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Adaptive Synchronization Control of Chaotic Nonlinear Systems in the Presence of Input Saturation and Actuator Faults

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Abstract: In this paper, the control problem is investigated for Jerk chaotic systems against unknown parameters, actuator faults and input saturation. The considered actuator fault covers both of the stuck faults and loss of effectiveness faults in actuators. The values, times and patterns of the considered faults are completely unknown. That is, during the system operation it is unknown when, by how much and which actuators fail. A robust adaptive controller is presented based on the backstepping design method to achieve complete synchronization of the identical Jerk chaotic systems. By introducing the new Lyapunov functions, it is proved that all the closed loop signals are bounded and the tracking error converges to a small neighborhood of the origin. The proposed adaptive method compensates the actuator faults without any need for explicit fault detection. Simulation results represent that the designed controller can synchronize the identical chaotic systems in the presence of actuator fault, input saturation and unknown parameters.

Index Terms: Input Saturation, Chaotic systems, Actuator fault, Adaptive control, Backstepping control method.

کنترل همزمان ساز تطبیقی سیستم‌های غیرخطی آشوب در حضور اشباع ورودی و عیب عملگر

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خلاصه: در این مقاله، یک کنترل‌کننده تطبیقی برای کنترل سیستم‌های غیرخطی جرک در معرض پارامترهای نامعین و محدودیت‌های کنترلی عیب عملگر و اشباع ورودی ارائه شده است. عیب عملگر در نظر گرفته شده عیوب کاهش کارایی و قفل‌شونده را پوشش می‌دهد. مقدار، زمان و الگوی عیوب در نظر گرفته شده کاملاً نامعین است یعنی مشخص نیست در چه زمانی، کدام عملگرها و با چه وضعیتی دچار عیب می‌شوند. کنترل‌کننده تطبیقی مقاوم پیشنهادی بر اساس روش کنترلی گام به عقب طراحی شده است. در این مقاله، با معرفی توابع لیاپانوف- کراسوسکی جدید، کرانداری سیگنالهای سیستم حلقه بسته و همگرایی خطای تعقیب به یک همسایگی نزدیک مبدأ تضمین شده است. روش تطبیقی پیشنهادی، عیوب عملگر را بدون نیاز به واحد تشخیص عیب جبران می‌کند. نتایج شبیه‌سازی، کارایی و صحت روش کنترلی ارائه شده را در همزمان‌سازی سیستم آشوب در حضور عیب عملگر، اشباع ورودی و نامعینی پارامتری نشان می‌دهد.

کلمات کلیدی: اشباع ورودی، سیستم‌های آشوب، عیب عملگر، کنترل تطبیق، روش کنترلی گام به عقب.

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1. Introduction

Chaos behaviour is a nonlinear performance of the complex nonlinear dynamical systems which is highly dependent on the initial conditions. The chaotic systems topic is an area of research that has attracted considerable attention in various fields such as information processing systems, secure communications, chemical and biological drew [1-9]. So far, varieties of control approaches have been developed for chaotic systems [10-17]. In [12], an adaptive feedback control scheme was provided for synchronization of the chaotic systems. The considered system in [12] was of a Vanderpol oscillator that is coupled to the linear oscillators with cubic term. The parameters of the master system in [13] were unknown and different from those of the slave system. In [14], an adaptive controller was presented for synchronization between two different chaotic systems with uncertainties, external disturbances, unknown parameters and input nonlinearities. In [15], an adaptive backstepping controller was designed to achieve complete chaos synchronization of the identical novel Jerk chaotic systems with unknown system parameters. In [16], a robust adaptive sliding mode controller was proposed to realize chaos synchronization between two different chaotic systems with uncertainties, external disturbances and unknown parameters. In [17], a sliding mode synchronization controller was presented for two chaotic systems by using radial basis function (RBF) neural networks. In [17], neural networks have been used to estimate the uncertainties of the considered system.

An important issue that recently has attracted considerable attention in the chaotic systems is the occurrence of faults. Faults often cause undesired system behaviour and sometimes, lead to instability. Indeed, fault is a deviation of the system structure or the system parameters from the nominal situation. Faults can occur due to the locking of an actuator, the loss of effectiveness in sensors or actuators and the disconnections of the system components. However, actuator faults are more serious than sensors or other components faults, because actuator faults could reduce some percentage or total of the control input effectiveness and lead to instability or even catastrophic accidents [18]. Until now, many researchhers have considered the actuator fault control problem in linear and nonlinear systems [19-30]. In [21], a direct adaptive control scheme was developed to solve the robust fault tolerant control problem for linear systems with mismatched parameter uncertainties, disturbances and actuator faults including loss of effectiveness, outage and stuck. In [22], an indirect adaptive state feedback control scheme was developed to solve the robust fault tolerant control design for linear time-invariant systems against actuator fault and perturbation. In [23], an adaptive control approach was developed to control a class of multi-input multi-output (MIMO) nonlinear systems in the presence of uncertain actuators faults. In [24], an adaptive controller was investigated for a class of MIMO nonlinear systems with unknown parameters, bounded time delays and in the presence of unknown time varying actuator failures. In [25], a robust adaptive control approach was presented based on the linear matrix inequality (LMI) approach and adaptive method to deal with the problem of flight tracking control problem in the presence of actuator faults. In [26], a fuzzy adaptive actuator fault compensation controller was presented for a class of uncertain stochastic nonlinear systems in the strict-feedback form. In [27], a class of unknown nonlinear systems was studied in the presence of uncertain actuator faults and external disturbances. The uncertainties of the considered system in [27] were approximated with the help of fuzzy approximation theory. In [28], a state feedback adaptive controller was presented for a class of parametric-strict-feedback nonlinear systems with multiple bounded timevarying state delays and in the presence of time varying actuator failures. In [29], an adaptive decentralized dynamic surface control (DSC) approach was proposed for a class of large-scale nonlinear systems with unknown nonlinear functions, unknown control gains, time varying delays and in the presence of unknown actuator failures. In [30], an adaptive DSC approach was developed for a class of MIMO nonlinear systems with unknown nonlinearrities, bounded time varying state delays, and in the presence of time varying actuator failures.

Furthermore, input saturation is another issue that is frequently encountered in many practical systems. Saturation is a potential problem for actuators of control systems. It severely limits system performance, giving rise to undesirable inaccuracy or leading instability. The development of adaptive control schemes for systems with input saturation has been a task of major practical interest as well as theoretical significance. So far, considerable atention has been devoted to the control of systems in the presence of input saturation [31-40]. In [31], a model reference adaptive control technique was used for linear plants in the presence of constraints on the input amplitude. In [32], an adaptive predictive regulator was proposed of single-input single-output (SISO) linear plants subject to saturations on both the control variable. In [33], an indirect adaptive regulator was presented for a class of linear systems in the presence of input saturation constraints and parametric uncertainties. In [34], a robust adaptive controller has been expressed for a class of nonlinear systems with unknown backlash-like hysteresis. In [35], an adaptive backstepping controller was designed for systems with unknown high-frequency gain. In [36], a decentralized adaptive neural control approach is presented for a class of interconnected large-scale nonlinear time-delay systems with input saturation. In [37], a direct adaptive fuzzy control scheme is proposed for uncertain nonlinear systems in the presence of input saturation. In [38], an adaptive tracking control is designed for a class of uncertain MIMO nonlinear systems with non-symmetric input constraints. In [39], an adaptive prescribed performance output feedback control scheme is proposed for a class of switched nonlinear systems with input saturation. In [40], a distributed neuro-adaptive control approach is developed for synchronisation of leader–follower multi-agent systems with non-linear dynamics in non-strict-feedback form in the presence of input saturation.

In this paper, a robust adaptive control approach is proposed for synchronization of the Jerk chaotic systems in the presence of unknown parameters, unknown actuator faults and input saturation. With the error between the control input and saturated input as the input of the constructed system, a number of signals are generated to compensate the effect of saturation problem. By the proposed adaptive backstepping design method, the tracking error is shown to approach to a signal generated by the constructed system. Also, the proposed adaptive method can compensate a large class of actuator faults without any need for explicit fault detection. The considered faults are modelled to cover loss of effectiveness faults as well as stuck at some unknown values. The values, times and patterns of the considered faults are unknown, that is, during the system operation it is unknown when, by how much, and which actuators fail. Compared with the existing results, the main contributions of this paper are as follows:

(i) The control problem is investigated for Jerk chaotic nonlinear systems with unknown parameters.

(ii) The proposed controller is designed in the presence of actuator faults and input saturation.

(iii) The considered actuator fault model can cover stuck faults as well as loss of effectiveness faults.

(iv) Appropriate Lyapunov-Krasovskii type functionals are introduced to design new adaptive laws to compensate the unknown actuator faults and saturation outcomes.

(v) The proposed method proves that without the need for explicit fault detection, not only are all the signals in the closed loop system bounded, but also the tracking error converges to a small neighbourhood of the origin.

The paper is organized as follows. In section 2, the system description is given along with the necessary assumptions. In section 3, design and stability analysis of the proposed controller is presented. In section 4, simulation examples are studied to illustrate the effectiveness of the proposed control scheme. Finally, the paper closes with some conclusions in section 5.

2. Problem Information

The master system, is considered as the following 3-D novel Jerk chaotic system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = 5 - a(x_2 + x_3) + b(x_1 x_2) - \exp(x_1) \end{cases} \qquad (1)$$

Where $x_1, x_2, x_3 \epsilon R$ are the states of the system, a and b are unknown constant parameters.

The slave system is considered as the following 3-D novel Jerk chaotic system:

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = y_3 \\ \dot{y}_3 = 5 - a(y_2 + y_3) + b(y_1 y_2) - exp(y_1 \end{cases} \qquad (2)$$

where

$$u(v^F) = \sum_{i=1}^{m} u_i\,(v_i^F) \qquad (3)$$

And $y_1, y_2, y_3 \epsilon R$are the states of the system, $u \in R^m$ is the control input.

The synchronization errors are defined as follows:

$$\begin{cases} e_1 = y_1 - x_1 \\ e_2 = y_2 - x_2 \\ e_3 = y_3 - x_3 \end{cases} \qquad (4)$$

The error dynamics is obtained as

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = e_3 \\ \dot{e}_3 = -a(e_2 + e_3) + b(e_1 e_2) - \exp(y_1) + \end{cases} \qquad (5)$$

In addition $u(v(t)) \in R^m$denotes the plant input subject to saturation which is described by:

$$u_i(v_i) = sat(v_i) = \begin{cases} sign(v_i) u_{M_i} |v_i| \geq u_{M_i} \\ v_i |v_i| < u_{M_i} \end{cases} \qquad (6)$$

Where $u_{M_i}$is the saturation bound of $u_i(v_i)(t)$.

The error of adaptive parameters is obtained as

$$\begin{cases} \tilde{a}(t) = a - \hat{a} \\ \tilde{b}(t) = b - \hat{b} \end{cases} \qquad (7)$$

The control objective is to design an adaptive controller for plant (6) in order to assure that all the closed loop signals are bounded and the tracking error converges to a small neighbourhood of the origin despite the presence of unknown plant parameters, actuator faults and input saturation. For this purpose, the following assumptions are considered.

To formulate the reliable control problem, the following actuator fault model is considered:

$$v^F(t) = \sum_{i=1}^{m} v_i^F(t) \qquad (8)$$

where

$$v_i^F(t) = (1-\rho_i)v_i(t) \quad 0 \le \underline{\rho_i} \le \rho_i \le \bar{\rho}_i \quad , \quad i = 1 \dots\dots m \tag{9}$$

Where ρ_i is an unknown constant and $v_i^F(t)$ represents the signal from the ith actuator that may fail. In the considered fault mode, $\underline{\rho_i}$ and $\bar{\rho}_i$ represent the lower and upper bounds of ρ_i, respectively. Note that, when $\underline{\rho_i} = \bar{\rho}_i = 0$, there is no fault in the ith actuator and when $\underline{\rho_i} = \bar{\rho}_i = 1$, the ith actuator u_i is outage. When $0 < \underline{\rho_i} \le \bar{\rho}_i < 1$, the actuator loses some fraction of its effectiveness.

Assumption 1. For plant (5) with known plant parameters and fault parameters, if any up to m -1 actuators outage as (9), the others may lose effectiveness; the closed loop system can still be driven to achieve a desired control objective.

3. Control design

In order to compensate the effect of the saturation, the following system is constructed to generate signals $\lambda(t) = [\lambda_1, \dots, \lambda_3]^T$ such that

$$\dot{\lambda}_1 = \lambda_2 - c_1\lambda_1 \quad \dot{\lambda}_2 = \lambda_3 - c_2\lambda_2 \quad \dot{\lambda}_3 = -c_3\lambda_3 + \Delta u \tag{10}$$

where c_i are positive constants and $\Delta u = u(v) - v^F$. Thus, the following change of coordinates has been made.

$$z_i = e_i - a_{n-1} - \lambda_i \quad i = 1,2,3 \tag{11}$$

where a_{i-1} is the virtual control at the ith step to be determined. Thus virtual control law α_i is designed as

$$\begin{cases} a_1 = -c_1 e_1 \\ a_2 = -c_2(e_2 - a_1) + \dot{a}_1(e_1, e_2) \end{cases} \tag{12}$$

The loss of effectiveness model of actuator fault to be considered is modelled as

$$v_i^F(t) = (1-\rho_i)v_i(t) \tag{13}$$

For unknown actuator faults, the adaptive control input becomes as:

$$v_i^F(t) = (1-\rho_i)(\hat{k}_{i1}v_0 + \hat{k}_{i2}) \; ; \; i = 1,2,\dots,m \tag{14}$$

where ρ_i is an unknown constant and $v_i^F(t)$ represents the signal from the ith actuator that has been failed and v_0 is the nominal control to be designed later and where $\hat{k}_{i1}$ and $\hat{k}_{i2}$ are the estimates of k_{i1} and k_{i2} where $k_{i1} \in R$ and $k_{i2} \in R$ are some constant parameters which satisfy

$$\begin{cases} \sum_{i=1}^{m}(1-\rho_i)k_{i1} = 1 \\ \sum_{i=1}^{m}(1-\rho_i)k_{i2} = 0 \end{cases} \tag{15}$$

In the following, the backstepping design method is explained. The design procedure is explained in the following three steps.

• *Step 1*: The z_1 subsystem is considered as

$$\dot{z}_1 = \dot{e}_1 - \dot{\lambda}_1 \tag{16}$$

Therefore

$$\dot{z}_1 = z_2 - c_1 z_1 \tag{17}$$

where $c_1 > 1/2$ is a positive design parameter. A positive Lyapunov function $V_1(t)$ is defined as

$$V_1 = \frac{1}{2}z_1^2 \tag{18}$$

Then by using (12) and (17), the time derivative of $V_1(t)$ satisfies

$$\dot{V}_1 = z_1\dot{z}_1 = z_1 z_2 - c_1 z_1^2 \tag{19}$$

By using the Young's inequality, the time derivative of $V_1(t)$ becomes

$$\dot{V}_1 \le -c_1 z_1^2 + \tfrac{1}{2}z_1^2 + \tfrac{1}{2}z_2^2 \tag{20}$$

Thus

$$\dot{V}_1 \le -\bar{c}_1\left(\tfrac{1}{2}z_1^2\right) + \tfrac{1}{2}z_2^2 \tag{21}$$

Eventually, the time derivative of $V_1(t)$ becomes

$$\dot{V}_1 \le -\bar{c}_1 V_1 + \mu_1 \tag{22}$$

where $\mu_1 = \frac{1}{2}z_2^2$ and $\bar{c}_1 = 2c_1 - 1 > 0$.

• *Step 2*: The z_2 subsystem is considered as

$$\dot{z}_2 = \dot{e}_2 - \dot{a}_1 - \dot{\lambda}_2 \tag{23}$$

Therefore

$$\dot{z}_2 = z_3 - c_2 z_2 \tag{24}$$

where $c_2 > 1$ is a positive constant. A Lyapunov function $V_2(t)$ is defined as:

$$V_2 = V_1 + \frac{1}{2}z_2^2 \tag{25}$$

By using (12) and (24), the time derivative of $V_2(t)$ becomes

$$\dot{V}_2 = z_1\dot{z}_1 + z_2\dot{z}_2 = z_1 z_2 - c_1 z_1^2 + z_2 z_3 - c_2 z_2^2 \tag{26}$$

By using the Young's inequality, the time derivative of $V_2(t)$ becomes

$$\dot{V}_2 \le -c_1 z_1^2 + \tfrac{1}{2}z_1^2 + \tfrac{1}{2}z_2^2 - c_2 z_2^2 + \tfrac{1}{2}z_2^2 + \tfrac{1}{2}z_3^2 \tag{27}$$

Consequently,

$$\dot{V}_2 \le -\bar{c}_1(\frac{1}{2}z_1^2) - \bar{c}_2(\frac{1}{2}z_2^2) + \frac{1}{2}z_3^2 \tag{28}$$

Eventually, the time derivative of $V_2(t)$ becomes

$$\dot{V}_2 \le -\acute{c}_2 V_2 + \mu_2 \tag{29}$$

where $\mu_2 = \frac{1}{2}z_3^2$, $\bar{c}_2 = 2c_2 - 2 > 0$ and $\acute{c}_2 = \min\{\bar{c}_1, \bar{c}_2\}$.

• *Step 3*: The z_3 subsystem is considered as

$$\dot{z}_3 = \dot{e}_3 - \dot{a}_2 - \dot{\lambda}_3 \tag{30}$$

Consequently,

$$\dot{z}_3 = -a(e_2 + e_3) + b(y_1 y_2 - x_1 x_2) - exp(y_1) + exp(x_1) + v^F + \left(\textstyle\sum_{j=1}^{3} g_j e_j\right) - c_3 z_3 \tag{31}$$

where

$$\begin{cases} g_1 = c_1 c_2 c_3 \\ g_2 = c_1 c_2 + c_1 c_3 + c_2 c_3 \\ g_3 = c_1 + c_2 + c_3 \end{cases} \tag{32}$$

in which $c_3 > 1/2$ is a positive constant.

The positive Lyapunov function $V_3(t)$ is defined as

$$V_3 = V_2 + \tfrac{1}{2}z_3^2 + \tfrac{1}{2}\tilde{a}^2 + \tfrac{1}{2}\tilde{b}^2 + \left(\tfrac{1}{2}\textstyle\sum_{j=1}^{3}\tilde{g}_j^{\,2}\right) + \left(\tfrac{1}{2}\textstyle\sum_{i=1}^{m}(1-\rho_i)(\tilde{k}_i^{\,T}\Gamma_i^{-1}\tilde{k}_i)\right) \tag{33}$$

Then by using (12) and (31), the time derivative of $V_3(t)$ satisfies

$$\dot{V}_3 = \dot{V}_2 + z_3\dot{z}_3 - \tilde{a}\dot{\hat{a}} - \tilde{b}\dot{\hat{b}} - \left(\sum_{j=1}^{3} \tilde{g}_j\dot{\hat{g}}_j\right) - \left(\sum_{i=1}^{m}(1-\rho_i)(\tilde{k}_i^{\,T}\Gamma_i^{-1}\dot{\hat{k}}_i)\right) \quad (34)$$

where

$$\begin{cases} \dot{\tilde{g}}_j = -\dot{\hat{g}}_j \\ \dot{\tilde{a}} = -\dot{\hat{a}} \\ \dot{\tilde{b}} = -\dot{\hat{b}} \\ \dot{\tilde{k}}_i = -\dot{\hat{k}}_i \end{cases} \quad (35)$$

Thus

$$\dot{V}_3 = z_1z_2 - c_1z_1^2 + z_2z_3 - c_2z_2^2 + z_3[-a(e_2+e_3) + b(y_1y_2 - x_1x_2) - exp(y_1) + exp(x_1) + (\sum_{i=1}^{m}(1-\rho_i)(\hat{k}_{i1}v_0 + \hat{k}_{i2})) + \left(\sum_{j=1}^{3} g_je_j\right) - c_3z_3] - \tilde{a}\dot{\hat{a}} - \tilde{b}\dot{\hat{b}} - \left(\sum_{j=1}^{3}\tilde{g}_j\dot{\hat{g}}_j\right) - \left(\sum_{i=1}^{m}(1-\rho_i)\left(\tilde{k}_i^{\,T}\Gamma_i^{-1}\dot{\hat{k}}_i\right)\right) \quad (36)$$

The updating laws are selected as:

$$\begin{cases} \dot{\hat{g}}_j = e_jz_3 - \sigma_j\hat{g}_j \\ \dot{\hat{a}} = -(e_2+e_3)z_3 - \alpha\hat{a} \\ \dot{\hat{b}} = -(y_1y_2 - x_1x_2)z_3 - \beta\hat{b} \\ \dot{\hat{k}}_i = \Gamma_i(-1)z_n[v_0\ \ 1]^T - \sigma_i\hat{k}_i \end{cases} \quad (37)$$

where $\Gamma_i > 0$ and σ_j, j=1,2,3,σ_i, i=1,…,m, α and β are small positive constants.

Therefore, the adaptive controller is selected as

$$v_0 = \hat{a}(e_2+e_3) - \hat{b}(y_1y_2 - x_1x_2) + \exp(y_1) - \exp(x_1) - \sum_{j=1}^{3}\hat{g}_je_j \quad (38)$$

Therefore, the time derivative of $V_3(t)$ becomes as follows:

$$\dot{V}_3 = z_1z_2 - c_1z_1^2 + z_2z_3 - c_2z_2^2 - c_3z_3^2 + z_3[-a(e_2+e_3) + b(y_1y_2 - x_1x_2) - exp(y_1) + exp(x_1) + \left(\sum_{j=1}^{3} g_je_j\right) + \hat{a}(e_2+e_3) - \hat{b}(y_1y_2 - x_1x_2) + \exp(y_1) - \exp(x_1) - (\sum_{j=1}^{3}\hat{g}_je_j) - \sum_{i=1}^{m}(1-\rho_i)(\tilde{k}_{i1}v_0 + \tilde{k}_{i2})] - \tilde{a}(-(e_2+e_3)z_3 - \alpha\hat{a}) - \tilde{b}(-(y_1y_2 - x_1x_2)z_3 - \beta\hat{b}) - \left(\sum_{j=1}^{3}\tilde{g}_j(e_jz_3 - \sigma_j\hat{g}_j\right) - \sum_{i=1}^{m}(1-\rho_i)(\tilde{k}_i^{\,T}\Gamma_i^{-1}\Gamma_i(-1)z_n[v_0\ \ 1]^T - \sigma_i\hat{k}_i)) \quad (39)$$

Where

$$\sum_{i=1}^{m}(1-\rho_i)(\hat{k}_{i1}v_0 + \hat{k}_{i2}) = \sum_{i=1}^{m}(1-\rho_i)(k_{i1}v_0 + k_{i2}) - \sum_{i=1}^{m}(1-\rho_i)(\tilde{k}_{i1}v_0 + \tilde{k}_{i2}) = v_0 - \sum_{i=1}^{m}(1-\rho_i)(\tilde{k}_{i1}v_0 + \tilde{k}_{i2}) \quad (40)$$

Thus, the time derivative of $V_3(t)$ becomes

$$\dot{V}_3 = z_1z_2 - c_1z_1^2 + z_2z_3 - c_2z_2^2 - c_3z_3^2 + \alpha\tilde{a}\hat{a} + \beta\tilde{b}\hat{b} + \left(\sum_{j=1}^{3}\sigma_j\tilde{g}_j\hat{g}_j\right) + \left(\sum_{i=1}^{m}\acute{\sigma}_i(\tilde{k}_i^{\,T}\hat{k}_i)\right) \quad (41)$$

where

$$\acute{\sigma}_i = (1-\rho_i)\sigma_i \quad (42)$$

Therefore, the time derivative of $V_3(t)$ becomes

$$\dot{V}_3 = z_1z_2 - c_1z_1^2 + z_2z_3 - c_2z_2^2 - c_3z_3^2 + \alpha\tilde{a}(a - \tilde{a}) + \beta\tilde{b}(b - \tilde{b}) + \left(\sum_{j=1}^{3}\sigma_j\tilde{g}_j(g_j - \tilde{g}_j)\right) + \left(\sum_{i=1}^{m}\acute{\sigma}_i(\tilde{k}_i^{\,T}(k_i - \tilde{k}_i)\right) \quad (43)$$

By using the Young's inequalities, the time derivative of $V_3(t)$ becomes as follows:

$$\dot{V}_3 \le -\bar{c}_1\frac{z_1^2}{2} - \bar{c}_2\frac{z_2^2}{2} - \bar{c}_3\frac{z_3^2}{2} - \frac{\alpha}{2}\tilde{a}^2 - \frac{\beta}{2}\tilde{b}^2 - \left(\frac{1}{2}\sum_{j=1}^{3}\sigma_j({\tilde{g}_j}^2)\right) - \left(\frac{1}{2}\sum_{i=1}^{m}\acute{\sigma}_i\tilde{k}_i^{\,T}\tilde{k}_i\right) + \frac{\alpha}{2}a^2 + \frac{\beta}{2}b^2 + \left(\frac{1}{2}\sum_{j=1}^{3}\sigma_j({g_j}^2)\right) + \left(\frac{1}{2}\sum_{i=1}^{m}\acute{\sigma}_ik_i^{\,T}k_i\right) \quad (44)$$

Eventually, the time derivative of $V_3(t)$ becomes

Where

$$\mu = \frac{\alpha}{2}a^2 + \frac{\beta}{2}b^2 + \left(\frac{1}{2}\sum_{j=1}^{3}\sigma_j({g_j}^2)\right) + \left(\frac{1}{2}\sum_{i=1}^{m}\acute{\sigma}_ik_i^{\,T}k_i\right) \quad (45)$$

$$\bar{c}_3 = 2c_3 - 1 > 0 \quad ,$$

$$\acute{c}_3 = min\{\bar{c}_1, \bar{c}_2, \bar{c}_3, \alpha, \beta, \sigma_j, \acute{\sigma}_i\}$$

The result of the proposed method is expressed in the form of the following theorem.

Theorem 1. Consider the closed loop system (5). Under assumption 1, the controller structure (14) with the parameter updating laws (37), assures that all the closed loop signals are globally bounded and the signal $z(t) = [z_1, z_2, z_3]$ converges to the following compact set.

$$A_z = \left\{\left(z \middle| \|z\| \le \sqrt{\frac{2\mu}{c}}\right)\right\} \quad (46)$$

Where $c = min\{\bar{c}_1, \bar{c}_2, \bar{c}_3\}$ and $\mu = \sum_{i=1}^{3}\mu_i$.

Proof: The following Lyapunov function is considered:

$$V = \sum_{i=1}^{3} V_i$$

Where $V_i(t)$ for i= 1,2,3 are defined in (18), (25) and (33). Therefore, the time derivative of $V(t)$ becomes

$$\dot{V} \le -cV + \mu$$

$$V \le \left[V(0) - \frac{\mu}{c}\right]e^{-ct} + \frac{\mu}{c}$$

$$\|z\| \le \sqrt{2(\left[V(0) - \frac{\mu}{c}\right]e^{-ct} + \frac{\mu}{c})}$$

Thus it can be concluded that $V(t)$ is bounded; accordingly all the closed loop signals are bounded and the signal $z(t)$ =$[z_1, z_2, z_3]$ converges to the compact set defined in (46). Thus, the synchronization error converges to a small neighbourhood of the origin.

Block diagram of the proposed approach is given in Fig.1.

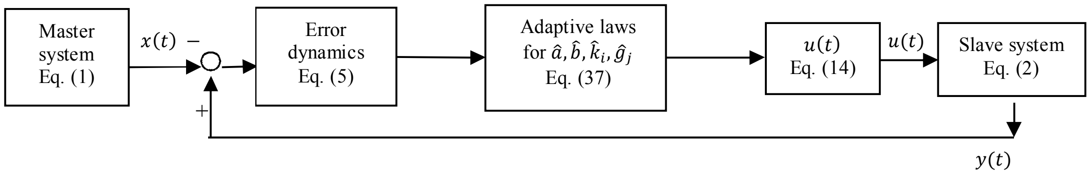

Fig. (1): Block diagram of the proposed adaptive controller, in which $y = [y_1, y_2, y_3]^T$ and $x = [x_1, x_2, x_3]^T$.

4. Simulation examples

In this section, the Jerk system is used as an example to verify the effectiveness of the proposed reliable synchronization controller designed method. onsider the following Jerk systems in the form of (47)

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = 5 - a(x_2 + x_3) + b(x_1 x_2) - exp(x_1 \end{cases} \quad (47)$$

The Jerk system (45) is chaotic when the parameter values are taken as:

$$a = 1 \quad , \quad b = 0.4 \quad (48)$$

and the design parameter values are taken as:

$$c_1 = c_2 = c_3 = 1 \quad (49)$$

For simulation purpose, the actuator faults and input saturation are considered as:

$$\begin{cases} \rho_1 = 0 & t \le 4 \\ \rho_1 = 0.2 & 4 < t \le 10 \\ \rho_1 = 1 & t > 10 \end{cases}, u_{M_1} = 5 \text{ and } \begin{cases} \rho_2 = 0 & t \le 8 \\ \rho_2 = 0.4 & t > 8 \end{cases}, u_{M_2} = 2 \quad (50)$$

The design parameter values and the parameter values of the novel Jerk chaotic systems are taken as in the chaotic case, i.e. a = 1, b = 0.4, $c_1 = c_2 = c_3 = 1$ and thus $g_1 = g_2 = g_3 = 1$.

Furthermore, the initial conditions of the master chaotic system (1) is selected as:

$$x_1(0) = 2.5 \quad , x_2(0) = 0.7 \quad , x_3(0) = -4.9 \quad (51)$$

The initial conditions of the master chaotic system (2) is selected as:

$$y_1(0) = -1.3 \, , y_2(0) = 4.2 \, , y_3(0) = 2.5 \quad (52)$$

Also, the initial conditions of the parameter estimates $\hat{a}$ (t), $\hat{b}$(t), $\hat{g}_1$(t), $\hat{g}_2$(t), $\hat{g}_3$(t) are selected as $\hat{a}$ (0) = 6.2, $\hat{b}$ (0) = 11.5, $\hat{g}_1(0) = 5$, $\hat{g}_2(0) = 3.6$, $\hat{g}_3(0) = -3.5$. The simulation results are shown in Figures 2-7. Figures 2-3 express the control inputs $u_1(v_1^F)$and $u_2(v_2^F)$. As can be seen from (50), the first actuator lost 80 percent of its effectiveness at $t = 4$ and stuck at $t = 10$ and the second actuator lost 60 percentage of its effectiveness at $t = 8$. In addition, due to the saturation problem the first input cannot become more than $u_{M_1} = 5$ and the second input cannot become more than $u_{M_2} = 2$. The oscillation of the control input is due to the saturation problem in the inputs. If the bounds of the saturation u_{M_1} and u_{M_2} increase, these oscillations will decrease. Figures 4-6 show the states $x_i(t)$, $i = 1, 2, 3$ that are completely synchronized with $y_i(t)$, $i = 1,2,3$. In addition, the time-history of the synchronization errors $e_1(t)$, $e_2(t)$, $e_3(t)$ are shown in Figure 7. Indeed, fig. 7 expresses the synchronization errors that converge to a small neighbourhood of the origin. The simulation results represent the efficiency of the proposed robust adaptive backstepping controller.

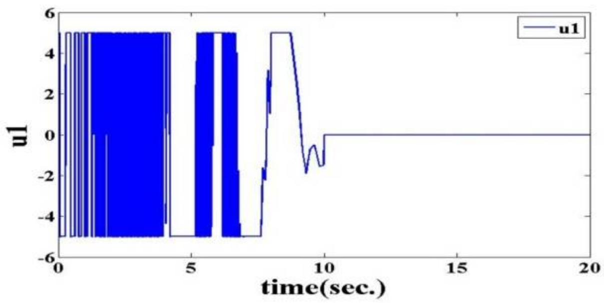

Fig. (2): Control input u_1

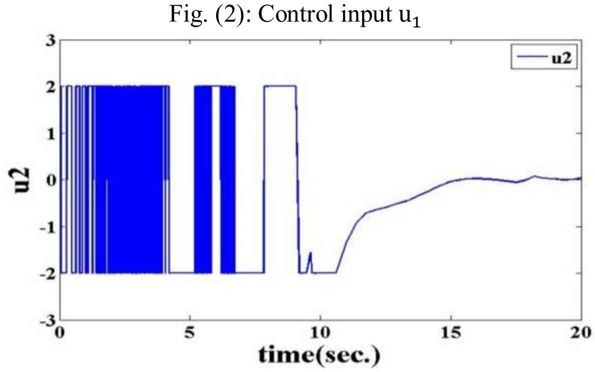

Fig. (3): Control input u_2

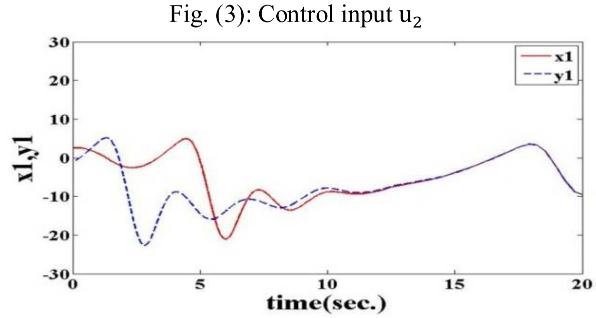

Fig. (4): The states $x_1(t)$ and $y_1(t)$

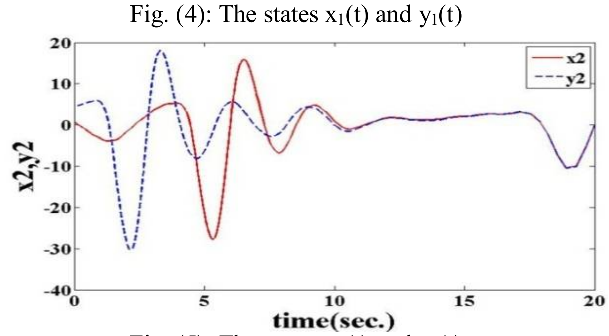

Fig. (5): The states $x_2(t)$ and $y_2(t)$

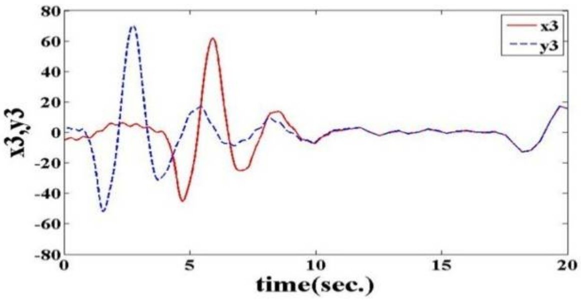

Fig. (6): The states $x_3(t)$ and $y_3(t)$

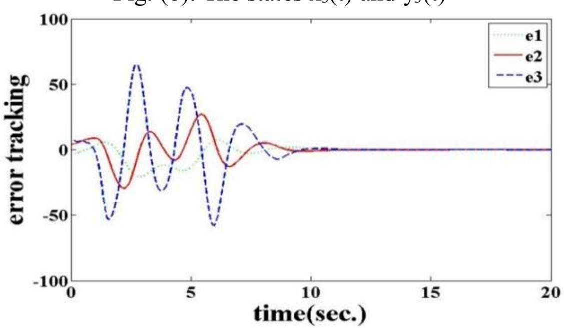

Fig. (7): Time-history of the synchronization errors $e_1(t)$, $e_2(t)$, $e_3(t)$

5. Conclusion

In this paper, a robust adaptive synchronization controller is proposed for the novel Jerk chaotic systems in the presence of unknown parameter, input saturation and actuator faults. The considered actuator faults are modelled to cover both loss of effectiveness and stuck at some unknown values. The offered controller compensates the actuator faults without any need for explicit fault detection. In this paper, the proposed adaptive control law compensates the effect of the unknown parameters, unknown actuator faults and saturation nonlinearities by using backstepping technique. Simulation results illustrate the effectiveness of the proposed scheme. The proposed adaptive design method proves that without the need for the explicit fault detection, all the closed loop signals remain bounded and the tracking error converges to a small neighborhood of the origin.

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