Finite Time Back-Stepping Algorithm to Control Permanent Magnet Synchronous Motor Speed

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Abstract–In this paper, the speed control of a permanent magnet synchronous motor is performed in a desired finite time. Due to the nonlinearity of the dynamics of this type of motors and the form of the state equations, a back-stepping strategy has been chosen to design the control system. In the proposed method, in each design step, the finite time stability condition is used, so the nonlinear controller has the ability to guarantee finite time convergence of output tracking error. The finite time stability of the proposed control method is proved based on Lyapunov theory. Adjusting the convergence time of system outputs can be done by changing the gain of the controllers. Furthermore, the proposed controller generates smooth control signal that can be implemented. The simulation results show that the proposed method is able to control the speed and current of a permanent magnet synchronous motor in desired finite time.

Keywords: Permanent magnet synchronous motor, Nonlinear control, Finite time convergence

1. Introduction

Nowadays, due to the dependence of different industrial sectors on electric motors such as automobiles, robotics and power generation industry, researchers have explored the need to develop digital control methods for these engines. Also, the use of processors in power electronic circuits related to motor drives has provided the conditions for the use of different algorithms to control motors. In [1] for controlling the permanent magnet synchronous motor (PMSM), the parameters of PID controller are adjusted using the optimal particle aggregation method. The PID controller in this reference is a linear method, so you must first linearize the system equations. When linearization is done around an operating point, the linear model obtained around the operating point is valid. The distance of the system operating conditions from this point reduces the compatibility of the original nonlinear system with the linearized system.

Reference [2] provides a method for controlling the speed of a PMSM based on parametric changes and adaptation of the controller under these conditions. In this reference, the system equations must also be linearized, but will lead to similar problems to the previous reference. In [3] using a fuzzy controller, the parameters of the PID controller is set online based on the change of motor speed from the standard state and parametric change. In this reference, the linear PID method is used like [1]. Also, the proposed fuzzy method does not have stability analysis and is designed with trial and error. In [4] using neural network, an algorithm for controlling and adjusting the parameters of PID controller is presented.

In [5] and [6], the control of the PMSM is based on the adaptive control of the reference model. In [7] using self-adjusting control to identify parameters, and also using nonlinear sliding mode method in [8], the speed of three-phase PMSMs is controlled. Chattering is one of the problems of the sliding mode method which makes it difficult to implement. In [9] the predictive control is used to control the speed of the three-phase PMSM. Reference [10] compares PID and fuzzy controllers in controlling a three-phase PMSM. Predictive control is used in [11] to control speed and in [12] to control current. In [13, 14] neural networks have been used to control the PMSM. In references [15, 16] the adaptive sliding mode control is used and in [17] the developed Kalman filter is used to estimate the shaft angle in a PMSM. Finally, in [18], a

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robust nonlinear predictive method is used to control the current in a PMSM. In many methods, the convergence of the finite time output to the desired value is very important. Finite time stability, in addition to the error convergence speed, provides resistance to uncertainties and possible disturbances [19]. The problem of finite time stabilization of nonlinear systems is mentioned in [20-22]. In some methods, such as the sliding mode method, in order to guarantee finite time convergence, the control input equation includes a discontinuous sign function of the signal that leads to high frequency oscillation in the control signal. It is not possible to apply these controllers due to the oscillation control signal [23, 24]. Therefore, these methods need to be modified to implement.

In the mentioned references, no equation for calculating the convergence time is presented and no proof of finite time stability are performed. The main purpose of this paper is to design a finite time nonlinear controller to control a PMSM.Due to the nonlinearity of the dynamic equations of this engine, in this paper, the back-stepping control theory will be used. Also, using the finite time stability theory, time convergence of motor speed is guaranteed

2. PMSM Model

PMSM state space model is presented as equation (1) [25].

$$\begin{cases} \dot{i}_d = -\frac{R_s}{L} i_d + P_n \omega i_q + \frac{u_d}{L} \\ \dot{i}_q = -P_n \omega i_d - \frac{R_s}{L} i_q - \frac{P_n \psi_f}{L}\omega + \frac{u_q}{L} \\ \dot{\omega} = \frac{P_n \psi_f}{J} i_q - \frac{B}{J}\omega - \frac{T_L}{J} \end{cases} \tag{1}$$

In this equation, i_d and i_q are the current components in the coordinates d-q, u_d and u_q of the voltages applied to the motor, ω is the rotor speed, L is the inductance, R_s is the stator resistance and P_n is the number of pairs of motor poles. T_L is the load torque. To design the controller according to equation (1), the input of the u_d control can be designed to control the i_d current and the input of the u_q control can be designed to control the angular speed of the shaft.

3. Design of the Control System

3-1- Back-stepping control with finite time stability

The concepts of asymptotic and exponential stability in dynamic systems guarantee the convergence of system paths over an infinite time horizon. In many applications, it is desirable for a dynamic system to converge to a stable equilibrium point in a finite time. Many researches has been done on the convergence finite time of system [22, 26]. For finite time stability, consider the following system.

$$\dot{x} = f(x), \qquad f : D \to \Re^n \tag{2}$$

The equilibrium point of the system (2) is stable for a finite time, if in the initial time t_0 the system moves from the initial condition x_0 and converges to the equilibrium point in the period t which depends on the initial condition. Consider the system (2) and the continuous Lyapunov function and the derivative $V : \mathbb{R}^n \to \mathbb{R}$ in the neighborhood of the equilibrium point, which is a definite positive. In this case, the origin will be finite time if:

$$\dot{V}(x) \le -cV^{\alpha}(x) \quad \forall \quad t \ge 0 \tag{3}$$

$0 < \alpha < 1$ and $c > 0$, the convergence time depends on the initial conditions as follows is:

$$t \le \frac{V\left(x(0)\right)^{1-\alpha}}{c\left(1-\alpha\right)} \tag{4}$$

Finite time stability will have all the benefits of asymptotic stability, and if the origin of a system is finite time stability, then it will be asymptotic stability [27, 28]. To describe a back-stepping controller with the ability to guarantee finite time convergence, consider the following second-order system:

$$\begin{cases} \dot{x} = f(x) + g(x)\xi \\ \dot{\xi} = u \end{cases} \tag{5}$$

Based on the backstepping design, we begin with the first part of equation (5):

$$\dot{x} = f(x) + g(x)\xi \tag{6}$$

In this equation, M is considered as the virtual input.

Then with this input, the virtual control is as follows:

$$\xi = \phi(x) \tag{7}$$

to bring the state variable x to the desired value in a finite time. For this purpose, consider the following Lyapunov function.

$$V_1(x) = \frac{1}{2}\left(x - x_d\right)^2 \tag{8}$$

x_d is the desirable value of the state variable x . The derivative of this Lyapunov function is as follows:

$$\begin{aligned} \dot{V}_1(x) &= \frac{\partial V_1}{\partial x}\dot{x} \\ &= \left(x - x_d\right)\left[f\left(x\right) + g(x)\phi(x) - \dot{x}_d\right] \end{aligned} \tag{9}$$

$$\phi(x) = \frac{1}{g(x)}\left[-f\left(x\right) + \dot{x}_d - \frac{1}{\left(x - x_d\right)}\left(c_1 V_1^{\alpha_1}\right)\right] \tag{10}$$

$$\dot{V}_1(x) \le -c_1 V_1^{\alpha_1} \tag{11}$$

Equation (11) guarantee that the state variable x converges to x_d in a finite time, and its convergence time is obtained from the following equation:

$$t_x \le \frac{V_1\left(x(0) - x_d(0)\right)^{1-\alpha_1}}{c_1\left(1-\alpha_1\right)} \tag{12}$$

Now we define a new variable as follows in the second step:

$$z = \xi - \phi(x) \tag{13}$$

$$\dot{z} = \dot{\xi} - \dot{\phi}(x) = \mathrm{u} - \dot{\phi}(x) \tag{14}$$

To stabilize this variable, consider the new state of the Lyapunov function as follows:

$$V_2 = \frac{1}{2}z^2 \tag{15}$$

The derivative of this Lyapunov function is:

$$\dot{V}_2 = z\dot{z} = z\left[\mathrm{u} - \dot{\phi}(x)\right] \tag{16}$$

By selecting the main control input as:

$$u = \dot{\phi}(x) - \frac{1}{z}\left(c_2 V_2^{\alpha_2}\right) \tag{17}$$

By using the equations (17) and (16) we have:

$$\dot{V}_2 = -c_2 V_2^{\alpha_2} \tag{18}$$

Therefore, the limited time stability of the system will be guaranteed in the second step, and the convergence time will be obtained from the following equation:

$$t_z \le \frac{V_2\left(z(0)\right)^{1-\alpha_2}}{c_2\left(1-\alpha_2\right)} \tag{19}$$

The total convergence time is obtained from the following equation:

$$t = t_x + t_z \tag{20}$$

3-2- Design of control system for PMSM

Considering Equation (1) as the dynamic equations of a PMSM, we will have a "finite time" to control id using the control theory:

$$\begin{aligned} \dot{i}_d &= -\frac{R}{L}i_d + P_n\omega i_q + \frac{u_d}{L} \\ V_1 &= \frac{1}{2}\left(i_d - i_{d_{ref}}\right)^2 \\ \dot{V}_1 &= \left(i_d - i_{d_{ref}}\right)\left(\dot{i}_d - \dot{i}_{d_{ref}}\right) \\ &= \left(i_d - i_{d_{ref}}\right)\left(-\frac{R}{L}i_d + P_n\omega i_q + \frac{u_d}{L} - \dot{i}_{d_{ref}}\right) \le -c_1 V_1^{\alpha_1} \\ u_d &= L\left(\frac{R}{L}i_d - P_n\omega i_q - \frac{c_1 V_1^{\alpha_1}}{\left(i_d - i_{d_d}\right)}\right) \end{aligned} \tag{21}$$

To control the speed of ω using finite time back-stepping theory, first consider the dynamic equation of speed as follows:

$$\dot{\omega} = \frac{P_n \psi_f}{J} i_q - \frac{B}{J}\omega - \frac{T_L}{J} \tag{22}$$

In the first step, the desired value of $i_{q_{ref}}$ is designed to bring the finite time of speed to the reference value as follows.

$$\begin{aligned}
&V_{21} = \frac{1}{2}(\omega - \omega_{ref})^2 \\
&\dot{V}_1 = (\omega - \omega_{ref})(\dot{\omega} - \dot{\omega}_{ref}) \\
&= (\omega - \omega_{ref})\left(\frac{P_n \psi_f}{J} i_q - \frac{B}{J}\omega - \frac{T_L}{J} - \dot{\omega}_{ref}\right) \le -c_{21} V_{21}^{\alpha_{21}} \\
&i_{q_{ref}} = \frac{J}{P_n \psi_f}\left(\frac{B}{J}\omega + \dot{\omega}_{ref} - \frac{c_{21} V_{21}^{\alpha_{21}}}{(\omega - \omega_{ref})}\right)
\end{aligned} \tag{23}$$

In the second step, the control input is designed to stabilize the finite time of current as follows.

$$\begin{aligned}
&V_{22} = \frac{1}{2}\left(i_q - i_{q_{ref}}\right)^2 \\
&\dot{V}_{22} = \left(i_q - i_{q_{ref}}\right)\left(\dot{i}_q - \dot{i}_{q_{ref}}\right) \\
&= \left(i_q - i_{q_{ref}}\right)\left(-P_n \omega i_d - \frac{R_s}{L} i_q - \frac{P_n \psi_f}{L}\omega + \frac{u_q}{L} - \dot{i}_{q_{ref}}\right) \\
&\le -c_{22} V_{22}^{\alpha_{22}} \\
&u_q = L\left(P_n \omega i_d + \frac{R_s}{L} i_q + \frac{P_n \psi_f}{L}\omega + \dot{i}_{q_{ref}} - \frac{c_{22} V_{22}^{\alpha_{22}}}{\left(i_q - i_{q_{ref}}\right)}\right)
\end{aligned} \tag{24}$$

4. Simulation Results

In this section, the simulation results are shown to evaluate the performance of the controller designed. The parameter values are in Table (1). Also, the load torque is considered as a variable and its changes are shown in Figure (1).

Table1- Parameters values

Unit	Value	Parameter
Ohm	2.875	R_s
H	0.085	L
Pair	4	P
W_b	0.0175	ψ
Nm	1	B
Kg*m^2	0.01	J

Figure (1) also shows the load torque changes.

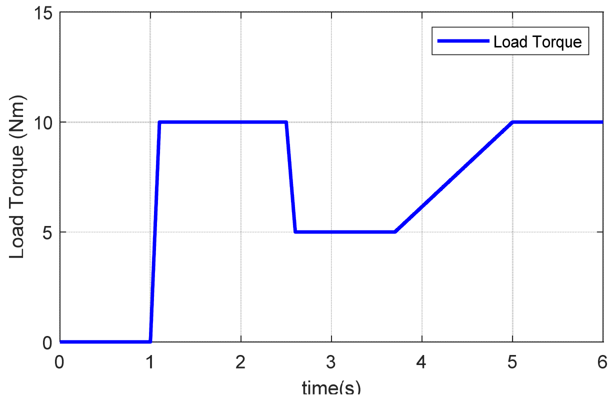

Fig. 1- curve of load torque changes

By applying the proposed controller to the motor model, in figure (2) the curve of the first output changes (i_d) and the control input changes with this output (u_d) in figure (3) are drawn. This output is stabilized at its desired value. Figures (4) and (5) show the second output changes curve which is the angular speed of the shaft and the tracking error of this variable. Figures (6) and (7) show the changes in the current i_q and the virtual control input of this variable ($i_{q\text{-}ref}$) and the tracking error of this state variable.The second output of the system and the state variable i_q are controlled with high accuracy in a finite time. Figure (8) shows the control input changes associated with this subsystem of the engine (u_q). The control signal issued using the proposed controller has a smooth behavior and can be implemented.

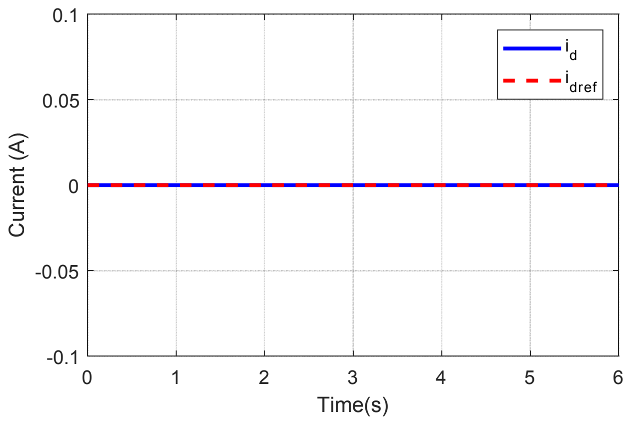

Fig. 2- Curve of current id changes

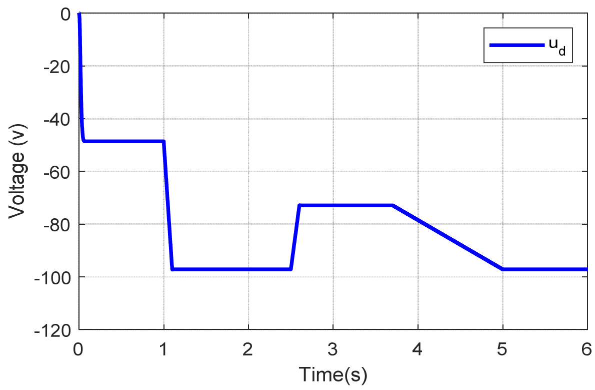

Fig. 3- Curve of control signal ud changes

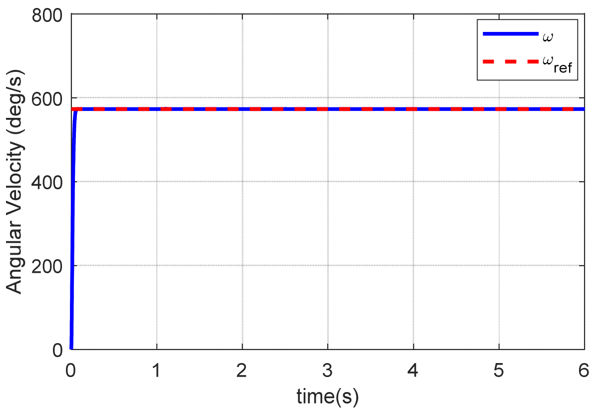

Fig. 4- Curve of Angular speed changes

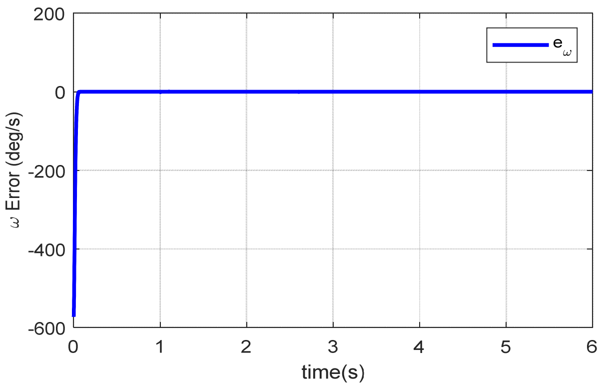

Fig. 5- Tracking error of angular speed

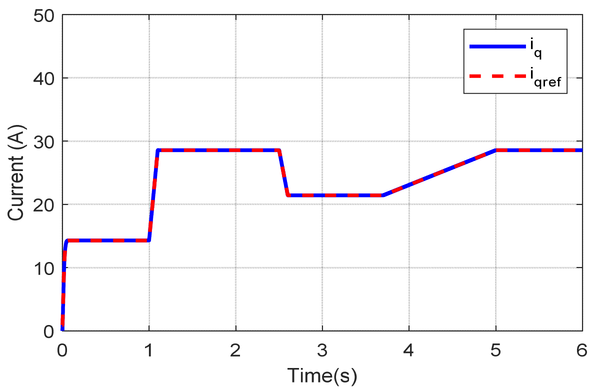

Fig. 6- Curve of current iq changes

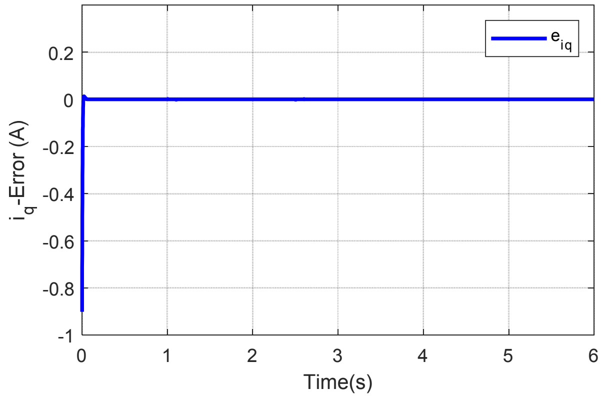

Fig. 7- Tracking error of current iq

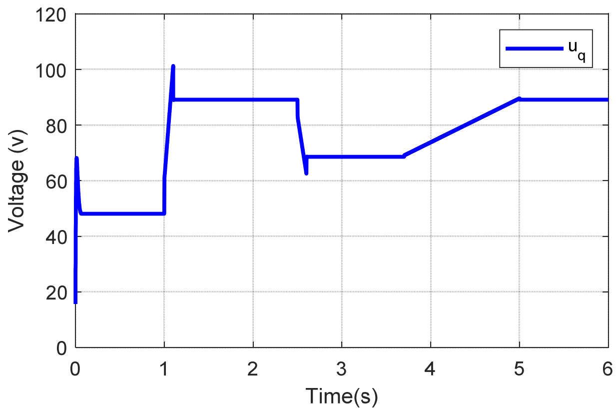

Fig. 8- Curve of control signal uq changes

To check the ability to adjust the convergence time in the proposed method, this time we performed the simulation for different control gains. In this case, for the gain $c_{22} = 200, 20, 2$ in Figures (9) and (10), i_q current changes and virtual control input tracking error has been shown by this variable. Figures (11) and (12) show the angular speed changes and the speed tracking error for gain $c_{21} = 100, 10, 2$. The convergence time of the variable state i_q can be adjusted to the desired value by changing the controller gain in this subsystem.

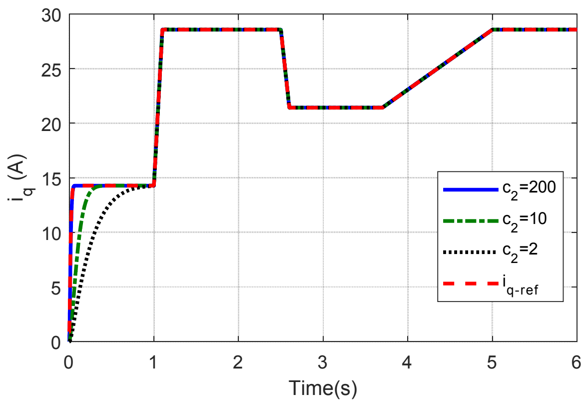

Fig. 9- Curve of current iq changes

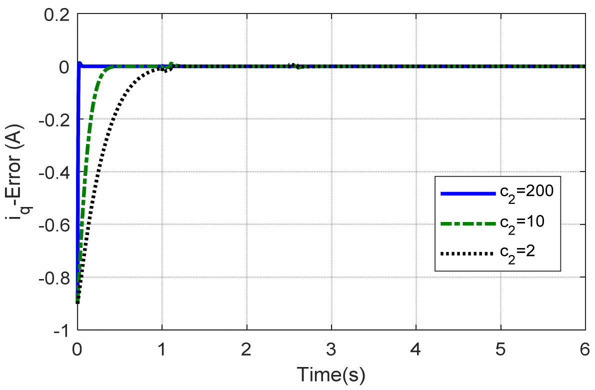

Fig. 10- Tracking error of current iq

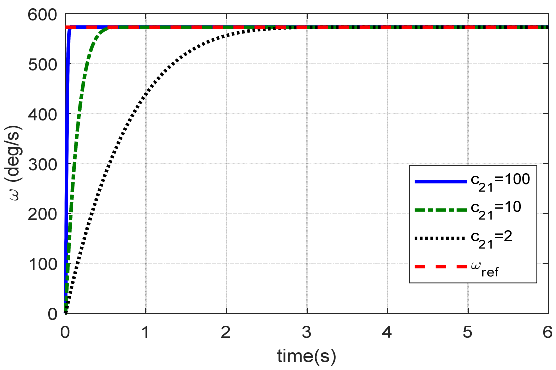

Fig. 11- Curve of angular speed changes

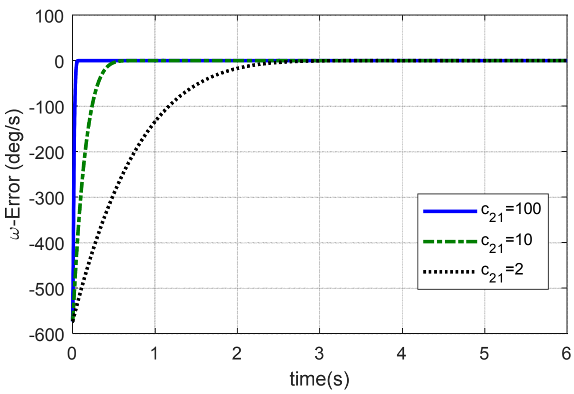

Fig. 12- Tracking error of angular speed

5. Conclusion

In this paper, a hybrid of back-stepping method and finite time control theory has been used to control the speed of a PMSM. The finite time stability of the proposed method is proved based on Lyapunov theory. The simulation results show that this method can control the speed of the PMSM. It also shows the ability to adjust the convergence time of the tracking error by changing the controller's gain.The control system proposed in this paper, although able to control the outputs in a finite time, but requires accurate value of parameters and is not robust to parametric uncertainties. Therefore, we propose combining robust theories with the method used in this paper.

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