Solving the Problem of Scheduling Unrelated Parallel Machines with Limited Access to Jobs

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Abstract

Nowadays, by successful application of on time production concept in other concepts like production management and storage, the need to complete the processing of jobs in their delivery time is considered a key issue in industrial environments. Unrelated parallel machines scheduling is a general mood of classic problems of parallel machines. In some of the applications of unrelated parallel machines scheduling, when machines have different technological levels and are not necessarily able to process each one of the existing jobs in the group of jobs and in many of the industrial environments, a sequence dependent setup time takes place during exchanging jobs on the machines. In this research, the unrelated parallel machines scheduling problem has been studied considering the limitations of sequence dependent setup time of processing of jobs and limited accessibility to machines and jobs with the purpose of minimizing the total weighting lateness and earliness times. An integer scheduling model is proposed for this problem. Also, a metaheuristically combined method consisting of Genetic algorithm and Particle swarm optimization (PSO) algorithm for its solutions is proposed. The obtained results of the proposed algorithm show that the proposed algorithm is very efficient especially in problems with large dimensions.

Keywords

Unrelated parallel machines scheduling, Sequence dependent setup time, Genetic algorithm, Particle swarm optimization algorithm.

1. Introduction

In the present competitive world, sequence and effective scheduling are essential for survival in business space. Scheduling is a tool that optimizes the application of available resources. Time is always a fundamental limitation; therefore, the scheduling of activities in order to minimize this limited resource is very essential. Scheduling and sequence of operations is one of the most important scheduling problems of production and have many applications in manufacturing and non-manufacturing units.

Scheduling in an organization uses models and mathematical methods and/or heuristic methods for allocating the limited resources to progressing jobs. The reason the importance of sequence of parallel machines with the purpose of focusing on delay is that in the present business environment, the competition of manufacturing companies is defined by their capability for immediate response to immediate changes in the business field and producing higher quality productions and lower costs. Manufacturing companies are attempting to obtain these capabilities by automation and heuristic concepts like Just In Time(JIT), Quick Response (QR), Group Technology (GT), and Total Quality Management (TQM) [1].

These concepts (for example, JIT and GT) have helped many companies to obtain economic benefits. In the JIT systems, the jobs should not be done neither sooner nor later, which leads to scheduling problems with earliness and late and of delivery times. In a competitive business the delay of jobs, given their delivery time, is a very important scale function for various manufacturing environments.

Problems with defining the delivery time in the previous 25 yearshave been taken into consideration, due to new methods of management like JIT concepts. Cheng, who helped a lotin scheduling problems and delivery time allocation, states that the completion of a job earlier than delivery time means imposition of unnecessary stock protecting costs, while the completion of a job later than delivery time leads to formal penalties and losing customer credibility [2].

The purpose of earliness and delay minimum problem (ET) is completely compatible with the JIT production control policy. The parallel machines problem is important from both theoretical and practical points of view. It has been considered theoretically since it is a generalization of single machine case and practically since in the real world it is very usual.

The parallel machines application problems have been taken into consideration because if scheduling on a machine leads to so much cost, it is possible that the consideration of more machines decreases costs. Also, the values of earliness and delay can be decreased by increasing the number of machines. In these conditions, the cost of applying the machine or protecting the machine will be increased. By solving the optimization problem, it can be specified that which kind of machines should be used and which jobs by which sequence should be performed on these machines.

In this paper, the unrelated parallel machines scheduling problem is investigated considering the limitations of sequence dependent setup processing time and access time of jobs with the purpose of minimizing the total weighting earliness and delay times. Theinnovations of the proposed model in this paper are related to the limitations of access times to jobs, limitation of access time to machines and also adding the earliness as the second purpose function of mathematical model. An integer scheduling model will be presented for the proposed problem. Also, a meta-heuristic method based on Genetic meta-heuristic algorithms and Particle swarm optimization algorithm has been presented for solving the proposed problem especially for problems with large dimensions.

2. Literature review

Recently, the study in the field of earliness and delay penalty in scheduling models has been extensively noticed by researchers. Previous studies were more focused on regular criteria like maximum completion time, maximum delay or mean delay. But nowadays, due to penalties produces by earliness, this irregular criterion has been noticed by researchers of scheduling science. All the manufacturers increasingly adopt themselves to the JIT attitude and minimum supply cost in order to contrast competitive pressures. For doing so, the earliness of works should be taken into consideration. In fact, in JIT the earliness of jobs is as important as their delay. Although it is assumed that the delivery time is an external variable and its control is out of the managers' responsibility, the importance of on time delivery is very clear. Canway [3] for the first time stated this theory that the delivery time is a part of the scheduling problem. Webster and Baker [4], Ragatz and Mabert [5] Christy and Kanet [6] are among the researchers who studied in this field. Many studies have been performed in the field of different scheduling problems with delay and earliness

criterions (E/T). Webster et al. [7] have solved single machine problem (E/T) considering the same and undefined delivery time as a decision variable, for two cases of independent and sequence dependent setup times by Genetic algorithm. Beck and Refalo [8] presented a combined method using linear programming and useful programming for scheduling problem with earliness and lateness criterion. Liman [9] presented a single machine scheduling problem by limited delivery time with the same interval and controllable processing time. They minimized the total penalty which consists of earliness and lateness penalties, the earliest delivery time, interval length related to delivery time and the real value of processing time reduction, by heuristic method.

Xiao and Chung [10] investigated approximate methods for allocating common delivery time to jobs and their scheduling in parallel machines problems. The purpose is allocating the delivery time of jobs so that total weight of delivery time, total earliness and total delay will be minimized.

CAI and Zhou [11] studied a single machine scheduling problem with probable processing time with an exponential distribution and probable delivery time, for the purpose of minimizing the total earliness and delay weight costs. In this problem, machine breakdown is a part of assumptions and the problem has been solved by dynamic programming.

Elapse [12] studied the optimized waiting time problem in partial manufacturing systems by earliness and delay casts criteria. The mentioned problem included steps with the waiting time being on the back of each probable step. Here, the purpose is finding the optimized waiting times, so that earliness and delay costs will be minimized. Radhakrishnan and Ventura [13] solved the scheduling problem of parallel machines with the criterion of earliness and delay costs with sequence dependent setup times by a simulated Annealing meta-heuristic approach.

Panwalkar and Liman [14] investigated a scheduling problem with n jobs each of which includes an operation on each machine by earliness and delay costs criteria. In this problem, the fracture of work is not allowable and delivery time and processing time parameters are determined. Here, the purpose is defining the number of machines so that the earliness and delay costs will be minimized.

3. Definition of the problem of study

The unrelated parallel machines scheduling problem with the limitation of sequence of jobs, dependent setup time and limited accessibility to jobs and the limitation of access time to jobs is presented in order to minimize the earliness and delay weight of jobs as follows:

A set of n distinct jobs on a set of m machines which are put together in parallel is processed so that each single job on a machine is processed and each machine can process only one job at every moment. Each work can be processed on a subset of machines set as $m_j \subseteq m$. This sub set is called processing sets and there are a number of processing sets in the problem. Each job in a given period of time is processed once and this period of time is called the processing time. The processing time of jobs on machines not only depends on the kind of job but it also depends on the kind of machine and there is no specific relation between the processing times of jobs on different machines. Before the beginning of the processing of a job on a machine, in order to prepare that machine for the processing of that job, some operations will be performed that are called installation of machine soperations, and the time period in which the installation of machine is performed is called installation of machine time. This time depends on the kind of job which is processed on the machine and the kind of the previously processed job and also the kind of machine and is called sequence dependent setup time of jobs. The access time to jobs is when a job entered the work shop and is ready to attain necessary services from manufacturing machines. This time is the earliest time the processing operations can be started on a job. Each job by considering its position in the processing sequence of jobs on the related machine is completed after finishing the setup and processing times. Each job has distinct delivery time and the time deviation of completion of jobs from delivery time is calculated as earliness and delay time. Each job is attributed weights as distinct coefficients that are representatives of earliness and delay times costs. The sum of these times is considered as the optimization criterion of the problem and consequently it is the purpose function of minimization function of earliness and delay costs of jobs. The assumptions which are assumed for the proposed problem are as follows:

1- The sequence dependent setup time is on machines during exchanging jobs.

2- Each of the jobs can be processed only on a subset of set of machines.

3- All jobs are not available at the beginning of the scheduling horizon. In another words, the processing of work cannot be initiated before the access time.

4- All machines are available continually and machine breakdowns are not possible.

5- Each machine can process only one job at each moment.

6- Each job is processed on only one machine during its processing time.

7- Processing, setup, delivery times and earliness and delay time coefficients and access time to jobs are determined.

8- The inaction of machines is permissible.

9- The zero kind virtual job is assumed. This work is always processed at the first position on all machines. The processing time of this job is assumed zero and its processing beginning does not need any machine installation operations.

4. The proposed mathematical model

In this section, the proposed linear mathematical model with an integer programming approach is presented for the proposed problem. Before presenting the model, the entrance parameter, the decision variables, the limitations and the purpose function are explained as follows:

4.1 The entrance parameters of the model

N: number of jobs

M: number of machines

P_{ij}: the processing time of job of kind j on machine i

 S_{ijk} : the preparation time of job k after job j on the machine i

 A_{ij} : if it is possible to process job of kind j on machine of kind I, it will be equal to 1; otherwise, it will be zero.

D_i: the delivery time of jobs

α: earliness penalties

 β : delay penalties

r_i: access time to jobs

4.2 Symbols, definitions, parameters and decision variables

j , k: job index (k=1,2,...,n and j=1,2,...,n)

i: machine index (i=1,2,...,k)

 $\begin{array}{l} C_i: \mbox{ the completion time of job j} \\ T_j: \mbox{ delay value of job j} \\ E_j: \mbox{ earliness value of job j} \\ X_{ij}: \mbox{ if job j goes on machine I, it will be equal to 1; otherwise, it is equal to zero.} \\ Y_{jk}: \mbox{ if job k comes after job j, it will be equal to 1; otherwise, it will be equal to zero.} \\ St_i: \mbox{ it is the indicator of beginning time of job j} \end{array}$

4.3 The mathematical model

$$Min \ Z = \sum_{j=1}^{n} \alpha E_j + \sum_{j=1}^{n} \beta T_j$$

$$s.t$$

$$\sum_{j=1,2,...,n} (2)$$

$$\sum_{i=1}^{n} x_{ij} = 1$$

$$\sum_{i=1}^{n} y_{jk} = 1$$
(3)

$$\sum_{i=1,2,...,n}^{j=0} j=0,1,2,...,n \quad (4)$$

$$\sum_{j=0,1,2,...,n}^{n} j=1,2,...,n \quad (5)$$

$$\frac{1}{x_{ij}} + x_{irj} \le 2 - y_{jk} \qquad j = 1,2...,n \quad i = 1,2...,m \quad \neq i \qquad (6)
S_k \ge \sum_{i=1}^m s_{jki} \cdot x_{ik} + C_j - M \cdot (1 - y_{jk}) \qquad (7)$$

$$S_{k} \leq \sum_{i=1}^{m} s_{jki} \cdot x_{ik} + C_{j} - M \cdot \left[\left(1 - y_{jk} \right) + \left(1 - z_{k} \right) \right]$$

$$j = 1, 2, ..., n; i = 1, 2, ..., m \quad (8)$$

$$k = 1, 2, ..., m \quad (9)$$

$$S_{k} \ge R_{k} + z_{k}.M$$

$$C_{j} = S_{j} + P_{j}$$

$$C_{j} + E_{j} - T_{j} = d_{j}$$

$$y_{jk}, x_{ij}, z_{j} = 0 \text{ or } 1$$

$$K = 1,2...,n$$

$$k = 1,2...,n$$

$$j = 0,1,2...,n$$

$$j = 0,1,2...,n$$

$$j = 1,2...,n$$

$$j = 1,2...,n$$

$$(13)$$

The relation (1) shows the purpose function of the problem which is equal to the sum of weighting earliness and delay. The limitation of (2) makes each job be performed only on one machine. The limitation of (3) assures that before each job there is one job, the first job after the assumed job is zero. The limitation of (4) is an indicator of limitation of accessibility to the jobs. The limitation of (5) shows that after each job there is at most one job. The limitation of (6) makes each two consecutively placed jobs perform on one machine. Limitations (7) to (10) determine the beginning time of jobs. The limitation of (11) calculates the completion time of jobs. The limitation of (12) calculates earliness and delay times of jobs. Based on these assumptions and by consideration of the limitations presented in this section, when the processing of a job on a machine is finished, the machine can be maintained inactive and prevented from processing of the next jobs on it provided thatrecovery in purpose function values is achieved. Maintaining the machine inactive causes the completion time of a job whose processing has been finished before the beginning of the inactive time and increases as much as inactivity time. As a result, the earliness and delay time of that job and the next jobs and consequently the purpose function value will be changed. Therefore, the

voluntary inactivity of machines can improve the model purpose function. The limitation of (13) represents that variables are binary.

5. The proposed algorithm for solving the problem

In this paper for solving the problem a meta-heuristic method consisting of Genetic algorithm and Particle swarm optimization has been proposed. The Particle swarm optimization algorithm or in short the PSO algorithm is an algorithm based on population and the optimization of this method has been inspired by flying swarm. This technique in addition to having an appropriate performance has affordable computational time. In the PSO algorithm some answers are put together simultaneously. Each answer is searched among a set of answers while the search course has been inspired by the flying swarm. In continuation, we will use an improved PSO algorithm which is produced by a combination of the PSO and the Genetic algorithms for updating and improving the particles. Different developments are presented for improving the performance of this combined algorithm as follows:

1. The display style is like the display style of the answers in the Genetic algorithm.

2. Most of the PSO algorithms use updating formulas based on the best answer (Pbest-Gbest-based). In the proposed algorithm, for updating the particles, Genetic operator is used.

The proposed combined PSO algorithm details are as follows:





Display style: The display style in this algorithm is like chromosome in the Genetic algorithm that is defined as follows. The display style in the algorithm includes two parts which one part is related to performing jobs on machines and the other is related to the sequence of jobs on machines. The proposed chromosome structure in this paper is like figure (1):

Each chromosome includes one matrix with the dimension of (number of jobs * number of machines) and each row of this matrix is related to a machine and its columns are representative of jobs. Each cell of this matrix includes two values, one value is the representative of the first part which defines whether the intended job is on the related machine or not, which is shown by a binary number. The second value is related to the second part that is an indicator of the priority of performing jobs on machines which is shown by a number in the interval of [0,1]. According to the shown chromosome in the above figure, jobs J_3 first and then J_2 are performed on the first machine and on the second machine at first job J_1 and then job J_4 is performed.

• Preparation: Primary particles are produced for beginning of the algorithm accidentally as follows:

We produce an accidental answer and then enter it into the primary particles. For completing this set after obtaining each feasible answer we compare it with the existing answers in the set; if the produced feasible answer is non-repetitive, we add it to the set of primary particles and continue this job until the set of primary particles is completed.

• Fitness function: The fitness function of this algorithm is calculated based on the purpose function value; therefore, a particle with a lesser purpose function has a better fitness value.

• Update process: The purpose of updating is reaching to a new point X_i^{k+1} . For updating of a particle, genetic operators are used. In the improved PSO algorithm in this paper, the following formula has been used:

$$X_{i}^{k+1} = \left(X_{Pbest,i}^{k} \otimes X_{i}^{k}\right) \vee \left(X_{Gbest}^{k} \otimes X_{i}^{k}\right) \vee \overline{X_{i}^{k}}$$

$$\tag{14}$$

In which X_{Pbest}^{k} is the best previous position of ith particle and X_{Gbest}^{k} is the best position in all of the particles and X_{i}^{k} s are positions of ith person in kth repetition. Since X_{i}^{k} , X_{Pbest}^{k} and X_{Gbest}^{k} are allocation arrays the sign \otimes is an indicator of intersection operator between two particles. Thev sign means that the best answer is obtained from selecting the child of $(X_{Pbest,i}^{k} \otimes X_{i}^{k})$, $(X_{Gbest}^{k} \otimes X_{i}^{k})$ and $\overline{X_{i}^{k}}$. $\overline{X_{i}^{k}}$ is the result of performing mutation operator on X_{i}^{k} . In continuation, it will be explained how crossover and mutation operators have been applied in this algorithm.

5.1 Crossover operator

The crossover operator which is used in the proposed algorithm has two types that we apply one of them each time with the same possibility. The first type of crossover is based on random number method. In this method, for producing the genes related to each job in the chromosome of the child, we produce a random number in the interval of [0,1]:if the produced number is less than or equal to 0.5, the related genes are from the first parent; otherwise, they are from the second parent.

The second type of crossover operator used in this paper is the single point crossover operator. In this method, a random integer is produced between 0 to 1 (1-number of jobs) which is called the cut point. For producing a child, the genes related to the jobs of 1 to cut point are selected from the first parent and the rest of the genes are selected from parent 2. Figure (2) shows how these crossover operators work.



Fig. 2. The performance of the proposed crossover operator

5.2 Mutation operator

Mutation operator searches a space of answers that is not found by crossover operator. In this section, swap mutation is used so that we select two jobs randomly and then swap their genes in each part of chromosomes. Figure (3) presents a display of this operator.



Fig. 3. The performance of the proposed mutation operator

5.3 Parameter adjustment and conditions of algorithm run

In order to show the appropriate performance of the designed hybrid algorithm, different problems in two forms of problems with small dimensions and problems with large dimension are randomly produced. For each two groups of the produced problems, the following assumptions should be noticed.

For running the PSO-GA hybrid meta-heuristic algorithm, Matlab programming software • has been used.

• Since for running the algorithm, it is assumed that the primary population is produced randomly, for removing the effects of randomness of the initial population on the final answer, the algorithm has been run 20 times for problems with small dimensions and 10 times for problems with large dimensions and the best answer is chosen among all of the produced answers.

• In order to run this algorithm for problems with small and medium dimensions, the number of population and the number of generation are considered 100 and 50, respectively, and for problems with large dimensions the number of population and the number of generation are considered 200 and 100, respectively.

5.4 the structure of the proposed problems

Problems	Type of problems	Specification of problems (jobs × machines)	Problems	Type of problems	Specification of problems (jobs × machines)					
1	small	5×2	5	large	50×10					
2	small	5×3	6	large	50×20					
3	small	8×2	7	large	70×10					
4	small	8×3	8	large	70×20					

Table 1.Specification of the proposed problems

It should be noted that in each two groups of the proposed problems, the following general assumptions are considered:

• Weights related to delay and earliness of all jobs are produced randomly in the interval of [0,1].

• Time related to processing of jobs is produced randomly and uniformly in the interval of [5,40].

• Setup times of machines for performing job reproduced randomly in the interval of [1,8].

• Access time to jobs is produced randomly and uniformly in the interval of [0,10].

• Delivery time of works is produced based on the presented method by Yano and Kim [15].

In this method the delivery time is produced in the interval of [[(SUMP/2)(1-F+RD/2)]

,[(SUMP/2)(1-F-RD/2)]]. In the stated phrase, $SUMP = \sum_{i=1}^{m} \left(\frac{\sum_{j=1}^{n} p_{ij}}{n} \right)$, F is related to delay and

RD is a factor related to the delivery time. In all of the produced problems, the value of F is considered as equal to 0.5 and RD as equal to 0.1.

6. Computational results

In this paper, Lingo 9 software is used for an exact solution of problems, especially in the small dimensions. Also,Matlab 2010 software is used for coding and running of the PSO-GA hybrid metaheuristic algorithm. The obtained results for the proposed problems are shown in table (2):

It should be noted that for the problem of 10 jobs and 3 machines, Lingo software did not yield an optimized solution after 10 hours. For this reason, this is a proof for NP-HARD of the problem and for larger problems of this problem (problem with large dimension) only the proposed hybrid metaheuristic algorithm is used for solving the problem.

Problems	Answers obtained from lingo software	Times obtained from lingo software	Stdev of answers obtained from lingo software	Average of answers obtained from suggested algorithm	Average of times obtained from suggested algorithm (second)	Stdev of answers obtained from suggested algorithm
1	10.36	0.49	0	10.36	2.82	0
2	13.48	1	0	13.48	3.56	0
3	46.46	255	0	46.46	4.21	0
4	28.86	344	0	28.89	7.07	0.02588
5	-	-	-	171.55	84.07	1.2226
6	-	-	-	271.7	102.83	3.3148
7	-	-	-	981.7	209.88	8.6487
8	-	-	-	1509	289.88	7.4659

Table 2. The obtained results for the proposed problems



Fig 4. The values of the obtained purpose function produced by the proposed meta-heuristic algorithm and Lingo 9 software for problems with small dimensions



Fig. 5. The values of the obtained purpose function produced by the proposed meta-heuristic algorithm for problems with large dimensions



Fig. 6. The values of the obtained times produced by the proposed meta-heuristic algorithm and Lingo 9 software for problems with small dimensions



Fig. 7. The values of the obtained times produced by the proposed meta-heuristic algorithm for problems with large dimensions

The above figures are indicators of the obtained values for the proposed problems in this paper. According to the above figures, it can be pointed out that the proposed algorithm is efficient for solving the proposed problem in this paper, especially for problems with large dimensions.

7. Conclusion

In this paper, a PSO-GA combined meta-heuristic algorithm is proposed for solving unrelated parallel machines scheduling problems with limitation of sequence dependent time of jobs and limited accessibility to machines and the limitation of access time to jobs in order to minimize weighting delay and earliness of jobs. For the exact solution of the problem, Lingo 9 software was used. Also, for coding the proposed meta-heuristic algorithm, Matlab 2010 software was used. The obtained results, especially for problems with small dimensions, show that the proposed meta-heuristic algorithm is efficient for solving the problem. Solving the proposed problem, considering

the probable delivery times and also the breakdown and unavailable for machines is suggested as futureworks.

8. References

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