JOURNAL OF SOUTHERN COMMUNICATION ENGINEERING
ISLAMIC AZAD UNIVERSITY BUSHEHR BRANCH

E-ISSN: 2980-9231
https://jce.bushehr.iau.ir

https://doi.org/10.30495/jce.2023.1996352.1281

Vol. 14/ No. 53/Autumn 2024

Research Article

Cooperative and Non-cooperative TDOA Based Source Localization with Copula Function Using Semidefinite Relaxation

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Received: 8 June 2024
Revised: 21 June 2024
Accepted: 25 June 2024

Abstract

The time difference of arrival based approach to wireless localization is perhaps one of the most interesting research subjects over the past decade. A method using copula function is proposed for source localization using TDOA measurements for both noncooperative and cooperative scheme. The proposed method is started with noncooperative localization, and finaly is extended to the cooperative localization problem. Because of the unknown measurement noise covariance matrix, by using the theory of copulas with Sklar's theorem, the joint likelihood function of TDOA measurements is coupled to the corresponding univariate marginal likelihood functions. Then, an attractive alternative using the method of inference functions for margings is applied to the maximum likelihood estimation. The procedure involving the maximization of univariate marginal likelihood functions and then estimation of copula parameter. The maximization suffers from noncovexity, so we apply semidefinite relaxation techniques to derive a convex estimator. Simulation results corroborate the performance of the proposed method as for sufficient signal to noise ratio, we observe one meter of improvement in source location accuracy.

Keywords: Time Difference of Arrival, Semidefinite Relaxation, Copula Functions, Source Localization, Correlated Noise.

Highlights

- A method using copula function is proposed for source localization using TDOA measurements for both noncooperative and cooperative scheme.
- The joint likelihood function is coupled to the corresponding univariate marginal likelihood functions with unknown measurement noise covariance matrix, by using the theory of copulas.
- An attractive alternative using the method of inference functions for margins is applied to the maximum likelihood estimation.

Citation: M. Dadkhah Tehrani, and H. Lohrasbipeydeh, "Cooperative and Non-cooperative TDOA Based Source Localization with Copula Function Using Semidefinite Relaxation," *Journal of Southern Communication Engineering*, vol. 14, no. 53, pp. 91–105, 2024, doi:10.30495/jce.2023.1996352.1281, [in Persian].

1. Introduction

Passive source localization with a wireless sensor network (WSN) is a classical problem for many applications in radar, sonar, navigation and public safety. Generally, in a WSN, the positions of several sensors are known (anchor nodes), while there are some sensors (source nodes) whose positions are unknown and thus must be estimated using sensor localization. The main purpose of a WSN is to determine the location of a signal emitted from a source node based on received noisy measurements at anchor nodes that are spatially distributed over a geographical area which is generally divided into two cases: non-cooperative and cooperative.
In the non-cooperative case, source nodes can communicate only with anchor nodes. The lack of accessible anchor nodes and also limited connectivity among anchor nodes and source nodes have led to the emergence of cooperative localization in which source nodes are able to communicate with both anchor nodes and other source nodes. Most source positioning techniques rely on distance estimates obtained from time of arrival (TOA), time difference of arrival (TDOA), angle of arrival (AOA), received signal strength (RSS) and received signal strength difference (RSSD) or a combination of these techniques.
In cooperative localization, anchor nodes can estimate the location of all source nodes simultaneously. Thus, both estimation performance and robustness are improved by employing cooperative localization. Therefore, not only are the RSS values between source nodes and anchor nodes (source-anchor measurements) measured, but also the source nodes themselves are involved and collect RSS measurements from each other (source-source measurements).
The TDOA-based source positioning in the presence of Gaussian measurement noise led to the minimization of the maximum likelihood (ML) cost function. Several ML-based estimators have been developed for TDOA based source localization. Solving the ML problem may be performed using iterative algorithms, where the final solution relies on the proper choice of initial estimate. This approach was improved by reformulating the original problem into a closed-form solution. To the improvement of the proposed method, TSWLS approach was considered. These methods are performed based on weighted least squares.

2. Innovation and contributions

The ML estimator is suboptimal due to the noise statistics are known, but is difficult to implement due to the highly nonlinear and nonconvex cost function. This paper considers the assumption of zero-mean Gaussian measurement noise with an unknown covariance matrix. To eliminate the unknown parameter from ML problem, we apply the central theory of copulas. Copulas are basically parametric functions for modeling dependencies among random variable that couple univariate marginal distribution functions to the corresponding multivariate distribution function. The functions play a key role in the fields of econometrics and finance. By utilizing an alternative to ML estimation and applying semidefinite programming technique, the estimation of source position can be found. The joint distribution based on copula function yields the estimation improvement due to the increase in the magnitude of Fisher information matrix diagonal elements and equivalently lower CRLB in comparison to the methods that employ conditional independence of measurements. This results in better performance for copula-based ML estimators in terms of mean squared error (MSE).

3. Materials and Methods

This section considers the problem of noncooperatvie correlated TDOA measurements for source location estimation. The target is a source with an unknown location while the sensor nodes have known positions The set of TDOA values between the first and ith anchor node in the presence of zero mean correlated Gaussian measurement noise can be formulated. Based on that the ML estimator can be developed and the corresponding nonconvex optimization problem can be formed based on the joint probability density function (PDF) with the unknown covariance matrix of Gaussian distribution with elements.
Note that the joint likelihood function is required to achieve an accurate solution based on ML estimator. The central theory of copulas for continuous distributions, known as Sklar's theorem, can be used to model the proposed dependency by reformulating the joint multivariate PDF, as marginal distributions. Several copula families were previously considered such as Archimedean, Gaussian and elliptic copulas which differ in their dependence representation. The Gaussian copula is of practical interest since it can be easily implemented and its dependence structure is intuitive, which can highlight the linear dependence based on correlation coefficients for correlated TDOA measurements.
Thus, the ML problem in (3) can be reformulated based on the Gaussian copula by employing Sklar's theorem. The ML problem involves simultaneous maximization over the dependence and marginal parameters. It has been shown that the marginal parameters could be estimated separately from the correlation matrix based on a computationally efficient method of inference functions for margins (IFM) which is asymptotically equivalent to ML. Thus, the above problem can be simplified by first estimating the marginals and then using those marginals to estimate the copula function and correspondingly the unknown covariance matrix. This results in a simple approach to obtain the unknown hyperparameters of the proposed nonconvex maximization problem.
The IFM estimation technique is motivated by the decomposition of the proposed problem in two stages. First, the unknown marginal parameter is estimated based on the sum of marginal log-likelihood functions. Then the corresponding first stage IFM optimization problem can be given as an SDP problem. Thus, the first stage IFM optimization problem is reformulated as a convex minimization problem utilizing semidefinite programming. This allows a globally optimum solution to be obtained efficiently using SDP solvers such as SDPT3 and SeDuMi that employ the interior point method.
Using the parameter estimated in the first stage and substituting it, we can begin the second stage. The goal of the second stage is to find copula parameter or correlation matrix which is related to the unknown covariance matrix. Note that the ML estimation and IFM procedures are equivalent in the special case of multivariate Gaussian distribution functions that have multivariate Gaussian copulas and univariate Gaussian margins. The Gaussian Copula density can be given using the multivariate normal distribution. The next section, considers the cooperative TDOA-based localization model in which there are more than two source nodes with unknown locations in 2D space, and moreover, source nodes can communicate with anchor nodes and each other as well.

4. Results and Discussion

In this section, simulation results are given to evaluate the performance of the proposed copula-based SDP method for both cooperative and noncooperative cases. A 2D geometry with M = 7 sensors is considered at known coordinates (0,0), (10,5), (5,10), (20,15), (15,10), (30,25) and (35,40) in meters to estimate an unknown source which is located at θ = (16, 24). The performance of the proposed correlated TDOA-based method is compared with several well-known methods for both cooperative and noncooperative scenarios where the corresponding CRLB is used as a performance benchmark. These comparisons examine the accuracy of the proposed methods in terms of root mean squares error and bias. The range difference measurements are generated by adding zero-mean Gaussian noise with the covariance matrix Q. The Choice of Q make arbitrary, here, Q is selected as a matrix with diagonal elements equal to all other elements. To solve the proposed problem and its constraints, the SDP method is implemented using the CVX toolbox with the Self-Dual-Minimization (SeDuMi) solver.

Figures show the placement of source nodes and anchor nodes in the first scenario and second scenario, respectively. By considering the first scenario, simulation results of comparison of these methods in terms of RMSE and bias are shown, respectively. In the experiment, variance of measurement noise varies from −60dB to 10dB and −60dB to 20dB, respectively. The results show that the proposed method performs better than other methods. The number of ensemble runs is equal to 10. In the second stage of IFM, we estimate copula parameter and then, we comprise with covariance matrix Q.

The experiment shows the results of the comparison in terms of mean square error. The simulation shows the effect of increasing the number of anchor nodes too. First, the experiment considers the first scenario without anchor node in [10, 10]T coordinate, then considers the first scenario with new anchor nodes in [15, 15]T and [15, 10]T coordinates. The number of ensemble runs is equal to 100. Other experiment examines the effect of increasing number of anchor nodes in terms of RMSE and bias for σ = 10−2. By considering the first scenario, a comparison is done between CRLB and RMSE of estimation. The simulation also shows the results of the comparison. Simulation results show that the RMSE of the proposed method achieves CRLB. In cooperative case, we use first scenario and consider four source nodes, where their coordinates have U(0, 10) distribution. These simulations show the results of the experiment in terms of RMSE and bias versus noise level.

5. Conclusion

In this paper, a method for TDOA based localization problem in noncooperative and cooperative cases was described. To eliminate the unknown covariance matrix from likelihood function, it utilized Sklar's theorem. Then, it applied IFM instead of ML estimation and found the source position and copula parameter. The proposed method was shown to reach the CRLB even under high level measurement noise for noncooperative case.

6. Acknowledgement

This paper is done based on my personal work at ECE department of Shahed University.

7. References

[1] A. Forster, Introduction to wireless sensor networks. John Wiley & Sons, 2016.

[2] L. Huang *et al.*, "Robust TDOA-Based Indoor Localization Using Improved Clock-Sync-Scheme and Multilevel Constrained ARPF," in *IEEE Sensors Journal*, vol. 23, no. 10, pp. 10633-10643, May 2023, doi: 10.1109/JSEN.2023.3259540.

[3] Y. Sun, K. C. Ho, Y. Yang and L. Chen, "An Asymptotically Optimal Estimator for Source Location and Propagation Speed by TDOA," in *IEEE Signal Processing Letters*, vol. 30, pp. 1037-1041, 2023, doi: 10.1109/LSP.2023.3302229.

[4] X. Qu, L. Xie and W. Tan, "Iterative Constrained Weighted Least Squares Source Localization Using TDOA and FDOA Measurements," in *IEEE Transactions on Signal Processing*, vol. 65, no. 15, pp. 3990-4003, Aug. 2017, doi: 10.1109/TSP.2017.2703667.

[5] K. C. Ho, X. Lu and L. Kovavisaruch, "Source Localization Using TDOA and FDOA Measurements in the Presence of Receiver Location Errors: Analysis and Solution," in *IEEE Transactions on Signal Processing*, vol. 55, no. 2, pp. 684-696, Feb. 2007, doi: 10.1109/TSP.2006.885744.

[6] A. Noroozi, A. H. Oveis, S. M. Hosseini and M. A. Sebt, "Improved Algebraic Solution for Source Localization From TDOA and FDOA Measurements," in *IEEE Wireless Communications Letters*, vol. 7, no. 3, pp. 352-355, June 2018, doi: 10.1109/LWC.2017.2777995.

[7] Y. Sun, K. C. Ho and Q. Wan, "Solution and Analysis of TDOA Localization of a Near or Distant Source in Closed Form," in *IEEE Transactions on Signal Processing*, vol. 67, no. 2, pp. 320-335, Jan. 2019, doi: 10.1109/TSP.2018.2879622.

[8] Z. Mao, H. Su, B. He and X. Jing, "Moving Source Localization in Passive Sensor Network With Location Uncertainty," in *IEEE Signal Processing Letters*, vol. 28, pp. 823-827, 2021, doi: 10.1109/LSP.2021.3074034.

[9] N. Vankayalapati, S. Kay and Q. Ding, "TDOA based direct positioning maximum likelihood estimator and the cramer-rao bound," in *IEEE Transactions on Aerospace and Electronic Systems*, vol. 50, no. 3, pp. 1616-1635, July 2014, doi: 10.1109/TAES.2013.110499.

[10] Y. Zou and H. Liu, "Semidefinite Programming Methods for Alleviating Clock Synchronization Bias and Sensor Position Errors in TDOA Localization," in *IEEE Signal Processing Letters*, vol. 27, pp. 241-245, 2020, doi: 10.1109/LSP.2020.2965822.

[11] X. Ma, T. Ballal, H. Chen, O. Aldayel and T. Y. Al-Naffouri, "A Maximum-Likelihood TDOA Localization Algorithm Using Difference-of-Convex Programming," in *IEEE Signal Processing Letters*, vol. 28, pp. 309-313, 2021, doi: 10.1109/LSP.2021.3051836.

[12] H. Chen, T. Ballal, N. Saeed, M. -S. Alouini and T. Y. Al-Naffouri, "A Joint TDOA-PDOA Localization Approach Using Particle Swarm Optimization," in *IEEE Wireless Communications Letters*, vol. 9, no. 8, pp. 1240-1244, Aug. 2020, doi: 10.1109/LWC.2020.2986756.

[13] Y. Sun, K. C. Ho, Y. Yang and L. Chen, "An Asymptotically Optimal Estimator for Source Location and Propagation Speed by TDOA," in *IEEE Signal Processing Letters*, vol. 30, pp. 1037-1041, 2023, doi: 10.1109/LSP.2023.3302229.

[14] W. Xiong, C. Schindelhauer and H. C. So, "Globally Optimized TDOA High-Frequency Source Localization Based on Quasi-Parabolic Ionosphere Modeling and Collaborative Gradient Projection," in *IEEE Transactions on Aerospace and Electronic Systems*, vol. 59, no. 1, pp. 580-590, Feb. 2023, doi: 10.1109/TAES.2022.3185971.

[15] Y. Sun, K. C. Ho, T. Xing, Y. Yang and L. Chen, "Projection-Based Algorithm and Performance Analysis for TDOA Localization in MPR," in *IEEE Transactions on Signal Processing*, vol. 72, pp. 896-911, 2024, doi: 10.1109/TSP.2024.3352923.

[16] M. R. Basheer and S. Jagannathan, "Localization of Objects Using Cross-Correlation of Shadow Fading Noise and Copulas," *IEEE Global Telecommunications Conference - GLOBECOM* , Houston, TX, USA, 2011, pp. 1-5, doi: 10.1109/GLOCOM.2011.6134101.

[17] N. Patwari, J. N. Ash, S. Kyperountas, A. O. Hero, R. L. Moses and N. S. Correal, "Locating the nodes: cooperative localization in wireless sensor networks," in *IEEE Signal Processing Magazine*, vol. 22, no. 4, pp. 54-69, July 2005, doi: 10.1109/MSP.2005.1458287.

[18] R. M. Vaghefi, M. R. Gholami, R. M. Buehrer and E. G. Strom, "Cooperative Received Signal Strength-Based Sensor Localization With Unknown Transmit Powers," in *IEEE Transactions on Signal Processing*, vol. 61, no. 6, pp. 1389-1403, March15, 2013, doi: 10.1109/TSP.2012.2232664.

[19] R. B. Nelsen, An introduction to copulas. Springer Science & Business Media, 2007.

[20] H. Joe, Multivariate models and multivariate dependence concepts. CRC press, 1997.

[21] H. Joe and J. J. Xu, "The estimation method of inference functions for margins for multivariate models," 1996.

[22] S. Boyd, S. P. Boyd, and L. Vandenberghe, Convex optimization. Cambridge university press, 2004.

[23] S. M. Kay, Fundamentals of statistical signal processing: estimation theory. Prentice-Hall, Inc., 1993.

[24] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming, version 2.1," ed, 2014.

Declaration of Competing Interest: Authors do not have conflict of interest. The content of the paper is approved by the authors.

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Author Contributions: All authors reviewed the manuscript.

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