# Optimal Sizing and Location of Distributed Generation Units to Improve Voltage Stability and Reduce Power Loss in the Distribution System

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**Abstract** – Voltage stability improvement, when overloading of current distribution network is leading it toward voltage collapse, has been deemed as an important criteria in networks including distributed generation units (DGs) as part of it. In this condition, DGs can be leveraged to improve reactive power through voltage control and thus, stabilizing network with respect to the voltage. In this paper, a forward-backward power flow with respect to a voltage stability index (VSI), optimal size and location of DGs within the network are to found. The proposed method are simulated on the 34-bus IEEE standard in order to cross validate the results.

**Keywords**: voltage stability, forward-backward power flow, distributed generation, distribution network.

## I. Introduction

In practice, voltage stability of a system is a sign for maximum supplied electrical demand in the system. Thus, researchers have found several measures to assess the voltage stability [1-4]. Recently, there has been presented a number of compensation of network passivity by adding electrical generators, such that the overall system reliability is adjusted and the voltages are kept within their permissible values [5]. These generators are parts of a DGs, whose plays and applications in the current and future networks are expected to be growing. Using optimal DGs have been shown that they are able to improve the network, from the viewpoints of cost and strategic utilization, in which problems related to low reliability are to be solved and enhancement of performance are expected [6].

The Electric Power Research Institute (EPRI) has defined a distributed generation unit (DG) as a generator with capacity from a few kW up to 50 MW [7]. Also, a few member of the technology has been recognized, such as, micro-turbines, small liquid or gas turbines, solar cells, wind and sun energy. Among the advantages of DGs we can refer to as power loss reduction, voltage profile enhancement, increased the total energy efficiency, increased power quality, reliability and security. In order to come up with all the advantages just mentioned, the size and location of each DG unit, as the two design optimization variables, worked on using methods such as analytical studies [8-9], heuristics [10-12], artificial intelligence (AI) and genetic algorithms (GA)[13-15].

In the related studies, the cost functions, generally, have been considered to be power loss, voltage profile improvement and reliability increase; most studies ignored voltage stability improvement of the network which are likely of high significant in diagnosing the loading conditions before occurrence of voltage collapses over the network and in the presence of DGs. In [9], localization methods such as Lagrangian, second-order gradient and sensitivity analysis methods have been used. Also, a few paper have used the voltage stability as a constraint in their cost functions. Hedayati *et al.*, have used a cost function with voltage stability in the constraints, and found the right location to install DGs with the right capacities in the related buses. Moreover, they have explained effects of the different DGs' technologies on the static voltage stability and emphasized that using the analytical tools for voltage stability studies is a must in the optimization programs [10].

In [12] , a combination of GA and PSO optimization methods have been applied to find the optimal location and size of the DGs, when constraints such as voltage stability, power loss, voltage profile improvement and regularization are considered in their objective function. In [13], also, there is another combinatorial method using GA and other techniques to optimally design DG, where the objective functions have been minimization of power loss and minimum acceptable levels of reliabilities. There are studies which have tried to survey varieties of methods based on GA in order to location and size optimization of DGs with

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different objective functions [14-15]. Besides, there is another study that tries optimization to locate DGs in the distribution networks based on analysis of the continuous power flow and recognition of the most sensitive buses to the voltage collapses [16].

In this paper, a new formula for voltage stability constraint has been proposed to find the critical lines ending to critical buses important in voltage collapse, when we are searching for optimal location and size of DGs in a distribution network. The solution of the problem is based on the iterative analysis of forward/backward power flow of the radial systems, trying to recognize the most critical lines ending to the lines which are exposed to the voltage collapse. The algorithm has been tested on an IEEE 34-bus standard system and results were shown using DIgSILENT software.

## II. An overview of the voltage stability

Voltage Stability, according to the IEEE power system stability subcommittee, is defined as [17]:

"the voltage stability of a system refers to the ability of that system to maintain voltage at acceptable levels, and when there is a sudden demand of load in the network, the system must be capable of concurrently control both voltage and power."

In order to simplify, the voltage stability concept will be analyzed using a system comprising 2 equivalent buses, an equivalent impedance $Z_s$ and electric motive force (EMF) $V_s$ (see fig. 1) [18].

The received voltage at the second bus can be stated as,

$$\vec{V}_r = \vec{V}_s - \vec{Z}_s * \vec{I}_r \tag{1}$$

where $\vec{Z}_s$ is the equivalent impedance of each line from the beginning of the feeder to the bus and can be calculated as,

$$\vec{Z}_s = \frac{(\vec{S}_s - \vec{S}_r) * |V_s|^2}{(P_s^2 + Q_s^2)} \tag{2}$$

When the load power increases, amplitude of $Z_r$ decreases and the current amplitude increases; thus, because of loading, which lead to decrease of voltage and the apparent power in the receiver bus, they can be written as,

$$V_r = \frac{Z_r}{Z_s} \frac{V_s}{\left[1 + (Z_r/Z_s)^2 + 2(Z_r/Z_s)\cos(\beta)\right]^{0.5}} \tag{3}$$

$$S_r = \frac{Z_r}{Z_s} \frac{(V_s)^2/Z_s}{\left[1 + (Z_r/Z_s)^2 + 2(Z_r/Z_s)\cos(\beta)\right]} \tag{4}$$

Where $\beta = \theta - \varphi$ is the phase difference between impedances $Z_s$ and $Z_r$.

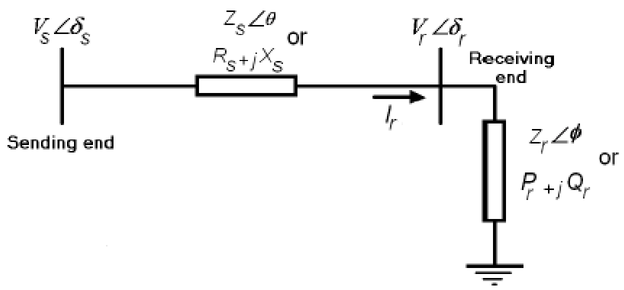


**Fig. 1:** Demonstration of the system as a 2-bus single line diagram

Figs 2 and 3 illustrate graphically the behavior of (3) and (4) for difference load states and $|Z_s| = 1$ and $|V_s| = 1$. As we can see from the figures, the maximum transmitted power is achieved when $|Z_r/Z_s| = 1$; this is also the primary condition for prevention of voltage collapse. In general, the fraction $Z_r/Z_s$ must be always greater than one. This means that abrupt increase in the size of $Z_s$ or rapid decrease in the size of $Z_r$ must be such that they do not lead to the instability of the whole network. Also, in order to have an acceptable response, amplitude of the $V_r$ should be always very high.

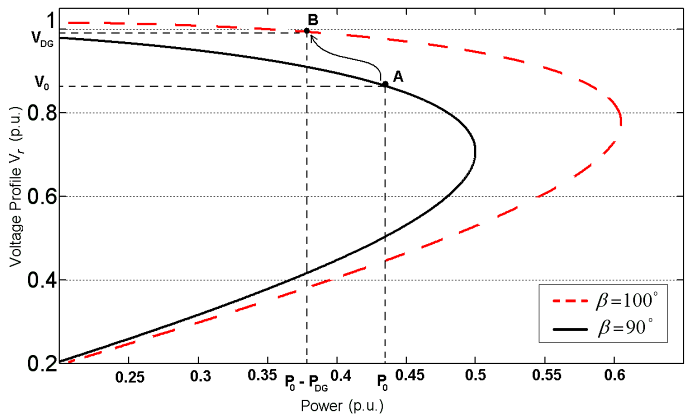


**Fig. 2:** Voltage vs. power curve in the 2-bus circuit system

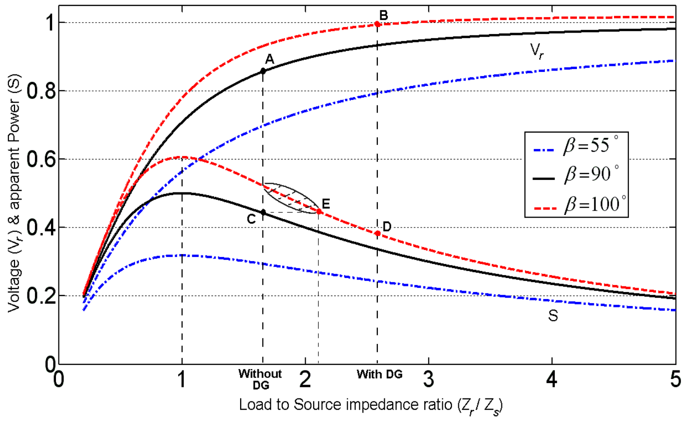


**Fig. 3:** voltage and apparent power curves vs. the fraction $\boldsymbol{Z_r/Z_s}$

In figure 2, we observe that injection of $P_{DG}$ moves voltage from point *A* to Point *B*, which is an improvement. However, there is still an open question: Is this selected size of DG right? Referring to figure 3, we notice that placement of an undesired value of DG moves down the voltage profile from point *C* to point *D*. furthermore, it can be noted that the best size of DG, effectively, is dependent on the power loss as well as the voltage stability index.

## III. forward-backward power flow analytical method

In the radial distribution systems, due to the radial structure of the networks and large value of the relation $R/X$ in the involving lines, usage of conventional approaches Newton-Raphson and Gauss-Seidel are not fast and efficient [19]. Therefore, in this study, a new technique based on forward-backward power flow has been proposed to speed up finding the solutions of power flows in distribution networks [20].

### *A. The backward propagation*

In the backward direction, assuming nominal voltage as the primary values, current of each branch of each section will be calculated (given that power consumption of each node is known). In this case, values of voltage will be kept constant and current of each branch leading to the bus will be calculated as a backward process until the beginning of the feeder using,

$$\vec{I}_k = \frac{\vec{V}_m - \vec{V}_k}{\vec{Z}} = \frac{\vec{P}_k - j\vec{Q}_k}{\vec{V}_k^*} \quad (5)$$

in which, $\vec{I}_k$ is the current of two buses *m* and *k*, and $V_m$ and $V_k$ are, respectively, voltages of the power transmitter bus *m* and power receiver bus *k*; $\vec{Z} = R_k + jX_k$ is the impedance between the two buses *m* and *k*; $P_k$ is the collective active power of loads connected to the bus k and thereafter plus the lines active power loss $Q_k$ , likewise, is the collective reactive power of the loads connected to the bus *k* and thereafter plus the lines reactive power loss.

### *B. The forward propagation*

In this approach, the goal is to find voltage of each node, starting from the beginning of the feeder, which is kept constant at a given value, up to the end of the network. The current which are already calculated will be maintained as before; as we move forward, the voltage of the next bus will be calculated based on,

$$\vec{V}_k = \vec{V}_m - \vec{Z} * \vec{I}_k \quad (6)$$

### *C. convergence condition*

In the proposed method, calculations will be repeated one after another until the difference between the current voltage amplitude and the previous one is less than a given threshold. When the power flow converges, voltage, and thus active and reactive powers and power losses in the network will be all calculated, respectively.

## IV. Voltage stability index formulation in presence of DG

For Although the voltage instability analysis is a dynamical problem in power systems, however, in order to estimate system performance under load with respect to knee point of the P-V curve, usage of static analyses are also acceptable.

Using the power flow *eq.* (1) and breaking it into real and imaginary parts, we have,

$$V_s V_r \cos(\delta) = |V_r|^2 + [R_s(P_r - P_{DG}) + X_s Q_r] \quad (7)$$

$$V_s V_r \sin(\delta) = [X_s(P_r - P_{DG}) - R_s Q_r] \quad (8)$$

in which, $\delta = \delta_s - \delta_r$ is the phase difference between the primary bus in feeder and the bus which receives the power. Eliminating the phase $\delta$ from both (7) and (8), we have,

$$|V_r|^4 + \{2[R(P_r - P_{DG}) + XQ_r] - |V_s|^2\}|V_r|^2 + \{(R^2 + X^2)[(P_r - P_{DG})^2 + Q_r^2]\} = 0 \quad (9)$$

and then

$$|V_r|^2 = \frac{\{|V_s|^2 - 2[R(P_r - P_{DG}) + XQ_r]\} \pm \sqrt{\Delta}}{2} \quad (10)$$

in which

$$\Delta = \{2[R(P_r - P_{DG}) + XQ_r] - |V_s|^2\}^2 - 4\{(R^2 + X^2)[(P_r - P_{DG})^2 + Q_r^2]\}$$

In order to have desired solution in the receiver bus, $\Delta \geq 0$ needs to be satisfied. With some mathematical manipulation, it can be seen that

$$\left\{|V_s|^2 - 2[R(P_r - P_{DG}) + XQ_r]\right\}^2 \geq 4\left\{(R^2 + X^2)\left[(P_r - P_{DG})^2 + Q_r^2\right]\right\} \quad (11)$$

Thus, we can write,

$$L_k = \frac{2\sqrt{\left\{(R^2 + X^2)\left[(P_r - P_{DG})^2 + Q_r^2\right]\right\}}}{\left||V_s|^2 - 2\{R(P_r - P_{DG}) + XQ_r\}\right|} \leq 1 \quad (12)$$

where $L_k$ is defined as voltage stability index for the weakest lines connected to the critical buses, and when this index equals one, voltage collapse will occur at the corresponding bus. The farther the index is away from one, the lower will be the effect of instability on the bus.

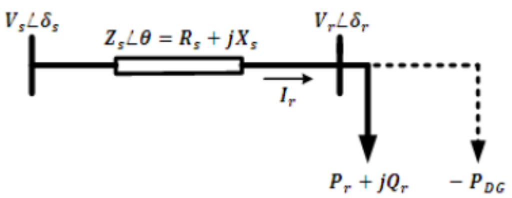


**Fig. 4:** schematic diagram of the 2-bus system in the presence of DG.

## V. Calculation of sizing and location of DG

In order to take the most out of them, choosing the best size and location of installation of the DGs is a necessity. Fig. 5 demonstrate flowchart the proposed algorithm in the MATLAB environment.

### *A. optimal location of DGs*

In According to figure 5, the suitable location of DGs in the network will be chosen based on an iterative algorithm. To find the best location, first, voltage stability index $L_k$will be calculated for the most critical bus with respect to the voltage collapse phenomenon and then, best size of the DG will be found based on the proposed algorithm for each iteration. In each iteration, in order to decide on the least value of $L_k$ over the existing buses with DG, value of the voltage stability will be determined based on the recommended algorithm. Then, the bus characterized with least $L_k$ , throughout the iterations, and connected to a DG, will be best suited for DG installation.

### *B. DG's optimal sizing algorithm*

In this work, the targeted objective function is to optimize the DGs' capacities using concurrent minimization of total power losses (TPL) and maximization of $L_k$ index in each iteration (in order to boost the voltage stability), *i.e.,*

$$f = Min\left\{\left(TPL = \sum_{i=1}^{b} |I_i|^2 \cdot R_i\right) + Max \quad of(L_k)\right\} \quad (13)$$

subject to

$$V_{i,\ \min} \leq V_i \leq V_{i,\ \max}, \quad i = 1,2,\ldots,n$$

and

$$0 \leq P_{DG} \leq \sum_{i=1}^{n} P_{load,\ i}$$

where b is the number of lines in each section, *n*, number of buses, $P_{DG}$, injected power by the DG, $P_{Load}$, connected loads to each node; the acceptable range of voltage for each node is within the interval of [0.9pu,1.05pu], too.

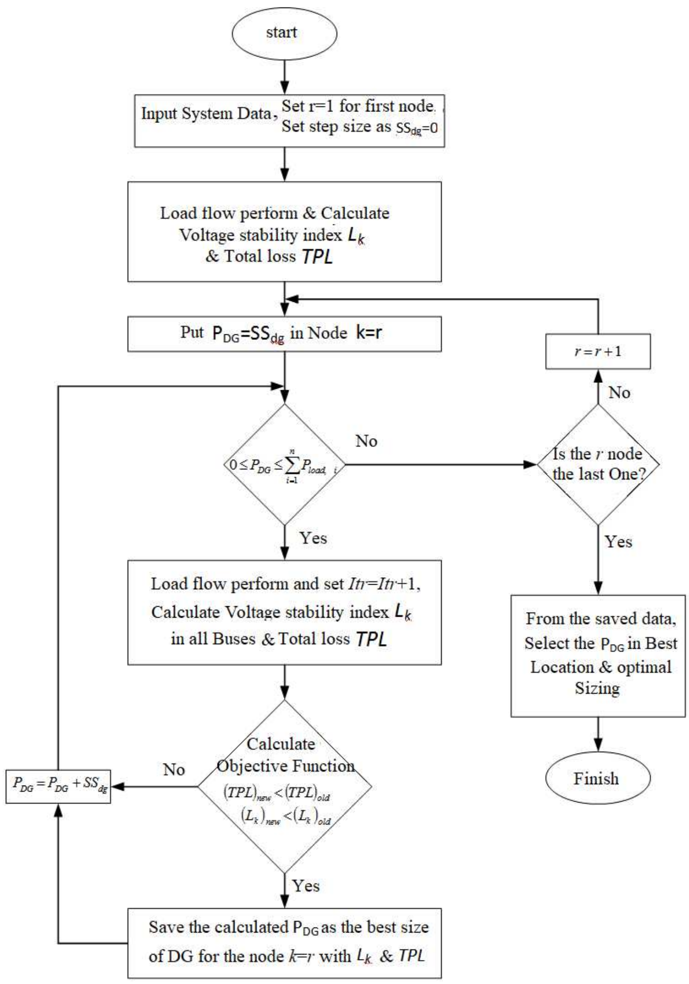


**Fig. 5:** The flowchart of the proposed method

## VI. Simulation Results

In order To validate the proposed algorithm to find best capacities and locations of DGs, the standard IEEE 34-bus testbed system was simulated in MATLAB programming environment. The characteristics of the testbed are as follows: an imbalanced 16.9*k*V system with a total centralized load with 1047*k*W and 677*k*VAR, and aggregate decentralized load with 722*k*W and 367*k*VAR, along with 2 voltage regulators and capacitor banks, as depicted in fig. 6 [21]. Fig. 7 illustrates the total active and reactive power loss of the network for different optimal capacities of DGs in the absence of voltage stability index. Although placement of DG at the node 834 would give us the least power losses, nevertheless, besides requiring a DG with higher size, there would be no improvements in the voltage stability indices, too.

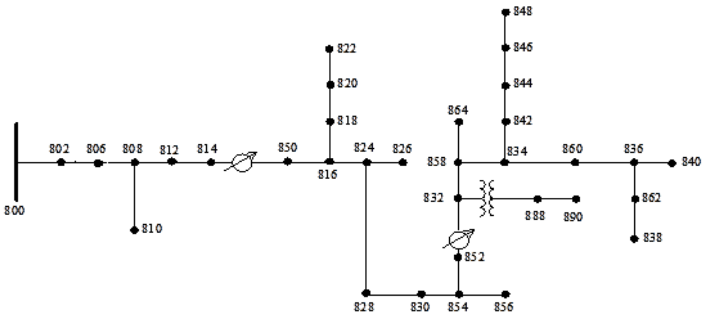


**Fig. 6:** Single line diagram of standard IEEE 34-bus testbed.

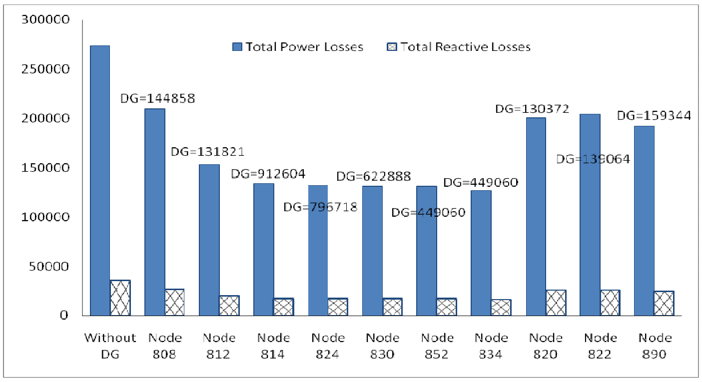


**Fig 7:** The total of active and reactive power losses in Watt for DGs with optimal sizes on 10 individually critical buses.

Figs. 8-11 show the voltage profiles of all three phases and that of the lowest voltage level in the network (phase A and only at the ending buses) 1) when there is no DG in the network and 2) when a DG has been placed at the critical bus 890. As it can be seen, optimization of location and size to 159.344*k*W has led to improvements in the moreover, voltage profile. Furthermore, in the most problematic phase (phase A), the voltage amplitude has been boosted from 0.83pu to 0.88pu.

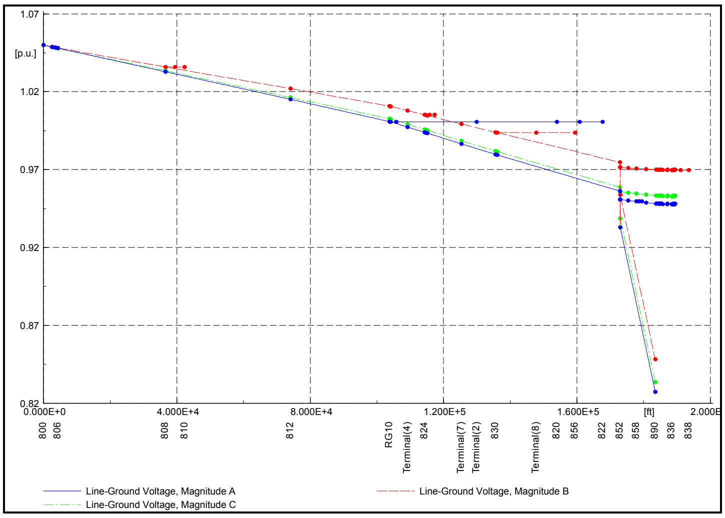


**Fig. 8:** The three phase voltage profile without DGs

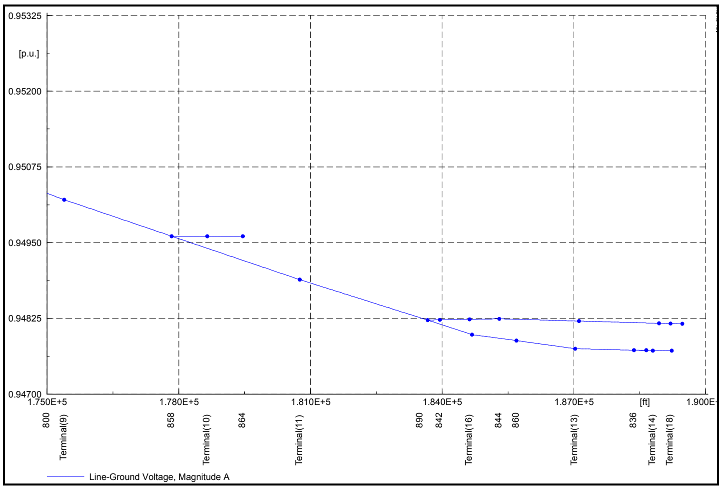


**Fig. 9:** The Voltage profile of phase A at terminating buses without DGs.

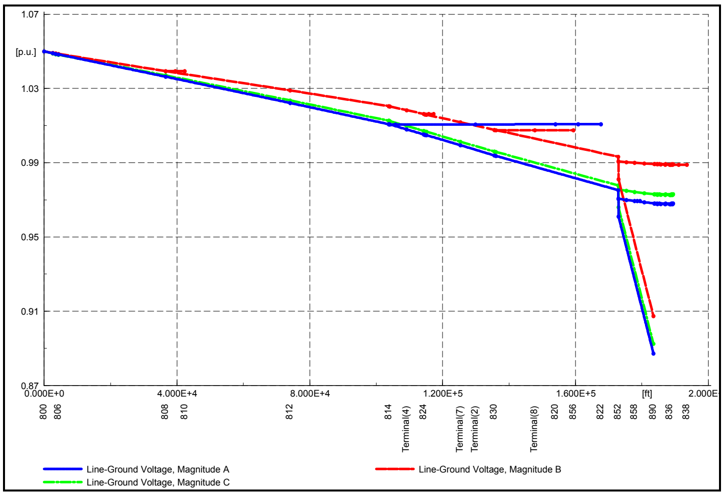


**Fig. 10:** The three phase voltage profile with placement of optimal sized DG at the bus 890.

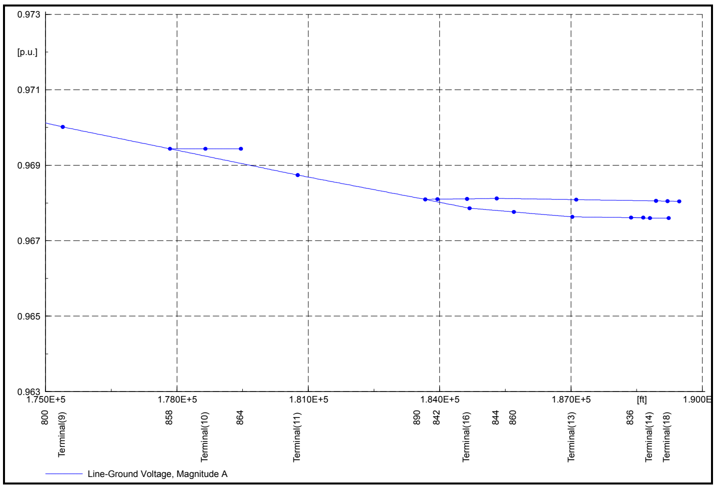


**Fig. 11:** Voltage of phase *A* at the terminating buses in the presence optimal sized DG at the bus 890

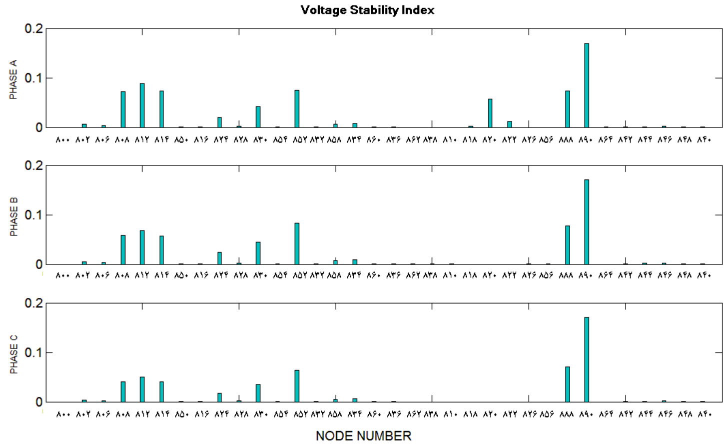


**Fig. 12**: Voltage stability index values without DGs

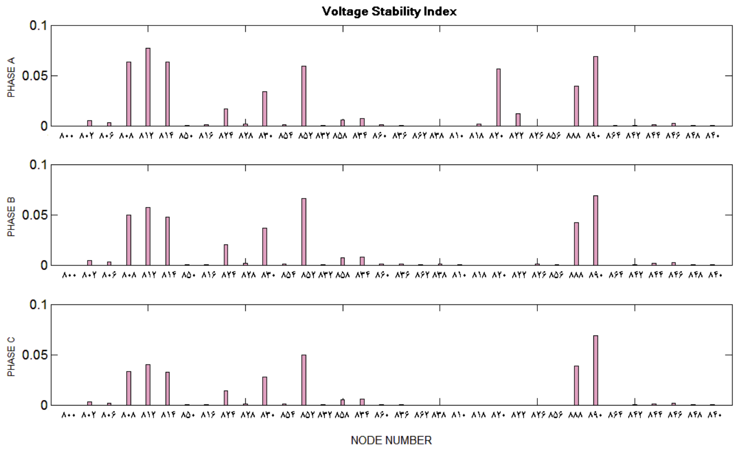


**Fig. 13:** Voltage stability index values with placement of optimal sized DG at the bus 890.

Figs. 12-13 show the voltage stability indices for 1) without DGs and 2) with placement an optimal sized DG at the critical bus 890. In addition to enhancement of voltage stability index, it can be seen that the total power losses of the system has also been decreased. Although other capacities of DG might decrease the power line losses more, however, it comes at the cost of larger and thus more expensive DGs, and lack of fitting voltage stability index which is a consequence of righteous loading in peak hour load demands.

**Table I:** Values of voltage stability indices for 5 critical buses

| Voltage stability index | | Node 890 | Node 812 | Node 852 | Node 814 | Node 888 | Node 808 |
|---|---|---|---|---|---|---|---|
| **Without DG** | **Phase A** | 0.1699 | 0.0885 | 0.0746 | 0.0737 | 0.0726 | 0.0723 |
| | **Phase B** | 0.1705 | 0.0684 | 0.0822 | 0.0567 | 0.0774 | 0.0580 |
| | **Phase C** | 0.1702 | 0.0495 | 0.0633 | 0.0408 | 0.0712 | 0.0409 |
| **With Optimal DG** | **Phase A** | 0.0687 | 0.0768 | 0.0595 | 0.0634 | 0.0395 | 0.0632 |
| | **Phase B** | 0.0690 | 0.0575 | 0.0661 | 0.0472 | 0.0421 | 0.0494 |
| | **Phase C** | 0.0689 | 0.0400 | 0.0496 | 0.0327 | 0.0390 | 0.0333 |

Table I shows the measured values of voltage stability index for 5 critical buses. It has been numerically illustrated that all the important factors have been improved with installation of optimal DGs.

## VII. Conclusion

Since In this paper, an analytically efficient method was proposed for optimal location and size of Distributed Generation Units (DGs) with the objective of power loss reduction and voltage stability index improvement. The suggested method, due to its computational complexities, i.e., high speed in run time and low iteration for convergence is well suited for practical applications. The results of simulations on the standard IEEE 34-bus testbed system have shown that taking the voltage stability index into account is of practical significance, in order to achieve the desired network's responses in terms of power losses reduction in the presence of DGs.

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