

Optical and Electrical Properties of Quantum Nano Ring at the Presence of Rasha Spin-orbit Interaction

Elmira Dehghan^{*1}, Azadeh Sadat Naeimi², Davod Sanavi Khoshnoud¹

Abstract — The significant information about the properties of matter can be described with the interaction between light and matter. On this subject, the effect of the applied magnetic field and structural variation on the optical and electrical properties of circular GaAs quantum rings at the existence of Rashba spin-orbit interaction (RSOI) have been investigated. Also the effect of Rashba spin-orbit interaction (SOI) on the gate response in a series of non-interacting one-dimensional rings connected to some leads are studied theoretically within the waveguide theory. The presence and absence of Rashba SOI is treated as the two inputs of the AND/NAND/NOT gates. Additionally the linear and nonlinear optical absorption coefficient (AC) and refractive index changes (RIC) are calculated by the density matrix.

Keywords: Rashba spin-orbit interaction, quantum ring, persistent spin, optical properties, gate response.

1. Introduction

Rapid progress in semiconductor manufacturing and associated technologies have increased the need for optical characterization techniques for materials analysis and in-situ monitoring/control applications. Optical properties have many unique and amazing features for studying and characterizing semiconductor properties [1-12]. For semiconductors, the optical properties are directly related to the electron energy band structure. Starting from the study of band structure, we will thus be able to figure out the interaction property between photons and electrons in semiconductors [13, 14]. Hitherto, have been studied the optical properties of semiconductor, both experimentally and theoretically. For instance, the third-order nonlinear susceptibility has been estimated in coupled quantum wells by Sartori et al. [15]. In 1999, the intraband harmonic conversion in InAs/GaAs self-assembled quantum dots has been investigated theoretically and experimentally by Sauvage and Boucaud [16]. It is worth mentioning, a lot of studies have been down on the optical properties of quantum ring and quantum dot. In 2010, G. Rezaei and et al [17] estimated the changing linear and the third-order

nonlinear optical properties at the presence of a hydrogenic impurity in an ellipsoidal quantum dot. They have shown that light intensity, size, geometry of the dot and aluminum concentration have a major impression on linear and nonlinear optical properties of the dot. In another work, Shijun Liang and Wenfang Xie [18] investigated theoretically the Optical properties of a two-dimensional quantum ring with pseudo potential in the presence of an external magnetic field and magnetic flux. Their results show that both of the pseudo potential and magnetic field can affect the third nonlinear susceptibility and oscillator strength. It is note that, in this study, the influence of SOI and combined effect of SOI and magnetic field have not been investigated. Recently Gumber and et al. [19] reported the effect of RSOI on the linear and nonlinear optical properties of two-dimensional mesoscopic ring in the presence of perpendicular magnetic field. They presented that the nonlinear absorption has been affected by RSOI and change in refractive index positively resulting in a decline of their total value.

In this article, we use the compact density matrix formalism to calculate the linear and nonlinear AC and RI of circular GaAs rings at the presence of RSOI and magnetic flux. In the next sections,

2. Theory and model

2.1. Effective Hamiltonian

1* Corresponding Author : Department of Physics, Semnan University, Semnan , Iran, Email:elmiradehqan@gmail.com

2 Department of Physics, Aliabad Katoul Branch, Islamic Azad University, Aliabad Katoul, Iran

Received 2020.02.12 ; Accepted 2020.06.05

One ring with finite width in x-y plan, in the presence of RSOI and electromagnetic field described by the vector potential $A=(0,0,A)$ are considered. The effective Hamiltonian in cylindrical coordinates can be written as [20]

$$H = \left[\frac{(-i\hbar\vec{\nabla} + eA)^2}{2m^*} + V_0 \right] + H_R + \frac{1}{2}gB\mu_B\sigma_z \quad (1)$$

where m^* is the electron effective mass, $\sigma_i (i=x,y,z)$ is the pauli matrices, α is the Rashbaco efficient and also V_0 , is confinement potential [21]. In Eq. 1 the second term $H_R = \alpha_R/\hbar(\sigma_x p_y - \sigma_y p_x)$ shown liner RSOI.

The 2D ring, such as elliptical and circular ring are demonstrated in Fig. 1. according to Fig 1(a), the semimajor axis and the semiminor axis of the ellipse are denoted by a_e and b_e respectively. For a circular ring as represented Fig. 1(b), a_e replace by the radius of the circle r_0 [22].

the Hamiltonian in Eq.1 rewrite in cylindrical coordinate.

$$H = \frac{\hbar^2}{2m^*} \left[-\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial}{\partial r} \right) - \frac{1}{r^2} \frac{\partial^2}{\partial \varphi^2} - i \frac{eB}{\hbar} \frac{\partial}{\partial \varphi} + \frac{e^2 B^2}{4\hbar^2} r^2 - \frac{\partial^2}{\partial z^2} \right] \\ + i\alpha_r (-\sin\varphi\sigma_x + \cos\varphi\sigma_y) \frac{\partial}{\partial r} - \frac{i\alpha_r}{r} (\cos\varphi\sigma_x + \sin\varphi\sigma_y) \left(\frac{\partial}{\partial \varphi} + i \frac{Ber^2}{2\hbar} \right) \\ + V_0 + \frac{1}{2}gB\mu_B\sigma_z \quad (2)$$

For solving the Hamiltonian and calculation the eigenvalues and eigenfunction, the method of exact diagonalization has been used. At first the Hamiltonian Eq. 1, will be written as a 2×2 matrix and Schrodinger equation, are as follows [23]

$$\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} \xi_{\uparrow} \\ \xi_{\downarrow} \end{bmatrix} = E \begin{bmatrix} \xi_{\uparrow} \\ \xi_{\downarrow} \end{bmatrix} \quad (3)$$

where the matrix elements are written as[23]

$$H_{11} = -\frac{\hbar^2}{2m^*} \left[\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{1}{r^2} \left(i \frac{\partial}{\partial \varphi} - \frac{m^* \omega_c r^2}{2\hbar} \right)^2 + \frac{\partial^2}{\partial z^2} \right] + v_0 + \frac{1}{2}gB\mu_B \quad (4)$$

$$H_{22} = -\frac{\hbar^2}{2m^*} \left[\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{1}{r^2} \left(i \frac{\partial}{\partial \varphi} - \frac{m^* \omega_c r^2}{2\hbar} \right)^2 + \frac{\partial^2}{\partial z^2} \right] + v_0 - \frac{1}{2} g B \mu_B \quad (5)$$

$$H_{12} = \alpha e^{-i\varphi} \frac{\partial}{\partial r} - i \alpha e^{-i\varphi} \left(\frac{1}{r} \right) \left(\frac{\partial}{\partial \varphi} + i \frac{m^* \omega_c r^2}{2\hbar} \right) \quad (6)$$

$$H_{21} = -\alpha e^{i\varphi} \frac{\partial}{\partial r} - i \alpha e^{i\varphi} \left(\frac{1}{r} \right) \left(\frac{\partial}{\partial \varphi} + i \frac{m^* \omega_c r^2}{2\hbar} \right) = H_{12}^* \quad (7)$$

Where $\omega_c = eB/m^*$.

In cylindrical coordinate eigenvalues can defined as

$$\xi(R, \varphi) = \Phi(\varphi) R(r) \quad (8)$$

where can be written $\Phi(\varphi) = e^{il\varphi}$ and l is orbital angular momentum. Thus the matrix elements are written for two lowest state ($l = 0, 1$)

$$H = \begin{bmatrix} \langle 0|H_{11}|0\rangle & 0 & 0 & \langle 0|H_{12}|1\rangle \\ 0 & \langle 1|H_{11}|1\rangle & \langle 1|H_{12}|0\rangle & 0 \\ 0 & \langle 0|H_{21}|1\rangle & \langle 0|H_{22}|0\rangle & 0 \\ \langle 1|H_{21}|0\rangle & 0 & 0 & \langle 1|H_{22}|1\rangle \end{bmatrix} \quad (9)$$

The matrix elements have been calculated as follows [23]

$$\langle 0|H_{11}|0\rangle = \frac{-\hbar^2}{2m^*} \left[\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \right] + \frac{1}{8} m^* \omega_c^2 r^2 + \frac{1}{2} g \mu_B B + V_0 \quad (10-11)$$

$$\langle 1|H_{11}|1\rangle = \frac{-\hbar^2}{2m^*} \left[\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \right] + \frac{\hbar^2}{2m^* r^2} + \frac{1}{2} \hbar \omega_c + \frac{1}{8} m^* \omega_c^2 r^2 + \frac{1}{2} g \mu_B B + V_0$$

$$\langle 0|H_{22}|0\rangle = \langle 0|H_{11}|0\rangle - g \mu_B B \quad (12)$$

$$\langle 1|H_{22}|1\rangle = \langle 1|H_{11}|1\rangle - g \mu_B B \quad (13)$$

$$\langle 0|H_{12}|1\rangle = \alpha \left(\frac{\partial}{\partial r} + \frac{1}{r} + \frac{m^* \omega_c r}{2\hbar} \right) \quad (14)$$

$$\langle 1|H_{12}|0\rangle = 0 \quad (15)$$

$$\langle 1|H_{21}|0\rangle = \alpha \left(\frac{m^* \omega_c r}{2\hbar} - \frac{\partial}{\partial r} \right) \quad (16)$$

$$\langle 0|H_{21}|1\rangle = 0 \quad (17)$$

According to Hamiltonian matrix elements Eq. (10-17), the eigenfunction and the eigenvalue of an electron confined in a quantum ring have been calculated by solving the Schrodinger equation. By using the finite difference method, 4 eigenfunctions ($\psi(l, \uparrow), \psi(l, \downarrow)$ with $l = 0, 1$) and 4 corresponding eigenvalues are calculating [24-28].

2.1. Calculation of the linear and thread-order nonlinear optical properties

the linear and the third-order nonlinear optical absorption and refractive coefficients can be appropriately described by formalism based on the density operator (or density matrix) and a perturbation expansion method. This formalism offers significant advantages in some contexts, particularly when resonant effects are involved and/or damping becomes important. Suppose the system is excited by a Laser field as

$$E(t) = E_0 \cos(\omega t) = \tilde{E} e^{i\omega t} + \tilde{E} e^{-i\omega t} \quad (18)$$

From the theoretical point of view, the linear and thread-order nonlinear optical absorption and refractive index can be written as [29]

$$\alpha^1(\omega) = \omega \sqrt{\frac{\mu}{\epsilon_R}} \frac{N_s |M_{if}|^2 \hbar \Gamma_{if}}{(E_{if} - \hbar\omega)^2 + (\hbar\Gamma_{if})^2} \quad (19)$$

$$\alpha^3(\omega, I) = -\omega \sqrt{\frac{\mu}{\epsilon_R}} \left(\frac{I}{2\epsilon_0 n_r c} \right) \times \frac{N_s |M_{if}|^2 \hbar \Gamma_{if}}{(E_{if} - \hbar\omega)^2 + (\hbar\Gamma_{if})^2} \tag{20}$$

$$\times \left[4|M_{if}|^2 - \frac{|M_{ff} - M_{ii}|^2 [3E_{if}^2 - 4E_{if} \hbar\omega + \hbar^2(\omega^2 - \Gamma_{if}^2)]}{E_{if}^2 + (\hbar\Gamma_{if})^2} \right]$$

where $E_{if} = E_f - E_i$ is the energy interval of two different electronic states, $I = 2n_r \epsilon_0 c |\tilde{E}|^2$ is the optical intensity depends on the applied laser field, $\Gamma_{if} = 1/T_{if}$ is called the damping rate, N_s is the carrier density, $M_{if} = \langle \psi_i | ez | \psi_f \rangle$ is the electric dipole moment of the transition from the ψ_i state to ψ_f state. The total absorption coefficient $\alpha(\omega, I)$ is

$$\alpha(\omega, I) = \alpha^{(1)}(\omega) + \alpha^{(3)}(\omega, I) \tag{21}$$

in addition, the refractive index is obtained as [29]

$$\frac{\Delta n^{(1)}(\omega)}{n_r} = \frac{N_s |M_{if}|^2}{2\epsilon_0 n_r^2} \frac{E_{if} - \hbar\omega}{(E_{if} - \hbar\omega)^2 + (\hbar\Gamma_{if})^2} \tag{22}$$

$$\frac{\Delta n^{(3)}(\omega, I)}{n_r} = -\left(\frac{\mu c I}{4\epsilon_0 n_r^3} \right) \frac{|M_{if}|^2 N_s}{[(E_{if} - \hbar\omega)^2 + (\hbar\Gamma_{if})^2]^2}$$

$$\times \left[4(E_{if} - \hbar\omega) |M_{if}|^2 - \frac{|M_{ff} - M_{ii}|^2 [(E_{if} - \hbar\omega)(E_{if}(E_{if} - \hbar\omega) - (\hbar\Gamma_{if})^2(2E_{if} - \hbar\Gamma_{if}))]}{E_{if}^2 + (\hbar\omega)^2} \right]$$

(23)

And the total refractive index change as follows

$$\frac{\Delta n(\omega)}{n_r} = \frac{\Delta n^{(1)}(\omega)}{n_r} + \frac{\Delta n^{(3)}(\omega, I)}{n_r} \tag{24}$$

3. Results and Discussions

In this section, we are determination to compare optical properties in different circular ring. The calculations are accomplished for GaAs one dimensional quantum ring in the presence of RSOI, magnetic field and geometrical sizes. The physical parameters apply in this calculation are the followings: The effective mass of an electron taken to be $m^* = 0.067 m_e$, where m_e is the free electron mass, $\epsilon_r = 12.74$, $N_s = 3 \times 10^{22} m^{-3}$ and $\tau = 0.2 ps$ [30]. The effect of geometry variation on the total optical

AC ($\alpha(\omega, I) = \alpha^{(1)}(\omega) + \alpha^{(3)}(\omega, I)$) and RI ($\frac{\Delta n(\omega)}{n_r} = \frac{\Delta n^{(1)}(\omega)}{n_r} + \frac{\Delta n^{(3)}(\omega, I)}{n_r}$) for circular ring have been displayed in Figure 1.

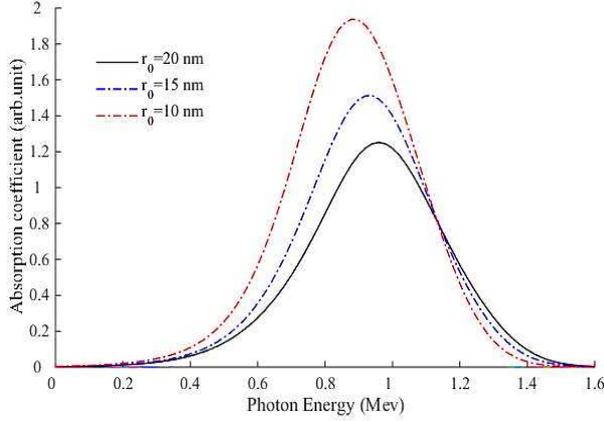


Figure 1: total absorption coefficient (AC) as a function of photon energy for circular ring of three different radius $r_0 = 10, 15, 20 nm$

The total RI for the circular ring are presented in Figure 2. The refractive index for both rings declined with increasing the radius (r_0).

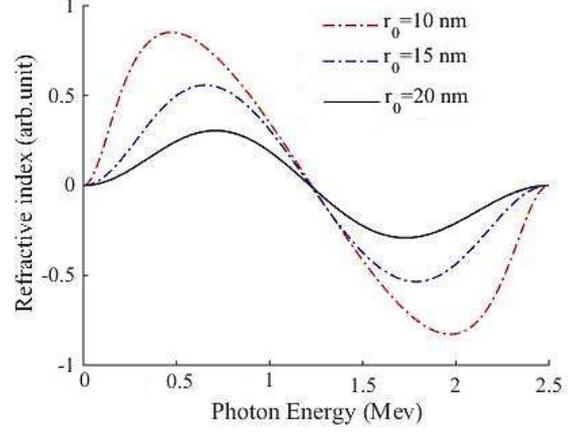


Figure 2: the total refractive index (RI) changes as a function of the photon energy for (a) circular ring of three different radius $r_0 = 10, 15, 20 nm$

The gate response of this series of quantum rings will be examined. For doing so, the presence and absence of Rashba SOI, are behaved as the two inputs of the NAND gates. Let us now describe how such a simple geometric model can be performed as universal gates. Figs. 3, reveals the current density-magnetic flux ($J-\frac{\phi}{\phi_0}$) characteristics

for the interferometric geometry. In view of the numerical simulations, and the values of α_L and α_R the interferometric geometry exhibits NAND gates response. These two (α_L and α_R) are treated as the two inputs of these gates. At first, the NAND gate responses for spin-up are obtained (Fig. 3). When both the two inputs to the gate are high (i.e., $\alpha_L = \alpha_R = 2.5$) the current density shows fine resonant peaks at $\frac{\phi}{\phi_0} = 2$ (Fig. 3(a)). On the

other hand, if any one of the two inputs is high (i.e., α_L or α_R) has a non-zero value, conductance exactly drops zero (see Fig. 3 (b) and (c)). Finally, when both the inputs to the gate are zero (i.e. $\alpha_L = \alpha_R = 0$), the current density again vanishes (Fig. 3(d)) for the special value of magnetic flux. This property of the system is indicated in Table 1.

Table 1: NAND gate behavior in the double quantum

ring geometry. The typical spin-up current density amplitude is determined at the magnetic flux $\phi/\phi_0 = 2$ for appropriate Rashba constant ($\alpha_L = \alpha_R = 2.5$).

Input-I (α_L)	Input-II (α_R)	Current density (J)
0	0	0.4
2.5	0	0.4
0	2.5	0.4
2.5	2.5	0

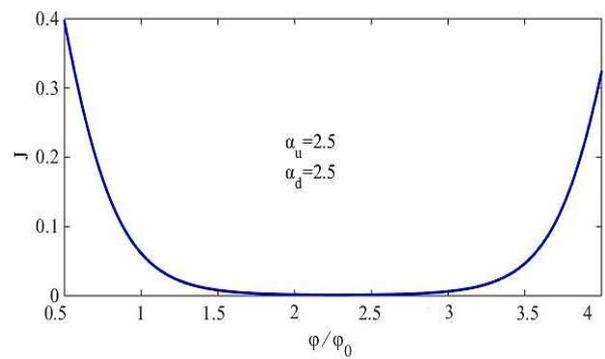
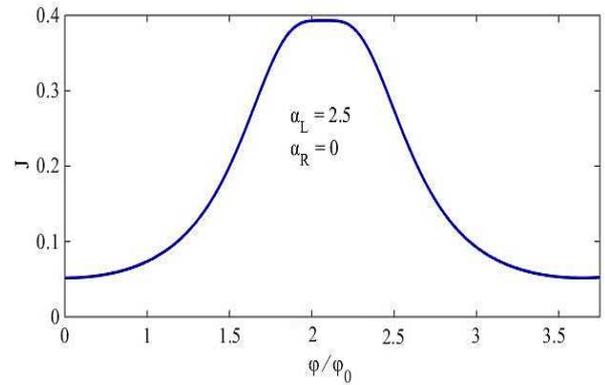
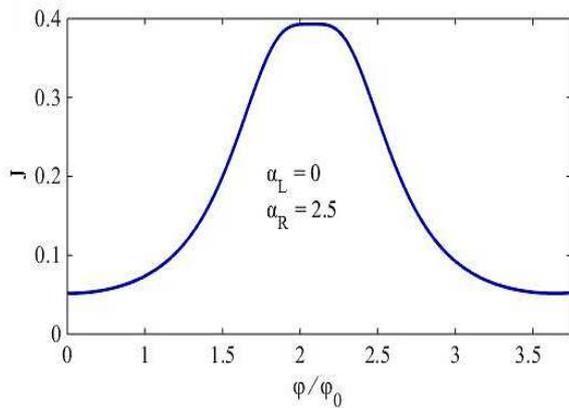
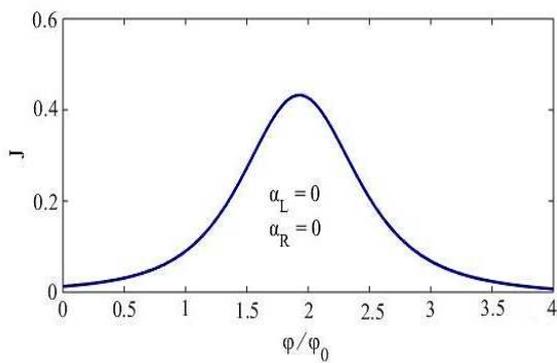


Figure 3: NAND gate behavior in the double quantum ring geometry. The typical spin-up current density amplitude is determined at the magnetic flux $\phi/\phi_0 = 2$ for appropriate Rashba constant ($\alpha_L = \alpha_R = 2.5$).

4-Conclusion

In summary, in the present work we have studied gate response and optical properties in GaAs quantum ring connected, while RSOI and magnetic flux are provided using the wave guide method and density matrix technique in the framework of effective mass approximation. According to the results, it is found that the linear and nonlinear parts of AC and RI of two-dimensional quantum rings are completely affected by the external magnetic field and geometry variation. The spin-dependent logic gates can be changed from NAND by changing the value of magnetic flux and gate voltage. It should be noted that two terminal double quantum rings have little power consumption because the electrons can transmit through the ring without collision (ballistic transmission).

Reference

- [1] M. A. Reed, J. N. Randall, R. J. Aggarwal, R. J. Martyi, T. M. Moore, and A. E. Wetsel, Observation of discrete electronic states in a zero-dimensional semiconductor nanostructure, *Phys. Rev. Lett.* 60 (1988) 535.
- [2] J. H. Davies, *The physics of low-dimensional semiconductors: an introduction*, Cambridge university press, (1998).
- [3] S. E. Harris, Y. Yamamoto, Photon Switching by Quantum Interference, *Phys. Rev. Lett.* 81 (1998) 3611.
- [4] J. Clarke, H. Chen, and W. A. van Wijngaarden, Electromagnetically induced transparency and optical switching in a rubidium cascade system, *Appl. Opt.* 40 (2001) 2047.
- [5] A. Andr e, M. D. Eisaman, R. L. Walsworth, A. S. Zibrov, and M. D. Lukin, Quantum control of light using electromagnetically induced transparency, *J. Phys. B: At. Mol. Opt. Phys.* 38 (2005) 589..
- [6] M. Sahrai, M. Mahmoudi, The effect of an incoherent pumping on the dispersive and absorptive properties of a four-level medium, *J. Phys. B: At. Mol. Opt. Phys.* 42 (2009) 185501.
- [7] S. M. Mousavi, L. Safari, M. Mahmoudi and M. Sahrai, Effect of quantum interference on the optical properties of a three-level V-type atomic system beyond the two-photon resonance condition, *J. Phys. B: At. Mol. Opt. Phys.* 43 (2010) 165501.
- [8] Y. Gu, L. Wang, K. Wang, C. Yang and Q. Gong, Coherent population trapping and electromagnetically induced transparency in a five-level M-type atom, *J. Phys. B: At. Mol. Opt. Phys.* 39 (2006) 463.
- [9] Z. Zhang, Y. Huang, P. J. Reece, Influence of GaAsSb structural properties on the optical properties of InAs/GaAsSb quantum dots, *Phys. E (Amsterdam, Neth.)* 94 (2017) 7.
- [10] A. Zamani, F. setareh, T. Azargoshasb, Spin-orbit coupling and applied magnetic field effects on electromagnetically induced transparency of a quantum ring at finite temperature, *Superlattices Microstruct.* 115 (2018) 40.
- [11] G. Rezaei, S. S kish, B. vaseghi, Electromagnetically induced transparency in a two-dimensional quantum dot: Effects of impurity, external fields, hydrostatic pressure and temperature, *Phys. E (Amsterdam, Neth.)* 62 (2014) 104.
- [12] A. S. Jahromi, G. Rezaei, Electromagnetically induced transparency in a two-dimensional quantum pseudo-dot system: Effects of geometrical size and external magnetic field, *Phys. B: Condens. Matter* 456 (2015) 103.
- [13] H. Haug, S. W. Koch, *Quantum Theory of the Optical and Electronic Properties of Semiconductors: Fifth Edition*. World Scientific Publishing Company, 2009.chapter 22
- [14] S. Liang, W. Xie, A. H. sarkisyan, Electronic and optical properties of a nanoring in the presence of external magnetic field, *Superlattices Microstruct.* 51.6 (2012) 868.
- [15] C. Sirtori, F. Capasso, D. L. Civco, *Phys. Rev. Lett.* 68 (1992) 1010.
- [16] S. Sauvage, P. Boucaud, F. Glotin, Third-harmonic generation in InAs/GaAs self-assembled quantum dots, *Phys. Rev. B* 59.15 (1999) 9830.
- [17] G. Rezaei, S. Shojaeian Kish, Linear and nonlinear optical properties of a hydrogenic impurity confined in a two-dimensional quantum dot: Effects of hydrostatic pressure, external electric and magnetic fields, *Superlattices Microstruct* 53 (2013) 99.
- [18] S. Liang, W. Xie, H. Shen, Optical properties in a two-dimensional quantum ring: Confinement potential and Aharonov–Bohm effect, *Opt. Commun.* 284.24 (2011) 5818.
- [19] S. Gumber, M. Gambhir, P. K. Jha, Optical response of a two dimensional quantum ring in presence of Rashba spin orbit coupling, *J. Appl. Phys.* 119.7 (2016) 073101.
- [20] Kh. Shakouri, B. Szafran, M. Esmailzadeh and F. M. Peeters, Effective spin-orbit interaction Hamiltonian for quasi-one-dimensional quantum rings, *Phys. Rev. B* 85 (2012) 165314.
- [21] G.A. Intronati, P. I. Tamborenea, D. Weinmann, R. A. Jalabert, Spin-orbit effects in nanowire-based wurtzite semiconductor quantum dots. *Phys. Rev. B* 88(4) (2013). 045303.
- [22] D. Y. Liu, J. Xia, Spin flip in single quantum ring with Rashba spin–orbit interaction. *Chin. Phys. B.* 27.3 (2018) 037201.
- [23] A. Zamani, T. Azargoshasb, E. Niknam, Second and third harmonic generations of a quantum ring with Rashba and Dresselhaus spin-orbit couplings: Temperature and Zeeman effects, *Phys. B: Condens. Matter.* 523 (2017) 85.
- [24] A. Zamani, Th. Estabar, Gh. Safarpour and M. Moradi, The binding energies of a bulged GaAs nanowire, *Superlattices Microstruct.* 76 (2014) 66.
- [25] A. Zamani, Gh. Safarpour, L. Safaei, E. Niknam and M. Novzari, The linear and nonlinear optical properties of a bulged GaAs nanowire, *Superlattices Microstruct.* 81 (2015)129.

[26] J. D. Cooper, A. Valavanis, Z. Ikonic, P. Harrison and J. E. Cunningham, Finite difference method for solving the Schrödinger equation with band nonparabolicity in mid-infrared quantum cascade lasers, *J. Appl. Phys.* 108 (2010) 113109.

[27] I. Wayan Sudiarta, D. J. Wallace Geldart, Solving the Schrödinger equation using the finite difference time domain method, *J. Phys. A: Math. Theor.* 40 (2007) 1885.

[28] G. M. Amiraliyev, The convergence of a finite difference method on layer-adapted mesh for a singularly perturbed system, *A. Math. Comput.* 162 (2005) 102.

[29] G. Rezaei, Z. Mousazadeh, and B. Vaseghi, Nonlinear optical properties of a two-dimensional elliptic quantum dot, *Phys. E (Amsterdam, Neth.)*. 42.5 (2010) 1477.

[30] E. Kasapoglu, C. A. Duque, M. E. Mora-Ramos, The effects of the intense laser field on the nonlinear optical properties of a cylindrical Ga_{1-x}Al_xAs/GaAs quantum dot under applied electric field, *Phys. B: Condens. Matter.* 474 (2015) 15.