring Efficiency of Stock, A case study: Tehran Stock Exchange

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Received Autumn 2015, Accepted Winter 2016

Abstract
Uncertainty is a prominent feature of real world problems and more especially financial markets; with this in mind, dealing with uncertainty becomes a necessary part of performance evaluation by means of data envelopment analysis. This paper presents three robust data envelopment analysis (DEA) models and their application for performance evaluation in Tehran Stock Exchange (TSE). Based on the results, the evaluated performance of stocks and the number of efficient stocks is decreased in all three models by increasing the level of uncertainty.

Keywords: Data Envelopment Analysis, Robust Optimization, Tehran Stock Exchange, Uncertainty.

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1. Introduction

Performance evaluation and efficiency measurement with DEA models have various applications. So in literature, researchers presented various models with respect to data type and different applications. Uncertainty and the way of dealing with it are unavoidable aspects, when researchers use DEA models under variable conditions. These aspects are important because DEA is so sensitive to little data changes. In fact, data uncertainty can dramatically change final results and unit performance classification. So DEA models must be robust against data uncertainty. Recent improvements in robust optimization, lead to new DEA models which are robust.

Sadjadi and Omrani (2008) used Ben-Tal and Nemirovski (2000) and Bertsimas and Sim (2004) approaches to present a robust DEA model for performance evaluation of Iranian power distribution companies. Also Sadjadi and Omrani (2010) used Bertsimas and Sim (2004) approach for performance calculation of Iranian telecommunication companies. Roghanian and Foroughi (2010) evaluated the performance of Iranian regional Airports using robust data envelopment analysis. Sadjadi et al. (2011) presented a robust super efficiency data envelopment analysis model for ranking of provincial gas companies in Iran. Lu (2015) used Ben-Tal and Nemirovski (2000) and Bertsimas and Sim (2004) approaches to present a robust DEA model for evaluating algorithmic performance.

With respect to the uncertainty of financial markets, the purpose of this article is to present robust DEA models for stock performance evaluation. So in section 2, DEA models are described and in section 3 models are presented. Next in section 4, real data of Tehran Stock market are presented and analyzed. Finally in section 5, final notes are discussed.

2. Data Envelopment Analysis

DEA developed by Charnes et al. (1978) based on Farrell (1957) idea. This approach estimate relative efficiency of sets of decision making units by use of inputs and outputs. DEA separates units in two different category efficient and inefficient units. Charnes et al. (1978) presented CCR as first model in DEA. CCR assumption is constant return to scale (CRS). A multiplier CCR input oriented model that is as model (1):

$$Max \quad \Theta = \sum_{r=1}^{s} y_{ro} u_r$$

$$S.t. \quad \sum_{r=1}^{s} y_{rj} u_r - \sum_{i=1}^{m} x_{ij} v_i \le 0 \qquad (1)$$

$$\sum_{i=1}^{m} x_{io} v_i = 1$$

$j = 1,\dots,\mathrm{n} \quad r = 1,\dots,\mathrm{s} \quad i = 1,\dots,\mathrm{m}$

$u_r \geq 0 \qquad v_i \geq 0$

Banker et al. (1984) presented BCC with the assumption of variable return to scale (VRS). This assumption makes BCC more real than CCR. A multiplier BCC input oriented model that is as model (2):

$$Max \quad \Phi = \sum_{r=1}^{s} y_{ro} u_r + w$$

$$S.t. \quad \sum_{r=1}^{s} y_{rj} u_r - \sum_{i=1}^{m} x_{ij} v_i + w \leq 0 \qquad (2)$$

$$\sum_{i=1}^{m} x_{io} v_i = 1$$

$j = 1,\dots,\mathrm{n} \quad r = 1,\dots,\mathrm{s} \quad i = 1,\dots,\mathrm{m}$

$u_r \geq 0 \qquad v_i \geq 0$

Another model which is used in the research is additive model (Charnes et al., 1995). Unlike two other models presented before, this model considers input reduction and output increasing simultaneously. Multiplier form of this model with variable return to scale (VRS) is presented as the model (3):

$$Min \quad \Psi = \sum_{i=1}^{m} x_{io} v_i - \sum_{r=1}^{s} y_{ro} u_r + w$$

$$S.t. \quad \sum_{r=1}^{s} y_{rj} u_r - \sum_{i=1}^{m} x_{ij} v_i - w \leq 0 \qquad (3)$$

$$\sum_{r=1}^{s} u_r \geq 1 \qquad \sum_{i=1}^{m} v_i \geq 1$$

$j = 1,\dots,\mathrm{n} \quad r = 1,\dots,\mathrm{s} \quad i = 1,\dots,\mathrm{m}$

$u_r \geq 0 \qquad v_i \geq 0$

Therefore CCR, BCC and additive models are selected in this research.

3. Robust Data Envelopment Analysis

After specifying models to evaluate efficiency, in this section robust modeling approach is examined: Soyster (1973), Ben-Tal and Nemirovski (2000) and Bertsimas and Sim (2004). First of all, their weaknesses and strengths are discussed. It is clear that Soyster approach usually find solutions which are conservative, i.e. in order to ensure solution robustness in this approach, we may be far away from nominal problem optimality. Ben-Tal and Nemirovski's approach, lead to nonlinear and conical robust formulations so their approach cannot be used directly for discrete optimization problems. Bertsimas and Sim's approach can operate in a reasonable conservative level and leads to a linear optimization pattern, so we can use this approach for discrete optimization patterns. With respect to this feature and linearity of robust solution in this approach, we use it for data envelopment analysis designing. For introduction of robust structure based on Bertsimas and Sim's approach, consider following linear optimization problem:

$$Max \ c^T x \quad s.t. \quad Ax \leq b \qquad l \leq x \leq u \quad (4)$$

Bertsimas and Sim (2004) proposed a new method for dealing with uncertainty in parameters in a model. Consider i^{th} constraint of a nominal problem as: $a_i'x \le b_i$. J_i is coefficient set of $a_{ij}, j \in J_i$ which has uncertainty. $\tilde{a}_{ij}, j \in J_i$ receives its values based on a symmetric distribution (which has an expected value of a_{ij}). $\tilde{a}_{ij}$ receives its values for each i in $[a_{ij} - \hat{a}_{ij}, a_{ij} + \hat{a}]$. Here, Γ_i parameter should be introduced. The value of Γ_i is restricted to $[0, |J_i|]$ and this is not necessary an integer number. Γ_i adjusts model robustness in response to solution conservative level. In other word, it is too unlikely that all a_{ij} values change, $a_{ij}, j \in J_i$ .the purpose is to protect model when its changing is more than $\lfloor \Gamma_i \rfloor$, so a_{it} would be a coefficent that changes in the form of $(\Gamma_i - \lfloor \Gamma_i \rfloor)\hat{a}_{it}$, i.e. only a subset of coeficients will change. So in this approach if all of changes are in $\lfloor \Gamma_i \rfloor$ limit, so solution is certainly feasible and also if these change are more than $\lfloor \Gamma_i \rfloor$, there is a high probability that the solution is still feasible. Uncertaint problem in normal form is as model (5):

$$Max\ c^T x$$

$$\text{st.}\quad \sum a_{ij}x_j + \underset{\{S_i \cup \{t_i\} | S_i \subseteq J_i|, |S_i| = \lfloor \Gamma_i \rfloor, t_i \in J_i \setminus S_i\}}{Max} \left\{\sum_{j \in S_i} \hat{a}_{ij}\eta_i + (\Gamma_i - \lfloor \Gamma_i \rfloor)\hat{a}_{it_i}\eta_t\right\} \le b_i \quad \forall i$$

$$\begin{aligned} & -\eta_j \le x_j \le \eta_j \\ & l_j \le x_j \le u_j \\ & f_j \ge 0 \end{aligned} \tag{5}$$

If $\lfloor \Gamma_i \rfloor$ is an integer number then i^{th} constraint is formulated as Equation (6):

$$B_i(x, \Gamma_i) = \underset{\{S_i : S_i, S_i \subseteq J_i\ |S_i| = \Gamma_i\}}{Max} \left\{\sum_{j \in S_i} \hat{a}_{ij}|x_j|\right\} \tag{6}$$

To reformulate robust cunterpart of a nominal problem, the bellow proposin is needed: the i^{th} constraint change as follow:

$$B_i(x^*, \Gamma_i) = \underset{\{S_i \cup \{t_i\} | S_i \subseteq J_i|, |S_i| = \lfloor \Gamma_i \rfloor, t_i \in J_i \setminus S\}}{Max} \left\{\sum_{j \in S_i} \hat{a}_{ij}|x_j^*| + (\Gamma_i - \lfloor \Gamma_i \rfloor\ \hat{a}_{it_i}|x_j^*|)\right\} \tag{7}$$

To find the maximum of function (7) DMs can solve below mathematical peoblem:

$$\begin{aligned} & B_i(x^*, \Gamma_i) = Max \left\{\sum_{j \in J_i} \hat{a}_{ij}|x_j^*|\ Z_{ij}\right\} \\ & s.t. \quad \sum_{j \in J_i} Z_{ij} \le \Gamma_i \\ & \qquad 0 \le Z_{ij} \le 1 \end{aligned} \tag{8}$$

Above formulation is not linear programming. Dual form of this problem is linear programming. Dual form is as model (9):

$$Min \quad \sum_{j \in J_i} P_{ij} + \Gamma_i Z_i$$

$$s.t. \quad Z_i + P_{ij} \ge \hat{a}_{ij}|x_j^*| \qquad \forall i, j \in J_i$$

$$P_{ij} \geq 0 \qquad \forall j \in J_i \qquad (9)$$
$$Z_i \geq 0 \qquad \forall i$$

Based on strong duality in optimality if primal problem is bounded then dual problem is bounded. In this case if $\Gamma_i \in [0, |J_i|]$ then primal and dual problem is bounded. In relations presented below, Z_i and P_{ij} are dual auxiliary variables which are used for problem linearity. Also η is used for $|x_j|$ transformation to linear form (Bertsimas and Sim, 2004).

$$\begin{aligned}
&Max\ c^T x \\
&s.t.\quad \sum_j a_{ij} x_j + Z_i \Gamma_i + \sum_{j \in J_i} P_{ij} \leq b_i && \forall i \\
&Z_i + P_{ij} \geq a_{ij} \eta_j && \forall i, j \in J_i \\
&-\eta_j \leq x_j \leq \eta_j && \forall j \\
&l_j \leq x_j \leq u_j && \forall j \qquad (10) \\
&P_{ij} \geq 0 && \forall i, j \in J_i \\
&\eta_j \geq 0 && \forall j \\
&Z_i \geq 0 && \forall i
\end{aligned}$$

It is noteworthy that we can show $\hat{a}_{ij}$ as $\hat{a}_{it} = \delta\, a_{ij}$, in which δ is deviation percentage. With respect to features of Bertsimas and Sim model and with our best knowledge, we can say that all of linear optimization models which use robust optimization methodology utilize this approach. As we said before, the most important feature of this approach is that when we use it, the dual of linear robust problem stays linear. Also we can say that this methodology has a good controllability over the solution robustness degree with respect to conservatism issues.

Now we can present robust CCR model based on Bertsimas and Sim's approach for considering outputs data uncertainty in model (11) as below:

$$\begin{aligned}
&Max\ \Theta \\
&S.t.\quad -(\sum_{r=1}^{s} y_{ro} u_r) + Z_0 \Gamma_0 + \sum_{r=1}^{s} P_{0r} \leq -\Theta \\
&\sum_{i=1}^{m} x_{io} v_i = 1 \\
&\sum_{r=1}^{s} y_{rj} u_r - \sum_{i=1}^{m} x_{ij} v_i + Z_j \Gamma_j + \sum_{r=1}^{s} P_{jr} \leq 0 \\
&\quad j = 1, \ldots, n \qquad r = 1, \ldots, s \\
&Z_0 + P_{0r} \geq \delta\, y_{ro}\, \eta_r \qquad (11) \\
&Z_j + P_{jr} \geq \delta\, y_{rj}\, \eta_r \quad j = 1, \ldots, n\ ,\ r = 1, \ldots, s \\
&-\eta_r \leq u_r \leq \eta_r \qquad r = 1, \ldots, s \\
&P_{jr} \geq 0 \qquad j = 0, \ldots, n\ ,\ r = 1, \ldots, s \\
&\eta_r \geq 0 \qquad r = 1, \ldots, s \\
&Z_j \geq 0 \qquad j = 0, \ldots, n \\
&u_r \geq 0 \qquad r = 1, \ldots, s \\
&v_i \geq 0 \qquad i = 1, \ldots, m
\end{aligned}$$

Robust BCC model based on Bertsimas and Sim's approach for considering outputs data uncertainty is as model (12):

$$\begin{aligned}
&Max\ \Phi \\
&S.t.\quad -(\sum_{r=1}^{s} y_{ro} u_r + w) + Z_0 \Gamma_0 + \sum_{r=1}^{s} P_{0r} \leq -\Phi \\
&\sum_{i=1}^{m} x_{io} v_i = 1 \\
&\sum_{r=1}^{s} y_{rj} u_r - \sum_{i=1}^{m} x_{ij} v_i + w + Z_j \Gamma_j + \sum_{r=1}^{s} P_{jr} \leq 0 \\
&\quad j = 1, \ldots, n \qquad r = 1, \ldots, s \\
&Z_0 + P_{0r} \geq \delta\, y_{ro}\, \eta_r \qquad (12) \\
&Z_j + P_{jr} \geq \delta\, y_{rj}\, \eta_r \quad j = 1, \ldots, n\ ,\ r = 1, \ldots, s
\end{aligned}$$

$$-\eta_r \leq u_r \leq \eta_r \qquad r = 1,\dots,s$$
$$P_{jr} \geq 0 \qquad j = 0,\dots,n \ , \ r = 1,\dots,s$$
$$\eta_r \geq 0 \qquad r = 1,\dots,s$$
$$Z_j \geq 0 \qquad j = 0,\dots,n$$
$$u_r \geq 0 \qquad r = 1,\dots,s$$
$$v_i \geq 0 \qquad i = 1,\dots,m$$

And final robust additive model With respect to Bertsimas and Sim's approach which considers inputs and outputs data uncertainty, is presented as model (13):

$$Min \quad \Psi$$
$$S.t. \quad \sum_{i=1}^{m} x_{io} v_i - \sum_{r=1}^{s} y_{ro} u_r + w + Z_0 \Gamma_0 + \sum_{h=1}^{s+m} P_{0h} \leq \Psi$$
$$\sum_{r=1}^{s} y_{rj} u_r - \sum_{i=1}^{m} x_{ij} v_i - w + Z_j \Gamma_j + \sum_{h=1}^{s+m} P_{jh} \leq 0$$
$$j = 1,\dots,n$$
$$-u_r \leq -1 \qquad r = 1,\dots,s$$
$$-v_i \leq -1 \qquad i = 1,\dots,m$$
$$Z_0 + P_{0h} \geq \delta x_{io} \eta_h \quad i = 1,\dots,m \ , \ h = i$$
$$Z_0 + P_{0h} \geq \delta \, y_{ro} \, \eta_h$$
$$r = 1,\dots,s \ , \ h = m + r$$
$$Z_j + P_{jh} \geq \delta \, y_{rj} \, \eta_h$$
$$j = 1,\dots,n \ , \ r = 1,\dots,s \ , \ h = r$$
$$Z_j + P_{jh} \geq \delta x_{ij} \eta_h$$
$$j = 1,\dots,n \ , \ i = 1,\dots,m \ , \ h = i + s$$
$$-\eta_h \leq u_r \leq \eta_h \qquad r = 1,\dots,s \ , \ h = r$$
$$-\eta_h \leq v_i \leq \eta_h \qquad i = 1,\dots,m \ , \ h = i + s$$
$$P_{jh} \geq 0 \quad j = 0,\dots,n \ , \ h = 1,\dots,s + m$$
$$\eta_h \geq 0 \qquad h = 1,\dots,s + m$$
$$Z_j \geq 0 \qquad j = 0,\dots,n$$
$$u_r \geq 0 \qquad r = 1,\dots,s$$
$$v_i \geq 0 \qquad i = 1,\dots,m$$

4. Case Study and Numerical Results

Numerical results are presented in this section. At first inputs and outputs of robust DEA are presented. Inputs and outputs are selected to evaluate 3 important stock aspects: stock risk, stock return and stock liquidity. The third aspect is important because sometimes stocks are in a good position from risk and return perspective, but selling them is not possible at the time. It must be mentioned that in this paper, two metrics (i.e. more efficient risk and better risk) are used as risk measures including semi-variance and value at risk. Researchers used monthly data of 15 stocks. Time period is between March 2012 and March 2015 and real-world data extracted from Tehran stock exchange (TSE).

The results of running Robust CCR Model, Robust BCC Model and Robust Additive Model based on Bertsimas and Sim's approach under different price of robustness and different level of uncertainty are presented in table (2), (3) and (4) respectively:

Table 1 : The inputs and the outputs of RDEA model

	Title	Description
Inputs	Semi Variance	average of the squared deviations of values that are less than the mean of returns
	Value at Risk (VaR)	Maximum expected loss during a specific period over a specific confidential level.
Outputs	Rate of Return	Proportion of gain or loss on an investment over a specified period
	Liquidity	degree which presents stock ability can be quickly bought or sold in the market

Table 2 : The Results of Robust CCR Model

DMUs	Γ_i=0	Γ_i=0.25%			Γ_i=0.50%			Γ_i=100%		
	δ=0	***δ=0.01***	***δ=0.02***	***δ=0.05***	***δ=0.01***	***δ=0.02***	***δ=0.05***	***δ=0.01***	***δ=0.02***	***δ=0.05***
Stock 01	0.673	0.666	0.660	0.640	0.660	0.647	0.609	0.660	0.647	0.609
Stock 02	0.487	0.483	0.478	0.464	0.478	0.468	0.441	0.478	0.468	0.441
Stock 03	0.794	0.786	0.778	0.755	0.778	0.763	0.718	0.778	0.763	0.718
Stock 04	0.728	0.721	0.714	0.693	0.714	0.700	0.672	0.714	0.700	0.659
Stock 05	0.693	0.686	0.679	0.659	0.679	0.666	0.627	0.679	0.666	0.627
Stock 06	0.535	0.530	0.525	0.509	0.525	0.514	0.484	0.525	0.514	0.484
Stock 07	0.662	0.656	0.649	0.630	0.649	0.636	0.599	0.649	0.636	0.599
Stock 08	0.780	0.773	0.765	0.742	0.765	0.750	0.706	0.765	0.750	0.706
Stock 09	1.000	0.995	0.990	0.975	0.990	0.980	0.951	0.980	0.961	0.905
Stock 10	0.739	0.731	0.724	0.702	0.724	0.710	0.668	0.724	0.710	0.668
Stock 11	0.605	0.599	0.593	0.576	0.593	0.581	0.547	0.593	0.581	0.547
Stock 12	0.874	0.865	0.857	0.831	0.857	0.840	0.791	0.857	0.840	0.791
Stock 13	0.662	0.655	0.649	0.629	0.649	0.636	0.599	0.649	0.636	0.599
Stock 14	0.600	0.594	0.589	0.571	0.589	0.577	0.543	0.589	0.577	0.543
Stock 15	0.886	0.877	0.869	0.843	0.869	0.851	0.802	0.869	0.851	0.802

Table 3 : The Results of Robust BCC Model

DMUs	***Γ$_i$=0***	***Γ$_i$=0.25%***			***Γ$_i$=0.50%***			***Γ$_i$=100%***		
	δ=0	***δ=0.01***	***δ=0.02***	***δ=0.05***	***δ=0.01***	***δ=0.02***	***δ=0.05***	***δ=0.01***	***δ=0.02***	***δ=0.05***
Stock 01	0.696	0.685	0.682	0.681	0.684	0.678	0.668	0.680	0.674	0.653
Stock 02	0.576	0.570	0.562	0.560	0.557	0.557	0.549	0.557	0.552	0.535
Stock 03	1.000	0.997	0.997	0.996	0.996	0.996	0.991	0.988	0.985	0.968
Stock 04	0.979	0.978	0.978	0.977	0.975	0.971	0.970	0.967	0.966	0.966
Stock 05	0.777	0.766	0.758	0.745	0.765	0.758	0.746	0.765	0.740	0.727
Stock 06	0.608	0.599	0.596	0.592	0.598	0.587	0.582	0.566	0.564	0.553
Stock 07	0.999	0.965	0.906	0.836	0.891	0.862	0.693	0.890	0.832	0.691
Stock 08	1.000	0.962	0.932	0.806	0.927	0.863	0.700	0.918	0.856	0.705
Stock 09	1.000	0.997	0.997	0.996	0.997	0.989	0.982	0.991	0.989	0.950
Stock 10	1.000	0.884	0.788	0.681	0.786	0.727	0.675	0.762	0.708	0.669
Stock 11	0.757	0.743	0.741	0.735	0.743	0.735	0.735	0.720	0.711	0.704
Stock 12	1.000	0.996	0.992	0.990	0.995	0.987	0.984	0.990	0.986	0.983
Stock 13	1.000	0.894	0.797	0.689	0.814	0.712	0.621	0.786	0.704	0.621
Stock 14	0.621	0.609	0.604	0.607	0.605	0.603	0.597	0.602	0.593	0.592
Stock 15	1.000	0.970	0.932	0.880	0.941	0.894	0.811	0.930	0.853	0.785

Table 4 : The Results of Robust Additive Model

DMUs	***Γ$_i$=0***	***Γ$_i$=0.25%***			***Γ$_i$=0.50%***			***Γ$_i$=100%***		
	δ=0	***δ=0.01***	***δ=0.02***	***δ=0.05***	***δ=0.01***	***δ=0.02***	***δ=0.05***	***δ=0.01***	***δ=0.02***	***δ=0.05***
Stock 01	15.57	15.94	16.24	17.25	16.15	16.52	18.58	16.40	17.07	19.54
Stock 02	20.19	20.72	21.28	22.39	21.21	21.81	23.15	21.33	22.25	24.55
Stock 03	0.00	1.94	3.44	8.74	2.04	4.04	10.10	2.40	4.61	11.61
Stock 04	6.90	7.45	7.68	8.56	7.68	7.86	9.31	7.68	8.33	10.56
Stock 05	14.94	15.34	15.99	17.18	15.53	16.22	18.23	15.80	16.78	19.41
Stock 06	26.29	27.02	27.32	28.73	27.06	27.74	29.52	27.27	28.08	30.71
Stock 07	0.05	2.17	4.32	10.33	2.56	4.99	12.32	3.68	5.96	14.59
Stock 08	0.03	2.13	4.10	9.96	2.40	5.03	11.49	3.03	5.62	12.98
Stock 09	0.00	0.35	0.82	2.27	0.68	1.38	2.63	0.95	1.62	3.89
Stock 10	0.00	6.85	8.84	11.44	7.26	9.37	12.51	7.60	9.82	14.09
Stock 11	14.69	15.22	15.76	16.29	15.39	15.84	17.43	16.00	16.26	18.80
Stock 12	0.00	2.36	4.83	10.06	2.82	5.42	12.10	3.65	6.46	13.33
Stock 13	0.00	12.52	14.64	20.25	12.71	15.45	22.40	13.36	15.81	23.61
Stock 14	17.56	18.16	18.23	19.46	18.41	18.75	20.88	18.60	19.88	22.11
Stock 15	0.01	3.82	5.50	9.39	4.05	6.20	10.67	4.28	6.59	12.39

From results, it is obvious that when the price of robustness increases or when data deviation is possible, stock efficiency can decrease. In some situations efficient stock can change to inefficient stock. The main reason of this event is data uncertainty. With the help of robust DEA model, the most pessimistic solution for stock efficiencies can be obtained. Thus a robust efficiency against data variance will be established.

5. Conclusion

In this paper, robust DEA models were used for stock performance evaluation, because they are powerful and suitable for considering data uncertainty. Neglecting this kind of uncertainty can influence on stock efficiency. After presenting robust DEA models, their solutions for Tehran Stock Exchange data were obtained and analyzed. Results show the effects of uncertainty on efficiency. In the extreme case, effects can be so dramatic. In fact they can change an efficient stock to inefficient stock and vice versa.

References

Farrell, M. J. (1957). The Measurment of Productive Efficiency. Journal of the Royal Statistical Society. Series A (General), 120(3): 253-290.

Charnes, A., Cooper W.W., Rhodes E. (1978). Measuring the efficiency of decision making units. European Journal of Operational Research, 2(4): 429-444.

Banker, R. D. Charnes, A. Cooper, W. W. (1984). Some Models for Estimation Technical and Scale in Efficiencies in Data Envelopment Analysis. Management Science, 30: 1078-1092.

Charnes, A., Cooper, W. W., Golany, B., Seiford, L., & Stutz, J. (1985). Foundations of data envelopment analysis for Pareto-Koopmans efficient empirical production functions. Journal of econometrics, 30(1), 91-107.

Soyster, A. L. (1973). Convex programming with set-inclusive constraints and applications to inexact linear programming. Operations research, 21(5): 1154-1157.

Ben-Tal, A., & Nemirovski, A. (2000). Robust solutions of linear programming problems contaminated with uncertain data. Mathematical programming, 88(3): 411-424.

Bertsimas, D., Sim, M., (2004). The price of robustness. Operations research. 52(1): 35-53.

Sadjadi, S. J., & Omrani, H. (2008). Data envelopment analysis with uncertain data: An application for Iranian electricity distribution companies. Energy Policy, 36(11), 4247-4254.

Sadjadi, S. J., & Omrani, H. (2010). A bootstrapped robust data envelopment analysis model for efficiency estimating of telecommunication companies in Iran. Telecommunications Policy, 34(4), 221-232.

Roghanian, E., & Foroughi, A. (2010). An empirical study of Iranian regional airports using robust data envelopment analysis. International Journal of Industrial Engineering Computations, 1(1), 65-72.

Sadjadi, S. J., Omrani, H., Abdollahzadeh, S., Alinaghian, M., Mohammadi, H. (2011). A robust super-efficiency data envelopment analysis model for ranking of provincial gas companies in Iran. Expert Systems with Applications, 38(9): 10875-10881.

Lu, C. C. (2015). Robust data envelopment analysis approaches for evaluating algorithmic performance. Computers & Industrial Engineering, 81, 78-89.