Robust Fractional-order Control of Flexible-Joint Electrically Driven Robots

Payam Kheirkhahan[1*]
[1]Department of Electrical Engineering, Garmsar branch, Islamic Azad University, Garmsar, Iran
[*]Email of Corresponding Author: kheirkhahan_payam@hotmail.com
Received: June 9, 2018 ; Accepted: August 25, 2018

Abstract
This paper presents a novel robust fractional PI^{λ} controller design for flexible joint electrically driven robots. Because of using voltage control strategy, the proposed approach is free of problems arising from torque control strategy in the design and implementation. In fact, the motor's current includes the effects of nonlinearities and coupling in the robot manipulator. Therefore, cancellation of motor current by feedback linearization in voltage control strategy can cancel the highly nonlinear dynamics of manipulators. Thus, it can guarantee robustness of control system to both structured and unstructured uncertainties associated with robot dynamics. As a result, the proposed control is simple, fast response and superior to torque control approaches. The control method is verified by stability analysis. Simulations on a two-link actuated flexible-joint robot show the effectiveness of the proposed control approach. Compared with ordinary controller, the fractional type shows a better tracking performance.

Keywords
Robust Fractional Control, Voltage Control Strategy, Electrically Driven Robots, Flexible-Joint Robots

1. Introduction

Although torque-level controllers are used more and more frequently for controlling robotic manipulators [1-7], the role of voltage-level controllers should not be undervalue. Indeed, torque-level controllers have some natural limitations coming from their practical implementation point of view, such as

1- A torque-level control law cannot be given directly to the torque inputs of an electrical manipulator, because physical control variables are not the torque vector applied to robot links but rather electrical signals to actuators.

2- The dynamics of motors and drives are excluded in the torque-level control strategies, while the actuator dynamics are often a source of uncertain, due e.g. to calibration errors, or parameter variation from overheating and changes in environment temperature [8].

3- The control problem becomes hypersensitive when faster trajectories (motions along specified paths at high speeds) are demanded. The main reason of this sensitivity refers to dynamic problems arising from high velocities. Therefore, robot's performance degrades quickly as speed increases.

4- The main discussion in the joint torque controller is that all of three customary approaches for transmitted torque's measurement through motors shaft, i.e. (I) torque measurement by using reaction force in shaft bearings, (II) the Prony brake method and (III) torque measurement through induced strain in rotating body, suffer from several inherent weakness. As a sample, the first method uses reaction force and the arm length for torque measurement indirectly. It also becomes touched from bearing friction and wind age torques which are not avoidable in practical robotic systems. The second approach measures the torque signal through a rope wounded around the shaft. This approach needs a standard mass, the measured force in the spring balance, shaft and Rope's parameters for torque calculation. It also utilizes few mathematical calculations for torque measurement. Furthermore, the produced heat arising from friction between the rope and shaft demands water cooling process, which is another weakness of this approach. Finally, the third and the most common method utilize bonding four strain gauges onto the shaft for torque measurement, where they have been arranged in a DC bridge circuit. The output from the bridge circuit is a function of the strain in the shaft and hence of the torque applied. It must be noted that, difficulty in precision of the positioning of the strain gauges on the shaft has made them relatively expensive. Also, this technique is ideal for measuring the stalled torque in a shaft before rotation commences, because a main problem is founding a suitable method to making electrical connections to the strain gauges in the case of rotating shafts. Although, a system of slip rings and brushes is a suitable choice in many commercial instruments, however it increases the cost of the instrument still further [9]. Recent developments in electronics and optical fiber technology have made the alternative methods for torque measurement possible, with relatively low costs and small physical size. However the lateral devices extend with the increase of DOF robot manipulators [9].

The considerable point is that, the control scheme proposed therein was confined to trajectory tracking control by integer order controllers and hence the trajectory tracking control of robots with fractional order controllers was remain as an open problem. Therefore, design of a fractional order controller that solves the above problems has been the subject of many researches over the two last decades. For instance, [10, 11] proposed a fractional order position/force control algorithm for rigid and flexible joint manipulators under the assumption of weak joint flexibility with 2×10^6 $Nm.rad^{-1}$ amount. [12] Investigated a fractional order PID controller for a position servomechanism control system considering actuator saturation and the shaft tensional flexibility. In addition, [13] proposed a feedback control for Direct Current (DC) motor speed with using the fractional-order $PI^{\lambda}D^{\sigma}$ controller. However, none stability of all these approaches were demonstrated in the literatures. [14] Proposed a fractional order sliding mode control for a polar 2DOF robot manipulator and utilized Genetic Algorithm for having optimized control parameters. [15] employed a fractional order fuzzy sliding mode controller for elastic joint robots although, there is yet problem arises from ignoring actuator dynamics in controller design as same as [10, 11] and [14]. Therefore, such a proper controller cannot be easily developed to implement robotic applications in experimental verification.

To tackle this problem, two good robust voltage control strategies were proposed aiming to prevent nominal performance degradations in presence of both parametric and unstructured uncertainties [16, 17]. The key issue of the proposed control schemes is ignoring the knowledge of manipulator dynamics in controller design, which makes it superior compared with torque-level controllers. Very recently, the extension to the rigid-link flexible joint robot manipulators was reported in [18, 19]. In the mind of authors, the aforementioned control strategies are the first results that can be deal with all the uncertainties in actuator dynamic using free model of the robot dynamics to the controller synthesis at the same time, although, the fractional order application of the voltage-level controllers is yet an open problem.

This paper is dedicated to design a robust fractional PI$^{\lambda}$ controller for the flexible joint electrically driven robots, using voltage control strategy. Control scheme presented here has several attractive features with respect to its design and implementation. First, overall complexity of the scheme is reduced to the motor dynamics control needless to any knowledge from robot dynamics. This is the main advantage, which makes this approach novel from theoretical point of view. The rest of this paper is organized as follows. In section 2 we recall some basic relationships for describing fractional order calculus. Further information about various approaches to fractional-order differentiation and integration can be found in the available literatures on this topic [20-22]. Nonlinear dynamic description is studied in section 3. The overall control structure of the robust fractional order controller will be outlined in section 4 and the closed-loop system stability is then established. In section 5 a simulation study will be presented to show the effectiveness of the proposed control approach. Finally, we give our concluding remarks in section 6.

2. Fractional Calculus

To study the fractional order controllers, the starting point is of course the fractional order differential equations using fractional calculus. Fractional calculus is a generalization of integration and differentiation to fractional order fundamental operator ${}_aD_t^{\alpha}$ which is defined as follows [23]

$$ {}_aD_t^{\alpha} = \begin{cases} \dfrac{d^{\alpha}}{dt^{\alpha}} & ,R(\alpha)>0 \\ 1 & ,R(\alpha)=0 \\ \displaystyle\int_a^t (d\tau)^{-\alpha} & ,R(\alpha)<0 \end{cases} \tag{1} $$

Where α is the order of the operation, which can be a complex number and the constant, a is related to the initial conditions. There are three definitions of fractional integration and differentiation. The most often used are the Grunwald-Letnikov (GL) definition, the Reimann-Liouville (RL) and the Caputo definition [22]. The GL definition is as:

$$ {}_aD_t^{\alpha} f(t) = \lim_{h\to 0} h^{-\alpha} \sum_{j=0}^{[(t-\alpha)/h]} (-1)^j \binom{\alpha}{j} f(t-jh) \tag{2} $$

Where $f(t)$ is an arbitrary differinte gralable function. In addition, the RL definition is formulated as

$$ {}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}} d\tau \tag{3} $$

For $(n-1<\alpha<n)$ with $\Gamma(x)$ denoting the famous Gamma function. Moreover, the Caputo's definition can be written as:

$$ {}_aD_t^{\alpha} f(t) = \frac{1}{\Gamma(\alpha - n)} \int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha - n + 1}} d\tau \tag{4} $$

For $(n-1<\alpha<n)$. For shortening of the paper, other important properties of the fractional derivatives and integrals can be found in the available literatures on this topic [20-22]. Nonlinear dynamic description is studied in the next section.

3. Nonlinear Dynamic Description

Let us consider an n-link electrical flexible joint robot manipulator, whose dynamics can be described as [24]

$$ D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = K(\theta - q) \tag{5} $$

$$ J_m\ddot{\theta} + B_m\dot{\theta} + K(\theta - q) = \tau \tag{6} $$

where $\theta \in \Re^n$ and $q \in \Re^n$ represent respectively the vectors of link positions and motor angular positions, $K \in \Re^{n\times n}$ is a diagonal positive definite matrix representing the joint stiffness, $D(q) \in \Re^{n\times n}$ is the symmetric positive definite manipulator inertia matrix, $C(q,\dot{q})\dot{q} \in \Re^n$ is a vector function containing Coriolis and centrifugal forces, $g(q) \in \Re^n$ is a vector function containing of gravitational forces, $J_m \in \Re^{n\times n}[kg.m^2]$ is a diagonal matrix of the lumped actuator rotor inertias, $B_m \in \Re^{n\times n}[N.m.s/rad]$ is diagonal matrix of the lumped actuator damping coefficients, and $\tau \in \Re^n[N.m]$ is the vector of actuator input torque. It must be noted that the link dynamics (5) and the mechanical subsystem of the motor dynamic are coupled only by the elastic torque term $K(\theta - q)$ and no any inertial coupling between actuators and links was considered in control system design. This assumption **is** true, since the off-diagonal terms of the inertia matrix and the associated Coriolis forces are considerably weaker compared with other source of interaction between degrees of freedom [25].

Now due to have motor voltages as the inputs for electrical flexible joint robot, some modifications are required. Toward this end, consider the electrical equation of permanent magnet DC motors in the matrix as

$$ L\dot{i} + Ri + k_b\dot{\theta} + \phi(t) = u \tag{7} $$

where $L \in \Re^{n\times n}$ is a constant diagonal matrix of electrical inductance, $R \in \Re^{n\times n}$ is diagonal matrix of armature resistances, $k_b \in \Re^{n\times n}[volt/rad/\sec]$ is a diagonal constant matrix for the back-EMF effects, $u \in \Re^n[volt]$ is the control input voltage applied for the joint actuators, $i \in \Re^n[A]$ is the vector of motor armature currents and $\phi(t)$ represents an external disturbance. It must be noted that, the motor torque vector as the input for dynamic equation (6) is produced by the motor current vector as

$$ \tau = k_m i \tag{8} $$

Where $k_m \in \Re^{n\times n}$ is an invertible constant diagonal matrix characterizing the electro-mechanical conversion between the current vector and the torque vector. As can be seen from (5)-(8), there exist strong couplings between the joint motions, since each element of $D(q)$, $C(q,\dot{q})$, and $g(q)$ is highly complicated nonlinear functions of the manipulator configuration q and the speed of motion $\dot{q}$, as well as the inertia parameters of the payload carried by the manipulator end-effecter.

4. Robust Control Design

In this section, we design a robust control for electrically driven flexible joint robots by applying the recursive procedure. It follows from (6) and (7) that the overall system of actuated robot manipulator can be viewed as two-cascaded dynamical system, if i is considered as the input signal to robot dynamics of rigid body. One consequence of this definition is that the rigid-link manipulator input i cannot be commanded directly, and instead it must be realized as the output of the actuator dynamics through proper specification of the actuator control input u. Hence, in order to control the robot manipulator to track the desired trajectory, first a robust control scheme is designed to generate the fictitious control input i_d required to ensure that the system (6) evolves as desired. The next control objective is, naturally, to generate a suitable control voltage u so that the motor current i can follow the desired current command i_d, and thus q will follow the desired trajectory q_d. Based on this observation, a recursive control scheme is developed. By the last definitions, the first attempt is define a current error as the form of

$$i_e = i - i_d \tag{9}$$

Or equivalently

$$i = i_d + i_e \tag{10}$$

By substituting (10) in to (6) we have,

$$J_m\ddot{\theta} + B_m\dot{\theta} + K(\theta - q) = k_m i_d + k_m i_e \tag{11}$$

Multiplying both sides of (11) in k_m^{-1} yields

$$k_m^{-1}\left(J_m\ddot{\theta} + B_m\dot{\theta} + K(\theta - q)\right) = i_d + i_e \tag{12}$$

Now, the problem is to design a desired current trajectory i_d so that a robust inner-controller u can be constructed to have $i \to i_d$ which further implies convergence of the output error as desired. To solve this problem, we define the desired current i_d as

$$i_d = \hat{k}_m^{-1}\left(\hat{J}_m\ddot{\theta} + \hat{B}_m\dot{\theta} + \hat{K}\theta - \hat{K}(q_d + K_1 D^{-\lambda_1} e)\right) \tag{13}$$

Where $e = q_d - q$ is the position error, $(\hat{\bullet})$ denotes an estimation of $(\bullet)$, K_1 is a diagonal positive definite gain matrix and $q_d \in \Re^n$ is a desired trajectory in joint-space. Now by definition of

$$\delta_1(t) = \hat{K}^{-1}\hat{k}_m \left(\Delta_{Jm}\ddot{\theta} + \Delta_{Bm}\dot{\theta} + \Delta_K(\theta - q)\right) \tag{14}$$

With

$$\begin{aligned} \Delta_{Jm} &= k_m^{-1}J_m - \hat{k}_m^{-1}\hat{J}_m, \\ \Delta_{Bm} &= k_m^{-1}B_m - \hat{k}_m^{-1}\hat{B}_m \\ \Delta_K &= k_m^{-1}K - \hat{k}_m^{-1}\hat{K} \end{aligned} \tag{15}$$

Substituting (13) into (12), rearranging with some manipulation this leads to dynamic of the output tracking loop as

$$e + K_1 D^{-\lambda_1} e = \hat{K}^{-1}\hat{k}_m i_e - \delta_1(t) \tag{16}$$

By the last result, the design procedure is now to design a control input u, to realize the perfect current vector in (13), such that, the current error can be bounded by a constant. Toward this end, we may construct the control input in the form

$$u = K_p i_e + K_i D^{-\lambda_2} i_e \tag{17}$$

Where K_p and K_i are diagonal positive definite matrices. Without loss of generality, assume that, the voltage of every motor is limited to protect the motor against over voltages. Therefore, control law (17) is modified as

$$u(t) = v \qquad \text{for } |v| \le v_{\max} \tag{18}$$

$$u(t) = v_{\max}\,\mathrm{sign(v)} \qquad \text{for } |v| > v_{\max} \tag{19}$$

Where $v_{\max}$ is a positive constant called as the maximum permitted voltage of motor and v is expressed as

$$v = K_p i_e + K_i D^{-\lambda_2} i_e \tag{20}$$

Stability Proof:

Here, we will prove stability of the proposed approach. Toward this end, we make the following assumptions.

A1. The actuator dynamics can be linearly parameterized as the multiplication of a constant bounded parameter matrix $P \in \Re^{n\times m}$ with a vector $W(\theta,\dot{\theta},\ddot{\theta},q) \in \Re^m$, i.e.

$$k_m^{-1}J_m\ddot{\theta} + k_m^{-1}B_m\dot{\theta} + k_m^{-1}K(\theta - q) = I = P.W(\theta,\dot{\theta},\ddot{\theta},q) \tag{21}$$

Where

$$W(\theta,\dot{\theta},\ddot{\theta},q)=\begin{bmatrix}\ddot{\theta}\\ \dot{\theta}\\ \theta-q\end{bmatrix} \quad , \quad P=\begin{bmatrix}k_m^{-1}J_m\\ k_m^{-1}B_m\\ k_m^{-1}K\end{bmatrix}^T \tag{22}$$

A2. The external disturbance $\phi(t)$ is bounded as

$$\|\phi(t)\|\leq\phi_{\max} \tag{23}$$

Where $\phi_{\max}$ is a positive constant.

A3. It is possible to show that u, $\dot{\theta}$, i, $\dot{i}$ and therefore the left hand side of equation (7) is bounded in compact sets for all $t\geq 0$ as stated in [18]. For shortening of the paper, the explicit description of calculating these bounding functions are ignored here.

A4. The parametric uncertainties are bounded as

$$\|\Delta_{Jm}\|\leq\eta_1, \quad \|\Delta_{Bm}\|\leq\eta_2, \quad \Delta_K\leq\eta_3, \tag{24}$$

A5. Assume that, there exists a positive scalar denoted by μ that

$$\left\|L\dot{i}_d+Ri+k_b\dot{\theta}+\phi(t)\right\|<\mu \tag{25}$$

As can be seen from (18)-(20), the control law operate in two areas of $|v|\leq v_{\max}$ and $|v|>v_{\max}$. The tracking performance should be evaluated in both areas.

(a) ***Area of*** $|v|\leq v_{\max}$

Substituting (18) into (7), one obtains the dynamics for the current tracking loop as

$$K_p i_e+K_i D^{-\lambda_2} i_e=L\dot{i}+Ri+k_b\dot{\theta}+\phi(t) \tag{26}$$

Since the variables i, $\dot{i}$, $\dot{\theta}$ and $\phi(t)$ are bounded, then the RHS of (26) and so i_e are bounded. Therefore, boundedness of i_d can be achieved. From (22) it can be easily shown that, the constant matrix P is bounded. Thus, the vector of $W(\theta,\dot{\theta},\ddot{\theta},q)$ is bounded. By the same manipulation, similar to those utilized for (22) we can obtain a bounding function for 3 first terms in (13) as

$$W(\theta,\dot{\theta},\ddot{\theta},q)=\begin{bmatrix}\ddot{\theta}\\ \dot{\theta}\\ \theta-q\end{bmatrix} \quad , \quad P=\begin{bmatrix}\hat{k}_m^{-1}\hat{J}_m\\ \hat{k}_m^{-1}\hat{B}_m\\ \hat{k}_m^{-1}\hat{K}\end{bmatrix}^T \tag{27}$$

Therefore (14) is bounded since $\hat{K}^{-1}\hat{k}_m$ is a constant. Besides, by using **A4**, all terms in the RHS of

(16) are bounded as stated by (26) and (27). Since the input of (16) is bounded and K_1 is positive, system (16) is stable and therefore, e is bounded. Since the desired joint angle q_d is bounded, the bounded variables e imply that $q = q_d - e$ is also bounded. Moreover, from (11) we have

$$J_m \ddot{\theta} + B_m \dot{\theta} + K\theta = k_m i + Kq \tag{28}$$

System (28) is a second order limited system with positive gains J_m, B_m, K , and a limited input $k_m i + Kq$. This system is stable based on the Routh-Hurwitz criterion and implies that θ, $\dot{\theta}$ and $\ddot{\theta}$ are bounded. Therefore, the robotic system has the Bounded Input-Bounded Output (BIBO) stability. It must be emphasized that, due to stability of the proposed control scheme, we use the following lemma for Equations (16) and (26).

Lemma: The system is BIBO stable, if it is asymptotic stable. This requires that

$$-\arg\left(\frac{-1}{K_1}\right) > \frac{\lambda\pi}{2} \tag{29}$$

Proof: For stability analysis, generally, we derive the transfer function and then we study the location of the roots of the denominator. The transfer function is stable if the roots are in LHP [26-27]. Hence, Due to stability we consider the transfer function as the form:

$$G(s) = \frac{e(s)}{\eta(s)} = \frac{1}{1 + K_1 s^{-\lambda}} \tag{30}$$

Where $\eta(s)$ is the bounded lumped input signals. Having $s = re^{j\theta}$ for Stability $\left(\frac{\pi}{2} < \theta < \frac{3\pi}{2}\right)$, and equating the denominator of equation (30), we have

$$\arg\left(\frac{-1}{K_1}\right) = -\lambda\theta \tag{31}$$

That is:

$$-\arg\left(\frac{-1}{K_1}\right) > \frac{\lambda\pi}{2} \tag{32}$$

(b) Area of $|v| > v_{\max}$

To consider the convergence of current tracking error i_e in the area that $|v| > v_{\max}$, a positive definite function is proposed as

$$V = \frac{1}{2} i_e^T L i_e \geq 0 \tag{33}$$

By taking the time derivative of V one can obtain

$$\dot{V} = i_e^T L \dot{i}_e \tag{34}$$

Substituting control law (19) into (7) forms the closed loop system

$$L\dot{i} + Ri + k_b\dot{\theta} + \phi(t) = v_{\max}\mathrm{sign(v)} \tag{35}$$

Substituting (9) into (34) yields

$$\dot{V} = -i_e^T\,(L\dot{i}_d + Ri + k_b\dot{\theta} + \phi(t) - v_{\max}\mathrm{sign(v)}) \tag{36}$$

Now, according to **A5**, the sufficient condition to establish the convergence of $\dot{V} < 0$ is

$$v_{\max}\mathrm{sign(v)}{=}\mu\mathrm{sign(i_e)} \tag{37}$$

Proof: Substituting (37) into (36) yields

$$\dot{V} = -i_e^T\left(L\dot{i}_d + Ri + k_b\dot{\theta} + \phi(t) - \mu\mathrm{sign(i_e)}\right) \tag{38}$$

Notice that, to satisfy $\dot{V} < 0$, the assumption (5) is required, that is

$$\dot{V} \le \left\|i_e^T\right\|\left\|L\dot{i}_d + Ri + k_b\dot{\theta} + \phi(t)\right\| - \mu i_e^T\mathrm{sign(i_e)} \tag{39}$$

Since $i_e^T\,\mathrm{sign(i_e)}{=}\mathrm{i_e}$, thus

$$\dot{V} \le \left\|i_e^T\right\|\left(\left\|L\dot{i}_d + Ri + k_b\dot{\theta} + \phi(t)\right\| - \mu\right) \tag{40}$$

Equation (37) means that

$$v_{\max}{=}\mu \tag{41}$$

Therefore, the maximum voltage of motor should satisfy (40) for the convergence of current tracking error. From the closed loop system (35), we can obtain

$$L\dot{i} + Ri = v_{\max}\mathrm{sign(v)} - k_b\dot{\theta} - \phi(t) \tag{42}$$

The RHS of (42) is bounded as

$$\left\|v_{\max}\mathrm{sign(v)} - k_b\dot{\theta} - \phi(t)\right\| \le v_{\max} + k_b\dot{\theta}_{\max} + \phi_{\max} \tag{43}$$

Thus, the linear system (42) under the bounded input $v_{\max} + k_b\dot{\theta}_{\max} + \phi_{\max}$ obtains the bounded output i. Since i is limited and considering boundedness of i_e, we have a limit for i_d. Therefore, the boundedness of all control signals can be achieved by the same manipulation as same as before. These conditions together with **A1-A5** complete the proof of the closed-loop system stability. The block diagram of the proposed approach has been shown in Fig. 1.

A main advantage of the proposed approach is that the PI^{λ} controllers in outer and inner control loops are less sensitive to changes of parameters of a controlled system. This is due to the two extra degrees of freedom to better adjust the dynamical properties of a fractional order control system [17].

5. Computer simulation

In this section, we present the simulation results for the validity of the proposed controller. The simulation task is carried out based on a two DOF planer flexible joint robot driven by permanent magnet dc motors.

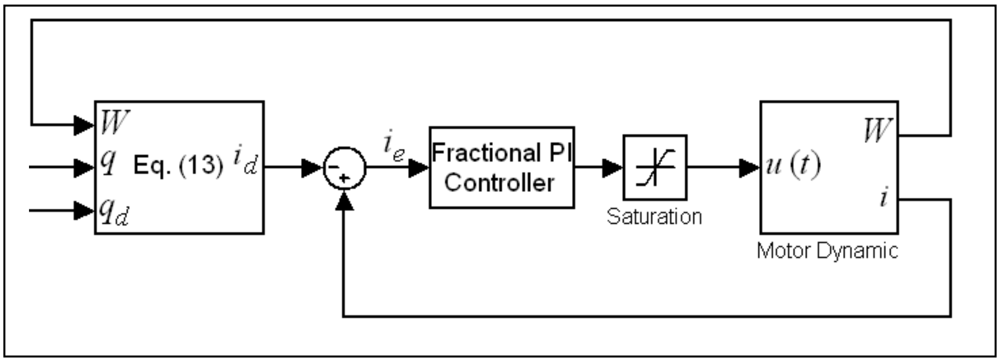

Figure1. The block diagram of proposed scheme

The dynamic model of the robot system can be described in the form of Equation (5) as

$$D(q)=\begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \ , \ C(q,\dot{q})\dot{q}=\begin{bmatrix} -2m_2l_1l_{c2}sin(q_2)\left(\dot{q}_1\dot{q}_2+0.5\dot{q}_2^2\right) \\ m_2l_1l_{c2}sin(q_2)\dot{q}_1^2 \end{bmatrix}$$

$$g(q)=\begin{bmatrix} (m_1l_{c1}+m_2l_1)\,gcos(q_1)+m_2l_{c2}gcos(q_1+q_2) \\ m_2l_{c2}gcos(q_1+q_2) \end{bmatrix} \tag{44}$$

$$d_{11}=m_2\left(l_1^2+l_{c2}^2+2l_1l_{c2}cos(q_2)\right)+m_1l_{c1}^2+I_1+I_2$$

$$d_{21}=d_{12}=m_2l_{c2}^2+m_2l_1l_{c2}cos(q_2)+I_2$$

$$d_{22}=m_2l_{c2}^2+I_2$$

where q_1 and q_2 are the angles of joints 1 and 2, m_1 and m_2 are the masses of links 1 and 2 respectively, l_1 and $l_2$are the lengths of links 1 and 2, I_i is the link's moment of inertia given in center of mass, l_{ci} is the distance between the center of mass of link and the ith joint, and g is the gravity acceleration. The manipulator dynamic parameters are defined as $l_1=l_2$=0.75m, $l_{c1}=l_{c2}$=0.375m, $m_1=m_2$=0.5kg and $I_1=I_2$=0.0234; Also, the exact-actuator dynamic model parameters are selected as J_m=diag(0.02,0.01), B_m=diag(5,4), R=diag(1,1), k_b=diag(1,1) and L=diag(0.025, 0.025), K=diag(1000, 1000) and k_m=diag (10,10). Due to comparison purpose, we will present the simulation results for both integer and fractional order control design. In order to keep fair comparison scenery, we have used the same numerical values for the proportional and integral gains. Due to observe the effect of the proposed controller, the robot is required to track a circle, characterized by 0.2m radius circle centered at (0.8m, 1.0m) in 10 seconds. We set the controller with $K_p=0.1,\ K_i=0.5$, $K_1=2,\ \lambda_1=0.5,\ \lambda_2=0.8$ and 80% parametric uncertainties in actuator dynamic model. The initial tracking error is considered zero in all simulations. By the last definitions, the desired and actual joint angles are drawn in Fig. 2, while the task-space trajectory tracking are observed in Fig. 3. Fig.4 shows the evolution of the link position error norm obtained for controllers (18)-(20). From Figure 4, it is observed that the fractional controller presents better tracking performance with respect to the integer one. The technical limits such as applied control voltage and performance in the current tracking loop are illustrated in Figs. 5 and 6, respectively. The simulation results clearly show the effectiveness of the proposed control scheme to robustly stabilize of the system, despite the imperfections in the actuator and robotic arm dynamics.

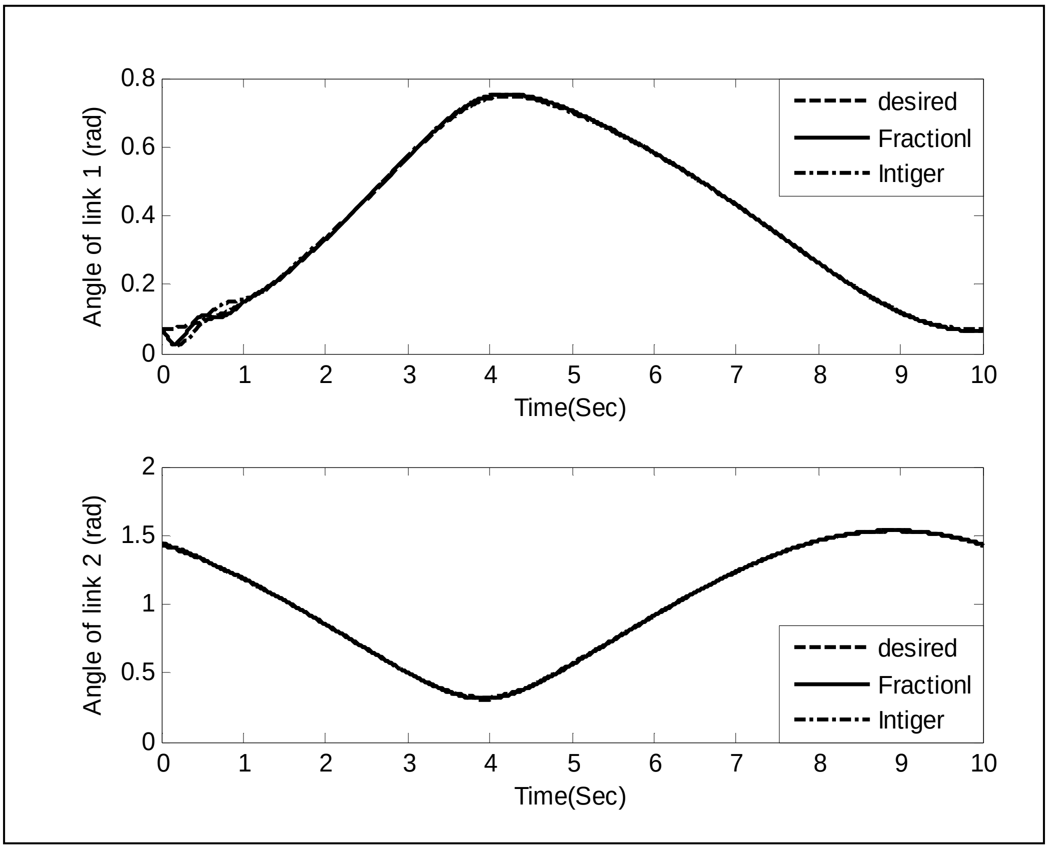

Figure2. The joint angular positions

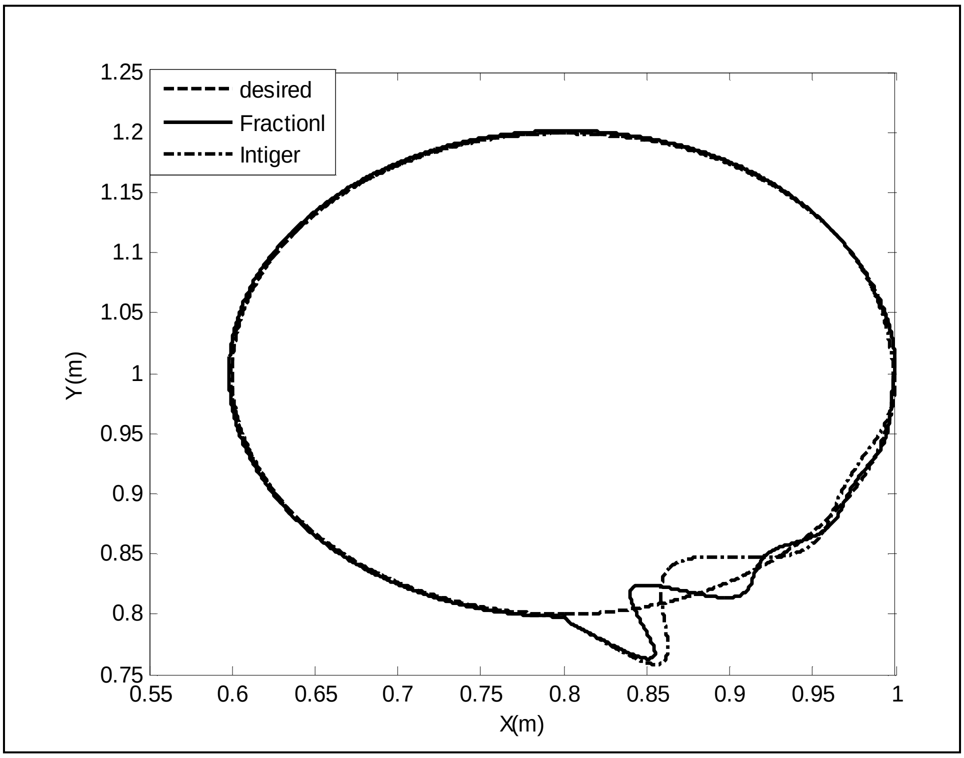

Figure3. Trajectory tracking in the task-space

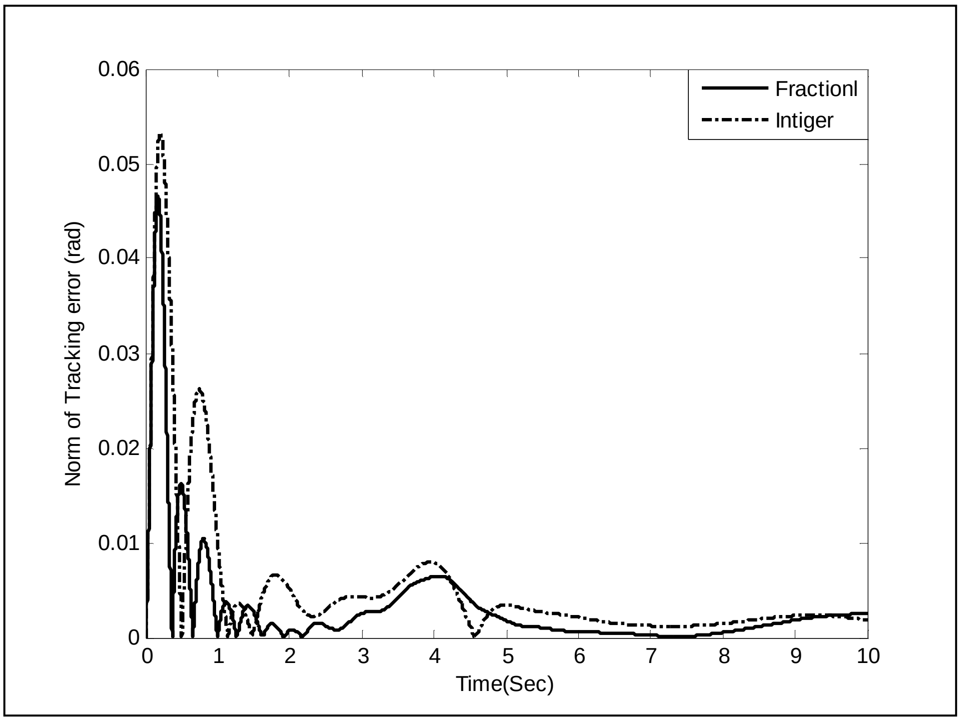

Figure4. Norm of tracking error

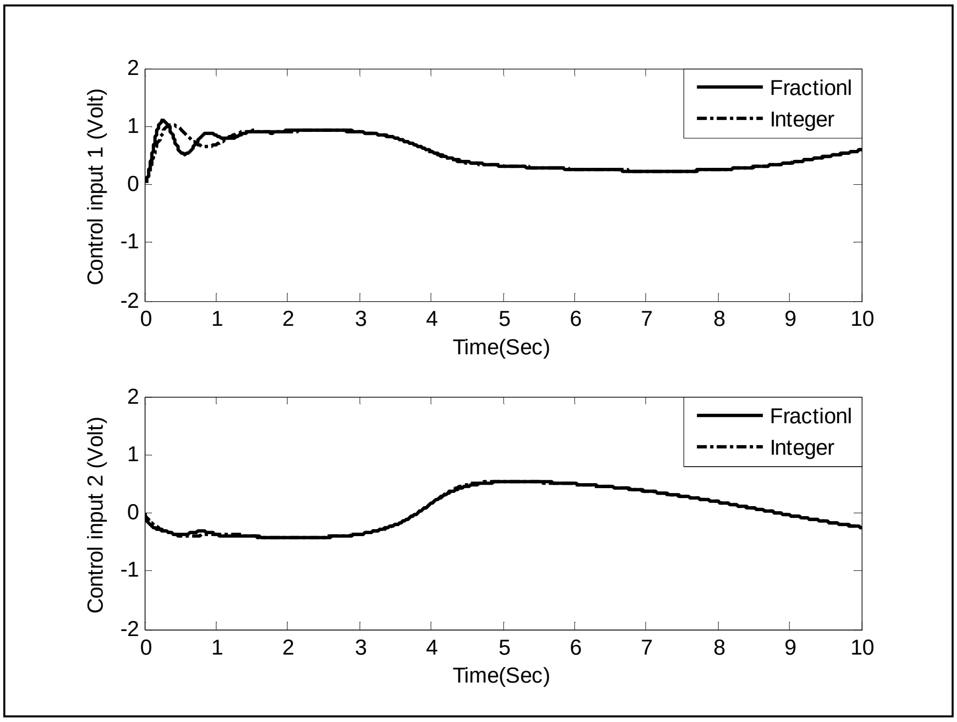

Figure5. The control input voltage

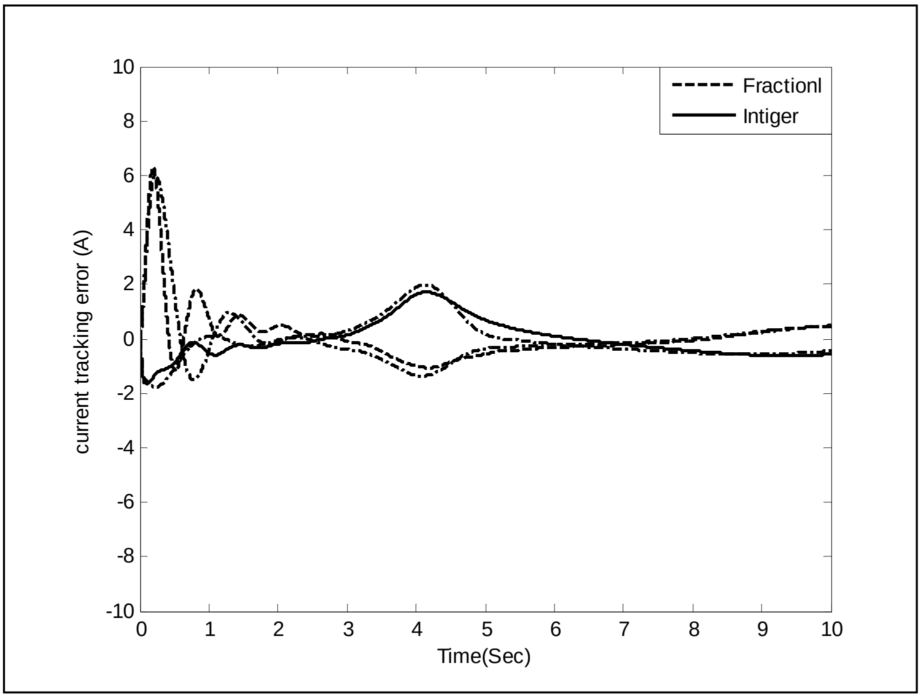

Figure6. Tracking error in current loop

6. Conclusion

This paper presents a robust fractional order control to cope with the tracking problem for robots with uncertainties in actuator dynamics, and needles to any knowledge about robot manipulator. It is shown that robotic system has the Bounded Input-Bounded Output (BIBO) stability. Simulation results show that the performance of the proposed controller is comparable with that of torque-level controllers.

7. References

[1] Chen, Y., Moore, K. L. and Bahl, V. 2004. Learning Feed Forward Control Using a Dilated B-Spline Network: Frequency Domain Analysis and Design. IEEE Trans. on Neural Networks. 15(2): 355-366.

[2] Jafarov, E. M., Parlakçı, M. N. A. and Istefanopulos, Y. 2005. A New Variable Structure PID-Controller Design for Robot Manipulators. IEEE Trans. on control systems Technology. 13(1): 122-130.

[3] Lin, C. K. 2006. Nonsingular Terminal Sliding Mode Control of Robot Manipulators Using Fuzzy Wavelet Networks. IEEE Trans. On Fuzzy Systems. 14(6): 849-859.

[4] Slotine, J. and Li, W. 1988. Adaptive Manipulator Control: A Case Study. IEEE Trans. On Automatic Control. 33(11): 995-1003.

[5] Assal, S. F. M., Watanabe, K. and Izumi, K. 2006. Neural Network-Based Kinematic Inversion of Industrial Redundant Robots Using Cooperative Fuzzy Hint for the Joint Limits Avoidance. IEEE/ASME Transaction on Mechatronics. 11(5): 593-603.

[6] Valavanis, K. P., Doitsidis, L., Long, M. and Murphy, R. R. 2006. A Case Study of Fuzzy-Logic-Based Robot Navigation. IEEE Robotic and Automation Magazine. 13(3): 93-107.

[7] Najim, K., Ikonen, E. and Gomez-Ramirez, E. 2008. Trajectory Tracking Control based on a Genealogical Decision Tree Controller for Robot Manipulators. International Journal of Innovative Computing, Information and Control. 4(1): 53-62.

[8] Liu, C., Cheah, C. C. and Slotine, J. J. E. 2006. Adaptive Jacobian Tracking Control of Rigid-Link-Electrically Driven Robots Based on Visual Task-Space Information. Automatica, 42: 1491-1501.

[9] Morris, A. S. 2001. Measurement and Instrumentation Principles. Butterworth-Heinemann. 3rd Edition. Elsevier.

[10] Fonseca Ferreira, N. M J. and Tenreiro Machado, A. 2003. Fractional-Order Hybrid Control of Robotic Manipulators. Proceedings of ICR. 393-398.

[11] Fonseca Ferreira, N. M., Tenreiro Machado, J. A. and Boaventura Cunha, J. 2005. Fractional-Order Position/Force Robot Control. Journal of Advance Computational Intelligence and intelligent informatics. 9(4): 379-386.

[12] Xue, D. and Zhao, Ch. and Chen, Y. Q. 2006. Fractional Order PID Control of a DC-Motor with Elastic Shaft: A Case Study. Proc. of the American Control Conference. 3182-3187.

[13] Petras, I. 2009. Fractional Order Feedback Control of a DC Motor. Journal of Electrical Engineering. 60(3): 117−128.

[14] Delavari, H., Ghaderi, R., Ranjbar, A. and Momani, S. 2010. Fuzzy Fractional Order Sliding Mode Controller for Nonlinear Systems. Communications in Nonlinear Science and Numerical Simulation. 15: 963-978.

[15] Fayazi, A. and Nabizadeh Rafsanjani, H. 2011. Fractional Order Fuzzy Sliding Mode Controller for Robotic Flexible Joint Manipulators. IEEE International Conference on Control and Automation. 1244-1249.

[16] Fateh, M. M. 2008. On the Voltage-Based Control of Robot Manipulators. International Journal of Control. Automation and Systems. 6(5): 702-712.

[17] Fateh, M. M. 2010. Robust Voltage Control of Electrical Manipulators in Task-Space. International Journal of Innovative Computing, Information and Control. 6(6): 2691-27000.

[18] Fateh, M. M. 2012. Nonlinear Control of Electrical Flexible-Joint Robots. Nonlinear Dynamic. 67(4): 2549-2559.

[19] Fateh, M. M. 2012. Robust Control of Flexible-Joint Robots Using Voltage Control Strategy. Nonlinear Dynamic. 67: 1525–1537.

[20] Oldham, K. B. and Spanier, J. 1974. The Fractional Calculus, Academic Press, New York.

[21] Miller, K. S. and Ross, B. 1993. An Introduction to the Fractional Calculus and Fractional Differential Equations. Wiley, New York.

[22] Podlubny, I. 1999. Fractional Differential Equations. Academic Press, San Diego, CA.

[23] Caldero, A. J., Vinagre, B. M. and Feliu, V. 2006. Fractional Order Control Strategies for Power Electronic Buck Converters. Signal Processing. 86: 2803-2819.

[24] Spong, M. 1980. Modeling and Control of Elastic Joint Robots. Journal of Dynamics Systems Measurement and Control. 109: 107-111.

[25] Oh, J. H. and Lee, J. S. 1998. Backstepping Control Design of Flexible Joint Manipulator Using Only Position Measurements. IEEE Conference on Decision and Control. 931-936.

[26] Bonnet, C. and Partington, J. R. 2000. Coprime Factorizations and Stability of Fractional Differential Systems. Systems and Control Letters. 41(3): 167-174.

[27] Rahimian, M. A. and Tavazoei, M. S. 2010. Comparing the Stability Regions for Fractionl-Order PI Controllers and Their Integer-Order Approximations. IEEE Conference on Decision and Control. 720-725.