Adaptive Control of Nonlinear Systems in the Presence of Actuator Failures

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Abstract

This paper presents an adaptive state feedback control scheme for a class of nonlinear systems with unknown parameters, variable control gains and in the presence of unknown time varying actuator failures. The designed controller compensates unknown loss of effectiveness failures as well as unknown time varying stuck failures in actuators. The considered actuator failure can cover most failures that may occur in actuators of the practical systems. The proposed adaptive controller is constructed based on a backstepping design method. Appropriate Lyapunov-Krasovskii functionals are introduced to design new adaptive laws to compensate the unknown actuator failures and unknown parameters. The offered method ensures the asymptotic output tracking and the boundedness of all the closed-loop signals. The proposed design approach is employed for a wing rock control of an aircraft in the presence of time varying actuator failures. The simulation results verify the effectiveness and correctness of the proposed adaptive control method.

I**ndex Terms:** Time varying actuator failure, Nonlinear systems, Adaptive control, Backstepping.

کنترل تطبیقی سیستم‌های غیرخطی در حضور خرابی عملگر

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خلاصه

در این مقاله، یک کنترل‌کننده تطبیقی برای کنترل یک کلاس از سیستم‌های غیرخطی در معرض پارامترهای نامعین، بهره کنترلی متغیر و با وجود خرابی عملگر ارائه شده است. کنترل‌کننده ارائه شده می‌تواند خرابی کاهش کارایی و خرابی قفل شونده در عملگر را کاملاً جبران کند. مدل خرابی عملگر در نظر گرفته شده قابلیت جبران غالب خرابی‌های قابل وقوع در سیستم‌های عملی و کاربردی را دارد. کنترل‌کننده تطبیقی پیشنهادی بر اساس روش کنترلی گام به عقب طراحی شده است. در این مقاله، با معرفی توابع لیاپانوف- کراسوسکی مناسب، قوانین تطبیقی جدیدی طراحی شده است که خرابی‌های نامعین و پارامترهای نامعلوم را جبران می‌کند. روش کنترلی ارائه شده، تعقیب مجانبی خروجی و کراداری تمامی سیگنال‌های سیستم حلقه بسته را تضمین می‌کند. روش پیشنهادی جهت کنترل بال هواپیما در حضور خرابی متغیر با زمان عملگر استفاده شده است. نتایج شبیه‌سازی، کارایی و صحت روش کنترلی ارائه شده را نشان می‌دهد.

کلمات کلیدی: خرابی متغیر با زمان عملگر، سیستم غیرخطی، کنترل تطبیقی، روش کنترلی گام به عقب.

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1. Introduction

Actuator failures often cause undesired system behavior and sometimes lead to instability or even catastrophic accidents. The problem of actuator failure compensation is of both practical and theoretical importance, especially for critical systems such as flight control systems. Actuator failure compensation problem is an area of research that has attracted considerable attention in the recent years. So far, varieties of fault compensation methods especially adaptive approaches had been developed [1]-[10]. Adaptive mechanisms show suitable performance in presence of uncertainties in failed actuators. Many valuable researches have been achieved in adaptive actuator failure compensation for linear systems. For example, in [6] direct adaptive state feedback controller scheme was proposed to solve tracking problems for linear systems with unknown system parameters and in the presence of stuck type actuator failures. In [7]-[10], output feedback model reference adaptive controllers were developed for linear systems with unknown parameters in the presence of actuator failures. The considered actuator failure in [7]-[9] were modeled as stuck type and in [10] the actuator failure was modeled to cover both loss of effectiveness and stuck at some unknown fixed values. In [11], the direct adaptive state feedback controller was presented for linear system with actuator failures. The asymptotical stability of all the closed loop signals in [11] was proved despite the presence of loss of effectiveness and stuck type failures in actuators. In [12]-[13], adaptive backstepping method was investigated for nonlinear systems. It was concluded that backstepping's advantages lies in its flexibility, due to its recursive use of Lyapunov functions and its robustness against unmodeled dynamic of the systems. Some valuable research and practical results have been achieved in actuator failure compensation for nonlinear systems based on the backstepping design method. For example in [14]-[23], adaptive actuator failure compensation schemes were proposed for a class of uncertain nonlinear systems based on the backstepping design method in the form of state feedback [14]-[15], [20]-[23] and output feedback [15]-[19]. The considered actuator failure in [14]-[19] were modeled as stuck at some unknown values. The considered faults in [20]-[23] were modeled to cover both loss of effectiveness and stuck at some unknown fixed values. In [24] adaptive observer was constructed to estimate the fault in a class of nonlinear systems, then a backstepping based active fault tolerant controller was designed for faulty system. In [25], an adaptive fuzzy controller based on the backstepping design method was proposed for a class of nonlinear systems with unknown parameters

and actuator failures. In [26], an adaptive fuzzy actuator failure compensator was proposed for a class of uncertain stochastic nonlinear systems in strict feedback form with known control gains. The considered faults in [25]-[26] were modeled to cover both loss of effectiveness and constant stuck failures. The proposed fuzzy adaptive actuator failure compensators in [25] and [26] promised the boundedness of all the signals in the closed loop system; however, the tracking problem was not considered.

In this paper, an adaptive compensator is proposed for a class of nonlinear systems with unknown parameters, unknown control gains and in the presence of actuator failures. The considered actuator failure covers both loss of effectiveness and time varying stuck failures which are uncertain in time, value, and pattern. In other words, during the system operation, it is unknown when, how much and which actuators fail. The proposed adaptive controller in this manuscript is constructed based on the backstepping design method.

The main contributions of this paper, compared with the existing results, are as follows:

(1) The control problem is investigated for a class of nonlinear systems with parameter uncertainties and in the presence of unknown actuator failures.

(2) The proposed design method does not require a priori knowledge of the bounds of the unknown parameters and actuator failures.

(3) The considered time varying actuator failures not only cause the system gain changes but also lead to system uncertainties.

(4) The considered unknown time varying actuator failure is more general than the failures considered in the existing results of [14-26].

(5) Appropriate Lyapunov-Krasovskii type functionals are introduced to design new adaptive laws with less complexity to compensate the unknown time varying actuator failures as well as uncertainties from unknown parameters.

(6) The proposed method ensures the asymptotic output tracking and the boundedness of all the closed loop signals.

The paper is organized as follows. In section 2, the system description is given along with the necessary assumptions. In section 3, the design and analysis of an adaptive actuator failure compensation scheme are explained. In section 4, the actuator failure compensation problem is considered for the F-18 HARV-like wing-rock model to illustrate the effectiveness of the proposed control scheme. Finally, the paper is concluded in section 5.

2. Problem Statement

Consider a class of strict-feedback nonlinear systems in the following form.

$$\dot{x}_i(t) = x_{i+1}(t) + \theta_{f_i}^T F_i(\bar{x}_i(t)), \qquad (1)$$
$$i = 1, \dots, n-1$$
$$\dot{x}_n(t) = \varphi_0(x(t)) + \beta^T(x)bu(t) + \theta_{f_n}^T F_n(x(t))$$
$$y(t) = x_1(t)$$

where $\bar{x}_i = [x_1, x_2, \dots, x_i]^T$, $x = [x_1, x_2, \dots, x_n]^T$, $u \in R^m$, $y \in R$ are the state variables, system input and output, respectively. $F_i(.)$ and $\beta(x) = [\beta_1(x(t)), \dots, \beta_m(x(t))]^T$ are smooth nonlinear function vectors, $b = diag\{b_1, \dots, b_m\}$ in which $b_j, j = 1, \dots, m$ are unknown constant parameters and $\theta_{f_i}, i = 1, \dots, n$, are unknown constant parameter vectors.

The control objective is to design a state feedback controller for plant (1) in order to assure that all the closed loop signals are bounded and the plant output $y(t)$ tracks a desired signal $y_d(t)$ despite the presence of unknown plant parameters, control gains and unknown time varying actuator failures. For this purpose, the following assumptions are considered:

Assumption 1 The signs of $b_j, j = 1, 2, \dots, m$ are known and $\beta_j(x(t)) \neq 0, j = 1, \dots, m$.

Assumption 2 The desired signal $y_d(t)$ and its first n-th order derivatives $y_d^{(i)}(i = 1, \dots, n)$ are known, bounded, and piecewise continuous.

The stuck type actuator failures to be considered are modeled as:

$$u_j(t) = \bar{u}_j(t), \quad t \geq t_j, \quad j = j_1, j_2, \dots, j_p, \qquad (2)$$
$$1 \leq p \leq m-1$$
$$\bar{u}_j(t) = \bar{\bar{u}}_j + \bar{d}_j(t)$$

where $\bar{\bar{u}}_j$ is an unknown constant and $\bar{d}_j(t)$ is given by

$$\bar{d}_j(t) = \sum_{l=1}^{h} \bar{d}_{jl} g_{jl}(t), h \geq 1, j = j_1, j_2, \dots, j_p. \qquad (3)$$

The failure time instant, t_j, the failure index, j, and the scalar constant, $\bar{d}_{jl}$, are unknown and the scalar bounded signals $g_{jl}(t), j = j_1, j_2, \dots, j_p, l = 1, 2, \dots, h, h \geq 1$ are known

The loss of effectiveness model of the actuator failure to be considered is modeled as

$$u_j(t) = \rho_j v_j(t), \rho_j \in [\underline{\rho}_j, \bar{\rho}_j], 0 < \underline{\rho}_j \leq 1, \bar{\rho}_j \qquad (4)$$
$$= 1,$$
$$j = \{1, \dots, m\} \cap \overline{\{j_1, j_2, \dots, j_p\}}$$

where ρ_j is an unknown constant parameter. For plant (1) with actuator failures (2)-(4), the input vector can be expressed as:

$$u(t) = \rho v(t) + \delta(\bar{u}(t) - \rho v(t))$$

$$\bar{u}(t) = [\bar{u}_1, \bar{u}_2, \dots, \bar{u}_m]^T, \rho \qquad (5)$$
$$= diag\{\rho_1, \rho_2, \dots, \rho_m\}$$
$$v(t) = [v_1(t), v_2(t), \dots, v_m(t)]^T$$
$$\delta = diag\{\delta_1, \delta_2, \dots, \delta_m\}, \quad \delta_i$$
$$= \begin{cases} 1, & u_i(t) = \bar{u}_i(t) \\ 0, & u_i(t) \neq \bar{u}_i(t) \end{cases}$$

where $v(t)$ is the applied control input that will be designed later. With this description, a general type of actuator failures including loss of effectiveness and time varying stuck failures are considered. Loss of effectiveness can occur due to loss of a part of a control surface, engine malfunction or icing. Variant stuck failures can occur for example due to hydraulic failures that can produce unintended movements in the control surfaces of an aircraft [15]. Table 1 describes different failure situations.

Table (1): Failure model

Failure model	ρ_i	δ_i
Normal	1	0
stuck	1	1
Loss of effectiveness	(0,1)	0

For systems in which actuators may fail during the operation of the system, one common way is to use actuator redundancy. In this way, when one actuator fails, some others could compensate for the effect [15].

Assumption 3: ([14-26]) In the plant (1) with known plant parameters and failure parameters, if any up to $m-1$ actuators stuck as (2), the others may lose effectiveness as (4), the closed loop systems can still be driven to achieve a desired control objective.

$$(3)$$

3. Controller design

In this section, the design procedure of the proposed compensator based on the backstepping method is explained for the system (1). The backstepping design method for system (1) contains n stages [12]. At each stage, the intermediate control function and updating laws are designed using an appropriate Lyapunov function. To design both the control laws and updating laws, the following state transformation is considered for system (1).

$$z_1 = x_1 - y_d$$
$$z_i = x_i - \alpha_{i-1}, \qquad i = 2, \dots, n$$

The transformed system in the new coordination is obtained as:

$$\dot{z}_1(t) = z_2(t) + \alpha_1(t) + \theta_{f_1}^T F_1(x_1(t)) - \dot{y}_d(t)$$
$$\dot{z}_i(t) = z_{i+1}(t) + \alpha_i(t) + \theta_{f_i}^T F_i(\bar{x}_i(t)) \qquad (6)$$
$$\quad - \dot{\alpha}_{i-1}(t),$$
$$1 < i < n$$

$$\dot{z}_n(t) = \varphi_0(x(t)) + \beta^T(x)b\delta\bar{u}(t)$$
$$+ \beta^T(x)b(I - \delta)\rho v$$
$$+ \theta_{f_n}^T F_n(x(t)) - \dot{\alpha}_{n-1}$$

The detailed design procedure is given as follows.

Step1: In the first step, the z_1 subsystem is considered and the controller will be designed for this subsystem.

For the $z_i, i = 1, \dots, n$ subsystems, the following Lyapunov functions are considered.

$$V_{z_i} = \frac{1}{2} z_i^2(t) \tag{7}$$

$$V_{\theta_i} = \frac{1}{2} \tilde{\theta}_i^T \Gamma_i^{-1} \tilde{\theta}_i \tag{8}$$

$$V_i = V_{z_i} + V_{\theta_i} \tag{9}$$

where $\tilde{\theta}_i = \hat{\theta}_i - \theta_i$ in which $\hat{\theta}_i$ is the estimate of θ_i that will be defined later. Along system (6) and by using (7)-(9), the time derivative of V_{z_1} becomes

$$\dot{V}_1 = \dot{V}_{z_1} + \dot{V}_{\theta_1} \le z_1(t)z_2(t) + z_1(t)\alpha_1(t) \tag{10}$$
$$+ z_1(t)\theta_1^T\varphi_1 - z_1(t)\dot{y}_d(t)$$
$$+ \tilde{\theta}_1^T \Gamma_1^{-1}\dot{\hat{\theta}}_1$$

where

$$\theta_1 = \theta_{f_1}, \quad \varphi_1 = F_1(x_1(t)) \tag{11}$$

Accordingly, the intermediate control input is selected as

$$\alpha_1(x_1(t)) = -\hat{\theta}_1^T\varphi_1 - \frac{1}{2}z_1 - \gamma_1 z_1 + \dot{y}_d \tag{12}$$

where γ_1 is a positive constant.

Therefore, the time derivative of $V_1(t)$ becomes

$$\dot{V}_1 \le \frac{1}{2}z_2^2 - \gamma_1 z_1^2 - z_1(t)\tilde{\theta}_1^T\varphi_1 + \tilde{\theta}_1^T \Gamma_1^{-1}\dot{\hat{\theta}}_1 \tag{13}$$

Accordingly the updating laws for parameters $\hat{\theta}_1$ is selected as

$$\dot{\hat{\theta}}_1 = \Gamma_1 z_1 \varphi_1 \tag{14}$$

where $\Gamma_1 = \Gamma_1^T > 0$. By using $-\sigma_1\tilde{\theta}_1^T\hat{\theta}_1 \le \frac{-1}{2}\sigma_1\|\tilde{\theta}_1\|^2 + \frac{1}{2}\sigma_1\|\theta_1\|^2$, the time derivative of $V_1(t)$ becomes

$$\dot{V}_1 \le -\gamma_1 z_1^2 + \frac{1}{2}z_2^2 \tag{15}$$

As can be seen from the above inequality, the time derivative of $V_1(t)$ is dependent on the boundedness of the z_2 signal that will be regulated in the following.

Step 2: Similar procedures are taken for each step when $i = 2, \dots, n-1$, as in step 1. The z_i subsystems for $i = 2, \dots, n-1$ are considered. The intermediate controllers $\alpha_{i-1}(t)$, $i = 2, \dots, n$, are functions of $\bar{x}_{i-1}(t)$, $\hat{\theta}_1, \dots, \hat{\theta}_{i-1}$, $y_d, y_d^{(1)} \dots, y_d^{(i-1)}$, hence the time derivative of $\alpha_{i-1}(t)$ becomes

$$\dot{\alpha}_{i-1}(t) = \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k}\{x_{k+1}(t) \tag{16}$$
$$+ \theta_{f_k}^T F_k(\bar{x}_k(t))\}$$
$$+ \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}_k}\dot{\hat{\theta}}_k$$
$$+ \sum_{k=1}^{i} \frac{\partial \alpha_{i-1}}{\partial y_d^{(k-1)}} y_d^{(k)}$$

The time derivative of $V_i(t)$ by using (7)-(9) becomes

$$\dot{V}_i = \dot{V}_{z_i} + \dot{V}_{\theta_i} \le z_i(t)z_{i+1}(t) + z_i(t)\alpha_i(t) \tag{17}$$
$$+ z_i(t)\theta_i^T\varphi_i$$
$$- z_i(t)\sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k}x_{k+1}(t)$$
$$- z_i(t)\sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}_k}\dot{\hat{\theta}}_k$$
$$- z_i(t)\sum_{k=1}^{i} \frac{\partial \alpha_{i-1}}{\partial y_d^{(k-1)}} y_d^{(k)} + \tilde{\theta}_i^T \Gamma_i^{-1}\dot{\hat{\theta}}_i$$

where θ_i and φ_i, $i = 2, \dots, n-1$, are defined as

$$\theta_i = [\theta_{f_i}^T, \theta_{f_{i-1}}^T, \dots, \theta_{f_1}^T]^T \tag{18}$$

$$\varphi_i = \left[F_i^T, -\frac{\partial \alpha_{i-1}}{\partial x_{i-1}}F_{i-1}^T, \dots, -\frac{\partial \alpha_{i-1}}{\partial x_1}F_1^T\right]^T \tag{19}$$

Therefore, the updating laws and the intermediate controllers are selected as

$$\dot{\hat{\theta}}_i = \Gamma_i z_i \varphi_i \qquad 1 < i < n \tag{20}$$

$$\alpha_i(t) = -\hat{\theta}_i^T\varphi_i - z_i - \gamma_i z_i \tag{21}$$
$$+ \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k}x_{k+1}(t)$$
$$+ \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}_k}\dot{\hat{\theta}}_k$$
$$+ \sum_{k=1}^{i} \frac{\partial \alpha_{i-1}}{\partial y_d^{(k-1)}} y_d^{(k)}$$

where $\Gamma_i = \Gamma_i^T > 0$ and γ_i is a positive constant.

Thus, the time derivative of $V_i(t)$ becomes as follows.

$$\dot{V}_i \le -\gamma_i z_i^2 - \frac{1}{2}z_i^2 + \frac{1}{2}z_{i+1}^2 \tag{22}$$

It can be seen from the above inequality that the stability of z_i subsystem in this region is dependent on z_{i+1}, which will be considered in stability analysis of z_{i+1} subsystem.

Step 3: In the final step, the z_n subsystem is considered. Assuming the knowledge of the uncertainty b and the actuator failures at time t, the structure of the ideal controller is :

$$v_j = \frac{k_j(t)v_0}{\beta_j(x)} \quad j = 1,2,\dots,m \tag{23}$$

where v_0 is the nominal control to be designed later and $k_{1,j} \in R$ is a constant parameter which satisfies

$$[k_1, k_2, \dots, k_m](I - \delta)\rho b[1, \dots, 1]^T = 1 \tag{24}$$

with the knowledge of b and actuator failures, k_j can be achieved from the above equation. Considering assumption 3, the above equation always has a solution.

For unknown b, δ and $\bar{u}(t)$, the adaptive control input is designed as

$$v_j = \frac{\hat{k}_j(t)v_0}{\beta_j(x)} \quad j = 1,2,\dots,m \tag{25}$$

where $\hat{k}_j(t)$ is the estimates of k_j.

For stability analysis of z_n subsystem, the following Lyapunov functions are considered.

$$V_{\bar{u}} = \sum_{j=1}^{m}(1 - \delta_j)\frac{|b_j|}{2L_j}\rho_j\tilde{k}_j^2(t) \tag{26}$$

$$V_n = V_{z_n} + V_{\theta_n} + V_{\bar{u}} \tag{27}$$

where $\tilde{k}_j = \hat{k}_j - k_j$, in which $\hat{k}_j$ is the estimates of k_j and V_{z_n} and V_{θ_n} are defined in (7)-(8).

The time derivative of $V_n(t)$ becomes

$$
\begin{aligned}
\dot{V}_n = \dot{V}_{z_n} + \dot{V}_{\theta_n} + \dot{V}_{\bar{u}} \qquad &(28)\\
\leq\ & z_n(t)\varphi_0(x)\\
&+ z_n(t)\sum_{j=1}^{m}[b_j(1\\
&- \delta_j)\rho_j\hat{k}_j(t)v_0] + z_n(t)\theta_n^{\ T}\varphi_n\\
&- z_n(t)\sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}x_{k+1}(t)\\
&- z_n(t)\sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}_k}\dot{\hat{\theta}}_k\\
&- z_n(t)\sum_{k=1}^{n}\frac{\partial\alpha_{n-1}}{\partial y_d^{(k-1)}}y_d^{(k)}\\
&+ \tilde{\theta}_n^{\ T}\Gamma_n^{-1}\dot{\hat{\theta}}_n\\
&+ \sum_{j=1}^{m}(1 - \delta_j)\rho_j\frac{|b_j|}{L_j}\tilde{k}_j(t)\dot{\hat{k}}_j
\end{aligned}
$$

and θ_n and φ_n are defined as

$$\theta_n = \tag{29}$$
$$[\theta_{fn}^T, \theta_{f(n-1)}^T, \dots, \theta_{f1}^T, |b_1|\delta_1\bar{u}_1, |b_2|\delta_2\bar{u}_2, \dots,$$
$$, |b_m|\delta_m\bar{u}_m, |b_1|\delta_1\Theta_1^T, |b_2|\delta_2\Theta_2^T, \dots, |b_m|\delta_m\Theta_m^T]$$
$$\varphi_n =$$
$$[F_n^T, -\frac{\partial\alpha_{n-1}}{\partial x_{n-1}}F_{n-1}^T,, \dots, -\frac{\partial\alpha_{n-1}}{\partial x_1}F_1^T, \beta_1(x)\text{sign}(1 \tag{30}$$

$$\beta_2(x)\text{sign}(b_2), \dots, \beta_m(x)\text{sign}(b_m),$$
$$\beta_1(x)\text{sign}(b_1)G_1^T,$$

$$\beta_2(x)\text{sign}(b_2)G_2^T, \dots, \beta_1(x)\text{sign}(b_m)G_m^T]^T$$

where $\Theta_j = [\bar{d}_{j1}, \bar{d}_{j2}, \dots, \bar{d}_{jh}]^T$ and $G_j = [g_{j1}, g_{j2}, \dots, g_{jh}]^T, j = 1, \dots, m, h \geq 1$.

Therefore, the updating laws and the controller are selected as

$$\dot{\hat{\theta}}_n = \Gamma_n z_n\varphi_n \tag{31}$$
$$\dot{\hat{k}}_j = -\text{sign}(b_j)L_j z_n v_0 \quad, j = 1, \dots, m \tag{32}$$

$$
\begin{aligned}
v_0 = &-\varphi_0(x) - \gamma_n z_n - \frac{1}{2}z_n - \hat{\theta}_n^T\varphi_n \qquad (33)\\
&+ \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_k}x_{k+1}(t)\\
&+ \sum_{k=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial\hat{\theta}_k}\dot{\hat{\theta}}_k\\
&+ \sum_{k=1}^{n}\frac{\partial\alpha_{n-1}}{\partial y_d^{(k-1)}}y_d^{(k)}
\end{aligned}
$$

where $\Gamma_n = \Gamma_n^T > 0$, $L_j, j = 1, \dots, m$ and γ_n are positive constants.

Thus, the time derivative of $V_n(t)$ becomes as follows

$$\dot{V}_n \leq -\gamma_n z_n^2 - \frac{1}{2}z_n^2 \tag{34}$$

The result shows that $V_n(t)$ is bounded.

Up to now, the design of adaptive actuator failure compensation approach has been completed. Now, the main result is summarized in the following theorem.

Theorem 1: Consider the closed loop system (1). Under assumptions 1-3, the proposed controller assures the asymptotic output tracking and the boundedness of all the closed loop signals.

Proof: The following Lyapunov function is considered:

$$V(t) = \sum_{i=1}^{n}V_i$$

where $V_i(t)$ for $i = 1, \dots, n$, is defined in (9) and (27). Therefore the time derivative of $V(t)$ becomes

$$\dot{V}(t) \leq -\sum_{i=1}^{n}\gamma_i z_i^2$$

Therefore all the closed loop signals are bounded. It can be seen that $z_i \in L^2, i = 1, \dots, n$ and by considering (6), $\dot{z}_i \in L^\infty$ because all the closed loop signals and the derivatives of the desired signal y_d are bounded. Thus $lim_{t\to\infty}z_i = 0, i = 1, \dots, n$, which implies that $lim_{t\to\infty}(y - y_d) = 0$.

4. Simulation results

In this section, the obtained results are simulated to verify the effectiveness of the proposed method. For this purpose, the actuator failure compensation problem is considered for the F-18 HARV-like wing-

rock model [15, Section 10.1.3]. The aircraft wing model is described as:

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = x_3(t) + \theta_{f_2}^T F_2(t)$$
$$\dot{x}_3(t) = \frac{1}{\tau} b^T u(t) - \frac{1}{\tau} x_3(t)$$
$$y(t) = x_1(t)$$

where the states x_1, x_2 and x_3 represent the roll angle, roll rate and aileron deflection angle respectively, $u(t) \in R^2$ is the control input and $\tau \in R$ is the aileron time constant which is unknown, $b \in R^2$ and $\theta_{f_2} \in R^5$ are unknown constant vectors and $F_2(t) = [1, x_1, x_2, |x_1|x_2, |x_2|x_2]^T$.

The control objective is to track the desired signal $y_d(t) = 0$.

For simulation purpose $\tau = \frac{1}{15}, b = [0.5, 0.2]^T$ and $\theta_{f_2} = [0, -2.667, 0.86485, -2.9225, 0]^T$.

This simulation example is considered for two actuator failure models in the form of two scenarios.

Scenario 1- The failure model in this scenario is considered as

$$u_1(t) = \begin{cases} v_1(t) & ,t < 20 \\ -8 & ,t \geq 20 \end{cases}'$$
$$u_2(t) = \begin{cases} v_2(t) & ,t < 30 \\ 0.5v_2(t) & ,t \geq 30 \end{cases}$$

The following design parameters are adopted in the simulation:

$[x_1(0), x_2(0)]^T = [0.1, -0.1, 0.1]^T, \gamma_1 = \gamma_2 = \gamma_3 = 10, \Gamma_2 = I, \Gamma_3 = 0.1I, \theta_2(0) = [0.5, 0.5, 0.5, 0.5, 0.5], \theta_3(0) = [0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5]^T, k_1(0) = 0.5, k_2(0) = 0.5, L_1 = 0.5, L_2 = 1.$

The simulation results are shown in Figs. 1-3. In all of the figures, '*' denotes the time occurrence of the actuator failures.

Scenario 1- The considered failure model in this scenario is considered as:

$$u_1(t) = \begin{cases} v_1(t) & t < 30 \\ 2 + \sin(t) & ,t \geq 30 \end{cases}'$$
$$u_2(t) = \begin{cases} v_2(t) & ,t < 10 \\ 0.9v_2(t), & t \geq 10 \end{cases}$$

The simulation results are shown in Figs. 4-6.

It can be seen from Figs. 1 and 4 that the asymptotic output tracking is ensured even though there are actuator failures during an operation whose failure time instants, values and patterns are unknown to the adaptive failure compensation controller. Figs. 2 and 5 represent the boundedness of the control inputs and Figs. 3 and 6 show the boundedness of the estimates of the parameters in the control loop system.

As can be seen from Fig. 2, the first input stuck at $t = 20$ at constant value and the second input lost 50% of its effectiveness at $t = 30$.

For the second scenario, as can be seen from Fig. 5, the first input stuck at $t = 30$ at time varying value and the second input lost 10% of its effectiveness at $t = 10$.

However, in both scenarios, all the states are asymptotically converged to the origin and all the closed loop signals remain bounded.

It can be seen from the results, that the proposed adaptive actuator failure compensator is feasible and effective for the unknown constant and time varying actuator failures of the nonlinear system (1). The above simulation results demonstrate the merits of the proposed design method.

5. Conclusion

In this paper, an adaptive actuator failure compensation scheme is proposed for a class of nonlinear systems with unknown parameters, variable control gains and unknown actuator failures. The considered actuator failure covers both loss of effectiveness and time varying stuck failures which are uncertain in time, value, and pattern. Appropriate Lyapunov-Krasovskii type functionals are introduced to design new adaptive laws to compensate the unknown actuator failures and unknown parameters. The proposed systematic backstepping design method can guarantee global
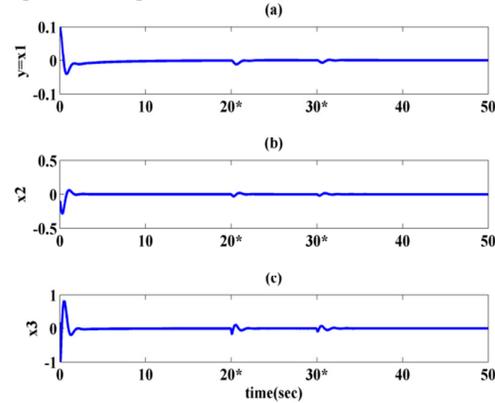
Fig.(1): The states responses of the aircraft wing system
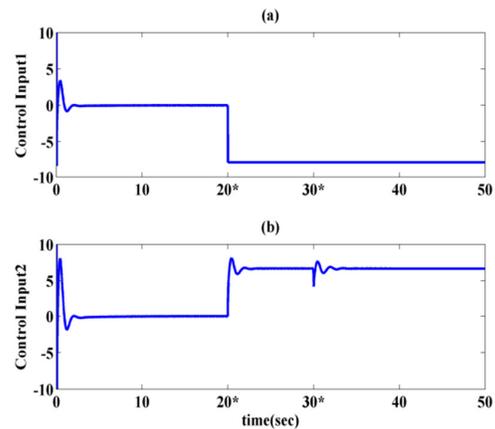
Fig.(2): (a) Control input $u_1(t)$. (b) Control input $u_2(t)$
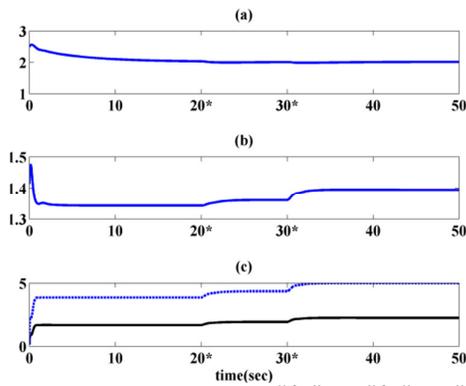
Fig.(3): Parameter estimates: (a) $\|\hat{\theta}_2\|$, (b) $\|\hat{\theta}_3\|$, (c) $\|\hat{k}_1\|$ (solid), $\|\hat{k}_2\|$ (dashed).

boundedness of all the closed loop signals in addition to the asymptotic convergence of the system output to the desired signal. Simulation results have been conducted to verify the effectiveness of the proposed method.
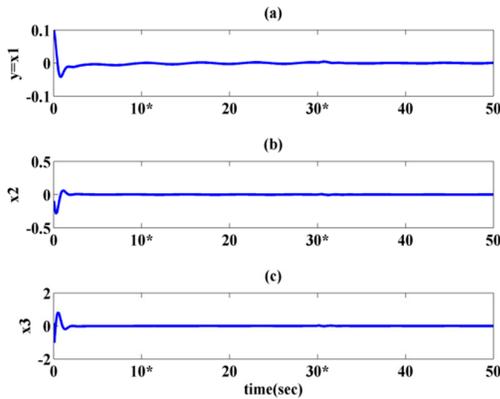
Fig.(4): The states responses of the aircraft wing system
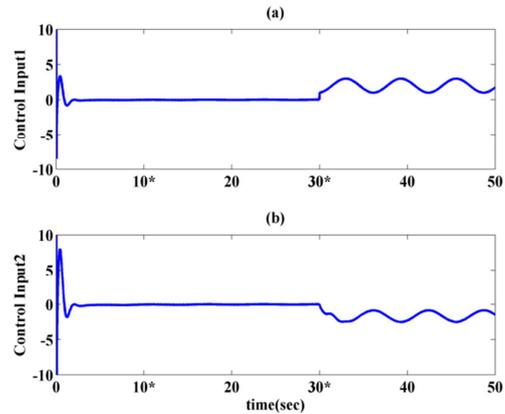
Fig.(5): (a) Control input $u_1(t)$. (b) Control input $u_2(t)$
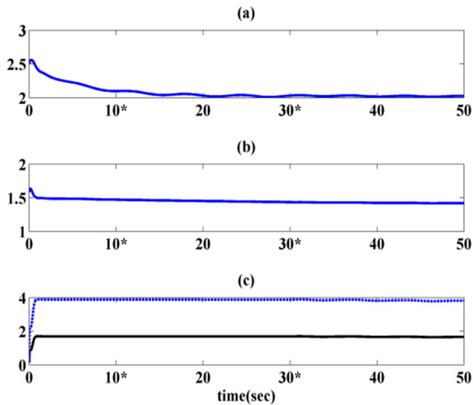
Fig.(6): Parameter estimates: (a) $\|\hat{\theta}_2\|$, (b) $\|\hat{\theta}_3\|$, (c) $\|\hat{k}_1\|$ (solid), $\|\hat{k}_2\|$ (dashed).

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