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Numerical and Analytical Investigation of a Cylinder Made of Functional Graded Materials under Thermo-Mechanical Fields

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Abstract
This research develops thermo-elastic analysis of a functionally graded cylinder under thermo-mechanical loadings. Heat conduction equation in cylindrical coordinate system is solved. Thermal conductivity coefficient is graded along the radial direction. By considering a symmetric distribution of temperature, loading and boundary conditions, strain-displacement and stress-strain relations can be developed. Material properties such as modulus of elasticity are graded along the radial direction. For validation of the obtained results; a complete numerical analysis using finite element approach is presented.

Keywords
Thermo-Elastic, Functionally Graded Materials, Cylinder, Stress, Strain

1. Introduction

Functionally graded materials (FGM's) have been developed during recent years. In 1891, Japanese material scientist, for the first time during the study, achieved this non-homogeneous material to obtain a material with high thermal resistance [1]. Chen and Lin [2] presented the Elastic analysis of thick cylinders and spherical pressure vessels stress distribution along the radial direction, made of FGM. The results show that non-homogeneity has significant influence on the stress distribution along the radial direction. Thermal stresses in a hollow circular cylinder made of functionally graded materials were given by Liew and et al. [3]. They offered the solution by a limiting process for analyzing homogeneous hollow cylinders without using thermo-elastic non-homogeneous equations. General analysis of one-dimensional uniform thermal stresses in thick-walled hollow cylinders made of FGM was provided by Jabbari et al. [4]. Nonlinear analysis of thermo-elastic hollow rotating disk made of FGM using the first and third-order shear deformation theories was examined by the Nokhandan and Jabbarzadeh [5]. These authors have achieved equilibrium equations using first-order and third-order shear deformation theory with a von Karman assumption and the principle of the least potential energy. Khoshgoftar et al. [6] presented the thermo-elastic analysis of a thick-walled cylinder made of functionally graded piezoelectric materials (FGPM). Thermal analysis of elastic-plastic and optimal design of a composite cylinder made of ceramic and metal FGM also were studied by Hang and colleagues [7]. Nonlinear thermal analysis of a thick-walled cylinder was made of FGM to the condition that temperature-dependent material properties are investigated by Azadi and Sharyat [8]. The authors investigated the effect under different boundary conditions and geometric parameters and volume fraction index for the temperature distribution. Vodenitcharova and Zhang [9] studied the pure bending and bending-induced local buckling of a nanocomposite beam reinforced by a SWCNT. They found that in thicker matrix layers the SWNT buckles are locally at smaller bending angles and greater flattening ratios. Asymmetric axial thermal analysis of a hollow cylinder made of the FGM has been investigated by Shao et al. [10]. Two-dimensional dynamic analysis of thermal stresses in a circular cylinder made of FGM subjected to thermal shock loading using an analytical method was investigated by Safary and Tahani [11]. They assumed that the thermal properties for the material in the radial direction along the cylinder are non-linear. These authors with the help of Laplace and Navier equations investigated the effects of thermal loading on a cylinder made of FGM. Nie and Batra have studied an analysis of the isotropic elastic and incompressible hollow cylinder. They presented an accurate solution for the deformation of the cylinder at different shear modulus [12]. Thermo-mechanic analysis of three-dimensional stress cracks in a cylinder that is made of FGM was investigated by Nami and Eskandari [13]. They presented the effect of the thermal expansion coefficient of the stress distribution around the cylinder with surface cracks. A new method for stress analysis of a cylinder under pressure made of FGM was presented by Tutuncu and Temel [14]. Salehi-Khojin and Jalili [15] studied the buckling of boron nitride nanotube reinforced piezoelectric polymeric composites subjected to combined electro-thermo-mechanical loadings. Their results indicated that the piezoelectric matrix enhanced the buckling resistance of composite significantly, and the supporting effect of elastic medium depended on the direction of applied voltage and thermal flow. Also, Salehi-Khojin and Jalili [16] proposed a semiactive control approach to obtain a composite structure with tunable mechanical properties ranging from stiffer structure to better damper. For this purpose, they proposed to apply an external electrical field to a piezoelectric polymeric matrix such as polyvinylidene fluoride (PVDF) reinforced with carbon nanotube. They showed that upon electrical loads to PVDF reinforced with nanotubes, the interfacial adhesion can be selectively controlled based on some desired characteristics.

In this paper, after performing a heat transfer analysis in cylindrical coordinate system, the thermo-elastic analysis can be presented. The symmetric strain-displacement and stress-strain can be used. For validation of the results, a comprehensive software analysis can be performed.

2. Basic Equations of the problem

If the cylinder is assumed to be of infinite length in the z direction, in cylindrical coordinate system, the relationship between strain and displacement can be written as follows [17]:

$$\varepsilon_{rr} = \frac{\partial u}{\partial r} \tag{1}$$

$$\varepsilon_{\theta\theta} = \frac{u}{r} \tag{2}$$

Where, "ε" show the strain, "u" represent the displacement in the radial direction of the cylinder. Stress-strain relations for plane stress condition can be written as following:

$$\sigma_{rr} = (\gamma + 2\eta)\varepsilon_{rr} + \gamma\varepsilon_{\theta\theta} - (3\gamma + 2\eta)\alpha T(r) \tag{3}$$

$$\sigma_{\theta\theta} = (\gamma + 2\eta)\varepsilon_{\theta\theta} + \gamma\varepsilon_{rr} - (3\gamma + 2\eta)\alpha T(r) \tag{4}$$

Where σ represents the stress and T(r) represents temperature distribution. The terms α, γ and η are physical relations. Such that, the term α Coefficient of thermal expansion, terms γ and η depend on the modulus of elasticity E and Poisson's ratio ν as follows:

$$\gamma = \frac{\nu E}{(1+\nu)(1-2\nu)} \tag{5}$$

$$\eta = \frac{E}{2(1+\nu)} \tag{6}$$

Equilibrium equation in the radial direction, regardless of body force and the inertia term is as follows:

$$\sigma'_{rr} + \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) = 0 \tag{7}$$

As mentioned before, the material properties are variable in terms of radial location of the cylinder. For presentation of variable properties, we can use power function as follows:

$$E(r) = E_0 r^{\beta_1}, \alpha(r) = \alpha_0 r^{\beta_2} \tag{8}$$

In this relation, the term β is called the non homogeneous index of the material.

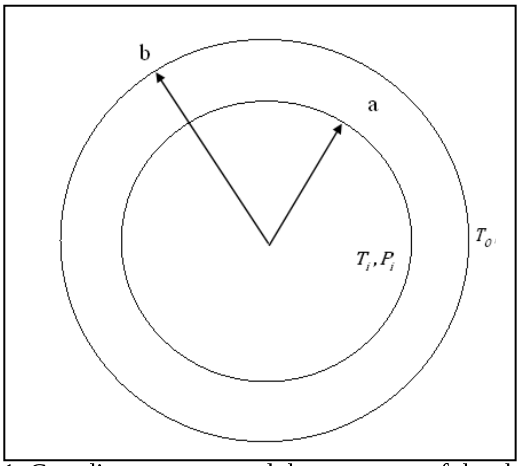

Figure1. Coordinate system and the geometry of the shell

By substitution of strain-displacement in stress-strain relation and consequently in equilibrium equation, we have:

$$d_1 r^{\beta} u'' + d_2 r^{\beta-1} u' + d_3 r^{\beta-2} u = d_4 r^{2\beta-1} T(r) + d_5 r^{2\beta} T'(r) \tag{9}$$

where:

$$\begin{aligned} & d_1 = \gamma, d_2 = \gamma(\beta+1), d_3 = \gamma\beta - (\gamma + 2\eta) \\ & d_4 = (3\gamma + 2\eta)\alpha(2\beta+1) + (3\gamma+2\eta)\alpha \\ & d_5 = (3\gamma+2\eta)\alpha \end{aligned} \tag{10}$$

3. The Heat Transfer Equations

One dimensional heat transfer equation in the steady state conditionin cylindrical coordinate system is:

$$\frac{1}{r}(rS(r)T'(r))' = 0$$
$$z_{11}T(a) + z_{12}T'(a) = q_1, \quad z_{21}T(b) + z_{22}T'(b) = q_2, \quad a \le r \le b \tag{11}$$

where, k=k(r) is thermal conductivity, a and b are the inner and outer radius of the cylinder and Z_{ij} is the parameters of the thermal conductivity which is thermally stable. q_1 and q_2 are constant inner and outer radiuses which are known. It is assumed that k(r) is thermal conductivity of FGM which is an inhomogeneous material, in equation (12):

$$S(r) = S_0 r^{\beta_3} \tag{12}$$

Where, S_0 and β_3 are the coefficients of the materials. Using equation (12), the heat transfer equation in the form of equation (13) can be written:

$$\frac{1}{r}\left[r^{\beta_3+1}T'(\mathrm{r})\right]' = 0 \tag{13}$$

Solution of above equation may be presented as follows:

$$T(r) = \frac{-M_1}{\beta_3} r^{-\beta_3} + M_2 \tag{14}$$

By using the boundary conditions (11), the constants M_1 and M_2 are obtained as follows:

$$M_1 = \frac{Z_{21}q_1 - Z_{11}q_2}{Z_{21}\left(z_{12}a^{-(\beta_3+1)} - \frac{c_{11}a^{-\beta_3}}{\beta_3}\right) - Z_{11}\left(Z_{22}b^{-(\beta_3+1)} - \frac{Z_{21}b^{-\beta_3}}{\beta_3}\right)}$$
$$M_2 = \frac{\left(Z_{12}a^{-(\beta_3+1)} - \frac{Z_{11}a^{-\beta_3}}{\beta_3}\right)q_2 - \left(Z_{22}b^{-(\beta_3+1)} - \frac{Z_{21}b^{-\beta_3}}{\beta_3}\right)q_1}{Z_{21}\left(Z_{12}a^{-(\beta_3+1)} - \frac{Z_{11}a^{-\beta_3}}{\beta_3}\right) - Z_{11}\left(Z_{22}b^{-(\beta_3+1)} - \frac{Z_{21}b^{-\beta_3}}{\beta_3}\right)} \tag{15}$$

4. Solving the Governing Equations of Problem

By solution of temperature distribution, we can complete equilibrium equation as follows:

$$d_1 r^{\beta} u'' + d_2 r^{\beta-1} u' + d_3 r^{\beta-2} u = M_3 r^{\beta_3-1} + M_4 r^{\beta_2-\beta_3-1} \tag{16}$$

Where:

$$M_3 = \frac{(1+\nu)(\beta_1+\beta_2)\alpha_0 M_2}{1-\nu}$$
$$M_4 = \frac{(1+\nu)(1-\frac{1}{\beta_1})\alpha_0 M_1}{(1-\nu)} \tag{17}$$

The general solution of this equation is given following:

$$u_n^g(r) = Br^{\zeta} \tag{18}$$

By inserting equation (18) into equation (16), the general solution of the problem of the equation (19) is provided that:

$$\zeta_{1,2} = -\frac{d_2 - d_1}{2d_1} \pm \sqrt{(\frac{d_2 - d_1}{16d_1})^2 - \frac{d_3}{d_1}} \tag{19}$$

The general solution is:

$$u^g(r) = B_1 r^{\zeta_1} + B_2 r^{\zeta_2} \tag{20}$$

And:

$$u^k(r) = W_1 r^{\beta_2+1} + W_2 r^{\beta_2-\beta_3+1} \tag{21}$$

By inserting equation (21) into equation (16) we have:

$$\begin{aligned} &W_1 r^{\beta_2-1}[(d_1 r^{\beta}(\beta_2+1)+(\beta_2+1)d_2 r^{\beta})] \\ &+W_2 r^{\beta_2-\beta_3-1}[(\mathrm{d}_1\, r^{\beta}(\beta_2-\beta_3+1)(\beta_2-\beta_3)) \\ &+((\beta_2-\beta_3+1)d_2 r^{\beta}) = A_3 r^{\beta_2-1} + A_4 r^{\beta_2-\beta_3-1} \end{aligned} \tag{22}$$

Equation (22) gives the result:

$$\begin{aligned} W_1 &= \frac{A_3}{(d_1 r^{\beta}(\beta_2+1))+((\beta_2+1)d_2 r^{\beta})} \\ W_2 &= \frac{A_4}{(d_1 r^{\beta}(\beta_2-\beta_3+1)(\beta_2-\beta_3))+((\beta_2-\beta_3+1)d_2 r^{\beta})} \end{aligned} \tag{23}$$

The general solution of this equation is:

$$\begin{aligned} u(r) &= u_n^g(r) + u^K(r) \\ u(r) &= \mathrm{B}_1\, r^{\zeta_1} + B_2 r^{\zeta_2} + W_1 r^{\beta_2+1} + W_2 r^{\beta_2-\beta_3+1} \end{aligned} \tag{24}$$

By inserting equation (24) in equation (1) and (2), the stress components can be obtained as:

$$\begin{aligned} \sigma_{rr} &= ((\gamma+2\eta)r^{\beta})(\zeta_1 B_1 r^{\zeta_1-1} + \zeta_2 B_2 r^{\zeta_2-1} + (\beta_2+1)W_1 r^{\beta_2} + (\beta_2+\beta_3+1)W_2 r^{\beta_2-\beta_3})) \\ &+ (\gamma r^{\beta})(B_1 r^{\zeta_1-1} + W_1 r^{\beta_2} + W_2 r^{\beta_2-\beta_3}) + ((3\gamma+2\eta)\alpha)(\frac{-A_1}{\beta_3} r^{-\beta_3} + A_2) \end{aligned} \tag{25}$$

$$\begin{aligned} \sigma_{\theta\theta} &= ((\gamma+2\eta)r^{\beta})(B_1 r^{\zeta_1-1} + B_2 r^{\zeta_2-1} + W_1 r^{\beta_2} + W_2 r^{\beta_2-\beta_3}) + (\gamma r^{\beta})(\zeta_1 B_1 r^{\zeta_1-1} + \zeta_2 B_2 r^{\zeta_2-1} + \\ &(\beta_2+1)W_1 r^{\beta_2} + (\beta_2-\beta_3+1)W_2 r^{\beta_2-\beta_3}) + ((3\gamma+2\eta)\alpha)(\frac{-A_1}{\beta_3} r^{-\beta_3} + A_2) \end{aligned} \tag{26}$$

By employing the required boundary conditions, we can complete solution procedure. For a cylinder of FGM, thermal and mechanical boundary conditions have been applied as:

$T(a) = 10^0 c, \mathrm{T}(\mathrm{b}) = 50^0 c$

And the mechanical boundary condition is presented as follows:

$\sigma_{rr}(a) = 1Mpa, \sigma_{rr}(b) = 0Mpa$

5. Numerical Results and Comparison

In this section, the numerical results along the radial direction and in terms of different values of non-homogeneous index can be presented. Furthermore, for validation of the obtained results, a comprehensive analysis using the finite element approach is performed. ABAQUS software is used for finite element analysis.The numerical constants are considered as follows:

$$a = 1.5, b = 2, \alpha = 1\times10^{-6} /^{0} C, E = 200GPa,$$

$$T_i = 50^0 C, T_o = 10^0 C$$

Figures 2 and 3 presented the temperature distribution in terms of different values of non homogenous index between ± 4 based on the exact and software analysis.

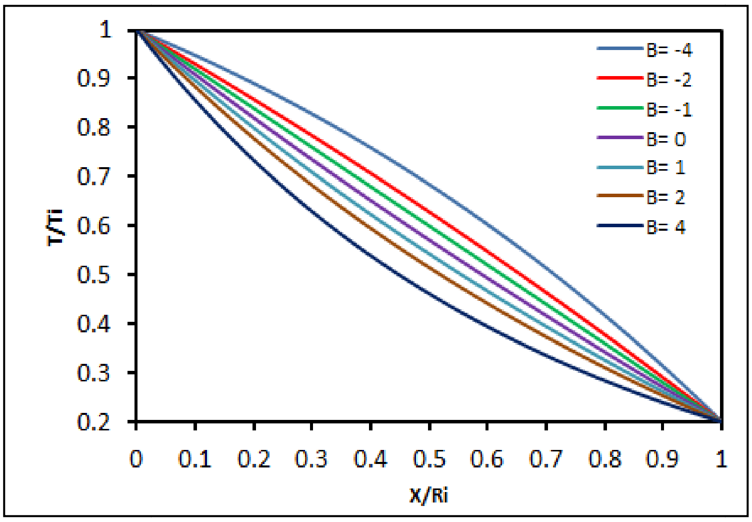

Figure2. Temperature distribution in terms of different values of non homogenous index between ± 4 based on the software analysis

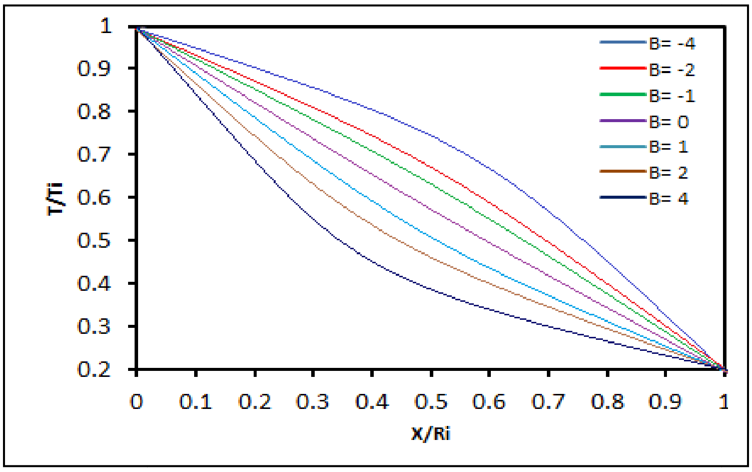

Figure3. Temperature distribution in terms of different values of non-homogenous index between ± 4 based on the exact analysis

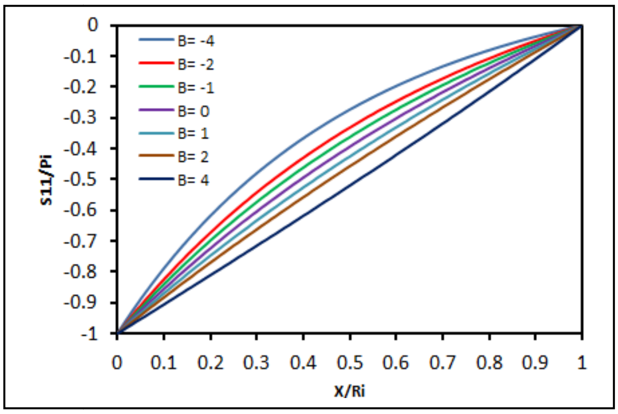

Figure4. The radial distribution of radial stress in terms of different values of non-homogenous index between ±4 based on the software analysis

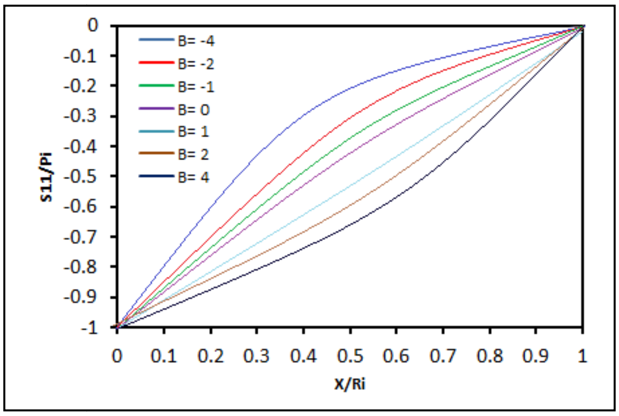

Figure5. Radial distribution of radial stress in terms of different values of non homogenous index between ±4 based on the exact analysis

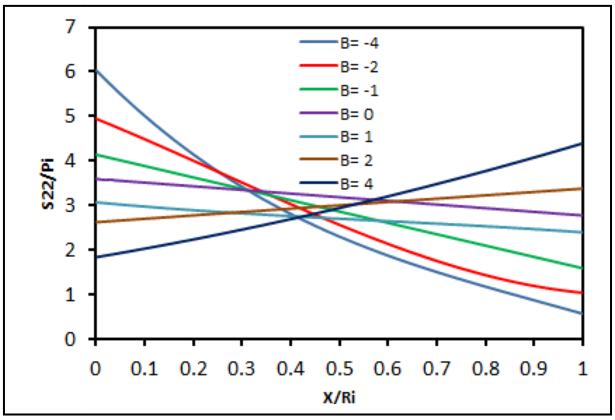

Figure6. Radial distribution of circumferential stress in terms of different values of non homogenous index between ±4 based on the software analysis

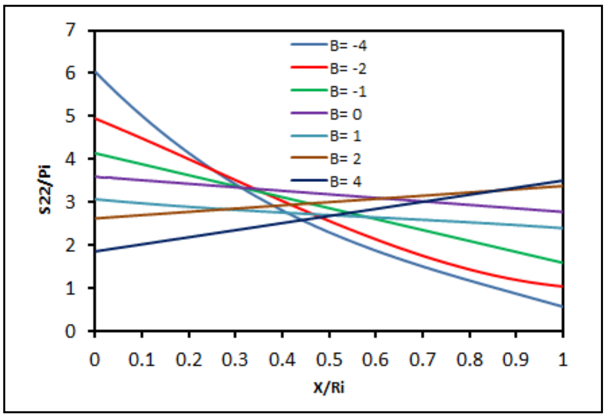

Figure7. Radial distribution of circumferential stress in terms of different values of non-homogenous index between ±4 based on the exact analysis

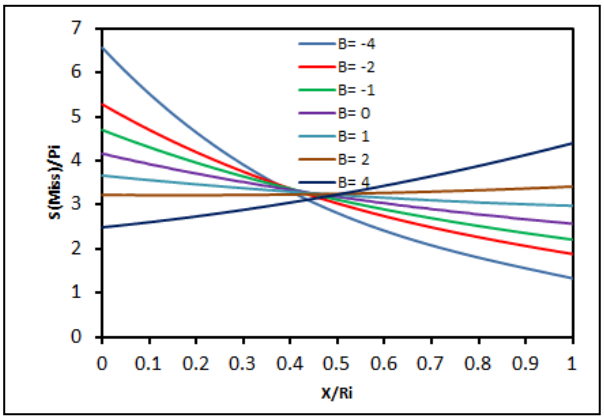

Figure8. The radial distribution of Von-Mises stress in terms of different values of non homogenous index between ±4 based on the software analysis

Figures 4 and 5 show the radial stress in the cylinder in terms of different values of non homogeneous index based on the exact and software analyses. Figures 6 and 7 show the circumferential stresses along the radial direction.

As presented in Figures 4 and 5, with increasing the non homogeneous index, the radial stresses are increasing. This increasing is due to increasing the stiffness of cylinder.

6. Conclusions

In this paper, thermo-elastic analysis of a functionally graded cylinder has been performed. Gradation of material properties have been considered for thermal and mechanical properties except Poisson ratio. Furthermore a software analysis using ABAQUS has been performed. The obtained results present good computability between both employed methods. The results of this analysis can direct researcher for finding the best and optimized condition and control of stress or displacement distribution in critical conditions.

7. References

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