Task-space Control of Robots Using an Adaptive Taylor Series Uncertainty Estimator Revisited

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Abstract
This paper presents an improved version of the article "task-space control of robots uses an adaptive Taylor series uncertainty estimator" by designing a more general framework for dealing with actuator saturation. There are four important issues about the aforementioned article. Firstly, the saturated and unsaturated regions have been discussed separately in that article, while this paper presents a unified approach for stability analysis. Secondly, the linear parameterization of unknown multi-variable vector-valued nonlinearities represented in the aforementioned article is not true. Consequently, it will affect the stability analysis significantly and the obtained results are doubtful. Thirdly, although the tracking error is bounded in the saturated area, it may be unacceptable due to undesirable performance. Thus, performance evaluation is needed to verify the satisfactory operation of the control system. However, in the aforementioned article, performance evaluation has not been presented. Fourthly, the aforementioned paper applies the Taylor series as a universal approximator without verifying the conditions of the universal approximation theorem. This paper proves that the Taylor series can satisfy the conditions of this theorem. All these four important issues are addressed in this paper and a modified version of the aforementioned article is presented.

Keywords
Adaptive Uncertainty Estimation, Stability Analysis, Actuator Saturation, Electrically Driven Robots

1.Introduction

As pointed out in the paper [1], function approximation methods such as neuro-fuzzy systems [2-4], Taylor series [5], differential equations [6, 7], Fourier series expansion and Legendre polynomials [6-14] have been utilized in robust adaptive control of many nonlinear systems. Among these uncertainty estimators, Taylor series expansion has the simplest structure [5] due to fewer tuning parameters. In [1], a third-order Taylor series expansion has been considered as an uncertainty estimator. The Taylor series coefficients are tuned based on the adaptation law obtained in the stability analysis.

Although designing a simple and powerful uncertainty estimator is of great importance, stability analysis and performance evaluation in the presence of input constraints (actuator saturation) are more crucial and challenging in control engineering. The considerable point is that the proposed approach in [1] does not give suitable stability analysis for the overall control system. It uses the boundedness of the saturated signal to prove the stability and boundedness of the closed-loop internal signals. It is worth emphasizing that in the saturated area of the control input, the controller operation does not influence the plant since the actuators (electrical motors in robotic systems) are driving the system by their maximum value. In this condition, although the tracking error is bounded [15], it may be unacceptable due to unsatisfactory performance. Nevertheless, the stability analysis presented in [1], does not address the saturated area properly. Another important issue is that in [1], stability is analyzed separately in saturated and unsaturated operation areas. However, the stability of the closed-loop system may not be guaranteed through these separate analyses, since transitions from saturation area to unsaturated area and vice versa are neglected. Furthermore, it must be noted that the linear parameterization of unknown multi-variable vector-valued nonlinearities represented in [1] is not true. The reason is that the respectable authors have considered them as single-variable functions.

The objective of this paper is to modify the previous results on the controller design and robust stability analysis of the work proposed by [1]. The overall closed-loop system composed by the full-actuated robotic manipulator for *n* degrees of freedom and the proposed controller is proved to be Uniformly Ultimately Bounded (UUB) stable, while the remained signals are bounded. Moreover, performance evaluation has been presented to verify that the norm of the error vector defined for the difference of actual and estimated Taylor series coefficients converges to small values.

This paper is organized as follows. Section 2 briefly presents dynamic modeling of the robotic system including the robot manipulator and the permanent magnet DC motors subjected to actuator saturation. Section 3 explains the function approximation technique using the Taylor series expansion. Section 4 presents the controller design scenario. The stability analysis and performance evaluation are presented in section 5. Finally, concluding remarks are drawn in section 6.

2. Dynamic Modeling

Consider an *n*-link manipulator driven by geared permanent magnet DC motors with voltages being inputs to amplifiers. As in ([1]), the dynamics are described by

$$\mathbf{D}(\mathbf{q})\ddot{\mathbf{q}}+\mathbf{C}(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}+\mathbf{g}(\mathbf{q})=\boldsymbol{\tau}_r-\boldsymbol{\tau}_f(\dot{\mathbf{q}}) \quad (1)$$

$$\mathbf{J}_m\mathbf{r}^{-1}\ddot{\mathbf{q}}+\mathbf{B}_m\mathbf{r}^{-1}\dot{\mathbf{q}}+\mathbf{r}\boldsymbol{\tau}_r=\mathbf{K}_m\mathbf{I}_a \quad (2)$$

$$\mathbf{R}\mathbf{I}_a+\mathbf{L}\dot{\mathbf{I}}_a+\mathbf{K}_b\mathbf{r}^{-1}\dot{\mathbf{q}}+\varphi=\mathbf{v}(t) \quad (3)$$

Where the parameters are defined exactly similar to [1]. Note that vectors and matrices are represented in bold form for clarity. Now, the substitution of (2) into (3) yields the following overall dynamic of electrically driven robot

$$\mathbf{R}\mathbf{K}_m^{-1}\mathbf{J}_m\mathbf{r}^{-1}\ddot{\mathbf{q}}+(\mathbf{R}\mathbf{K}_m^{-1}\mathbf{B}_m+\mathbf{K}_b)\mathbf{r}^{-1}\dot{\mathbf{q}}+\mathbf{R}\mathbf{K}_m^{-1}\mathbf{r}\boldsymbol{\tau}_r+\mathbf{L}\dot{\mathbf{I}}_a+\varphi=\mathbf{v}(t) \quad (4)$$

For practical situations, the actuator input voltages are subjected to some constraints, called motor saturation limits. This occurs usually between the output of the controller and the PWM module [16, 17]. Following the same notation as in [1], for the development of the controller in this paper, we assume that the relation between the actual actuator's input $(\mathbf{v}(t)\in\Re^n)$ and the control signal produced by the controller $(\mathbf{u}(t)\in\Re^n)$ is given by

$$\mathbf{v}(t)=\mathbf{h}(\mathbf{u}(t)) \quad (5)$$

Where $\mathbf{h}(\mathbf{u}(t))\in\Re^n$ is a continuous nonlinear function representing the saturation nonlinearity or its approximation. As shown in [17], the non-implemented control signal of the actuators can be expressed as

$$\Im(\mathbf{u}(t)) = \mathbf{u}(t) - \mathbf{h}(\mathbf{u}(t)) \tag{6}$$

Now, substituting (5) into (4), and using (6) we have

$$\mathbf{RK}_m^{-1}\mathbf{J}_m\mathbf{r}^{-1}\ddot{\mathbf{q}} + (\mathbf{RK}_m^{-1}\mathbf{B}_m + \mathbf{K}_b)\mathbf{r}^{-1}\dot{\mathbf{q}} + \mathbf{RK}_m^{-1}\mathbf{r}\boldsymbol{\tau}_r + \mathbf{L}\dot{\mathbf{I}}_a + \varphi = \mathbf{u}(t) - \Im(\mathbf{u}(t)) \tag{7}$$

Remark 1: The control input given by (5) indicates that the motor voltage is limited, that is

$$|v(t)| \le u_{\max} \tag{8}$$

Where $v(t)$ stands for the i[th] entry of vector $\mathbf{v}(t)$ and $u_{\max}$ is a positive constant representing the maximum permitted voltage of the i[th] motor. As a result, $\mathbf{q} \in \Re^n$, $\dot{\mathbf{q}} \in \Re^n$, and $\mathbf{I}_a \in \Re^n$ are bounded. This is a result of BIBO stability ([6]).

2.1 Kinematic Analysis

Concerning n-joint coordinates $\mathbf{q}$, and m-task coordinates $\mathbf{x}$, the kinematics of the manipulator can be described with the following equations ([7]):

$$\mathbf{x} = \phi(\mathbf{q}) \tag{9}$$

$$\dot{\mathbf{x}} = \mathbf{J}(\mathbf{q})\dot{\mathbf{q}} \tag{10}$$

$$\ddot{\mathbf{x}} = \mathbf{J}(\mathbf{q})\ddot{\mathbf{q}} + \dot{\mathbf{J}}(\mathbf{q})\dot{\mathbf{q}} \tag{11}$$

Where ϕ is an m-dimensional vector function representing direct kinematics, $\mathbf{J}(\mathbf{q}) \in \Re^{m\times n}$ is the Jacobian matrix defined as $\partial\phi/\partial\mathbf{q}$, and the upper dot denotes its time derivative. With this in mind, the equation for robot system motion in the joint space, (7), can then be represented as Cartesian space coordinates based on the following relationship:

$$\dot{\mathbf{q}} = \mathbf{J}_s(\mathbf{q})\dot{\mathbf{x}} \tag{12}$$

$$\ddot{\mathbf{q}} = \mathbf{J}_s(\mathbf{q})\left(\ddot{\mathbf{x}} - \dot{\mathbf{J}}(\mathbf{q})\dot{\mathbf{q}}\right) \tag{13}$$

Where $\mathbf{J}_s(\mathbf{q}) \in \Re^{n\times m}$ represents the generalized inverse of the Jacobian matrix and it is defined as

$$\mathbf{J}_s(\mathbf{q}) = \mathbf{J}^T(\mathbf{q})(\mathbf{J}(\mathbf{q})\mathbf{J}^T(\mathbf{q}))^{-1} \tag{14}$$

Now, substituting (12) and (13) into (7) yields

$$\mathbf{M}(\mathbf{x})\ddot{\mathbf{x}} + \mathbf{N}(\mathbf{x},\dot{\mathbf{x}})\dot{\mathbf{x}} + \mathbf{G}(\mathbf{x}) = \mathbf{w}(t) \tag{15}$$

Where

$$\begin{aligned}
\mathbf{M}(\mathbf{x}) &= \mathbf{J}_s^T(\mathbf{q})\mathbf{RK}_m^{-1}\mathbf{J}_m\mathbf{r}^{-1}\mathbf{J}_s(\mathbf{q}) \\
\mathbf{N}(\mathbf{x},\dot{\mathbf{x}}) &= \mathbf{J}_s^T(\mathbf{q})\left((\mathbf{RK}_m^{-1}\mathbf{B}_m + \mathbf{K}_b)\mathbf{r}^{-1} - \mathbf{RK}_m^{-1}\mathbf{J}_m\mathbf{r}^{-1}\mathbf{J}_s(\mathbf{q})\dot{\mathbf{J}}(\mathbf{q})\right)\mathbf{J}_s(\mathbf{q}) \\
\mathbf{G}(\mathbf{x}) &= \mathbf{J}_s^T(\mathbf{q})\left(\mathbf{RK}_m^{-1}\mathbf{r}\boldsymbol{\tau}_r + \mathbf{L}\dot{\mathbf{I}}_a + \varphi + \mathbf{dzn}(\mathbf{u}(t), u_{\max})\right)
\end{aligned} \tag{16}$$

And $\mathbf{w}(t) \in \Re^m$ represents the new control input in the task space. To develop our control scheme, assume that Equation (15) can be represented by a second-order nonlinear differential equation, called "available model" as

$$\ddot{\mathbf{x}} + \mathbf{F} = \mathbf{w}(t) \tag{17}$$

Where $\mathbf{F} = ((\mathbf{M}(\mathbf{x}) - \mathbf{I})\ddot{\mathbf{x}} + \mathbf{N}(\mathbf{x},\dot{\mathbf{x}})\dot{\mathbf{x}} + \mathbf{G}(\mathbf{x})) \in \Re^m$ is referred to as the lumped uncertainty, also

$\mathbf{I} \in \Re^{m\times m}$ and $\mathbf{0} \in \Re^{m\times m}$ are the identity and zero matrices, respectively.

Remark 2: The last representation (Eq. (17)) does not simplify the control problem. It can be interpreted as a standard computed torque-controlled system, when there is no knowledge about the controlled robotic manipulator i.e. $\hat{\mathbf{M}}(\mathbf{x}) = \mathbf{I}$ and $\hat{\mathbf{N}}(\mathbf{x}, \dot{\mathbf{x}}) = \hat{\mathbf{G}}(\mathbf{x}) = \mathbf{0}$. This is the most conservative choice with $(\hat{\cdot})$ denoting an estimated value of $(\cdot)$ ([8]).

Remark 3: Some previous valuable published works have exploited the universal approximation property of Neural Network (NN) to actuator nonlinearities compensation, ([18-20]) although the problems originated by NN and Fuzzy approaches still exist, as mentioned in ([21, 22]).

3. Function Approximation Using Taylor Series

Uncertainty estimators are not confined to fuzzy systems and neural networks. In the calculus courses, it is well known that, given a function *f*(x) and a point *a* in the domain of *f*, suppose the function is *n*-times differentiable at *a*, then we can construct a polynomial

$$f_l(x) = \sum_{p=0}^{l} \frac{f^{(p)}(a)}{p!}(x-a)^p \qquad (18)$$

Where $f_l(x)$ is called the *l*th-degree Taylor polynomial approximation of *f* at *a*. It is interesting to investigate the capability of the last Equation, Equation (18), from a function approximation capability point of view. Herein, we will prove that Equation (18) has the universal approximation capability. In the following, we suppose that the input universe of discourse *T* is a convex set in $\Re$. First, we need the following useful theorem.

3.1 Stone-Weierstrass Theorem [23]

Let o be a set of real continuous functions on a convex set *T*. If

1. The set o is algebra, that is the set o is closed under multiplication, addition, and scalar multiplication;

2. The set o separates points of *T*, i.e.

$$\forall x_1, x_2 \in T, \; x_1 \neq x_2, \; \exists \; f_l(x) \in o : \; f_l(x_1) \neq f_l(x_2) \qquad (19)$$

3. The set o vanishes at no point of *T*, that is,

$$\forall x \in T, \exists f_l(x) \in o : \; f_l(x) \neq 0 \qquad (20)$$

Then for any real continuous function $f(x)$ on *T* and arbitrary $\varepsilon > 0$, there exists a function $f_l(x)$ in o such that

$$\sup_{x\in T} \left| f_l(x) - f(x) \right| < \varepsilon \qquad (21)$$

Proposition1. (Universal Approximation Theorem)

Let $f(x)$ be a continuous real function on the convex set *T* in $\Re$. Then for each arbitrary $\varepsilon > 0$, there exists a function in the form of

$$f_l(x)=\sum_{p=0}^{l}\frac{f^{(p)}(a)}{p!}(x-a)^p \tag{22}$$

Such that

$$\sup_{x\in T}\left|\sum_{p=0}^{l}\frac{f^{(p)}(a)}{p!}(x-a)^p - f(x)\right|<\varepsilon \tag{23}$$

Proof of proposition1: Let o to be a set of continuous functions on T in which T is a Convex set in the form of (18). Now, suppose $f_{l,1}(x)$ and $f_{l,2}(x)$ are given as

$$\begin{aligned} f_{l,1}(x)&=\sum_{i=0}^{l}\frac{f_1^{(i)}(a)}{i!}(x-a)^i \\ f_{l,2}(x)&=\sum_{j=0}^{l}\frac{f_2^{(j)}(\bar{a})}{j!}(x-\bar{a})^j \end{aligned} \tag{24}$$

We have

$$f_{l,1}(x)+f_{l,2}(x)=\sum_{i=0}^{l}\frac{f_1^{(i)}(a)}{i!}(x-a)^i+\sum_{j=0}^{l}\frac{f_2^{(j)}(\bar{a})}{j!}(x-\bar{a})^j \tag{25}$$

$$f_{l,1}(x).f_{l,2}(x)=\left(\sum_{i=0}^{l}\frac{f_1^{(i)}(a)}{i!}(x-a)^i\right).\left(\sum_{j=0}^{l}\frac{f_2^{(j)}(\bar{a})}{j!}(x-\bar{a})^j\right) \tag{26}$$

Hence, $f_{l,1}(x)+f_{l,2}(x)\in o$ and $f_{l,1}(x).f_{l,2}(x)\in o$. Furthermore, for any arbitrary $\kappa\in\Re$, we can get

$$\kappa.f_l(x)=\sum_{i=0}^{l}\kappa\frac{f^{(i)}(a)}{i!}(x-a)^i \tag{27}$$

Which is also in the form of (18). So, according to (25) to (27), we can conclude that o is an algebra. Therefore, the first condition of the Stone-Weierstrass theorem is satisfied for o. Now, we show that o separates various points on T. Choose the parameters of $f_l(x)$ in (18) as:

$$l=1,\ a=0 \tag{28}$$

Since $x_1\neq x_2$, then $f(0)+f^{(1)}(0)x_1\neq f(0)+f^{(1)}(0)x_2$, which can be simplified to $x_1\neq x_2$. Therefore, the second condition is also verified. To show that o vanishes at no point of T, we simply observe that any function in the form of (18) with $f(a)\neq 0$ and $f^{(p)}(a)=0$ for $p=1,\dots,l$ has the property of

$$\forall x\in T\ ,\ f_l(x)>0 \tag{29}$$

Hence, o vanishes at no point of T. Thus, the three conditions of the Stone-Weierstrass theorem are satisfied. Therefore, the result follows by the Stone-Weierstrass theorem.■

4. Adaptive Uncertainty Estimator

In this section, actuator saturation compensation is considered to achieve satisfactory tracking control of robots as an extended form of ([1]). For this purpose, the robust control law is proposed as

$$\mathbf{w}(t) = \ddot{\mathbf{x}}_d + \mathbf{k}_d(\dot{\mathbf{x}}_d - \dot{\mathbf{x}}) + \mathbf{k}_p(\mathbf{x}_d - \mathbf{x}) + \hat{\mathbf{F}} \quad (30)$$

where $\mathbf{x}_d \in \Re^m$, $\dot{\mathbf{x}}_d \in \Re^m$, and $\ddot{\mathbf{x}}_d \in \Re^m$ are desired position, velocity and acceleration in the task space, respectively; $\mathbf{k}_p \in \Re^{m\times m}$ and $\mathbf{k}_d \in \Re^{m\times m}$ are the feedback gain matrices usually selected as diagonal, and $\hat{\mathbf{F}} \in \Re^m$ is the estimated value of $\mathbf{F}$. Substituting (30) into (17) and some simple manipulation lead to

$$\ddot{\tilde{\mathbf{x}}} + \mathbf{k}_d\dot{\tilde{\mathbf{x}}} + \mathbf{k}_p\tilde{\mathbf{x}} = \mathbf{F} - \hat{\mathbf{F}} \quad (31)$$

Where $\tilde{\mathbf{x}} \in \Re^m$ is the tracking error defined by

$$\tilde{\mathbf{x}} = \mathbf{x}_d - \mathbf{x} \quad (32)$$

It must be emphasized that the development of the proposed control law is under the assumption that complete information of the actuator and robot dynamic is not available (i.e., we have not any knowledge of plant parameters or datasheet which are provided usually by the manufacturer). Such an assumption has been previously utilized in ([24, 25]). With this in mind, a first-order Taylor series expansion, neglecting the higher-order terms, will be used to represent the uncertainty estimator $\hat{\mathbf{F}}$ as ([1])

$$\hat{\mathbf{F}} = \hat{\mathbf{F}}_0 + \left.\frac{\partial\hat{\mathbf{F}}}{\partial\tilde{\mathbf{x}}}\right|_{(0,0)} \tilde{\mathbf{x}} + \left.\frac{\partial\hat{\mathbf{F}}}{\partial\dot{\tilde{\mathbf{x}}}}\right|_{(0,0)} \dot{\tilde{\mathbf{x}}} \quad (33)$$

To estimate the matrix of coefficients, (33) is represented as

$$\hat{\mathbf{F}} = \hat{\mathbf{\Lambda}}^T \xi \quad (34)$$

Where $\hat{\mathbf{\Lambda}}$ and ξ are expressed as

$$\hat{\mathbf{\Lambda}}^T = \begin{bmatrix} \hat{\mathbf{\Lambda}}_1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & \hat{\mathbf{\Lambda}}_2 & 0 & \cdots & 0 & 0 \\ \vdots & 0 & \hat{\mathbf{\Lambda}}_3 & \vdots & \vdots & \vdots \\ \vdots & \vdots & \cdots & \cdots & \hat{\mathbf{\Lambda}}_{m-1} & 0 \\ 0 & 0 & \cdots & \cdots & 0 & \hat{\mathbf{\Lambda}}_m \end{bmatrix} \in \Re^{m\times(2m^2+m)} \quad (35)$$

$$\xi = [\underbrace{\mathbf{y}^T \quad \cdots \quad \mathbf{y}^T}_{m}]^T \in \Re^{(2m^2+m)} \quad (36)$$

and $\mathbf{y} = \begin{bmatrix} 1 & \tilde{\mathbf{x}}^T & \dot{\tilde{\mathbf{x}}}^T \end{bmatrix}^T \in \Re^{2m+1}$.

Remark 4: The 2nd order term can no longer be expressed in matrix form, as it requires tensor notation. This is the main weakness of Taylor series expansion for multi-variable vector-valued functions.

Suppose that $\mathbf{F}$ can be modeled as

$$\mathbf{F} = \mathbf{\Lambda}^T \xi + \boldsymbol{\varepsilon} \quad (37)$$

Where $\boldsymbol{\varepsilon} \in \Re^m$ is the approximation error and matrix $\mathbf{\Lambda} \in \Re^{(2m^2+m)\times m}$ is a block diagonal constant matrix. The dynamics of tracking error can then be expressed by substituting (34) and (37) in (31) to have

$$\ddot{\tilde{\mathbf{x}}}+\mathbf{k}_d\dot{\tilde{\mathbf{x}}}+\mathbf{k}_p\tilde{\mathbf{x}}=\tilde{\mathbf{\Lambda}}^T\boldsymbol{\xi}+\boldsymbol{\varepsilon} \quad (38)$$

Where $\tilde{\mathbf{\Lambda}}=\mathbf{\Lambda}-\hat{\mathbf{\Lambda}}\in\Re^{(2m^2+m)\times m}$ is the parametric estimation error. Define $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{E}$ as

$$\mathbf{A}=\begin{bmatrix}\mathbf{0} & \mathbf{I}\\ -\mathbf{k}_p & -\mathbf{k}_d\end{bmatrix}\in\Re^{2m\times 2m}\quad,\ \mathbf{B}=\begin{bmatrix}\mathbf{0}\\ \mathbf{I}\end{bmatrix}\in\Re^{2m\times m}\quad,\ \mathbf{E}=\begin{bmatrix}\tilde{\mathbf{x}}\\ \dot{\tilde{\mathbf{x}}}\end{bmatrix}\in\Re^{2m} \quad (39)$$

Hence, equation (38) can be written in the following state equation form.

$$\dot{\mathbf{E}}=\mathbf{AE}+\mathbf{B}\tilde{\mathbf{\Lambda}}^T\boldsymbol{\xi}+\mathbf{B}\boldsymbol{\varepsilon} \quad (40)$$

5. Stability Analysis and Performance Evaluation

To proceed with subsequent stability analysis, the following assumption is required.

Assumption 1: The desired task-space trajectories and their time derivatives are in L_∞ space, i.e. $(\mathbf{x}_d, \dot{\mathbf{x}}_d \in L_\infty)$.

5.1 Stability Analysis

To study the stability and analyze the performance of the closed-loop system, choose the following positive definite function:

$$\mathrm{V}(\mathbf{E},\tilde{\mathbf{\Lambda}})=\mathbf{E}^T\mathbf{SE}+Tr(\tilde{\mathbf{\Lambda}}^T\mathbf{\Gamma}^{-1}\tilde{\mathbf{\Lambda}}) \quad (41)$$

Where $\mathbf{S}\in\Re^{2m\times 2m}$ is the solution of Lyapunov Equation $\mathbf{A}^T\mathbf{S}+\mathbf{SA}+\mathbf{Q}=0$, $\mathbf{Q}$ is a positive definite matrix, and $\mathbf{\Gamma}\in\Re^{(2m^2+m)\times(2m^2+m)}$ is a positive definite weighting matrix related to the adaption laws. The last function has the following upper and lower bounds which are crucial within the analytical setting in this work:

$$\mathbf{V}(\mathbf{E},\tilde{\mathbf{\Lambda}})\le\lambda_{\max}(\mathbf{S})\|\mathbf{E}\|^2+\lambda_{\max}(\mathbf{\Gamma}^{-1})Tr(\tilde{\mathbf{\Lambda}}^T\tilde{\mathbf{\Lambda}}) \quad (42)$$

$$\mathbf{V}(\mathbf{E},\tilde{\mathbf{\Lambda}})\ge\lambda_{\min}(\mathbf{S})\|\mathbf{E}\|^2+\lambda_{\min}(\mathbf{\Gamma}^{-1})Tr(\tilde{\mathbf{\Lambda}}^T\tilde{\mathbf{\Lambda}}) \quad (43)$$

Where $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the smallest and the largest eigenvalues of $(\cdot)$, respectively. The time derivative of (41) along the trajectory of system (40) yields

$$\dot{\mathrm{V}}(\mathbf{E},\tilde{\mathbf{\Lambda}})=-\mathbf{E}^T\mathbf{QE}+2\mathbf{E}^T\mathbf{SB}\tilde{\mathbf{\Lambda}}^T\boldsymbol{\xi}+2\mathbf{E}^T\mathbf{SB}\boldsymbol{\varepsilon}-2Tr(\tilde{\mathbf{\Lambda}}^T\mathbf{\Gamma}^{-1}\dot{\hat{\mathbf{\Lambda}}}) \quad (44)$$

Now, select the updated law using σ-modification as

$$\dot{\hat{\mathbf{\Lambda}}}=\mathbf{\Gamma}(\boldsymbol{\xi}\mathbf{E}^T\mathbf{SB}-\sigma\hat{\mathbf{\Lambda}}) \quad (45)$$

Where σ is a positive scalar. Consequently, substituting Equation (45) into (44) and using some mathematical calculation result in

$$\dot{\mathrm{V}}(\mathbf{E},\tilde{\mathbf{\Lambda}})=-\mathbf{E}^T\mathbf{QE}+2\mathbf{E}^T\mathbf{SB}\boldsymbol{\varepsilon}+2\sigma Tr(\tilde{\mathbf{\Lambda}}^T\hat{\mathbf{\Lambda}}) \quad (46)$$

To obtain definiteness of (46), one can simply prove that the following inequalities are hold

$$-\mathbf{E}^T\mathbf{QE}+2\mathbf{E}^T\mathbf{SB}\boldsymbol{\varepsilon}\le-\frac{1}{2}\lambda_{\min}(\mathbf{Q})\|\mathbf{E}\|^2+\frac{2\sigma_{\max}^2(\mathbf{SB})}{\lambda_{\min}(\mathbf{Q})}\|\boldsymbol{\varepsilon}\|^2 \quad (47)$$

$$2Tr(\tilde{\mathbf{\Lambda}}^T\hat{\mathbf{\Lambda}}) \le Tr(\mathbf{\Lambda}^T\mathbf{\Lambda}) - Tr(\tilde{\mathbf{\Lambda}}^T\tilde{\mathbf{\Lambda}}) \quad (48)$$

Where $\sigma_{\max}(\cdot)$ is the maximum singular value of $(\cdot)$. Together with these relationships, (46) may be rewritten as:

$$\dot{V}(\mathbf{E},\tilde{\mathbf{\Lambda}}) \le -\frac{1}{2}\lambda_{\min}(\mathbf{Q})\|\mathbf{E}\|^2 - \sigma Tr(\tilde{\mathbf{\Lambda}}^T\tilde{\mathbf{\Lambda}}) + \frac{2\sigma_{\max}^2(\mathbf{SB})}{\lambda_{\min}(\mathbf{Q})}\|\boldsymbol{\varepsilon}\|^2 + \sigma Tr(\mathbf{\Lambda}^T\mathbf{\Lambda}) \quad (49)$$

One can easily relate (49) to V by considering (42). Then, (49) can be further rewritten as

$$\begin{aligned}\dot{V}(\mathbf{E},\tilde{\mathbf{\Lambda}}) \le & -\delta V + (\delta\lambda_{\max}(\mathbf{S}) - \frac{1}{2}\lambda_{\min}(\mathbf{Q}))\|\mathbf{E}\|^2 \\ & + (\delta\lambda_{\max}(\mathbf{\Gamma}^{-1}) - \sigma)Tr(\tilde{\mathbf{\Lambda}}^T\tilde{\mathbf{\Lambda}}) + \frac{2\sigma_{\max}^2(\mathbf{SB})}{\lambda_{\min}(\mathbf{Q})}\|\boldsymbol{\varepsilon}\|^2 + \sigma Tr(\mathbf{\Lambda}^T\mathbf{\Lambda})\end{aligned} \quad (50)$$

Where δ is a constant that can be selected as

$$\delta \le \min\left\{\frac{\lambda_{\min}(\mathbf{Q})}{2\lambda_{\max}(\mathbf{S})}, \frac{\sigma}{\lambda_{\max}(\mathbf{\Gamma}^{-1})}\right\} \quad (51)$$

Then, (50) simplifies to

$$\dot{V}(\mathbf{E},\tilde{\mathbf{\Lambda}}) \le -\delta V + \frac{2\sigma_{\max}^2(\mathbf{SB})}{\lambda_{\min}(\mathbf{Q})}\|\boldsymbol{\varepsilon}\|^2 + \sigma Tr(\mathbf{\Lambda}^T\mathbf{\Lambda}) \quad (52)$$

This implies $\dot{V}(\mathbf{E},\tilde{\mathbf{\Lambda}}) < 0$, which is satisfied whenever

$$(\mathbf{E},\tilde{\mathbf{\Lambda}}) \in \Omega \equiv \left\{(\mathbf{E},\tilde{\mathbf{\Lambda}}) \middle| \; V > \frac{2\sigma_{\max}^2(\mathbf{SB})}{\delta\lambda_{\min}(\mathbf{Q})}\sup_{t_0 \le \tau}\|\boldsymbol{\varepsilon}(\tau)\|^2 + \frac{\sigma}{\delta}Tr(\mathbf{\Lambda}^T\mathbf{\Lambda})\right\} \quad (53)$$

Hence, we have proved that $(\mathbf{E},\tilde{\mathbf{\Lambda}})$ are uniformly ultimately bounded. Using the Assumptions (1) and boundedness of $\mathbf{E}$, it can be concluded from the stability of the closed-loop system that the task-space velocity vector $\dot{\mathbf{x}}$ is bounded. From (12), it follows that $\dot{\mathbf{q}} = \int_0^t \mathbf{J}_s(\mathbf{q})\dot{\mathbf{x}}dt + \mathbf{q}(0)$. Therefore, for finite operational times, the joint position $\mathbf{q}$ is bounded. These results together and also remark 1 prove the stability of the closed-loop system. Note that the size of the set Ω is adjustable by proper selections of the parameters of δ, σ, $\mathbf{S}$, and $\mathbf{Q}$.

5.2 Performance Evaluation

The above derivation only demonstrates the boundedness of the closed-loop system, but in practical applications, the transient performance is also of great importance. For further development, the upper bound for $V(t)$ can be computed by solving the differential inequality of $V(\mathbf{E},\tilde{\mathbf{\Lambda}})$ in (52) as

$$V(t) \le e^{-\delta(t-t_0)}V(t_0) + \frac{2\sigma_{\max}^2(\mathbf{SB})}{\delta\lambda_{\min}(\mathbf{Q})}\sup_{t_0<\tau<t}\|\boldsymbol{\varepsilon}(\tau)\|^2 + \frac{\sigma}{\delta}Tr(\mathbf{\Lambda}^T\mathbf{\Lambda}) \quad (54)$$

Using the inequality (43), the upper bound for $\|\mathbf{E}\|^2$ can be calculated as

$$\|\mathbf{E}\|^2 \le \frac{\mathbf{V}(\mathbf{E},\tilde{\mathbf{\Lambda}})}{\lambda_{\min}(\mathbf{S})} \quad (55)$$

With (54), this can be further written as

$$\|\mathbf{E}\| \le \sqrt{\frac{V(t_0)}{\lambda_{\min}(\mathbf{S})}e^{\frac{-\delta(t-t_0)}{2}}} + \sqrt{\frac{2\sigma_{\max}^2(\mathbf{SB})}{\delta\lambda_{\min}(\mathbf{S})\lambda_{\min}(\mathbf{Q})}} \sup_{t_0<\tau<t}\|\boldsymbol{\varepsilon}(\tau)\| + \sqrt{\frac{\sigma Tr(\boldsymbol{\Lambda}^T\boldsymbol{\Lambda})}{\delta\lambda_{\min}(\mathbf{S})}} \tag{56}$$

This implies that the magnitude of $\|\mathbf{E}\|$ is bounded by an exponential function plus some constants. This also implies that by adjusting controller parameters, the output error convergence rate can be improved. As a consequence,

$$\lim_{t\to\infty}\|\mathbf{E}\| \le \sqrt{\frac{2\sigma_{\max}^2(\mathbf{SB})}{\delta\lambda_{\min}(\mathbf{S})\lambda_{\min}(\mathbf{Q})}} \sup_{t_0<\tau<t}\|\boldsymbol{\varepsilon}(\tau)\| + \sqrt{\frac{\sigma Tr(\boldsymbol{\Lambda}^T\boldsymbol{\Lambda})}{\delta\lambda_{\min}(\mathbf{S})}} \tag{57}$$

Considering the Frobenius norm definition, ($\|\tilde{\boldsymbol{\Lambda}}\|_F^2 = Tr(\tilde{\boldsymbol{\Lambda}}^T\tilde{\boldsymbol{\Lambda}})$), one can also obtain the following bound for the weighting vector $\tilde{\boldsymbol{\Lambda}}$.

$$\lim_{t\to\infty}\|\tilde{\boldsymbol{\Lambda}}\|_F \le \sqrt{\frac{2\sigma_{\max}^2(\mathbf{SB})}{\delta\lambda_{\min}(\boldsymbol{\Gamma}^{-1})\lambda_{\min}(\mathbf{Q})}} \sup_{t_0<\tau<t}\|\boldsymbol{\varepsilon}(\tau)\| + \sqrt{\frac{\sigma Tr(\boldsymbol{\Lambda}^T\boldsymbol{\Lambda})}{\delta\lambda_{\min}(\boldsymbol{\Gamma}^{-1})}} \tag{58}$$

6. Conclusion

This paper improves stability results of the robust adaptive controller proposed by "task-space control of robots using an adaptive Taylor series uncertainty estimator" considering actuator voltage input constraint. A general stability analysis has been presented that considers the saturated and unsaturated regions of the control input simultaneously. It is shown that the joint position and velocity tracking errors is UUB stable in agreements with Lyapunov direct method in any finite region in the state space, while the other signals in the system remain bounded.

7. References

[1] Ahmadi, S. M. and Fateh, M. M. 2018. Task-space control of robots using an adaptive Taylor series uncertainty estimator. International Journal of Control. 1: 1-29.

[2] Li, Y., Tong, S. and Li, T. 2013. Adaptive fuzzy output feedback control for a single-link flexible robot manipulator driven DC motor via back-stepping. Nonlinear Analysis: Real World Applications. 14: 483-494.

[3] Chaoui, H., Gueaieb, W., Biglarbegian, M. and Yagoub, M. C. 2013. Computationally efficient adaptive type-2 fuzzy control of flexible-joint manipulators. Robotics. 2: 66-91.

[4] Zhou, Q., Li, H., Wu, C., Wang, L. and Ahn, C. K. 2017. Adaptive fuzzy control of nonlinear systems with unmodeled dynamics and input saturation using small-gain approach. IEEE Transactions on Systems, Man, and Cybernetics: Systems. 47: 1979-1989

[5] Ahmadi, S. M. and Fateh, M. M. 2016. Robust control of electrically driven robots using adaptive uncertainty estimation. Computers and Electrical Engineering.

[6] Izadbakhsh, A. 2017. FAT-based robust adaptive control of electrically driven robots without velocity measurements. Nonlinear Dynamics. 89: 289-304.

[7] Izadbakhsh, A. 2017. A note on the nonlinear control of electrical flexible-joint robots. Nonlinear Dynamics. 89: 2753-2767.

[8] Izadbakhsh, A. 2017. Robust adaptive control of voltage saturated flexible joint robots with experimental evaluations. AUT Journal of Modeling, and simulation. 50(1): 31-38.

[9] Chien, M. C. and Huang, A. C. 2004. Adaptive impedance control of robot manipulators based on function approximation technique. Robotica. 22: 395-403.
[10] Chien, M. C. and Huang, A. C. 2012. Adaptive impedance controller design for flexible-joint electrically-driven robots without computation of the regressor matrix. Robotica. 30: 133-144.
[11] Kai, C. Y. and Huang, A. C. 2013. A regressor-free adaptive controller for robot manipulators without Slotine and Li's modification. Robotica. 31: 1051-1058.
[12] Izadbakhsh, A. and Masoumi, M. 2017. FAT-based robust adaptive control of flexible-joint robots: singular perturbation approach. Annual IEEE Industrial Society's 18th International Conf. on Industrial Technology.
[13] Khorashadizadeh, S. and Majidi, M. H. 2017. Synchronization of two different chaotic systems using Legendre polynomials with application to secure communications. Frontiers of Information Technology & Electronic Engineering. 19(9): 1180-1190.
[14] Huang, A. C., Wu, S. C. and Ting, W. F. 2006. A FAT-based adaptive controller for robot manipulators without regressor matrix: theory and experiments. Robotica. 24: 205-210.
[15] Fateh, M. M. 2012. Robust control of flexible-joint robots using voltage control strategy. Nonlinear Dynam. 67: 1525-1537.
[16] Izadbakhsh, A. 2016. Robust control design for rigid-link flexible-joint electrically driven robot subjected to constraint: theory and experimental verification. Nonlinear Dynamic. 5: 751-765.
[17] Izadbakhsh, A., Akbarzadeh Kalat, A., Fateh, M. M. and Rafiei, S. M. R. 2011. A robust anti-windup control design for electrically driven robots-Theory and Experiment. International Journal of Control, Automation, and Systems. 9: 1005-1012.
[18] Gao, W. and Slemic, R. R. 2005. Adaptive neural network output feedback control of nonlinear systems with actuator saturation. 44th IEEE Conferance on. Decision and Control and the European Control Conference: 5522-5527.
[19] Gao, W. and Selmic, R. R. 2006. Neural Network Control of a Class of Nonlinear Systems with Actuator Saturation. IEEE Trans. On Neural Networks. 17: 147-156.
[20] Slemic, R. R., Phoha, V. V. and Lewis, F. L. 2003. Intelligent compensation of actuator nonlinearities. 42th International Conferance on. Decision and Control. 4327-4332.
[21] Izadbakhsh, A. and Rafiei, S. M. R. 2009. Endpoint Perfect Tracking Control of Robots – A Robust Non Inversion-Based Approach. International Journal of Control, Automation, and Systems. 7: 888-898.
[22] Izadbakhsh, A. and Fateh, M. M. 2014. Real-time robust adaptive control of robots subjected to actuator voltage constraint. Nonlinear Dynamics. 78: 1999-2014.
[23] Izadbakhsh, A. and Khorashadizadeh, S. 2017. Robust Impedance Control of Robot Manipulators Using Differential Equations as Universal Approximator. International Journal of Control: 1-17.
[24] Izadbakhsh, A. and Khorashadizadeh, S. 2017. Robust Task-Space Control of Robot Manipulators Using Differential Equations for Uncertainty Estimation. Robotica. 35: 1923-1938.
[25] Izadbakhsh, A. and Rafiei, S. M. R. 2008. Robust Control Methodologies for optical Micro Electro Mechanical System-New Approaches and Comparison. 13th Power Electronics and Motion Control Conference. EPE-PEMC: 2102-2107.