

# Closed Loop Identification of Multi-Rate System by Expectation-Maximization

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**Abstract**– Closed-loop identification of multi-rate system with unknown parameters, that including prevalent non-uniform sampling data, is considered. The purpose is to identify a multi-rate closed loop model to approximate the parameters varying system. As far as the research has been done, the identification of multi-rate closed loop model with unknown parameters by using the expectation-maximization algorithm has not been done. To address this challenge, the two-stage method and expectation-maximization algorithm are applied in this paper to identify unknown parameters of system. In this case, by introducing the hidden variable, an EM is utilized to estimate the unknown model parameters. And also, it will be demonstrated that, to estimate of system parameters, Instead of the point estimate of the time variable, the full probability distribution of the time variable estimate is required. The performance of this procedure represents by simulation, and obtain consequences affirm that method has high precision and also has a high convergence speed. These simulations express that the performance of this algorithm is good, as the identified parameters accede the true parameters after several iterations. The Monte Carlo simulation with different noise realizations at SNR=26dB and SNR=46 dB are performed for showing the effectiveness of mentioned algorithm.

**Keywords:** Closed loop system, Identification, Multi-rate system, Expectation-maximization

## 1. Introduction

In many industrial processes, variables are sampled at different rates. The manipulated variables are taken at fast rate while the measurement variables can be taken at slow rate after several minutes which lead to multi-rate (MR) system [1-3]. In addition, if the sampling intervals are different for each variables, the sampling is called as non-uniform [4, 5]. Early research into such systems(multi-rate sampling) began in the 1950s. Multi-rate systems have attracted a lot of attention from researchers to controller design [6, 7], system identification [8, 9], and fault detection [10, 11]. Various methods have been described for modeling MR systems and inferring unmeasured or missing outputs [12, 13]. Kranc presented the first significant investigation for the multi-rate system on the switch decomposition method, which was later named as the lifting technique, which is the standard finding for converting a

periodically time-varying system into a time-invariant system. For example, Zamani et al. proposes the discrete-time linear systems with multi-rate outputs [14], Zhang et al. considers the finite-time filtering issue for a class of wireless networked multi-rate systems with fading channels [15]. Some estimation methods have been presented for linear systems [16], [17], pseudo-linear systems [18], bilinear systems [19], [20] and bilinear-parameter systems [21].

To solve the problem of incomplete data, Dempster et al. [22] proposed the expectation-maximization (EM) algorithm. An expectation-maximization (EM) algorithm is an iterative method which calculates maximum-likelihood estimates of parameters in models statistically, that models contain latent variables [23]-[25]. In this article, the EM algorithm is employed and it computes a maximum-likelihood estimate.

The identification of the closed-loop system is done by three well-known techniques, which are direct, indirect, and joint input-output methods. in direct method, by using linear controller, open- loop parameters are specified and the closed-loop transfer function of the system is also identified, in indirect identification, awareness of the controller structure is mandatory, the data is collected from the input-output signals and system is defined by the

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controller structure, in joint input-output approach, there is no need to know the system and noise model. In this way, the transfer function of the system is utilized for identifying the system and input-output of the system is used for defining the transfer function of data, after that the system can be estimated [26]-[30]. Estimation of the parameters of the closed-loop model, is the main stimulants. To solve the above problem, we will use two-stage method and the EM algorithm. In this case, it will be demonstrated that, to estimate of system parameters, the complete statistically distribution for timing variable estimate is needed, rather than the point estimation. The states estimation is given by their expectation. Parameter identification methods can be used in many fields including engineering[31]-[36]. This article considers a multi-rate closed loop system and identifies the unknown process that outputs sampled irregularly. Identifying scheme is based on the two-stage method and expectation-maximization algorithm. In the two-stage computation the sensitivity function is estimated first by using the identification algorithm then by converting the dynamic of unknown process to state space model, the parameters of continuous model are obtained by expectation-maximization algorithm. This procedure of identifying parameters is repeated until the convergence condition is satisfied. This investigation is different from the procedure in [37], which is done with iterative identification idea; and also, recent research in [38], which is done according to the Wiener system identification.

The main assignments of this paper are as follows: First, this paper presents closed-loop model identification with unknown parameters, which includes non-uniformly sampled regular data systems and multi-rate systems as special cases. Then the sensitivity function is estimated using the two-step method, then by introducing the hidden variables, this paper proposes the EM algorithm to estimate the parameters of the unknown model. Finally, this paper demonstrates the performance of the proposed algorithm using a numerical example.

This paper is organized as follow. Identifying the closed-loop presents in Section 2, Section 3 describes an EM algorithm. Simulation experiments have been carried out in Section 4. Finally, the conclusion is given in Section 5.

## 2. Approaches to closed loop identification

Fig. 1 shows a general system in closed-loop form, where the controller and transfer functions of the noise, the process are defined by C, H and G, respectively, input and output signals are r and Y, e is an additional input which entered on the controller output, the unmeasured disturbances is denoted by J, n represents the Gaussian

white noise whose power spectral density is constant,  $\sigma^2$ . The closed loop system described by:

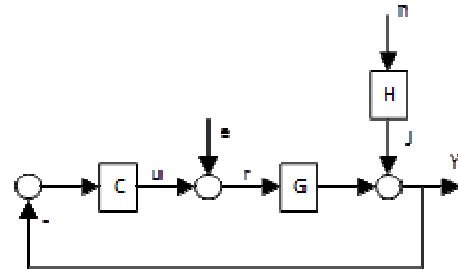


Fig. 1.closedloop identification Schema.

$$Y = Gr + Hn \tag{1}$$

$$r = e - CYu \tag{2}$$

The input-output signals are independent of exciting external signals as follows:

$$Y = GSe + HS n \tag{3}$$

$$r = Se - SCn \tag{4}$$

The sensitivity function  $S = (1 + GC)^{-1}$  can be estimated from Eq. (4). As can be seen from Eq. (4), the correlation between  $n$  and  $r$  leads to  $\hat{S}$ , which is an open-loop identification problem.  $S^{-1}$  is used for filtering  $Y$ , in which case, the relationship between  $Y$  and  $e$  in Eq. (3) is as follows:

$$\frac{Y}{S} = y^f = Ge + Hn \tag{5}$$

According to Eq. (5),  $G$ , which is an openloop problem, is identified. The form of Eq. (3) and Eq. (5) are similar, That in Eq. (5),  $e$  and  $y^f$  are as input and output data, in this case, by using expectation maximization which is described in section 4, this problem will be solved.

## 3. Problem statement

The mathematics of the process model is considered as follows:

$$x_{t+1} = \alpha x_t + \beta u_t + z_t \tag{6}$$

$$y_{N_i} = Cx_{N_i-d_i} + p_{N_i} \tag{7}$$

Where  $x(t)$  is the immeasurable state; input has fast rate, denotes by  $\{u_t = e, \quad t = 1, 2, \dots, N\}$  and can be measured in each period (t), N indicates the collected data.  $\{y_t = y^f, \quad t = N_1, N_2, \dots, N_T\}$  Is output and has slow rate that has been sampled irregularly, the output is available at time constant  $t = N_i \cdot t$ .  $z_t \in R_{n \times 1}$  Represents the noise of process which has zero mean and covariance matrix  $P_0$ ,  $P_{N_i}$  indicates the noise of measurement with covariance matrix  $R_0$ . The parameters to be estimated are indicated by  $\alpha \in R_{n \times n}$ ,  $\beta \in R_{n \times 1}$  and  $\gamma \in R_{1 \times n}$ . The term multirate here refers to multiple sampling rates, Input data is measured at a fast rate and is available at every sampling period, and Output data is available irregularly and at a slow rate.

The Q function of the EM calculations is as follow:

$$Q(\delta | \delta^k) = E_{c_{mi} | c_{ob}, \delta^k} \{\log(p(c_{ob}, c_{mi} | \delta))\} \quad (8)$$

$$Q(\delta | \delta^k) = \int_{c_{mi}} \log[p(c_{ob}, c_{mi} | \delta)] p(c_{mi} | c_{ob}, \delta^k) dc_{mi} \quad (9)$$

Where  $\delta$  Represents the parameters of system,  $\alpha, \beta, \gamma$ .  $\delta^k$  Illustrates the parameters estimated from the previous iteration.  $c_{ob}$  Is observed data which is shown by  $\{y_{N_1}, \dots, y_{N_T}\}, \{u_1, \dots, u_N\}$ , and  $c_{mi}$  represents latent data  $X = \{x_1, \dots, x_N\}, \{d_{N_i}\}$ . Under the EM framework, to obtain  $\delta^{k+1} = \arg \max_{\delta} Q(\delta | \delta^k)$ , iteration alternates between performing an expectation (E-step), which computes the Q function, and a maximization (M-step), which maximizes the Q function with respect to  $\delta$ . A diagram of EM algorithm is shown in Fig. 2. By defining the Q function as the conditional expectation, we have:

$$\begin{aligned} Q(\delta | \delta^k) = & C_1 \sum_{i=1}^T \sum_{k=0}^q E_{x_{N_i-d_{N_i}}} [\log p(y_{N_i} | x_{N_i-d_{N_i}}, d_{N_i} = k, \delta)] \\ & + C_1 \sum_{i=1}^T \sum_{k=0}^q E_{x_{N_i-d_{N_i}} \rightarrow x_{N_i-d_{N_i}-1}} [\log p(x_{N_i-d_{N_i}} | x_{N_i-d_{N_i}-1}, d_{N_i} = k, \delta)] \\ & + C_1 \sum_{i=1}^T \sum_{k=0}^q E_{x_{N_i-d_{N_i}+1} \rightarrow x_{N_i-d_{N_i}}} [\log p(x_{N_i-d_{N_i}+1} | x_{N_i-d_{N_i}}, d_{N_i} = k, \delta)] \\ & + C_1 \sum_{t \neq N_i-d_{N_i}, N_i-d_{N_i}+1} E_{x_t, x_{t-1}} [\log p(x_t | x_{t-1}, \delta)] + C_3 \end{aligned} \quad (11)$$

$$\begin{aligned} Q(\delta | \delta^k) = & E_{c_{mi} | c_{ob}, \delta^k} \{\log[p(c_{ob}, c_{mi} | \delta)]\} \\ = & E_{x_{1:N}, d_{N_1:N_T} | c_{ob}, \delta^k} \{\log[p(y_{N_1:N_T}, x_{1:N}, d_{N_1:N_T} | \delta)]\} \end{aligned} \quad (10)$$

Where  $\delta^k$  is the estimation of  $\delta$  after k iteration. Finally, we can get the Q function as follow:

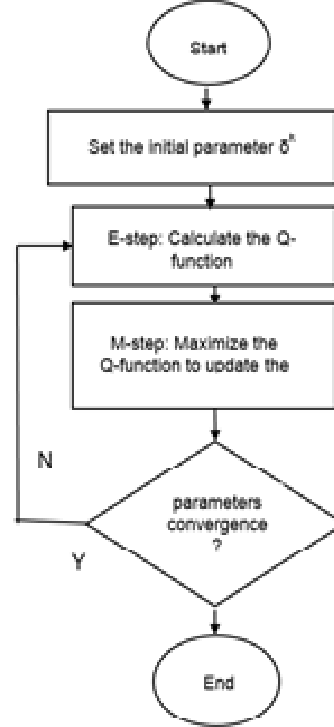


Fig. 2. A block diagram of EM algorithm.

By taking the gradient of the Q function, parameter estimates are calculated [39].

$$\alpha^{k+1} = \frac{1}{12} P_0 \sum_{i=1}^T \sum_{k=0}^2 \sum_{t=1}^N W_{ikN} (P_4^1 + P_4^2 + P_4^3 + P_4^4 + P_4^{2T} + P_4^{3T} + 2\zeta_{1,N_i-k-1}^T \zeta_{1,N_i-k-1}^T + 2\zeta_{2,N_i-k}^T \zeta_{2,N_i-k}^T + 2\zeta_{3,t-1}^T \zeta_{3,t-1}^T)^{-1} \quad (12)$$

Where

$$W_{ikN} = P_0^1 P_2^T + P_0^1 P_3^1 + 2P_0^1 \zeta_{1,N_i-k} \zeta_{1,N_i-k}^T - 2P_0^1 b^{k+1} \zeta_{1,N_i-k}^T u_{N_i-k-1} + P_0^1 P_2^{2T} + P_0^1 P_3^2 + 2P_0^1 \zeta_{2,N_i-k-1} \zeta_{2,N_i-k}^T - 2P_0^1 b^{k+1} \zeta_{2,N_i-k}^T u_{N_i-k} + P_0^1 P_2^{3T} + P_0^1 P_3^3 + 2P_0^1 \zeta_{3,t} \zeta_{3,t-1}^T - 2P_0^1 b^{k+1} \zeta_{3,t-1}^T u_{t-1} \quad (13)$$

$$\beta^{k+1} = \sum_{i=1}^T \sum_{k=0}^2 \sum_{t=1}^N (u_{N_i-k-1}^2 + u_{N_i-k}^2 + u_{t-1}^2)^{-1} (\zeta_{1,N_i-k} u_{N_i-k-1} - \alpha^{k+1} \zeta_{1,N_i-k} u_{N_i-k-1} + \zeta_{2,N_i-k} u_{N_i-k} - \alpha^{k+1} \zeta_{2,N_i-k} u_{N_i-k} + \zeta_{3,t} u_{t-1} - \alpha^{k+1} \zeta_{3,t-1} u_{t-1}) \quad (14)$$

$$\gamma^{k+1} = -2 \sum_{i=1}^T \sum_{k=0}^2 y_{N_i} (P_{new}^T + P_{new} + 2\zeta_{new} \zeta_{new}^T)^{-1} \zeta_{new} \quad (15)$$

#### 4. Simulation

There are some methods such as direct method, indirect method, joint input-output method, and two-stage method which used to identify the closed-loop system. In this article we use two-stage method and EM algorithm for identification of closed-loop parameters.

Following the multi-rate system with time varying are considered (as process model),

$$\begin{aligned} x_{t+1} &= 3.6x_t - u_t + z_t \\ y_{N_i}^f &= 0.2x_{N_i-d_{N_i}} + p_{N_i} \end{aligned} \quad (16)$$

Where  $\{u_t = e\}$  have Gaussian distribution  $N(0,1)$  shown in Fig. 3, input has fast rate and can be measured in each sampling period, slow rate output  $\{y_{N_i}^f\}$  is available at time instant  $N_i t (N_i = 5i)$  that has been sampled

irregularly.  $\delta_z^2 = 0.01$  Is the variance of the process noise  $\{z_t\}$  and  $\delta_p^2 = 0.01$  is measurement noise  $\{p_t\}$ ; the fast rate input and slow rate output that has been collected is 300 and 100, respectively.

Applying the two-stage method and EM algorithm to identify closed loop system, the estimated model is shown in Fig 4 and Q function is shown in Fig5. The validation result is presented in Fig. 6. These simulations express that the performance of this algorithm is good, as the identified parameters accede the true parameters after several iterations. The Monte Carlo simulation with different noise realizations at SNR=26dB and SNR=46 dB are performed for showing the effectiveness of mentioned algorithm. The EM estimates and variances based on 15 Monte Carlo simulations are shown in table 1. The mean and standard deviation (Std) of the parameter estimates from the Monte Carlo simulation are calculated and results are shown in

table2. in order to compare, Liu and Gao [61] method has been used with the same identification setting. The results in terms of mean and std deviation of estimated parameters and are shown in Table 3. It is stated from the results that the mentioned method, it has more robust and keeps the identification accuracy and produces a better parameter estimation compared to Liu and Gao method.

**Table 1.**The EM estimates and variances based on 15 Monte Carlo simulations

$t$	$\alpha$	$\beta$	$\gamma$
1	3,3890 ± 0.2346	-0,8389 ± 0.1531	0,2245 ± 0.3565
3	3,6016 ± 0.4323	-0,9621± 0.5177	0,2213 ± 0.4747
5	3,5870 ± 0.4732	-1,0176± 0.5354	0,2081 ± 0.4672
7	3,6284 ± 0.4021	-0,9976± 0.5345	0,2242 ± 0.4856
9	3,5754 ± 0.4670	-1,0080 ± 0.5047	0,2048± 0.4798
11	3,5940 ± 0.4655	-1,0198 ± 0.4850	0,2154 ± 0.4834
13	3,6218 ± 0.3907	-1,0220 ± 0.4967	0,2059 ± 0.5032
15	3,5796 ± 0.4281	-0,9980 ± 0.5178	0,2094 ± 0.4821
True value	3.60000	-1.00000	0,200000

**Table 2.**The mean and standard deviation of parameter estimates from Monte Carlo simulations

True value	SNR=26dB		SNR=46dB	
	mean	std	mean	std
$\alpha = 3.6$	3,5895	0,1683	3,6070	0,1511
$\beta = -1$	-0,9023	0,1938	-0,9073	0,0967
$\gamma = 0.2$	0,2251	0,1965	0,2001	0,1132

## 5. Conclusion

Identification of multi-rate closed-loop systems with time-varying is considered in this paper. For addressing this challenge, two-stage method and expectation maximization is used for identifying parameters of system. The efficiency of this method has been considered by a simulation example and the results show that the accuracy of the mentioned method, and the speed of convergence are high. Method which mentioned in this paper can be applied to multivariate systems with different structures [40]-[47], and to investigate the efficiency of parameters estimation, this method can synthesize mathematical implements [48]-[52] and statistical methods [53]-[60]. In the future research, the identification of multirate closedloop system in the presence of unknown time-delay and modeling uncertainties can be investigated.

**Table 3.**The mean and std deviation with different SNRs (by different algorithm).

snr	Estimated Parameters(mentioned)			Estimated Parameters[61]		
	$\alpha$	$\beta$	$\gamma$	$\alpha$	$\beta$	$\gamma$
26	3,5995 ± 0.1683	-0,9023± 0.1938	0,2251 ± 0.1965	3,3890 ± 0.2346	-0,8389 ± 0.1531	0,2265 ± 0.3565
46	3,6070± 0.1511	-0,9073± 0.0967	0,2001± 0.1132	3,4890 ± 0.3128	-0,8621± 0.5177	0,2213 ± 0.4747
True Value	3.6	-1	0.2	03,6	-1	0.2

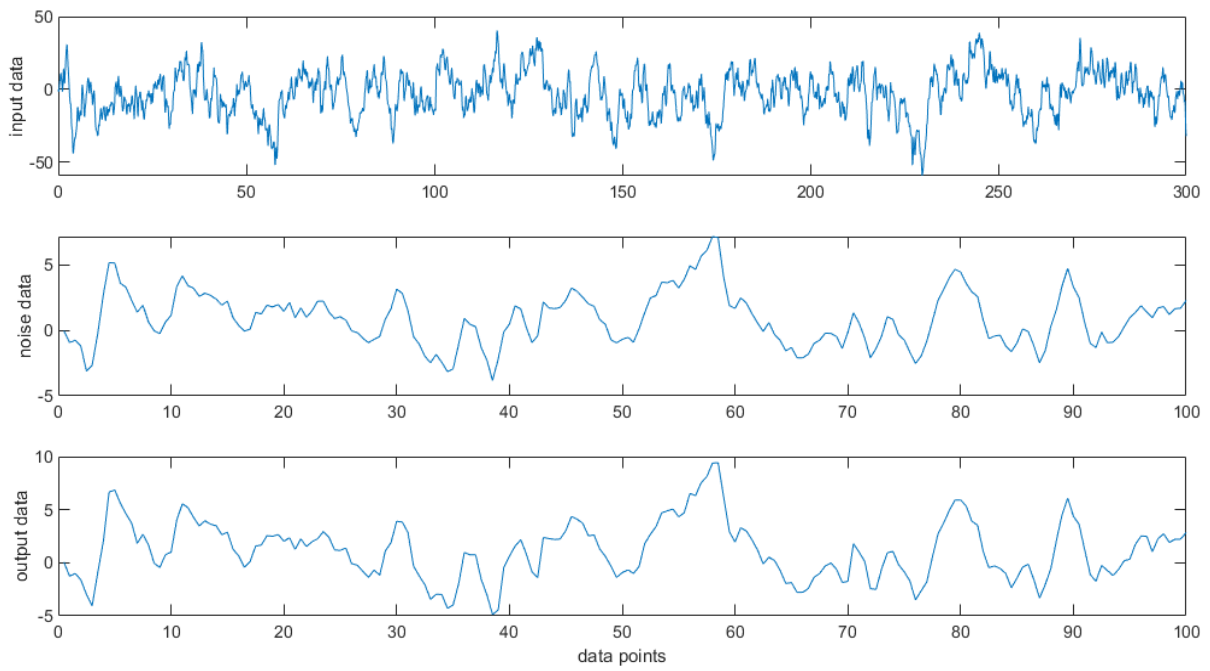


Fig. 3. The sampled inputs and outputs in multirate system .

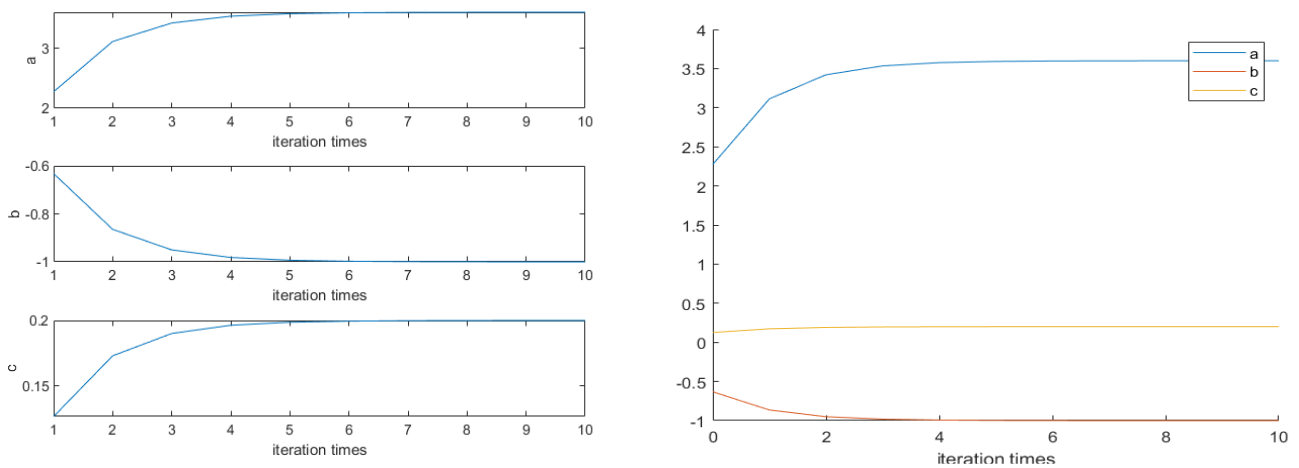


Fig. 4. Estimation of the parameters .

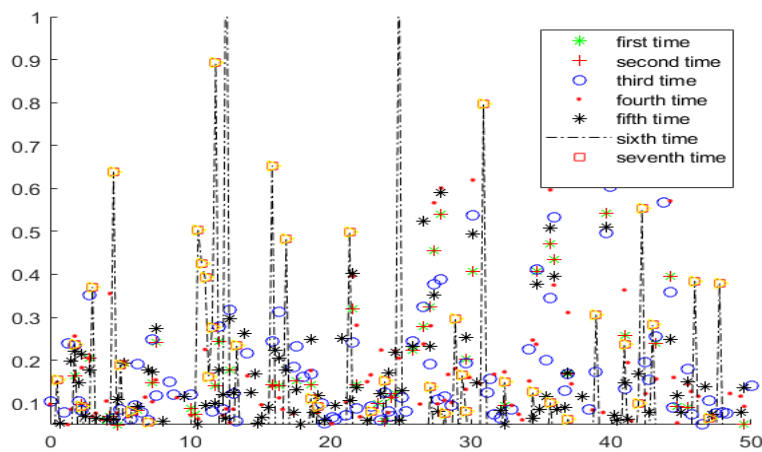


Fig. 5. The Q function .

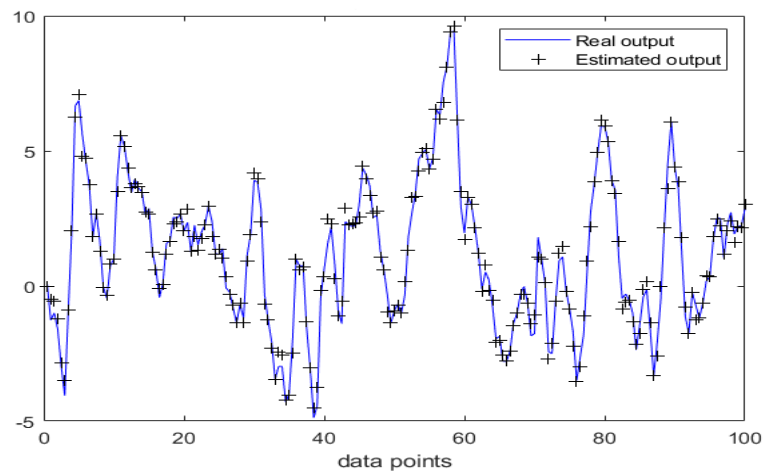


Fig. 6. Validation result.

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