Intelligent Multimedia Processing & Communication Systems Journal

IMPCSJ

J IMPCS (2025) 19: 49-56
DOI 10.71856/IMPCS.2025.1203068

Research Paper

Fully-Distributed Leader-Follower Flocking Protocol for Time-Depend Lagrange systems with External Disturbance

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Article Info

Article history:
Received: 26 Dec 2024
Accepted: 5 Feb 2025
DOR:

Keywords:
fully-distributed control,
Leader-follower flocking,
multi-agent systems,
time-depend Lagrange systems,
Time-varying uncertainty.

ABSTRACT

This paper designs a protocol for leader-follower flocking problem, for multi-agent time-depend Lagrange systems (MATDLS) with external disturbances problem. The mechanical systems with varying mass can be modeled with time-depend Lagrange systems. The variable mass causes to uncertainty in the model. We suppose that the virtual leader has a bounded acceleration, and only its neighbor agents can receive its information. The study of leader-follower flocking with dynamic leader/virtual leader (LFFDL) for MATDLS under external disturbance problem is very challengeable.
An adaptive estimator is allocated to each agent for estimation of the leader's velocity, and an adaptive control is proposed to solve LFFDL problem. The benefits of our protocol are both fully-distributed and continuous. Under the proposed control protocol, each follower's velocity asymptotically tends to that of the virtual leader, the group's network stays connected, and no collision happens between agents. In the end, we perform some simulations to show the theoretical results validation.

NOMENCLATURE

η_i	position of agent i
$\dot{\eta}_i$	velocity of agent i
η_r	virtual leader's position
$\dot{\eta}_r$	virtual leader's velocity
u_{cop_i}	is the cooperative control of each agent
$\phi(\|\eta_{ij}\|)$	potential function
$\hat{k}_{n_\theta+1_i}$	adaptive gain of the i-th agent
σ_i	controller of agent i
η_{ij}	relative position of agents i and j

I. Introduction

Flocking is a branch of cooperative control that has a wide application in the robots group and automobile vehicles' control. For achieving a flocking motion, while agents endeavor to stay in the group and avoid the collision, they should reach their velocity to the neighbors. Many mathematical models were proposed to solve flocking motion challenges over the years, such as the obstacle avoidance problem, the target-tracking problem, and the connectivity-preserving problem [1-10]. The leaderless flocking algorithm was studied in [2], and the theoretical framework of the leader-follower flocking was presented in [3]. The authors of [4] introduced an algorithm for the case that the states of the virtual leader are not measured by all agents. Because, under the mentioned algorithms, the group was fragmented, the connectivity-preserving flocking algorithms were investigated by the researcher [5]. Furthermore, the flocking with the dynamic leader was studied in [6].

The flocking study under practical problems such as uncertainties and external disturbance was performed in [11-19]. The existence of uncertainty and external disturbance in the agents' dynamics may cause the group to fragment or collide the agents together. Authors in [11-15] studied the multi-agent systems having unknown parametric uncertainty and no external disturbance problem, while in [16], they did not consider uncertainty problems. Whereas, it is of great challenge to design distributed controllers for the flocking problem of multi-agent systems, in the presence of both of uncertainty and external disturbance problem, especially for the time-variant uncertainty.

Furthermore, since most of the results mentioned above required a global information of the virtual leader for designing of controller, especially for the dynamic leader, if both the uncertainty and external disturbance present in agents' dynamics, the flocking problem with a virtual leader will be more challengeable than leaderless flocking. In this contest, the proposed protocols in refs. [11-12, 14, 17-19] were leader-follower.

Moreover, the agents' dynamics considered in these works included linear second-order dynamics [15, 18], nonlinear double-integrator [13-14, 16-17], and Lagrange dynamics [11, 12, 19]. Among the aforementioned dynamics, the Lagrange dynamics covering the nonlinear double-integrator dynamic and a specific class of linear second-order dynamics were more applicable [20-21]. On the other hand, there are some mechanical systems with varying mass, [27, 28], that are modeled by Lagrange systems. These systems are called time-depend Lagrange systems. The variation of mass into such systems can cause uncertainty in the dynamic of agents.

In recent years, researchers mainly paid attention to the design of fully-distributed and continuous cooperative controllers. The fully-distributed protocol needs no global information of agents' network, the virtual leader, or the initial agents' network in its structure. Also, as we know, the continuous protocols' advantage is that they have no chattering potential.

In the aforementioned works, only the proposed protocol in ref. [11, 19] was fully-distributed and those in refs. [14, 17-18] were continuous, but except of ref. [19], there was no both continuous and fully-distributed protocol. To tackle the demand to the global information of the leader's velocity in the leader-follower protocols, authors in ref. [22] proposed an estimator for estimating the leader's velocity to synchronize the multi-agent systems with Lagrange dynamic. Ref. [11], adding the derivative of the potential function to the estimator, extended this estimator to the flocking problem.

Therefore, here we aim to investigate a continuous fully-distributed adaptive control (CFDAC) for leader-follower flocking of MATDLS with external disturbances problem. The main contributions are as follows.

1. We consider a time-depend Lagrange dynamic, is general form Lagrange dynamic of this dynamic covers various physical and engineering systems and is more practical. Also, we suppose the virtual leader moves with a bounded acceleration, and only the informed agents can receive its information. We assume that the agents are subject to the TVUED problem. To the author's best knowledge, there are very few works that study uncertainty and external disturbance problems together, especially for time-varying uncertainty. Compared to the existing works [x], our problem is more challenging; we detail it in Remarks 5.

2. Unlike plenty of works performed on adaptive control of time-invariant systems, there are few works on time-varying systems [22-25]. The non-zero derivative of uncertain parameters causes many challenges; some assumptions are considered on the parameter variations' rate to handle this problem. Contrary to these methods limiting the variation rate of parameters, some works do not restrict the parameters' derivative, [22-23], and only suppose a compact set for them. This paper also assumes that the changes of uncertain parameters are inside an arbitrarily unknown compact set and put no limitation on the uncertain parameters' variation rate.

3. To solve the above challenges, this paper considers an estimator for each agent to estimate the virtual leader's velocity. By standard adaptive laws and deriving Lemma 2, we propose a continuous and fully-distributed protocol for flocking multi-agent systems. In the fully-distributed controller, there is no need for global information of the virtual leader for whole agents and the initial neighboring graph in the controller structure. It should be noted that though the controller [11] is fully-distributed, it did not apply to the MALS with TVUED problem. Moreover, the protocol in ref. [11] utilizing 1-norm signals led to an unbounded increase in the adaptive gains in the presence of external disturbance. Also, since that protocol was designed for constant parametric uncertainty with the non-zero derivative of uncertain parameters, it could not provide an asymptotic flocking motion. Besides, since it is discontinuous, it led to the chattering phenomenon subject to external disturbance. Therefore, compared with the protocol in ref. [11], our proposed protocol is more applicable and useful.

The remains of the paper are given as follows. Sections 2, 3, and 4 are organized to represent the problem statement, the main theoretical results, and numerical simulations. The conclusions of this paper are provided in Section 5.

II. Problem Statement

Suppose a group of N agents intend to reach a flocking motion. Each agent can sense neighbors by communicating equipment. The neighboring graph of agents is undirected and time-varying, denoted by graph $g(t) = (\nu, e(t))$. Vertices in the graph $g(t)$ indicate the agents, and edges $e(t)$ indicate the links between neighbor agents. Also, all agent's communication radius, r is assumed the same. Consider two positive constants $\hat{\varepsilon}$ and $\check{\varepsilon}$ such $\hat{\varepsilon} \leq \check{\varepsilon} < r$, then $e(t) = \{(i,j)|i,j \in \nu\}$ which is the set of links is generated as

1. The initial links, $e(0)$ is generated such that $e(0) = \{(i,j)|\hat{\varepsilon} < \|\eta_i(0) - \eta_j(0)\| < r - \check{\varepsilon}, i,j \in \nu\}$, where $\eta_i(0)$ and $\eta_j(0)$ are respectively initial positions of agents i and j.
2. If $(i,j) \notin e(t^-)$ and $\|\eta_i(t) - \eta_j(t)\| < r - \check{\varepsilon}$, then (i,j) is a newly added link to $e(t)$.
3. If $\|\eta_i(t) - \eta_j(t)\| \geq r$, then $(i,j) \notin e(t)$.

In view of the definition proposed for the links' set, each agent's neighbor nodes are specified by the set: $n_i(t) = \{j|(i,j) \in e(t), j \in \nu\}$.

Consider the definition of the adjacent matrix of the graph $g(t)$, $\mathcal{A}(t) = [a_{ij}]$, where $a_{ij} = a_{ji} = 1$ if agents i and j are neighbor, otherwise $a_{ij} = a_{ji} = 0$. Also, $L(t) = [l_{ij}]$ denotes the Laplacian matrix of graph $g(t)$where $l_{ii} = \sum_{j\neq i} a_{ij}$, $l_{ij} = -a_{ij}$ for $i \neq j$, [3].

Consider the agents as the following Lagrange dynamics

$$M_i(\eta_i)\ddot{\eta}_i + C_i(\eta_i, \dot{\eta}_i)\dot{\eta}_i + G_i(\eta_i) = \sigma_i + \varrho_i(t), \quad (1)$$

where $i = 1, \dots, N$, $\eta_i, \dot{\eta}_i \in \mathbb{R}^n$ are the position and velocity of agent i, respectively. $M_i(\eta_i)$ is the $n \times n$ symmetric inertia matrix, $C_i(\eta_i, \dot{\eta}_i)$ is the Coriolis and centrifugal force, $G_i(\eta_i)$ is the vector of gravitational force. Also, $\sigma_i \in \mathbb{R}^n$ is the input of agent i, and $\varrho_i \in \mathbb{R}^{n_d}$ is the unknown acted external disturbance on agent i. The Lagrange dynamics have the following properties

Prop. 1: There exist positive constants $\underline{m}$, $\overline{m}$, $\overline{c}$, $\overline{g}$ such that $\underline{m} \leq \|M_i(\eta_i)\| \leq \overline{m}$, $\|C_i(\eta_i, \dot{\eta}_i)\| \leq \overline{c}\|\dot{\eta}_i\|$, $\|G_i(\eta_i)\| \leq \overline{g}$.

Prop. 2: matrix $\dot{M}_i(\eta_i) - 2C_i(\eta_i, \dot{\eta}_i)$ is skew symmetric.

Prop. 3: The linearly parameterization property of the Lagrange system (1) is defined as $M_i(\eta_i)x + C_i(\eta_i, \dot{\eta}_i)y + G_i(\eta_i) = Y_i(\eta_i, \dot{\eta}_i, x, y)\vartheta_i$ for all vectors $z, v \in \mathbb{R}^n$, where $Y_i(\eta_i, \dot{\eta}_i, z, v)$ is the regressor matrix and $\vartheta_i \in \mathbb{R}^{n_\theta}$ is considered as a unknown time-varying vector.

Remark 1: In most physical systems modeled by Lagrange dynamics, the parameter ϑ_i is a known/unknown constant, but in some systems the parameter θ_i is time-varying, [26-28]. These systems include time-depend Lagrange systems such as mechanical systems with varying mass [27, 28].

Assumption 1: Consider unknown constants $\bar{\varrho}$ and $\bar{\vartheta}$ such that $\|\vartheta_i(t)\| \leq \bar{\vartheta}$ and $\|\varrho_i(t)\| \leq \bar{\varrho}$.

Remark 2: Assumption 1 is standard in the adaptive control of the time-varying systems. Authors in ref. [22] demonstrated by simulation results, even under the significant variation in parameters, the system can remain stable in many cases. Also, they proved that under the accessibility assumption of states, the existence of a compact set for parameters, and using a standard adaptive law, one can guarantee states to be bounded.

Therefore, here we also assume that the system's states are measurable. Our controller is constructed a feedback of states of each agent and a cooperative protocol. The cooperative protocol in flocking contains the gradient of an artificial potential function to avoid losing the existing links and collision between agents, a term for velocity consensus of all agents, and a feedback of virtual leader's state. If an agent can measure the virtual leader's states, it is informed; otherwise, it is uninformed. Suppose the virtual leader's dynamic as

$$\ddot{\eta}_r = \sigma_r, \quad (2)$$

where $\eta_r, \dot{\eta}_r \in \mathbb{R}^n$ are respectively virtual leader's position and velocity vectors, and σ_r is its input.

Assumption 2: The virtual leader's velocity $\dot{\eta}_r$ is bounded.

Assumption 3: There is a positive constant $\overline{\sigma}$ such that $\|\sigma_r\| \leq \overline{\sigma}$.

The following potential function is used in the cooperative protocol

$$\phi(\|\eta_{ij}\|) = \frac{r}{\|\eta_{ij}\|(r - \|\eta_{ij}\|)}, 0 < \|\eta_{ij}\| < r, \quad (3)$$

where $\eta_{ij} \in \mathbb{R}^n$ is the relative position of agents i and j, that is, $\eta_{ij} = \eta_i - \eta_j$. Note that since potential function (3) tends to infinity when the distance among two agents tends to r or zero, it could maintain the initial links connected and avoid collision among agents.

In the following, we define the objective control of flocking.

Definition 1: Consider a group of agents pursuing a virtual leader. Suppose that their initial network graph $g(0)$ is connected and initial energy $V(0)$ is finite. When we say that the group achieve a flocking motion, under the controller $\sigma_i(t)$ the following properties are realized:

1- The initial links of graph $g(0)$ are preserved during the motion, that is if $(i,j) \in e(0)$, $i,j = 1, \dots, N$, then $\|\eta_i(t) - \eta_j(t)\| < r$ for all $t > 0$.

2- No pair of agents collide together, that is $\|\eta_i(t) - \eta_j(t)\| > 0, i,j = 1, \dots, N$ for all $t > 0$.

3- All agents' velocity asymptotically tends to the virtual leader's velocity, η_r, that is $\lim_{t\to\infty}(\eta_i(t)-\eta_r(t))=0, i=1,\dots,N$.

Consider the following Lemmas that we will use for proof of results. Note that in them, we put the eigen values of matrix $P\in\mathbb{R}^{N\times N}$ in the increasing order as $Re\{\lambda_1(P)\}\le Re\{\lambda_2(P)\}\le\cdots\le Re\{\lambda_N(P)\}$.

Lemma 1 [29]: Consider the graphs g_1 and g_2, which are undirected and of order N. Denote their Laplacian matrices by L_1 and L_2, respectively. Suppose by adding some edge(s) into the graph g_1, g_2 is generated. Consider non-zero matrix $\Gamma=diag(\gamma_1,\dots,\gamma_N)$, where $\gamma_i\ge 0$, $i=1,2,\dots,N$. Then, $\lambda_1(L_2+\Gamma)\ge\lambda_1(L_1+\Gamma)$, for all $i=1,2,\dots,N$.

Lemma 2: Consider a network graph $G(t)$ of order N, whose adjacent matrix elements are a_{ij} and Laplacian matrix is. Also, matrix Γ, defined in Lemma 1. Denote $x(t)=[x_1(t),\dots,x_N(t)]^T$, where $x_i\in\mathbb{R}^n$ is a state variable of node i, $i=1,2,\dots,N$. Then, $\sum_{i=1}^N\left\|\sum_{j\in N_i(t)}a_{ij}(x_i(t)-x_j(t))+\gamma_i x_i\right\|\ge\sqrt{\lambda_1((L+\Gamma)^2)}\|x\|$,

where $N_i(t)$ indicates the neighboring space of node i at each instant t.

Proof: Break up matrices L and Γ to N row matrices respectively as $L=[L_1{}^T\ \cdots\ L_N{}^T]$ and $\Gamma=[\Gamma_1{}^T\ \cdots\ \Gamma_N{}^T]$, where $L_i\in\mathbb{R}^{1\times n}$, $\Gamma_i\in\mathbb{R}^{1\times n}$, $i=1,\dots N$. Also, L_i and Γ_i represent respectively the i^{th} row of matrix L and matrix Γ. Then, $\sum_{j\in N_i(t)}a_{ij}(x_i(t)-x_j(t))+\gamma_i x_i=((L_i+\Gamma_i)\otimes I_n)x$. Thus,

$$\sum_{i=1}^N\left\|\sum_{j\in N_i(t)}a_{ij}(x_i(t)-x_j(t))+\gamma_i x_i\right\|=$$
$$\sum_{i=1}^N\|((L_i+\Gamma_i)\otimes I_n)x\|\ge\left\|\begin{bmatrix}((L_1+\Gamma_1)\otimes I_n)x\\ \vdots\\ ((L_N+\Gamma_N)\otimes I_n)x\end{bmatrix}\right\|=$$
$$\|((L+\Gamma)\otimes I_n)x\|=\sqrt{x^T((L+\Gamma)\otimes I_n)((L+\Gamma)\otimes I_n)x}\ge$$
$$\sqrt{\lambda_1((L+\Gamma)^2)\|x\|^2}=\sqrt{\lambda_1((L+\Gamma)^2)}\|x\|.$$

III. Main Results

In this section, we design a fully-distributed adaptive control for the systems (1). Our protocol comprises the continuous adaptive control input and estimator. Since all agents could not sense the virtual leader's information, for estimation of the virtual leader's velocity, an estimator is designed for each agent.

Before moving on, denote the velocity error by $\dot{\tilde{\eta}}_i=\dot\eta_i-\dot\eta_\gamma$, where $\dot{\tilde{\eta}}_i-\dot{\tilde{\eta}}_j=\dot\eta_i-\dot\eta_j$. Also, consider the auxiliary variables $\tilde\xi_i=\xi_i-\dot\eta_\gamma$ and $z_i=\dot\eta_i-\xi_i=\dot{\tilde{\eta}}_i-\tilde\xi_i$. Consider the estimator of each agent as

$$\begin{aligned}\dot\xi_i&=-\sum_{j\in N_i(t)}\nabla\phi_{\tilde\eta_i}(\|\eta_{ij}\|)\\&-d_1\sum_{j\in N_i(t)}a_{ij}(\dot{\tilde{\eta}}_i-\dot{\tilde{\eta}}_j)-d_2\gamma_i\dot{\tilde{\eta}}_i-tanh(\hat k_{n_\theta+1_i}\xi_i).\end{aligned}\quad(4)$$

where d_1 and d_2 are positive constants, and $\gamma_i=1$ is for informed agents, and $\gamma_i=0$, otherwise. Also, $\hat k_{n_\theta+1_i}, i=1,\dots,N$, are the adaptive gains.

Remark 3: In [21], to estimate the leader's velocity, an estimator idea was introduced for synchronization of the MALS. In [11], the authors extended it for the flocking problem and added the partial derivative of the potential function. Both of these protocols were based on the Sign function. Here, since we aim to design a continuous controller, we add the adaptive term $tanh(\hat k_{n_\theta+1_i}\xi_i)$ to estimator (4).

The controller of each agent $i=1,2,\dots,N$, is as

$$\sigma_i=Y_i(\eta_i,\dot\eta_i,\xi_i,\dot\xi_i)^T\boldsymbol{e}^{\hat{\boldsymbol{k}}_i}-tanh(\hat k_{n_\theta+1_i}\xi_i)-\mu z_i+\sigma_{cop_i},\quad(5)$$

where μ is an positive arbitrary design parameter, u_{cop_i} is the cooperative control of each agent, and $\boldsymbol{e}^{\hat{\boldsymbol{k}}_i}=\begin{bmatrix}e^{\hat k_{1_i}}\\ \vdots\\ e^{\hat k_{n_{\theta_i}}}\end{bmatrix}$, $\hat k_{p_i}, p=1,\dots,n_\theta, i=1,\dots,N$, are adaptive gains given by

$$\begin{aligned}\dot{\hat k}_{p_i}&=-m_{p_i}\|z_i\|\|Y_i^p(\eta_i,\dot\eta_i,\xi_i,\dot\xi_i)\|, p=1,\dots,n_\theta,\\ \dot{\hat k}_{n_\theta+1_i}&=-m_{n_\theta+1_i}(\|\textstyle\sum_{j\in n_i(t)}a_{ij}(\dot\eta_i-\dot\eta_j)+\gamma_i\dot{\tilde{\eta}}_i\|+\|z_i\|),\end{aligned}\quad(6)$$

where $Y_i^p(\eta_i,\dot\eta_i,\xi_i,\dot\xi_i)$ is the p^{th} column of matrix $Y_i(\eta_i,\dot\eta_i,\xi_i,\dot\xi_i)$, and $m_{p_i}, p=1,\dots,n_\theta, i=1,\dots,N$ and $m_{n_\theta+1_i}$ are some positive parameters chosen arbitrarily. Also,

$$\sigma_{cop_i}=-\sum_{j\in n_i(t)}\nabla\phi_{\tilde\eta_i}(\|\eta_{ij}\|)-d_1\sum_{j\in n_i(t)}a_{ij}(\dot{\tilde{\eta}}_i-\dot{\tilde{\eta}}_j)-d_2\gamma_i\dot{\tilde{\eta}}_i.\quad(7)$$

By using Prop. 2, one can get $M_i(\eta_i)\dot\xi_i+C_i(\eta_i,\dot\eta_i)\xi_i+G_i(\eta_i)=Y_i(\eta_i,\dot\eta_i,\xi_i,\dot\xi_i)\vartheta_i$. Then, by using $z_i=\dot\eta_i-\xi_i$ and (1), we can obtain that

$$M_i(\eta_i)\dot z_i+C_i(\eta_i,\dot\eta_i)z_i=\sigma_i+\varrho_i-Y_i(\eta_i,\dot\eta_i,\xi_i,\dot\xi_i)\vartheta_i.\quad(8)$$

Consider the following Theorem.

Theorem 1: Consider dynamic (1), which represent the agents' dynamic of a multi-agent group tracking a leader with dynamic (2). Suppose that Assumptions 1-3 hold, the initial graph $g(0)$ is connected, and the initial energy $V(0)$ is finite. Then, the control protocol (4)-(7) solves the flocking problem in Definition 1.

We define the following energy function for the group

$$V=e^{V_1+V_2+V_3},\quad(9)$$

where

$$\begin{aligned}V_1&=\tfrac{1}{2}\textstyle\sum_{i=1}^N\sum_{j=1,j\ne i}^N\phi(\|\eta_{ij}\|),\\ V_2&=\tfrac{1}{2}(\textstyle\sum_{i=1}^N z_i{}^T M_i(q_i)z_i+\sum_{i=1}^N\tilde\xi_i{}^T\tilde\xi_i),\\ V_3&=\textstyle\sum_{p=1}^{n_\theta}\beta_p\sum_{i=1}^N\hat k_{p_i}+\beta_{n_\theta+1}\sum_{i=1}^N\hat k_{n_\theta+1_i},\end{aligned}\quad(10)$$

with β_p and $\beta_{n_\theta+1}$ being some positive constants chosen by

$$\beta_p\ge max\{\frac{(\bar\vartheta+e^{\hat k_{p_i}(0)})}{m_{p_i}}|i=1,\dots,N\}, p=1,\dots,n_\theta,$$

$$\beta_{n_\theta+1}\ge max\{\{\frac{\sqrt N(\sqrt n+\bar\sigma))}{\bar m_{n_\theta+1}\sqrt{\lambda_1((L(0)+\Gamma)^2)}},\frac{\sqrt N(\bar\sigma+\bar\varrho)}{\bar m_{n_\theta+1}}\}|i=1,\dots,N\},$$

$$\bar m_{n_\theta+1}=min\{m_{n_\theta+1_i}|i=1,\dots,N\}.$$

Clearly, V is positive.

Proof: Let the network graph $g(t)$ switch at times t_k ($k = 1,2, \dots$), and it is fixed in intervals $[t_{k-1}, t_k), k = 1,2, \dots$. Then, the derivative of $V(t)$ in these interval is given by $\dot{V} = e^{V_1+V_2+V_3}(\dot{V}_1 + \dot{V}_2 + \dot{V}_3)$, where

$$\begin{aligned}\dot{V}_1 &= \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} \nabla\phi_{\tilde{\eta}_i}(\|\eta_{ij}\|)\,\dot{\tilde{\eta}}_i \\ &= \sum_{i=1}^{N}(-\sigma_{cop_i} - d_1\sum_{j\in n_i(t)} a_{ij}(\dot{\tilde{\eta}}_i - \dot{\tilde{\eta}}_j) - d_2\dot{\tilde{\eta}}_i)^T\dot{\tilde{\eta}}_i.\end{aligned} \tag{11}$$

From (2), (4) and (7), it follows that $\dot{\tilde{\xi}}_i = \dot{\xi}_i - \ddot{\eta}_\gamma = \sigma_{cop_i} - tanh(\hat{k}_{n_\theta+1_i}\xi_i) - \sigma_\gamma$. Thus, by recent relation and (5)-(8), we have

$$\begin{aligned}\dot{V}_2 &= (\sum_{i=1}^{N} z_i^T M_i(\eta_i)\dot{z}_i + \tfrac{1}{2} z_i^T \dot{M}_i(\eta_i) z_i) + \sum_{i=1}^{N}\tilde{\xi}_i^T\dot{\tilde{\xi}}_i \\ &= \sum_{i=1}^{N} z_i^T(-C_i(\eta_i,\dot{\eta}_i)z_i + \sigma_i + \varrho_i - Y_i(\eta_i,\dot{\eta}_i,\xi_i,\dot{\xi}_i)\vartheta_i + \tfrac{1}{2}\dot{M}_i(\eta_i)z_i) \\ &\quad + \sum_{i=1}^{N}\tilde{\xi}_i^T(\sigma_{cop_i} - tanh(\hat{k}_{n_\theta+1_i}\xi_i) - \sigma_r) \\ &= \sum_{i=1}^{N} z_i^T((-C_i(\eta_i,\dot{\eta}_i) + \tfrac{1}{2}\dot{M}_i(\eta_i)) + Y_i(\eta_i,\dot{\eta}_i,\xi_i,\dot{\xi}_i)^T \boldsymbol{e}^{\hat{\boldsymbol{k}}_i} \\ &\quad - tanh(\hat{k}_{n_\theta+1_i}\xi_i)) - \mu_1 z_i + \sigma_{cop_i} + \varrho_i - Y_i(\eta_i,\dot{\eta}_i,\xi_i,\dot{\xi}_i)\vartheta_i) \\ &\quad + \sum_{i=1}^{N}\tilde{\xi}_i^T(\sigma_{cop_i} - tanh(\hat{k}_{n_\theta+1_i}\xi_i) - \sigma_r).\end{aligned} \tag{12}$$

From Prop. 2, $z_i^T(-C_i(\eta_i,\dot{\eta}_i) + \frac{1}{2}\dot{M}_i(\eta_i))z_i$ is a skew matrix, and by using that $z_i = \dot{\tilde{\eta}}_i - \dot{\tilde{\xi}}_i$, we get $\left(\sum_{i=n_{l1}}^{N} z_i^T\sigma_{cop_i} + \sum_{i=1}^{N}\tilde{\xi}_i^T\sigma_{cop_i}\right) = \sum_{i=n_l}^{N}\dot{\tilde{\eta}}_i^T\sigma_{cop_i}$, and $\sum_{i=1}^{N}(z_i + \dot{\tilde{\xi}}_i)^T tanh(\hat{k}_{n_\theta+1_i}\xi_i) = \sum_{i=1}^{N}\dot{\tilde{\eta}}_i^T tanh(\hat{k}_{n_\theta+1_i}\xi_i)$. Thus, one can obtain

$$\begin{aligned}\dot{V}_2 &\le \sum_{i=1}^{N}(\|\vartheta_i\|\|Y_i(\eta_i,\dot{\eta}_i,\xi_i,\dot{\xi}_i)\| + \left\|Y_i(\eta_i,\dot{\eta}_i,\xi_i,\dot{\xi}_i)^T\boldsymbol{e}^{\hat{\boldsymbol{k}}_i}\right\| \\ &\quad - \mu_1\|z_i\|)\|z_i\| + \sum_{i=1}^{N} z_i^T\varrho_i \\ &\quad + \sum_{i=1}^{N}\dot{\tilde{\eta}}_i^T(\sigma_{cop_i} - tanh(\hat{k}_{n_\theta+1_i}\xi_i)) - \sum_{i=1}^{N}\tilde{\xi}_i^T\sigma_r,\end{aligned} \tag{13}$$

where $\sum_{i=1}^{N}\dot{\tilde{\eta}}_i^T tanh(\hat{k}_{n_\theta+1_i}\xi_i) = [\dot{\tilde{\eta}}_1^T(t)\cdots\dot{\tilde{\eta}}_N^T(t)]\begin{bmatrix} tanh(\hat{k}_{n_\theta+1_1}\xi_1) \\ \vdots \\ tanh(\hat{k}_{n_\theta+1_N}\xi_N)\end{bmatrix} \le \sqrt{nN}\|\dot{\tilde{\eta}}\|$ where $\dot{\tilde{\eta}}(t) = [\dot{\tilde{\eta}}_1^T(t)\cdots\dot{\tilde{\eta}}_N^T(t)]^T$. Note that from $|tanh(.)| \le 1$, the recent inequality has been obtained. Also, $\sum_{i=1}^{N}\tilde{\xi}_i^T\sigma_r = [\tilde{\xi}_1^T \ \dots \ \tilde{\xi}_N^T]\begin{bmatrix}\sigma_r \\ \vdots \\ \sigma_r\end{bmatrix} \le \sqrt{N}\bar{\sigma}\|\tilde{\xi}\| = \sqrt{N}\bar{\sigma}\left\|\dot{\tilde{\xi}} - z\right\| \le \sqrt{N}\bar{\sigma}(\left\|\dot{\tilde{\xi}}\right\| + \|z\|)$, and from Assumptions 1, we have $\sum_{i=1}^{N} z_i^T\varrho_i = [z_1^T \ \dots \ z_N^T]\begin{bmatrix}\varrho_1 \\ \vdots \\ \varrho_N\end{bmatrix} \le \sqrt{N}\bar{\varrho}\|z\|$, where $\tilde{\xi} = \left[\tilde{\xi}_1^T \cdots \tilde{\xi}_N^T\right]^T, z = [z_1^T \ \dots \ z_N^T]^T$. Moreover, from definition of $\boldsymbol{e}^{\hat{\boldsymbol{k}}_i}$ and $Y_i^p(\eta_i,\dot{\eta}_i,\xi_i,\dot{\xi}_i)$, we get $\|Y_i(\eta_i,\dot{\eta}_i,\xi_i,\dot{\xi}_i)^T\boldsymbol{e}^{\hat{\boldsymbol{k}}_i}\| \le \sum_{p=1}^{n_\theta} e^{\hat{k}_{p_i}}\|Y_i^p(\eta_i,\dot{\eta}_i,\xi_i,\dot{\xi}_i)\|$, and $\|Y_i(\eta_i,\dot{\eta}_i,\xi_i,\dot{\xi}_i)\| \le \sum_{p=1}^{n_\theta}\|Y_i^p(\eta_i,\dot{\eta}_i,\xi_i,\dot{\xi}_i)\|$. Furthermore, from (6), it is concluded that $\hat{k}_{p_i} \le \hat{k}_{p_i}(0)$, $\hat{k}_{n_\theta+1_i} \le \hat{k}_{n_\theta+1_i}(0)$. Thus, by some manipulation, we get

$$\begin{aligned}\dot{V}_2 &\le \sum_{p=1}^{n_\theta}\sum_{i=1}^{N}(\bar{\vartheta} + e^{\hat{k}_{p_i}(0)})\|Y_i^p(\eta_i,\dot{\eta}_i,\xi_i,\dot{\xi}_i)\|\,\|z_i\| \\ &\quad + \sum_{i=1}^{N}\dot{\tilde{\eta}}_i^T\sigma_{cop_i} + \sqrt{N}(\sqrt{n} + \bar{\sigma})\|\dot{\tilde{\eta}}\| + \sqrt{N}(\bar{\sigma} + \bar{\varrho})\|z\|.\end{aligned} \tag{14}$$

From Lemma 2, we have

$$\begin{aligned}\dot{V}_3 &= \sum_{p=1}^{n_\theta}\beta_p\sum_{i=1}^{N}\dot{\hat{k}}_{p_i} + \beta_{n_\theta+1}\sum_{i=1}^{N}\dot{\hat{k}}_{n_\theta+1_i} = \\ &- \sum_{p=1}^{n_\theta}\beta_p\sum_{i=1}^{N} m_{p_i}\|Y_i^p(\eta_i,\dot{\eta}_i,\xi_i,\dot{\xi}_i)\|\|z_i\| \\ &- \beta_{n_\theta+1}\sum_{i=1}^{N} m_{n_\theta+1_i}(\|\sum_{j\in N_i(t)} a_{ij}(\dot{\tilde{\eta}}_i - \dot{\tilde{\eta}}_j) + d_2\dot{\tilde{\eta}}_i\| + \|z_i\|) \le \\ &- \sum_{p=1}^{n_\theta}\beta_p\sum_{i=1}^{N} m_{p_i}\|Y_i^p(\eta_i,\dot{\eta}_i,\xi_i,\dot{\xi}_i)\|\|z_i\| \\ &- \beta_{n_\theta+1}\bar{m}_{n_\theta+1}\left(\sqrt{\lambda_1((L(t_{k-1}) + \Gamma)^2)}\|\dot{\tilde{\eta}}\| + \|z\|\right),\end{aligned} \tag{15}$$

where $\bar{m}_{n_\theta+1} = min\{m_{n_\theta+1_i}|i = 1, \dots, N\}$. Thus, using Lemma 1 that $-\sqrt{\lambda_1((L(t_{k-1}) + \Gamma)^2)} \le -\sqrt{\lambda_1((L(0) + \Gamma)^2)}$, one has

$$\begin{aligned}\dot{V}(t) &= e^{V_1+V_2+V_3}(\dot{V}_1 + \dot{V}_2 + \dot{V}_3) = V(t)(\dot{V}_1 + \dot{V}_2 + \dot{V}_3) \le \\ &- V(t)(\dot{\tilde{\eta}}^T((d_1 L(t_{k-1}) + d_2\Gamma)\otimes I_n)\dot{\tilde{\eta}} \\ &+ \sum_{p=1}^{n_\theta}\sum_{i=1}^{N}(\beta_p m_{p_i} - (\bar{\vartheta} + e^{\hat{k}_{p_i}(0)}))\|Y_i^p(\eta_i,\dot{\eta}_i,\xi_i,\dot{\xi}_i)\|\|z_i\| \\ &+ (\beta_{n_\theta+1}\bar{m}_{n_\theta+1}\sqrt{\lambda_1((L(0) + \Gamma)^2)} - \sqrt{N}(\sqrt{n} + \bar{\sigma}))\|\dot{\tilde{\eta}}\|) \\ &+ (\beta_{n_\theta+1}\bar{m}_{n_\theta+1} - \sqrt{N}(\bar{\sigma} + \bar{\varrho}))\|z\|) \le 0.\end{aligned} \tag{16}$$

If the coefficients $\beta_p, \beta_{n_\theta+1}$ of energy function are chosen such that

$$\beta_p \ge max\{\frac{(\bar{\vartheta} + e^{\hat{k}_{p_i}(0)})}{m_{p_i}}|i = 1, \dots, N\}, p = 1, \dots, n_\theta,$$

$$\beta_{n_\theta+1} \ge max\{\{\frac{\sqrt{N}(\sqrt{n}+\bar{\sigma}))}{\bar{m}_{n_\theta+1}\sqrt{\lambda_1((L(0)+\Gamma)^2)}}, \frac{\sqrt{N}(\bar{\sigma}+\bar{\varrho})}{\bar{m}_{n_\theta+1}}\}|i = 1, \dots, N\},$$

then (16) is guaranteed. From (10) and (16), it follows that $V(t) \le V(t_{k-1})$ for $t \in [t_{k-1}, t_k)$. Without loss of generality, Assume $t_0 = 0$. From (23), it follows that $V(t) \le V(0)$ for $\forall t \in [0, t_1)$. Also, since $g(0)$ is connected and $lim_{\|\eta_{ij}\|\to r}\phi(\|\eta_{ij}\|) = \infty$, then none of the links will tend to r for $t \in [0, t_1)$ and no edge is lost. Thus, the newly added links to the neighboring graph at t_1 cause to switch in $g(t)$. Therefore, the finiteness of initial energy $V(0)$, and the number of the newly added edges implies that $V(t_1)$ will remain finite. Similarly, for each $[t_{k-1}, t_k)$, from (10) and (16), it is concluded that $V(t) \le V(t_{k-1}) < \infty$. Therefore, since $g(t_{k-1})$ is connected and $lim_{\|\eta_{ij}\|\to r}\phi(\|\eta_{ij}\|) = \infty$, then none of the existing edges will tend to r for $t \in [t_{k-1}, t_k)$. Thus, the newly added links to the neighboring graph at t_k cause to switch in $g(t)$. Therefore, the finiteness of the number of the newly added edges implies that, $V(t_k)$ will be finite. Also, since $g(0)$ is connected and no edges in $e(t)$ was lost, $g(t)$ will remain connected for all $t \ge 0$.

Denote the number of the newly added links to the interaction network at t_k by ϵ_k. Clearly, $1 \le \epsilon_k \le \Xi, 1 \le k \le \Xi$ and $\Xi \triangleq 0.5(N-1)(N-2)$. From (9), it follows that $V(t_k^+) = e^{\epsilon_a V(r-\breve{\varepsilon})}V(t_k^-)$, and then

$$\begin{aligned}V(t_k^+) &= e^{\epsilon_k\phi(r-\breve{\varepsilon})}V(t_k^-) \le e^{\epsilon_a\phi(r-\breve{\varepsilon})}V(t_{k-1}^+) \\ &= e^{(\epsilon_{k-1}+\epsilon_k)\phi(r-\breve{\varepsilon})}V(t_{k-1}^-) \le e^{(\epsilon_{k-1}+\epsilon_k)\phi(r-\breve{\varepsilon})}V(t_{k-2}^+) \\ &\le \cdots \le e^{(\epsilon_1+\epsilon_2+\cdots+\epsilon_k)\phi(r-\breve{\varepsilon})}V(0).\end{aligned} \tag{17}$$

Thus, by using (17), one has $V(t_k^+) \leq e^{(\epsilon_1+\epsilon_2+\cdots+\epsilon_k)\phi(r-\check{\varepsilon})}V(0) \leq V_{max}$, and then $V(t) \leq V(t_k^+) \leq V_{max}$ for all $t \geq 0$, where $V_{max} \triangleq V(0)e^{\Xi\phi(r-\check{\varepsilon})} < \infty$.

Here we divide two sides of (16) to $V(t)$, hence we integrate them and get for left side

$$\begin{aligned}&\int_0^\infty \frac{\dot V(t)}{V(t)}dt = \\ &\int_0^{t_1^-} \frac{\dot V(t)}{V(t)}dt + \int_{t_1^+}^{t_2^-} \frac{\dot V(t)}{V(t)}dt + \cdots + \int_{t_{k-1}^+}^{t_k^-} \frac{\dot V(t)}{V(t)}dt + \int_{t_k^+}^{\infty} \frac{\dot V(t)}{V(t)}dt = \\ &\ln(V(t_1^-)) - \ln(V(0)) + \ln(V(t_2^-)) - \ln(V(t_1^+)) \\ &+\cdots + \ln(V(t_k^-)) - \ln(V(t_{k-1}^+)) + \ln(V(\infty)) - \ln(V(t_k^+)) \\ &= -\ln\big(V(0)\big) - \ln\left(\frac{V(t_1^+)}{V(t_1^-)}\right) - \cdots - \ln\left(\frac{V(t_k^+)}{V(t_k^-)}\right) + \ln(V(\infty)) \\ &\geq -\ln\big(V(0)\big) - (\epsilon_1+\epsilon_2+\cdots+\epsilon_k)\phi(r-\check{\varepsilon}) + \ln(V(\infty)) \\ &\geq -\ln\left(e^{(\epsilon_1+\epsilon_2+\cdots+\epsilon_k)\phi(r-\check{\varepsilon})}V(0)\right) \geq -V(0)e^{\Xi\phi(r-\check{\varepsilon})} = -V_{\max}\,.\end{aligned} \tag{18}$$

Also, for right side of (16), we have

$$\begin{aligned}&+\sum_{p=1}^{n_\theta}\sum_{i=1}^{N}(\beta_p m_{p_i} - (\bar\vartheta + e^{\hat k_{p_i}(0)}))\int_0^\infty\left\|Y_i^p(\eta_i,\dot\eta_i,\xi_i,\dot\xi_i)\right\|\|z_i\| \\ &+(\beta_{n_\theta+1}\bar m_{n_\theta+1}\sqrt{\lambda_1((L(0)+\Gamma)^2)} - \sqrt{N}(\sqrt{n}+\bar\sigma))\int_0^\infty\|\dot{\tilde\eta}\| \\ &+(\beta_{n_\theta+1}\bar m_{n_\theta+1} - \sqrt{N}(\bar\sigma+\bar\varrho))\int_0^\infty\|z\| \\ &+\lambda_1(d_1L(0)+d_2\Gamma)\int_0^\infty\left\|\dot{\tilde\eta}\right\|^2 \leq -\int_0^\infty\frac{\dot V(t)}{V(t)}dt \leq V_{max}\,.\end{aligned} \tag{19}$$

From (19), it follows that $\int_0^\infty\left\|Y_i^p(\eta_i,\dot\eta_i,\xi_i,\dot\xi_i)\right\|\|z_i\|$, $\int\|z\|dt$, $\int_0^\infty\|\dot{\tilde\eta}\|$, and $\int_0^\infty\|\dot{\tilde\eta}\|^2$ are finite. Thus, from (6) and (10), it follows that $\hat k_{1_i} \in L_\infty$, $\hat k_{2_i} \in L_\infty$, $\hat k_{3_i} \in L_\infty$. Furthermore, from $V(t) \leq V_{max}$ and (10), it follows that $V_1 \in L_\infty$, $\tilde\xi_i \in L_\infty$, $z_i \in L_\infty$, and then $\dot{\tilde\eta}_i \in L_\infty$.

Moreover, from $V_1 \in L_\infty$ and (3), it follows that the maximum and minimum distances of agents from each others are respectively $max\{\phi^{-1}(V_{max})\}$ and $min\{\phi^{-1}(V_{max})\}$. This guarantees that no collision is happened among agents and $\nabla\phi_{\tilde\eta_i}(\|\eta_{ij}\|) \in L_\infty$. Thus, from (1), (5), and (7), it follows that $\sigma_{cop_i} \in L_\infty$, and then $\sigma_i \in L_\infty$.

Also, from Assumption 2 and $\dot{\tilde\eta}_i \in L_\infty$, it is concluded that $\dot\eta_i \in L_\infty$. Thus, from (1) and Assumptions 1, 3, it follows that $\ddot\eta_i \in L_\infty$, and then $\ddot{\tilde\eta}_i \in L_\infty$ or $\ddot{\tilde\eta} \in L_\infty$ $\ddot{\tilde\eta} \in L_\infty$. Therefore, $\dot{\tilde\eta}$ is uniformly continues. Also, from (19), it obtains $\int_0^\infty\|\dot{\tilde\eta}\|^2 \leq \frac{V_{max}}{\lambda_1(d_1L(0)+d_2\Gamma)}$. Thus, by using Barbalat lemma, it is concluded that $\lim_{t\to\infty}\dot{\tilde\eta} = 0$ i.e. $\dot{\tilde\eta}_1 = \cdots = \dot{\tilde\eta}_N = 0$, then $\dot\eta_1 = \dot\eta_2 = \cdots = \dot\eta_N = \dot\eta_r$.

Remark 4: Here we investigate a new adaptive control for MALS, which is both fully-distributed and continuous. Our proposed protocol in (4)–(7) guarantees the velocity convergence of all followers with the Lagrange dynamic and in the presence of the TVUED to a leader with bounded acceleration. Moreover, it guarantees a connectivity-maintaining and collision-free flocking motion.

Remark 5: The flocking problem with the external disturbance and uncertain parameters causes many challenges in the controller design process, especially with time-variant parameters and a dynamic leader. Most difficulties are due to the design of a controller being fully-distributed and continuous. In the fully-distributed controllers, there is no need to global information of neighbors, the virtual leader, and the initial network graph, and a continuous controller causes no chattering phenomenon in the performance. In the existing references [11-18], there is no reference to consider both external disturbance and uncertainty problem, besides [17, 18]. However, in these works, the uncertainty is not supposed as time-varying in these works, and the designed controller is not fully distributed. Also, there is no reference, with the controller being both fully-distributed and continuous. The only reference with the fully-distributed controller is ref. [11], where an adaptive protocol for the leader-follower flocking of MALS with the unknown constant uncertainty, has been proposed. The controller, estimator, and adaptive laws in this work were defined based on the discontinuous Sign function and 1-norm of the system's states. Besides the chattering problem, the other drawback of this protocol is that using the 1-norm of the signals results in infinitely increasing the adaptive gains in the presence of disturbances or measurement errors. In Remark 3.9 of [11], authors claimed that in order to cope the unboundedly increasing problem of adaptive gains, an alternation is to introduce the small bounds for the norm of error signals in the adaptive laws that after interring the norm of error signals within these predefined bounds, increasing of adaptive gains are stopped. However, this solution does not causes an asymptotic flocking motion due to the time-varying uncertainty, and only a stable flocking motion involving the chattering phenomenon is achievable. Also, because of depending of the predefined bounds to global information of the initial network graph, it is not fully-distributed. Therefore, unlike to our protocol, the protocol [11] is not applied to the MALS with TVUED problem.

IV. Simulation Study

In In this section, we perform a simulation to illustrate the effectiveness of the results. We assume that the agents move in a 2-dimensional space (X, Y), and the first agent is informed. The multi-agent group includes five standard two-DOF robot manipulators. Assume that each manipulator carries a time-varying mass. The dynamic of each agent is modeled as

$$\begin{bmatrix} m_i & 0 \\ 0 & m_i \end{bmatrix}\ddot\eta_i + \begin{bmatrix} 0 & -\beta_i \\ \beta_i & 0 \end{bmatrix}\dot\eta_i = \sigma_i + \varrho_i,$$

where $\eta_i \in R^2$ is the vector of joint displacements, $\sigma_i \in R^2$ is the control torques, ϱ_i is the acted external disturbance on ith manipulator, and m_i, β_i are its time-varying mass and damping constants, respectively. Suppose $m_i = (1 + 0.1\,cos(4it))m_0$, $\beta_i = (1 + 0.1\,cos(4it))\beta_0$, where m_0 and β_0 are nominal value. Also, the dynamic of leader is $\ddot\eta_r = \begin{bmatrix} sin(3t) \\ cos(3t) \end{bmatrix}$, and external disturbance of each agent is that $\varrho_i = \begin{bmatrix} sin(2it) \\ cos(2it) \end{bmatrix}$. Here $\vartheta_i = \begin{bmatrix} m_i \\ \beta_i \end{bmatrix}$, $Y_i(\eta_i,\dot\eta_i,\xi_i,\dot\xi_i) = [\dot\xi_i \quad \xi_i]$, where $\dot\xi_i$ defined in (4).

In the first simulation for the protocol (4)-(7), we consider $r = 10$, $\varepsilon_0 = 0.1$, $\varepsilon = 0.5$, $m_{1_i} = m_{2_i} = m_{3_i} = 10, i = 1, \ldots, N$, $\mu = 6000$, $d_1 = 400$, $d_2 = 3000$.

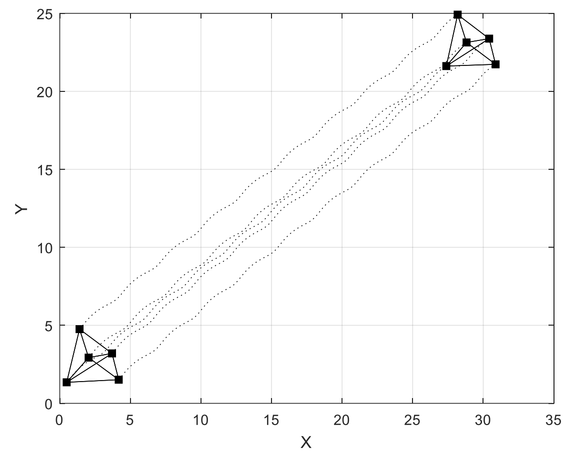

Fig. **1**: The position and trajectory of agents under protocol (4)-(7).

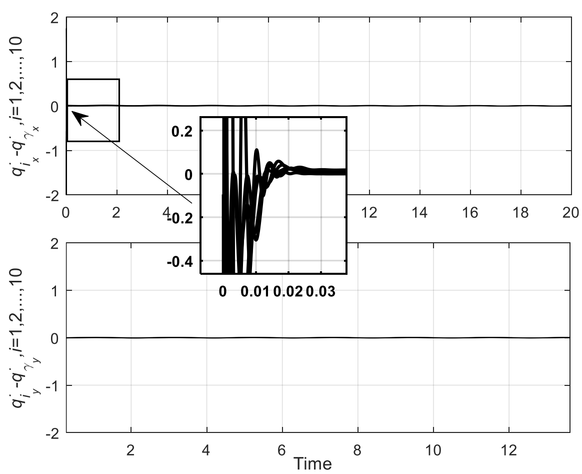

Fig. 2: Agents' velocity over the X-Y axes under protocol (4)-(7).

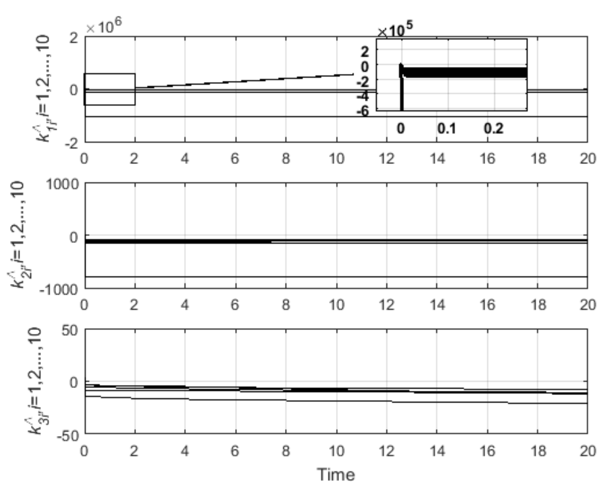

Fig. 3: The trajectory of $\hat{k}_{1_i}\hat{k}_{2_i}, \hat{k}_{3_i}, i = 1, \ldots, 5$ in protocol (4)-(7).

Figs. 1-3 show the results for our protocol. In the fig. 1 the squares represent the uninformed agents, and the star depicts the informed agent. Solid lines represent neighboring links, and the dash-dotted lines depict the agents' trajectory.

Fig. 2 indicates the velocity of all agents over the X-axis and the Y-axis. Fig. 3 depicts the adaptive gain of $\hat{k}_{1_i}\hat{k}_{2_i}, \hat{k}_{3_i}, i = 1, \ldots, 5$. As shown in the figs. 1-3 under our protocol, the network's initial connectivity is preserved during the motion, and no collision happens among the agents. Also, the whole group's velocity asymptotically converges to that of the virtual leader, and all adaptive gains remain bounded.

V. Conclusion

This paper proposed a controller for Lagrange systems' leader-follower flocking with the time-varying uncertainty and external disturbances problems. It was assumed, the virtual leader moves with a time-varying velocity and bounded acceleration. By designing a new adaptive law and an estimator, we proposed a continuous fully-distributed adaptive protocol, which achieve all flocking motion goals.

It is remarkable that our results also can be extended to the connectivity-preserving consensus problem.

For future work, we aim to design a fully-distributed adaptive protocol for the flocking of Lagrange systems under input saturation or actuator fault problem.

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