Task-space Control of Electrically Driven Robots

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Abstract

Actuators of robot operate in the joint-space while the end-effect or of robot is controlled in the task-space. Therefore, designing a control system for a robotic system in the task-space requires the jacobian matrix information for transforming joint-space to task-space, which suffers from uncertainties. This paper deals with the robust task-space control of electrically driven robot manipulators. In conventional robust control approaches, the uncertainty upper bound is required to design the control law. This type of controller design is conservative that may increase the amplitude of the control signal and damage the system. Moreover, calculation of this bound requires some feedbacks of the system states which may be expensive. The novelty of this paper is addressing a robust control law in which the lumped uncertainty is modeled by a differential equation. The control design is simple, robust against uncertainties, and less computational. Simulation results verify the effectiveness of the proposed control approach applied on a two-link robot manipulator driven by geared permanent magnet DC motors.

Keywords

Model-free Control, Electrically Driven Robot, Differential Equations, Robust Control

1 Introduction

In the last few decades, task space adaptive/robust controls of robot manipulators have been the focus of widespread researches [1–3]. The reason for the importance of robust and adaptive control may be their efficiency in overcoming the uncertainty originated from mismatch between the nominal and actual models. External disturbances, un-modeled dynamics and parametric uncertainty are the main sources of uncertainty in control engineering which can seriously degrade the controller performance.

In earlier methods of robust control [4-5], the controller is designed based on the nominal model of the system. Then, a robustifying term is added to the control law to compensate the uncertainty and the value of this term is determined using a Lyapunov stability analysis. Variable structure control (VSC) is such a controller type. In these approaches, the uncertainty upper bound is required to the controller design and guarantee the system stability. Usually, this bound is a function of the system states and the upper bound of external disturbance. Thus, all the required feedbacks should be available and the upper bounds of parametric uncertainty and external disturbances should be known in advance. In addition to these, there is yet another problem. Variable structure control is a powerful strategy against external disturbances, quickly varying parameters and un-modeled dynamics. However, a conservative design may be obtain, since the VSC scheme should be design to treat the worst situation of uncertainties [6].

In the case of adaptive control, linear parameterization of the robotic motion equations is necessary [1-5]. Therefore, the manipulator motion equation should be completely modeled in order to identify the regressor matrices. Furthermore, it requires persistent excitation condition of the reference input signal due to the convergence of the parameter's vector, and slow behavior of the dynamic system. This problem becomes hypersensitive especially for higher degree of freedom (DOF) robot manipulators. Furthermore, they are unable to handle unstructured uncertainty and external disturbances adequately [7].

Alternatively, fuzzy/neural network based control methods are also known as effective and robust approaches for uncertain systems [8-12]. Neural networks provide powerful abilities such as adaptive learning, parallelism, fault tolerance, and generalization to the fuzzy controller. However, it is very difficult to guarantee the stability and robustness of neural network control systems. In addition, some fuzzy/neural networks methods require predefined and fixed fuzzy rules or NN structure, which reduce the flexibility of the controller [13].

Recently, some regressor-free adaptive approaches have been presented [14-21] in which uncertainties have been approximated using the Fourier series. On the basis of Lyapunov stability, some adaptation laws are derived for adjustment of the Fourier series coefficients. According to [15], other orthogonal functions such as Legendre and lagure polynomials can also approximate continuous time functions with an arbitrary accuracy.

The rest of this paper is organized as follows. In section 2 we recall nonlinear dynamic description of the robotic manipulator actuated by permanent magnet DC motor. The overall control structure of the proposed controller will be studied in section 3 and the closed-loop system stability is then established. In section 4 a simulation study will be presented to show the effectiveness of the proposed control approach. Finally, concluding remarks are given in section 5.

2 Mathematical Models

The dynamic equations of the rigid-link electrically driven robot manipulator with n degrees of freedom can be written as follows

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau_l \tag{1}$$

$$RK_{m}^{-1}J_{m}r^{-1}\ddot{q} + (RK_{m}^{-1}B + K_{b})r^{-1}\dot{q} = u(t) - RK_{m}^{-1}r\tau_{l}$$
⁽²⁾

where $q \in \Re^n$ is the vector of join positions, $D(q) \in \Re^{n \times n}$ is the symmetric positive definite inertia matrix, $C(q, \dot{q})\dot{q} \in \Re^n$ is the vector of Coriolis and centrifugal torques, $g(q) \in \Re^n$ denotes the gravitational torque vector, $\tau_l \in \Re^n$ is the vector of load torques applied on motor shaft, $R \in \Re^{n \times n}$ is the actuator resistance matrix, $K_m \in \Re^{n \times n}$ is the motor torque constant matrix, $J_m \in \Re^{n \times n}$ is a diagonal matrix of the lumped actuator rotor inertia, $B \in \Re^{n \times n}$ denotes a diagonal matrix of the lumped actuator state, $K_b \in \Re^{n \times n}$ is the matrix characterizing the back EMF constants of the actuator, $u(t) \in \Re^n$ denotes the vector of armature voltages, and r is an $n \times n$ transmission matrix given as follows

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$$\dot{q} = r\dot{\theta} \tag{3}$$

With $\dot{\theta} \in \Re^n$ denoting the vector of motor angular velocity. Combining equations (1) and (2) yields the following overall dynamic of electrically driven robot

$$\overline{D}(q)\ddot{q} + \overline{C}(q,\dot{q}) = u(t) \tag{4}$$

Where

$$\overline{D}(q) = RK_m^{-1}(J_m r^{-1} + rD(q))$$

$$\overline{C}(q, \dot{q}) = (RK_m^{-1}(Br^{-1} + rC(q, \dot{q})) + K_b r^{-1})\dot{q} + RK_m^{-1}r(g(q) + F(\dot{q}) + T_l(t))$$
(5)

2.1 Kinematic analysis

With respect to *n*-joint coordinates q, and *m* task coordinates h, the kinematics of the manipulator can be described with the following equations [22]:

$$h = \phi(q) \tag{6}$$

$$\dot{h} = J(q)\dot{q} \tag{7}$$

Where ϕ is an *m*-dimensional vector function representing direct kinematics, $J(q) \in \Re^{m \times n}$ is the Jacobian matrix from joint space to task space. With this in mind, the robotic equation (4) can be represented in task space coordinates based on the following relationship:

$$\ddot{q} = J(q)^{-1}(\ddot{h} - \dot{J}(q)J(q)^{-1}\dot{h})$$
(8)

Substituting (8) into (4) yields

$$M(h)\dot{h} + H(h,\dot{h}) = v(t) \tag{9}$$

Where

$$M(h) = J(q)^{-T} \overline{D}(q)J(q)^{-1}$$

$$H(h,\dot{h}) = (J(q)^{-T} \left(\overline{C}(q,\dot{q}) - \overline{D}(q)J(q)^{-1}\dot{J}(q) \right) J(q)^{-1} \dot{h} + J(q)^{-T} \overline{G}(q)$$
(10)

And $v(t) = J(q)^{-T}u(t)$ represent the control input in the task-space. Please note that in this paper we assume that m = n.

3 Control Design

Concerning the former, a robust task-space control strategy is developed. Suppose that $M_{q,k}(h)$ and $H_{q,k}(h,\dot{h})$ represent the known terms of M(h) and $H(h,\dot{h})$, respectively. Furthermore, $M_{q,u}(h)$ and $H_{q,u}(h,\dot{h})$ represent the unknown terms of M(h) and $H(h,\dot{h})$, respectively. With this in mind, the inner nonlinear control law is proposed as follows:

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$$v(t) = M_{q,k}(h)v'(t) + H_{q,k}(h,h)$$
(11)

Substituting (11) into (9) yields

$$\ddot{h} = v'(t) + \eta(h, \dot{h}, v'(t))$$
(12)

Where

$$\eta(h,\dot{h},v'(t)) = (M^{-1}(h)M_{q,k}(h) - I)v'(t) + M^{-1}(h)(H_{q,k}(h,\dot{h}) - H(h,\dot{h}))$$
(13)

Represents the lumped uncertainty, and I denotes the identity matrix. Equation (12) can be stabilized using the following linear control law

$$v'(t) = a_0 V - a_1 h - a_0 h \tag{14}$$

By substituting (14) in to (12) we have

••

$$h + a_1 h + a_0 h = a_0 V + \eta(h, h, v'(t))$$
(15)

Now, we design an algorithm to adjust the control input V such that the task-space tracking error is reduced in the presence of uncertainties. To that end, a desired closed-loop differential equation is defined as

$$\ddot{h}_{d} + a_{1}\dot{h}_{d} + a_{0}h_{d} = a_{0}V_{d}$$
(16)

Where V_d and h_d are desired path for the end-effector and desired output in the task space, respectively. As a result, using (16), the control gains can be selected in a more systematic manner. Once the motion specification is given in terms of the desired trajectory V_d in the task space, then the motion control objective in operational space is to achieve

$$\lim_{t \to \infty} \xi(t) = 0 \tag{17}$$

Where

$$\xi(t) = h(t) - h_d(t) \tag{18}$$

Denotes the task space position error. Now, the system error dynamics can be obtained by deducing (16) from (15) as below

$$\ddot{\xi}(t) + a_1 \dot{\xi}(t) + a_0 \xi(t) = a_0 V' + \eta(h, \dot{h}, v'(t))$$
⁽¹⁹⁾

Where

$$V'(t) = V(t) - V_d(t) \tag{20}$$

The linear differential equation (19) can be represented as the following state-space form

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$$\dot{x}(t) = Ax(t) + BV' + \Delta(h, \dot{h}, v'(t)))$$
(21)

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$$y(t) = Cx(t) \tag{22}$$

Where $x = [\xi^T(t) \quad \dot{\xi}^T(t)]^T$ indicates the state vector,

$$A = \begin{bmatrix} 0 & I \\ -a_0 & -a_1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ a_0 \end{bmatrix}, \quad C = \begin{bmatrix} I & 0 \end{bmatrix}$$
(23)

And $\Delta(h, \dot{h}, v'(t))) = \begin{bmatrix} 0\\ \eta(h, \dot{h}, v'(t)) \end{bmatrix}$ represents the lumped uncertainty injected to the system. Three

assumptions may be summarized as follows:

Assumption 1. The manipulator is operating away from any singularity.

Assumption 2. The desired reference trajectory V_d is assumed to be uniformly continuous, and has bounded and uniformly continuous derivatives up to a necessary order.

Assumption 3.If $\Delta(h, \dot{h}, v'(t))$ represents uncertainties that include inertia, coriolis/centrifugal forces, gravity force and kinematic uncertainties, $\Delta(h, \dot{h}, v'(t))) \in T$, where *T* is compact set.

Under these assumptions, and to determine the influence of input V' on the tracking error, (21) and (22) are differentiated p-times to obtain

$$\dot{\wp}(t) = A\wp(t) + B\vartheta(t) + \delta(t) \tag{24}$$

$$y^{(p)}(t) = C_{\mathcal{O}}(t) + \sum_{j=1}^{p} \alpha_j y^{(p-j)}$$
(25)

Where

$$\wp(t) = x^{(p)} - \sum_{j=1}^{p} \alpha_j x^{(p-j)}$$
(26)

$$\vartheta(t) = V^{\prime(p)} - \sum_{j=1}^{p} \alpha_j V^{\prime(p-j)}$$
⁽²⁷⁾

And $\delta(t)$ is the lumped approximation errors, which can be assumed to be negligible. In other words, it has been supposed that, uncertainties can be approximated by a *p*- th order ordinary differential equation of the form

$$\Delta^{(p)}(h,\dot{h},v'(t))) - \sum_{j=1}^{p} \alpha_{j} \Delta^{(p-j)}(h,\dot{h},v'(t))) = 0$$
(28)

Where order *p* reflects the dynamic structure of $\Delta(h, \dot{h}, v'(t)))$, and α_j 's are zero or positive real scalars to be designed. This approximation opens the possibility to obtain the corrective input $\vartheta(t)$ to include it into the controller to compensate the perturbation present in the plant. Now, we can define a coordinate transformation represented by

$$Z = \begin{bmatrix} y & \dot{y} & \dots & y^{(p-1)} & \wp \end{bmatrix}^T$$
(29)

Then, the system state equation (24) and (25), in new coordinates will be

$$Z = \Lambda Z + \Psi \vartheta(t) + \Xi \tag{30}$$

Where

$$\Lambda = \begin{bmatrix}
0 & 1 & 0 & \cdots & 0 & 0 \\
0 & 0 & 1 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \cdots & \cdots & 1 & 0 \\
\alpha_{p} & \alpha_{p-1} & \cdots & \cdots & \alpha_{1} & C \\
0 & 0 & \cdots & \cdots & 0 & A
\end{bmatrix},$$

$$\Psi = \begin{bmatrix}
0 & \cdots & 0 & B^{T}
\end{bmatrix}^{T},$$

$$\Xi = \begin{bmatrix}
0 & \cdots & 0 & \delta(t)
\end{bmatrix}^{T}$$
(31)

And $\vartheta(t)$ is defined as

$$\vartheta(t) = -\mu Z \tag{32}$$

Such that the satisfactory performance of task space trajectory tracking is obtained. Now, substituting (32) into (30) yields

$$\dot{Z} = (\Lambda - \Psi \mu) Z + \Xi \tag{33}$$

3.1 Stability Analysis

Choose the following Lyapunov candidate function:

$$V = Z^T P Z \tag{34}$$

Where $P = P^T$ is a positive definite matrix satisfying the Lyapunov equation $(\Lambda - \Psi \mu)^T P + P(\Lambda - \Psi \mu) + Q = 0$ with *Q* representing some positive definite matrix. Taking the time derivative of (34) along the trajectories of (33), we have

$$\dot{V} \leq -\lambda_{\min}(Q) \left\| Z \right\|^2 + 2\lambda_{\max}(P) \left\| Z \right\| \left\| \Xi \right\|$$
(35)

Where $\lambda_{\min}(Q)$, and $\lambda_{\max}(P)$ denote the minimum, and maximum eigen values of Q and P, respectively.

Remark 1: Suppose appropriate models are used and the approximation error can be ignored. Then \dot{V} is a negative definite matrix, and asymptotically stability of the closed-loop system can be approved.

Remark 2: If the approximation error cannot be ignored, utilizing further manipulations of (35) we have

$$\dot{V} \leq \|Z\| \left(-\lambda_{\min}(Q) \|Z\| + 2\lambda_{\max}(P) \|\Xi\| \right)$$
(36)

The last inequality will be negative definite whenever

$$\|Z\| > \frac{2\lambda_{\max}(P)\|\Xi\|}{\lambda_{\min}(Q)}$$
(37)

This implies that $(y, \dot{y}, \dots, y^{(p-1)}, \wp)$ is uniformly bounded.

Using Assumptions (1) and (2) and boundedness of ||Z||, it can be concluded from the closed-loop stability that the task-space velocity vector \dot{h} is bounded. Since $\dot{q} = J(q)^{-1}\dot{h}$, it follows that $q = \int_{0}^{t} J(q)^{-1}\dot{h}dt + q(0)$. Therefore, for finite operational times, the joint position q is bounded.

4 Simulation Results

In this section, we present the simulation results for the proposed control scheme. The simulation task is carried out based on a two degree-of-freedom planer robot driven by geared permanent magnet dc motors. The dynamic model of the robot system can be described in the form of (1) as

$$D(q) = \begin{bmatrix} m_{1}l_{1}^{2} + m_{2}\left(l_{1}^{2} + l_{2}^{2} + 2l_{1}l_{2}cos(q_{2})\right) & m_{2}\left(l_{2}^{2} + l_{1}l_{2}cos(q_{2})\right) \\ m_{2}\left(l_{2}^{2} + l_{1}l_{2}cos(q_{2})\right) & m_{2}l_{2}^{2} \end{bmatrix},$$

$$C(q, \dot{q})\dot{q} = \begin{bmatrix} -2m_{2}l_{1}l_{2}sin(q_{2})(\dot{q}_{1}\dot{q}_{2} + 0.5\dot{q}_{2}^{2}) \\ m_{2}l_{1}l_{2}\dot{q}_{1}^{2}sin(q_{2}) \end{bmatrix},$$

$$G(q) = \begin{bmatrix} (m_{1} + m_{2})l_{1}gcos(q_{1}) + m_{2}l_{2}gcos(q_{1} + q_{2}) \\ m_{2}l_{2}gcos(q_{1} + q_{2}) \end{bmatrix}$$
(38)

The exact dynamic model parameters of the actuator and manipulator are selected as $B = diag(10^{-4}, 10^{-4})[N.m.s/rad], R = diag(1,1)[\Omega], l_1 = 0.4318m, l_2 = 0.6491m m_1 = 3kg, m_2 = 1kg$, $J_m = diag(10^{-6}, 10^{-6})[kg.m^2], K_b = K_m = diag(0.03, 0.03)[N.m/A],$ and r = diag(0.01, 0.01). Furthermore, the Jacobean matrix of the robotic manipulator are given as follows:

$$J(\theta) = \begin{bmatrix} -l_1 \sin(q_1) - l_2 \sin(q_1 + q_2) & -l_2 \sin(q_1 + q_2) \\ l_1 \cos(q_2) + l_2 \cos(q_1 + q_2) & l_2 \cos(q_1 + q_2) \end{bmatrix}$$
(39)

The desired end-effector trajectory is a circle with the radius of 0.15m, as:

$$\begin{cases} x_d(t) = 0.35 + 0.15\cos(\pi t) \\ y_d(t) = 0.35 + 0.15\sin(\pi t) \end{cases}$$
(40)

The control parameters were selected as $a_0 = [\operatorname{diag}_{2\times 2}(8000)], a_1 = [\operatorname{diag}_{2\times 2}(80)]$, p = 1 and $\mu = [\operatorname{diag}_{2\times 2}(236.465) \operatorname{diag}_{2\times 2}(5.6916) \operatorname{diag}_{2\times 2}(0.0245)]$. These choices guarantee well-damped behavior of the closed-loop system. It is further assumed that there exist 30% uncertainties in the system parameters. To see the performance of the proposed controller, Figure 1 shows the actual motions in the x-y planes tracked by the proposed method. Figure 2 indicates tracking error in the task space. Figure 2 shows that the proposed control strategy can effectively attenuate the lumped uncertainties, and improve the tracking accuracy of the end-effector. The control efforts acting on the related joints are shown in Figure 3.



Figure1. Following trajectory of the circle motion



Figure3. Motor voltages

As shown in this Figure, motor voltages are smooth and do not include high frequency vibrations. From the above simulation results, it may be concluded that the control strategy can achieve a favorable control performance and has high robustness.

5. Conclusion

A robust task-space control strategy has been proposed for electrically driven robots. The control law has two parts. The inner control loop deletes the nonlinear dynamics of integrated actuated

manipulator dynamics and the outer loop stabilizes the overall control system. The system stability has been verified by the lyapunov method. Simulation results have demonstrated the effectiveness of the control. The advantage of the proposed schemes is simplicity of the control laws, which do not contain explicit information on the system, actuator and Jacobean matrix.

6. References

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