

A Green Closed-loop Supply Chain with Allocating Retailer to Probabilistic Customers by Considering Electric Converter of CO₂ to O₂ in Vehicles

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Abstract—In this paper, a closed-loop supply chain is modeled to obtain the best allocation and location of retailers including production centers, retailers' centers, probabilistic customers, collection and disposal centers. In this study, by considering electric conversion of CO₂ to O₂ in vehicles, the amount of environmental pollution is minimized. Furthermore, two strategies are considered to find the best places for retailers by focusing on: 1- the type of expected movement (Rectangular, Euclidean, Euclidean Square, and Chebyshev); 2- expected coverage (distance and time). To this end, a bi-objective nonlinear programming model is proposed. This model concurrently compares strategies 1 and 2 and selects the best competitor. Based on the selected strategy, the best allocation is made by employing a heuristic algorithm and the locations of the best retailers are determined. As the proposed model is NP-hard in its nature of the problem, a meta-heuristic, namely, a non-dominated sorting genetic algorithm is employed for the solution process. Eventually, to authenticate and confirm the effectiveness of the suggested model, a numerical example is given and solved utilizing optimization software, and the results are analyzed.

Keywords: Green Closed-loop supply chain, Probabilistic customer, Expected distance, Expected coverage, Electric conversion

1. Introduction

Driven by the global campaign against climate change, the market of electric vehicles has boomed across the world in recent years[1]. Many big companies such as Xerox, Canon, Kodak, Dell, and Acer have put efforts into green operations[2]. The reduction of CO₂ emissions was partly due to the use of green energy from Solar Power Generation and Electrical System. In a closed-loop supply chain (CLSC), the customers are considered the last member of a chain [3]. These sets are growing and becoming more complex as demands increase. Moreover, consumers tend to require higher-quality products. This leads to a large number of returns, translating directly to increased environmental impacts. Thus, the urgent need to reduce those impacts has aroused broad attention from governments, academia, and

industries. Various measures are developed to meet the trade-off between environmental protection and cost reduction by many large economic entities[4]. CLSC management can be used to achieve a competitive advantage and attain sustainable development[5]. Note that in various situations different supply chain (SC) processes do not experience certainty, but rather probable events in one or more divisions. That is why most decision-makers further face the difficulty of optimizing uncertain models[6]. A smart cyber-physical multi-source energy system for electric vehicle applications introduced. This system is realized to increase the autonomy of the vehicle as well as a good self-dispatch energy system [7]. To minimize the loss of electrical equipment and tools which one of the most important parts in terms of waste management stable network is used to design a CLSC network with a system dynamics model. The proposed model is visualized with the program Any Logic[8]. The significance of the factors that comprise the environmental sustainability strategies and the operational features of the CLSC, their interactions and the type of their impact on the environmental, and economic sustainability of electrical and electronic equipment is very much [9]. The use of a complex logistics system for

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recovering batteries after use in its end-of-life(EOL)stage shows that the system dynamics (SD) is very important for modeling costs, income and strategic decisions are very important[8,10]. Comprehensive literature review of recent papers published at different scientific journals in Reverse logistics RL/CLSC issues that considered Waste Electrical and Electronic Equipment(WEEE) or E-waste is an EOL product[11]. The impact of ecological motivation and technological innovations on the long-term behavior of a CLSC chain with recycling activities is a developed model. This model was implemented to a real-world supply chain of electrical equipment in Greece[12].China is expected to realize the complete electrification of traditional internal combustion engine vehicles (ICEVs) by 2050[13].In this paper, a two-objective mathematical model is presented which covers the production, retailers, probabilistic customers, collection, and disposal. Considering the amount of CO₂ gas production and emission by transportation machines and convert it to O₂ by considering electric convertor, in the first step a comparison and optimal allocation to reduce distance is performed through two heuristic algorithms among the different types of mobility (Rectilinear, Euclidean, Euclidean Square, and Chebyshev), the Maximum expected coverage distance (MECD) in the retail center to provide services, and the Maximum expected coverage time (MECT) among customers to receive services. Moreover, in the second step, by taking into account the location of probabilistic customers, the general model of the chain is solved by employing thenon-dominated sorting genetic algorithm(NSGA-II)and the best location for retailers is determined. Since earlier studies have paid less attention to the type of dislocation between probabilistic customers and retailers, the current study addresses this issue such that each can readily use its desired motion.

2. Mathematical Model

2.1. Network structure

The formation of the CLSC network is presented in Figure 1 and involves production centers, retail centers, probabilistic customers, collection centers, and disposing centers.

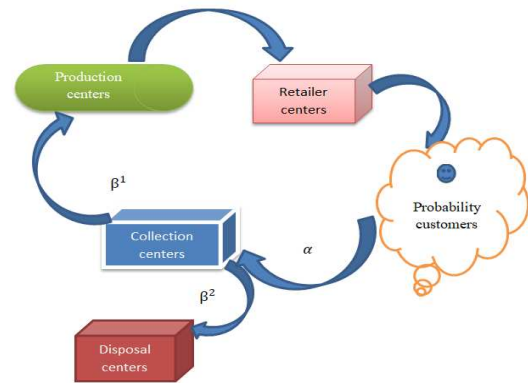


Figure 1.Schematic diagram of the modeled CLSC

3.2. Model Assumption

1. The single-period model is recognized;
2. Insufficiency is not allowed in any section;
3. Transportation costs are fixed over a period;
4. Products returned by the customer are subject to penalties;
5. All customers must take their service;
6. Every customer can visit more than one retail center to receive services;
7. The customers' and retailers' motion type is Rectangular, Euclidean, Euclidean Square, or Chebyshev;
8. The motion happens on a page;
9. Based on the relocation distance, the transportation time is constant and invariable;
10. All chain parameters and variables are definite, excluding the customer place, customer coverage time, and retailer coverage distance;
11. Raw materials are provided by production centers. Hence, in this model, supply centers are not counted;
12. No cost is considered for keeping the goods;
13. The retail coverage distance for random customers is not constant at each stage;
14. The probabilistic customer time coverage of retailers is not consistent at every stage.
15. Via electric converters, transportation devices can convert CO₂ gas to O₂.

3.3. Sets

In this section, all the indexes used in modeling the problem are included.

$p = 1, 2, \dots, P$	Index of collection centers that have the potential to produce.
$j = 1, 2, \dots, J$	Index of collection of retail centers that have the selling potential.
$i = 1, 2, \dots, I$	Index of a collection of probabilistic customers.
$cc = 1, 2, \dots, CC$	Index of collection centers that have the potential to collect.
$dis = 1, 2, \dots, DIS$	Index of collection of disposal centers that have the elimination potential.
$r = 1, 2, \dots, R$	Index of the produced goods.
$e, e' = \text{set of all echelons}$	$(e, e' \in \{p, j, i, cc, dis\})$
$k, k' = \text{Set of facilities in echelon}$	$(k_e, k'_e \in \{1, \dots, K_e\})$

3.4. Model parameters

In this section, all the parameters associated with the problem are presented.

Table 1. Parameters used in model

Parameter	Description
cap_e	capacity of facility $e \in \{p, j, i, cc, dis\}$
cap_{k_e}	capacity of facility $k_e e \in \{p, j, cc, dis\}$
τ_{k_e}	The Amount of facility $k_e e \in \{p, j, cc, dis\}$
v_r	The amount of the type r production
FC_{k_e}	Fixed cost of the facility $k_e e \in \{p\}$
FC_j	Fixed cost of the j potential retailer center
FC'_{k_e}	Fixed cost of the facility $k_e e \in \{cc, dis\}$
L_j	Standard radius of the service distance for the j retailer
T_i	Standard radius of time of service receiving for the i probabilistic customer
$l_{j,i,r}$	Distance covered by the j retailer transfer to provide service for the i probabilistic customer due to send the type r product
$t_{i,j,r}$	Time covered by the i probabilistic customer spend to gain service for the j retailer due to receive the type r product
E_j	Upper and lower limit values from the standard distance radius
θ_i	Upper and lower limit values from the standard time radius
μ_{i1}	The average horizontal coordinates of the i probabilistic customer
μ_{i2}	The average vertical coordinates of the i probabilistic customer
σ_{i1}^2	The variance of horizontal coordinates of the i probabilistic customer
σ_{i2}^2	The variance of vertical coordinates of the i probabilistic customer
d_{j1}	Spatial horizontal coordinates of the j retailer
d_{j2}	Spatial vertical coordinates of the j retailer
α	The percentage of the costumers returned goods. ($\alpha \leq 1$)
$\beta^1 \beta^2$	The percentage of the products that can be revived in the collection center and eliminated in the disposal center. ($\beta^1 + \beta^2 = \alpha$)
$Cost_{k_e k'_e, r}$	The relocation cost of the type r product from the facility center k_e to facility center $k'_e (e, e' \in \{p, j\})$
$Cost_{j,i,r}$	The relocation cost of the type r product from the j potential retailer center to the i probabilistic customer
$Cost'_{i,j,r}$	The relocation cost of the type r product from the i probabilistic customer to the j potential retailer center
$Cost'_{k_e k'_e, r}$	The relocation cost of the type r product from the facility center k_e to facility center $k'_e (e, e' \in \{i, cc, dis, p\})$

$dic_{k_e k'_e, r}$	The transferring distance of the type r product from the facility center k_e to facility center $k'_e (e, e' \in \{p, j\})$
$dic_{j,i,r}$	The transferring distance of the type r product from the j potential retailer center to the i probabilistic customer
$dic'_{i,j,r}$	The transferring distance of the type r product from the i probabilistic customer to the j potential retailer center
$dic'_{k_e k'_e, r}$	The transferring distance of the type r product from the facility center k_e to facility center $k'_e (e, e' \in \{i, cc, dis, p\})$
$x_{k_e k'_e, r}$	The amount of the type r product send from the facility center k_e to facility center $k'_e (e, e' \in \{p, j\})$
$x_{j,i,r}$	The amount of probabilistic demand of the type r product send from j potential retailer center to the i probabilistic customer
$x'_{i,j,r}$	The amount of probabilistic demand of the type r product received the i probabilistic customer from, the j potential retailer center
$x'_{k_e k'_e, r}$	The amount of the type r product send from the facility center k_e to facility center $k'_e (e, e' \in \{i, cc, dis, p\})$
$dicR$	Rectangular motion
$dicED$	Euclidean motion
$dicEDS$	Euclidean Square motion
$dicCH$	Chebyshev motion
$COVER^1$	Expected distance coverage
$COVER^2$	Expected time coverage
$CO2AIR_{k_e k'_e, r}$	The amount of CO ₂ produced due to relocation of the type r from the facility center k_e to facility center $k'_e (e, e' \in \{p, j\})$
$Co2AIR_{j,i,r}$	The amount of CO ₂ produced due to relocation of the type r from the j potential retailer center to the i probabilistic customer in time or meters
$CO2AIR'_{i,j,r}$	The amount of CO ₂ produced due to relocation of the type r from the i probabilistic customer to the j potential retailer center in time or meters
$CO2AIR'_{k_e k'_e, r}$	The amount of CO ₂ produced due to the relocation of the type r from the facility center k_e to facility center $k'_e (e, e' \in \{i, cc, dis, p\})$
$AIRO_2$	The Percentage of carbon dioxide to oxygen by electric conversion
$O_2STANDARD$	The amount of O ₂ standard emissions

3.5. Decision variable

In this article, only one decision variable is used as 0 and 1 to designate customers to retailers or vice versa.

$$Q_1 = \begin{cases} 1 & \text{if the retailer is allocated to the customer} \\ 0 & \text{otherwise} \end{cases}$$

3.6. Model formulation

The mathematical model of this chain is in two stages. In the first step, the mathematical model is formulated among probabilistic customers and retailers, and in the second stage, the entire problem is formulated.

Step 1: Given that calculations are quite influential among retailers and probabilistic customers, first, assuming the type of movement by retailers to customers or vice versa and calculating the distance coverage radius of the retailers and the time coverage radius of customers and comparing them with each other according to the heuristic

algorithms, their minimum value is chosen and considered as the output of this section.

$$f = \min \left(\left\{ \sum_j^J FC_j + \sum_{i=1}^I \sum_{j=1}^J \sum_{r=1}^R x_{j,i,r} \right. \right. \quad (1)$$

$$\times Cost_{j,i,r}$$

$$\times \min(f_{1(j,i)}, f_{3(j,i)}) \left. \right\} \times Q_1 \quad (2)$$

$$+ \left\{ \sum_j^J FC_j \right. \quad (3)$$

$$+ \sum_{j=1}^J \sum_{i=1}^I \sum_{r=1}^R x'_{i,j,r} \quad (4)$$

$$\times Cost'_{i,j,r}$$

$$\times \min(f_{2(i,j)}, f_{4(i,j)}) \left. \right\}$$

$$\times (1 - Q_1)$$

S.t.

$$\theta_i < T_i \forall i \in I$$

$$e_j < L_j \forall j \in J$$

$$Q_1 + (1 - Q_1) = 1$$

Equation (1) presents the objective function of the first step of the process among retailers and probabilistic customers within a parenthesis, which has two parts. The parts are distinguished by "{}". The first part of the selection is the calculation of the minimum coverage of the expected distance and the expected motion (Rectangular, Euclidean, Euclidean Square, and Chebyshev) of retailers to grant services to customers. The second part of the selection includes the minimum calculation of the amount of expected time coverage and the expected motion (Rectangular, Euclidean, Euclidean Square, and Chebyshev) of customers to receive services from retailers. Lastly, by comparing the chosen minimum costs, the lowest value is selected as the output. The output of this part explains whether the retailers convey the goods or the customers come to receive them. Constraint (2) Gives the maximum amount of time coverage radius of the customer. Constraint (3) gives the maximum motion radius of retailers. Equation (4) displays choosing the decision variable.

Step 2: In this step, according to Figure 1 of the modeling process, each of the input and output parts of the desired chain is calculated. Finally, it shows the value of the first target function (displacement cost) and the second objective function (the carbon dioxide emissions generated by the means of transport). See the related equation $f_{1(j,i)}$, $f_{2(i,j)}$, $f_{3(j,i)}$ and $f_{4(i,j)}$ in appendix.

$$TOTAL OBJECT1 = \min \left(\left\{ \sum_{k_e \in \{p\}} FC_{k_e} + \sum_{k_e \in \{p,j\}} \sum_r x_{k_e k'_e, r} \times Cost_{k_e k'_e, r} \times \right. \right.$$

$$dic_{k_e k'_e, r} \left. \right\} + \min\{f\} + \left\{ \sum_{k_e \in \{cc, dis\}} FC'_{k_e} + \sum_{k_e \in \{i, cc, dis, p\}} \sum_r x'_{k_e k'_e, r} \times Cost'_{k_e k'_e, r} \times \right. \quad (5)$$

$$dic'_{k_e k'_e, r} \left. \right\}$$

Equation (5) is the objective function of the process, which consists of three parts, each separated by a "{}" from the others. Part one includes the sum of the fixed and the variable costs of the transfer of goods, which are shipped from the production centers to the retailer centers. Part two is calculated using Equation (1). Part three includes the entire fixed and variable cost of products that are returned by customers to the collection centers. After, this stage is calculated fixed and variable costs of the transfer of goods from collection centers to repair and disposing center and finally are calculated fixed and variable costs of the transfer of goods from repair center to distribution and warehouse and disposing centers.

$$TOTAL OBJECT2 = \max \left(\begin{aligned} &AIRO_2 \\ &\times \left(\min \left(\sum_{k_e \in \{p,j\}} \sum_r CO2AIR_{k_e k'_e, r} \right. \right. \\ &\times dic_{k_e k'_e, r} \\ &+ \left(\sum_{i=1}^I \sum_{j=1}^J \sum_{r=1}^R (CO2AIR_{j,i,r} \right. \\ &\times \min(f_{1(j,i)}, f_{3(j,i)}) \times Q_1 \\ &+ \sum_{j=1}^J \sum_{i=1}^I \sum_{r=1}^R (CO2AIR'_{i,j,r} \\ &\times \min(f_{2(i,j)}, f_{4(i,j)}) \times (1 - Q_1) \left. \right) \left. \right) \\ &+ \sum_{k_e \in \{i, cc, dis, p\}} \sum_r CO2AIR'_{k_e k'_e, r} \\ &\times dic'_{k_e k'_e, r} \left. \right) \end{aligned} \right) \quad (6)$$

Equation (6) shows the objective function of carbondioxide emissions and converted to oxygen on the type of movement or coverage selection.

$$\sum_{k_e | e \in \{p, j, cc, dis\}} \tau_{k_e, r} \leq cap_{e \in \{p, j, cc, dis\}} \quad \forall r \quad (7)$$

$$\sum_r \tau_{k_e, r} \leq cap_{k_e} \quad \forall k_e | e \in \{p, j, cc, dis\} \quad (8)$$

$$\sum_{k_e | e \in \{p, j, cc, dis\}} \tau_{k_e, r} \leq v_r \quad \forall r \quad (9)$$

$$x'_{k_e k'_{e'}, r} \sum_{e, e' \in \{cc, p\}} x'_{k_e k'_{e'}, r} \quad \forall r \quad (10)$$

$$+ \sum_{k_e, k'_{e'} | e, e' \in \{p, j\}} x_{k_e k'_{e'}, r} \leq \sum_{k_e | e \in \{p\}} \tau_{k_e, r} \quad (11)$$

$$\sum_r x_{k_e k'_{e'}, r} \leq \sum_r \tau_{k_e, r} \quad \forall x_{k_e k'_{e'}, r} \in (e, e' \in \{p, j\})$$

$$\forall \tau_{k_e} | e \in \{j\} \quad \sum_{x_{k_e k'_{e'}, r} \in (e, e' \in \{p, j\}), r} x_{k_e k'_{e'}, r} = \quad \forall r \quad (12)$$

$$\left(\sum_{i=1}^I \sum_{j=1}^J E(x_{j,i,r}) \right) \times Q_1 + \left(\sum_{j=1}^J \sum_{i=1}^I E(x'_{i,j,r}) \right) \times (1 - Q_1) \quad \forall r \quad (13)$$

$$\left(\sum_i \sum_j \alpha_{j,i,r} \times \sum_{i=1}^I \sum_{j=1}^J E(x_{j,i,r}) \right) \times Q_1 + \left(\sum_j \sum_i \alpha_{i,j,r} \times \sum_{j=1}^J \sum_{i=1}^I E(x'_{i,j,r}) \right) \times (1 - Q_1) = \sum_{x'_{k_e k'_{e'}, r} \in (e, e' \in \{i, cc\})} x'_{k_e k'_{e'}, r} \quad \forall r \quad (14)$$

$$\sum_i \sum_j \alpha_{j,i,r} \times Q_1 + \sum_j \sum_i \alpha_{i,j,r} \times (1 - Q_1) \leq 1 \quad \forall r \quad (15)$$

$$\sum_{x'_{k_e k'_{e'}, r} \in (e, e' \in \{i, cc\})} x'_{k_e k'_{e'}, r} \leq \sum_{k_e | e \in \{cc\}} \tau_{k_e, r} \quad \forall r \quad (16)$$

$$\sum_{cc=1}^{CC} \sum_{i=1}^I \beta_{i,cc,r}^1 \times \sum_{x'_{k_e k'_{e'}, r} \in (e, e' \in \{i, cc\})} x'_{k_e k'_{e'}, r} = \sum_{x'_{k_e k'_{e'}, r} \in (e, e' \in \{cc, p\})} x'_{k_e k'_{e'}, r} \quad \forall r \quad (17)$$

$$\sum_{x'_{k_e k'_{e'}, r} \in (e, e' \in \{cc, p\})} x'_{k_e k'_{e'}, r} \leq \sum_{x'_{k_e k'_{e'}, r} \in (e, e' \in \{i, cc\})} x'_{k_e k'_{e'}, r} \quad \forall r \quad (18)$$

$$\sum_{cc} \sum_i \beta_{i,cc,r}^2 \times \sum_{x'_{k_e k'_{e'}, r} \in (e, e' \in \{i, cc\})} x'_{k_e k'_{e'}, r} = \sum_{x'_{k_e k'_{e'}, r} \in (e, e' \in \{cc, dis\})} x'_{k_e k'_{e'}, r} \quad \forall r \quad (18)$$

Constraint (7) gives the maximum capacity of all facility centers Constraint (8) gives the maximum capacity of each facility. Constraint (9) declares the maximum amount of production r in each facility centers. Constraint (10) shows the maximum capacity of goods joining the production centers. Constraint (11) shows the maximum entry capacity of each retailer center. Constraint (12) gives the balance of entry and exit of goods in retail centers. In this restriction, according to algorithms 1 and 2, it is

determined whether the retailers send the goods to the customers or the customers refer to the retailers to receive the goods. Constraint (13) Depending on the decision variable shows the return percentage of goods from customers. Constraint (14) gives the total percentage of goods returned by customers, which is a maximum of 1. Constraint (15) gives the maximum capacity of goods joining the collection centers by all customers. Constraint (16) shows the percentage of goods that are shipped from the collection center to the production centers. Constraint (17) states that the maximum number of collection center goods is equal to the number of returned goods. Constraint (18) gives the percentage of goods sent from the total collection center to the disposal centers.

$$\sum_{x'_{k_e k'_{e'}, r} \in (e, e' \in \{cc, dis\})} x'_{k_e k'_{e'}, r} \leq \sum_{k_e | e \in \{dis\}} \tau_{k_e, r} \quad \forall r \quad (19)$$

$$\sum_{x'_{k_e k'_{e'}, r} \in (e, e' \in \{cc, dis\})} x'_{k_e k'_{e'}, r} \quad \forall r \quad (20)$$

$$\leq \sum_{x'_{k_e k'_{e'}, r} \in (e, e' \in \{i, cc\})} x'_{k_e k'_{e'}, r} \quad \forall r \quad (21)$$

Constraint (19) shows that the maximum number of goods sent from the collection centers to the disposal centers is equal to the maximum capacity of the disposal centers. Constraint (20) asserts that the maximum number of destroyed goods is equal to the number of returned goods. Constraint (21) presents the balance of goods entering and leaving in all collection centers.

$$\begin{aligned}
 \text{AIRO}_2 \times \left(\min \min \left(\sum_{k_e \in \{p,j\}} \sum_r \text{CO2AIR}_{k_e k'_e, r} \times \text{dic}_{k_e k'_e, r} \right. \right. \\
 + \left(\sum_{i=1}^I \sum_{j=1}^J \sum_{r=1}^R (\text{CO2AIR}_{j,i,r} \right. \\
 \times (\min(f_{1(j,i)}, f_{3(j,i)})) \times Q_1) \\
 + \sum_{j=1}^J \sum_{i=1}^I \sum_{r=1}^R (\text{CO2AIR}'_{i,j,r} \\
 \times (\min(f_{2(i,j)}, f_{4(i,j)})) \times (1 - Q_1)) \left. \right) \\
 + \left. \sum_{k_e \in \{i,cc,dis,p\}} \sum_r \text{CO2AIR}'_{k_e k'_e, r} \right. \\
 \left. \times \text{dic}'_{k_e k'_e, r} \right) \geq \text{O}_2 \text{STANDAR} \quad (22)
 \end{aligned}$$

Constraint (22) gives the maximum standard of carbon dioxide.

Parameters ≥ 0 and $Q_1 = 0,1$.

3.7. Solution approach

The proposed model, in addition to decreasing costs by choosing the best place for retailers, also reduces the amount of carbon dioxide in retailers and customers by allowing the best place for retailers. In this model, in addition to the fact that retailers can be selected to send services, customers can also refer to receive services. Due to the probabilistic nature of customers, first of all, the expected distances between customers and retailers are calculated per movement methods performed (Rectangular, Euclidean, Euclidean Square, and Chebyshev). These values are compared with MECD of retailers, which is displayed in algorithm 1 heuristically, and the minimum value is picked. Also, to allocate customers to retailers, considering the customer's movement methods and comparing it with MECT, which is presented in Algorithm 2 heuristically, the minimum value is chosen. At the end, by choosing the minimum cost from the two mentioned methods, the allocation and how to provide the service is determined Algorithm.

Algorithm 1: Assigning retailers to customers

Step 1: Initialization

-Generate the average longitudinal and transverse to the number of probabilistic customers

$$\mu_{11}, \mu_{21}, \dots, \mu_{11}$$

$$\mu_{12}, \mu_{22}, \dots, \mu_{12}$$

Step 2: Computing expected distance and cost

-Depending on the type of sending the goods from retailers to probabilistic customers (Rectangular, Euclidean, Euclidean Square, Chebyshev) Compute expected distance

-Calculate the cost of sending goods from retailers to potential customers using the second step

Step3: Computing distance coverage radius

-using the second step

Computes the maximum distance maximum distance

$$= \max(E[(\text{dicR}_{j,i}), (\text{dicED}_{j,i}), (\text{dicEDS}_{j,i}), (\text{dicCH}_{j,i})])$$

$$l_{j,i,r} = \text{rand}(1, \text{maximum distance})_{J \times I \times R}$$

Step4: Computing maximum and the minimum distance coverage radius

-For all retailers to provide services, calculate the minimum and the maximum distance coverage radius separately;

$$\text{Min and Max} = [L_j - E_j \quad L_j + E_j]$$

Step5: assign retailer

If:

$$(l_{j,i,r}) \leq [L_j - E_j] \text{ Then calculate } \{\sum_j^J FC_j +$$

$$\sum_{i=1}^I \sum_{j=1}^J \sum_{r=1}^R x_{j,i,r} \times \text{Cost}_{j,i,r} \times \min(E[\text{dic}_{j,i,r}], E[\text{COVER}^1(l_{j,i,r})])\}$$

and assign the retailer to the max(E[dic_{j,i,r}], E[COVER¹(l_{j,i,r})])

$$L_j - E_j < (l_{j,i,r}) \leq L_j, \text{ then calculate } \{\sum_j^J FC_j +$$

$$\sum_{i=1}^I \sum_{j=1}^J \sum_{r=1}^R x_{j,i,r} \times \text{Cost}_{j,i,r} \times \min(E[\text{dic}_{j,i,r}], E[\text{COVER}^1(l_{j,i,r})])\}$$

and assign the retailer to the max(E[dic_{j,i,r}], E[COVER¹(l_{j,i,r})])

$$L_j < (l_{j,i,r}) \leq L_j + E_j, \text{ then calculate } \{\sum_j^J FC_j +$$

$$\sum_{i=1}^I \sum_{j=1}^J \sum_{r=1}^R x_{j,i,r} \times \text{Cost}_{j,i,r} \times \min(E[\text{dic}_{j,i,r}], E[\text{COVER}^1(l_{j,i,r})])\}$$

and assign the retailer to the max(E[dic_{j,i,r}], E[COVER¹(l_{j,i,r})])

$$(l_{j,i,r}) > L_j + E_j, \text{ then calculate}$$

$$\{\sum_j^J FC_j + \sum_{i=1}^I \sum_{j=1}^J \sum_{r=1}^R x_{j,i,r} \times \text{Cost}_{j,i,r} \times \min(E[\text{dic}_{j,i,r}], E[\text{COVER}^1(l_{j,i,r})])\}$$

and assign the retailer to the max(E[dic_{j,i,r}], E[COVER¹(l_{j,i,r})])

Algorithm 2: Assigning customers to retailers

Step 1: Initialization

Generate the average longitudinal and transverse to the number of probabilistic customers

$$\mu_{11}, \mu_{21}, \dots, \mu_{11}$$

$$\mu_{12}, \mu_{22}, \dots, \mu_{12}$$

Step 2: Computing expected distance and cost

Depending on the type of sending the goods from retailers to probabilistic customers (Rectangular, Euclidean, Euclidean Square, Chebyshev) Compute expected distance

Calculate the cost of sending goods from retailers to potential customers using the second step

Step3: Computing time coverage radius

using the second step

computesthe maximum distance

MaxTime

$$= \max(E[(dicR_{i,j}), (dicED_{i,j}), (dicEDS_{i,j}), (dicCH_{i,j})])$$

$$t_{i,j,r} = rand([1, MaxTime])_{I \times J \times R}$$

Step4: Computing maximum and the minimum time coverage radius

For all retailers to provide services, calculate the minimum and the maximum time coverage radius separately;

$$Min \text{ and } max = [T_i - \theta_i T_i + \theta_i]$$

Step5: assign customer

if:

$(t_{i,j,r}) \leq [T_i - \theta_i]$ then calculate

$$\{\sum_j^J FC_j + \sum_{j=1}^J \sum_{i=1}^I \sum_{r=1}^R x'_{i,j,r} \times Cost'_{i,j,r} \times$$

$min([dic_{i,j,r}], E[COVER^2(t_{i,j,r})])\}$ And assign

the customer to the max

$$([dic_{i,j,r}], E[COVER^2(t_{i,j,r})])$$

$T_i - \theta_i < (t_{i,j,r}) \leq T_i$ then calculate

$$\{\sum_j^J FC_j + \sum_{j=1}^J \sum_{i=1}^I \sum_{r=1}^R x'_{i,j,r} \times Cost'_{i,j,r} \times$$

$min([dic_{i,j,r}], E[COVER^2(t_{i,j,r})])\}$ And assign

the customer to the max

$$([dic_{i,j,r}], E[COVER^2(t_{i,j,r})])$$

$T_i < (t_{i,j,r}) \leq T_i + \theta_i$, then calculate

$$\{\sum_j^J FC_j + \sum_{j=1}^J \sum_{i=1}^I \sum_{r=1}^R x'_{i,j,r} \times Cost'_{i,j,r} \times$$

$min([dic_{i,j,r}], E[COVER^2(t_{i,j,r})])\}$ And assign

the customer to the max

$$([dic_{i,j,r}], E[COVER^2(t_{i,j,r})])$$

$(t_{i,j,r}) > T_i + \theta_i$ then calculate

$$\{\sum_j^J FC_j + \sum_{j=1}^J \sum_{i=1}^I \sum_{r=1}^R x'_{i,j,r} \times Cost'_{i,j,r} \times$$

$min([dic_{i,j,r}], E[COVER^2(t_{i,j,r})])\}$ And assign

the customer to the max

$$([dic_{i,j,r}], E[COVER^2(t_{i,j,r})])$$

By comparing the outputs of algorithm 1 and algorithm 2, we choose the minimum cost from them.

3.7.1. Non-Sorting-Genetic Algorithm II

General problem solving: The use of meta-heuristic algorithms will have highly profitable results in solving complex, difficult problems [14]. Therefore, the use of the NSGA-II in solving unrestricted multi-objective problems is expanding swiftly [15]. In this paper, due to the double-objective nature and multiplicity of constraints, this algorithm is employed to solve the general model of the SC. Figure 2 gives the flowchart of this type of algorithm [16].

4. Numerical example

In this paper, to understand the problem model, a numerical example for the CLSC model presented in Figure 1 and considering the normal distribution is solved using MATLAB R2018b coding.

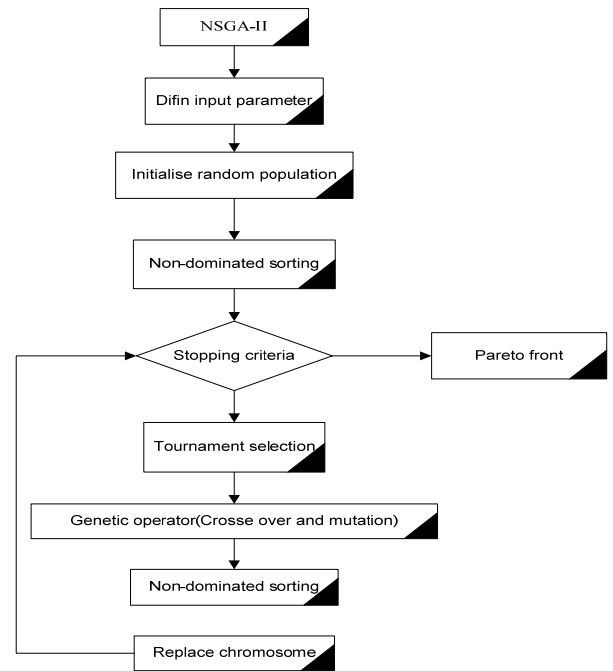


Figure 2. Flow chart of the NSGA-II

4. Numerical example

In this paper, to understand the problem model, a numerical example for the CLSC model presented in Figure 1 and considering the normal distribution is solved using MATLAB R2018b coding.

Table 2. Probabilistic customer coordinates and time coverage radius and tolerance

Probabilistic customer number	probabilistic customer coordinates	Standard time coverage radius (Second)	Upper and lower limit values (Second)
1	[4361,4254]	3100	500
2	[4294,6191]	3200	1000
3	[6793,8863]	3000	1000
4	[5232,5483]	3100	1000
5	[6867,6340]	3200	1000
6	[5247,5683]	2300	1000
7	[4145,4788]	2500	1100
8	[5409,6065]	2500	1200

Table 3. Moving type from Retailer to probabilistic customer

j	i=1	i=2	i=3	i=4	i=5	i=6	i=7	i=8
1	dicED	dicED	dicR	dicCH	dicEDS	dicCH	dicED	dicR
2	dicEDS	dicCH	dicR	dicED	dicED	dicCH	dicR	dicR
3	dicR	dicED	dicEDS	dicED	dicCH	dicED	dicR	dicED

Table 4. Retailers coordinate and standard distance coverage radius

Retailer number	Retailers coordinates	Standard distance coverage radius(meter)	Upper and lower limit values (meter)
1	[3200,5000]	3000	1200
2	[5500,6000]	3500	1500
3	[7000,5000]	2800	1500

Table 5. Number facility center and every production capacity

	Production centers	Retailer centers	Collection centers	Disposing center
unit numbers	4	3	3	2
The Capacity of each product type	r1	400	236	33
	r2	400	153	31
Total Capacity	800	389	64	31

Table 6. The variance of the probabilistic costumers

	j=1	j=2	j=3
i=1	0.7195	0.3020	0.0860
i=2	0.5003	0.1604	0.6629
i=3	0.5271	0.8269	0.4002
i=4	0.4717	0.3054	0.6324
i=5	0.5632	0.0610	0.7476
i=6	0.3680	0.5619	0.3076
i=7	0.3289	0.4198	0.1270
i=8	0.2158	0.8125	0.2429

Table7. Transportation costs between retailer centers and customer

	i=1	i=2	i=3	i=4	i=5	i=6	i=7	i=8
j=1	3	4	4	1	3	4	1	13
j=2	3	2	4	5	1	2	5	4
j=3	2	4	5	2	3	4	1	2

Table 8. The amount of goods and the distance and cost between production and retailer centers

		production			Dic(km)			Cost(dollar)		
		r=1	r=2	r=3	r=1	r=2	r=3	r=1	r=2	r=3
p=1	v=1	30	34	30	1	3	3	2	3	3
	v=2	25	30	28						
p=2	v=1	18	26	30	2	3	4	2	5	5
	v=2	22	25	32						
p=3	v=1	18	30	15	4	4	2	4	2	2
	v=2	21	22	24						
p=4	v=1	17	30	20	3	4	3	3	4	4
	v=2	22	18	20						

Table 9. The Percentage of goods returned and distance and between collection centers and disposal centers.

		Percentage of goods returned		dic (km)		cost (dollar)	
		dis1	dis2	dis1	dis2	dis1	dis2
cc=1	r=2	0.1	0.2				
	r=1	0	0.2	6	5	2	5
cc=2	r=2	0	0				
	cc=3	r=1	0	0	4	5	6
r=2		0	0.3				

Table 10: Fixed cost and maximum capacity of each facility center

Facility	Fixed cost	Maximum capacity	Facility	Fixed cost	Maximum capacity
Production centers	91	200	p=1	91	200
	90	200	p=2	90	200
	105	200	p=3	105	200
Retailer centers	93	200	r=1	93	200
	100	300	r=2	100	300
	99	300	r=3	99	300
Probability Customer	92	41	c=1	92	41
	.	44	c=2	.	44
	.	53	c=3	.	53
	.	58	c=4	.	58
	.	58	c=5	.	58
	.	26	c=6	.	26
	.	50	c=7	.	50
	.	59	c=8	.	59
Collection centers	85	25	cc=1	85	25
	91	27	cc=2	91	27
	86	12	cc=3	86	12
Disposing center	55	13	dis=1	55	13
	60	18	dis=2	60	18

Table 11. Customer demand and distance cost

		demand			distance cost(cent)		
		j=1	j=2	j=3	j=1	j=2	j=3
i=1	r=1	10	8	8	4	2	4
	r=2	6	9	0			
i=2	r=1	8	10	10	4	4	5
	r=2	8	0	8			
i=3	r=1	13	11	12	1	5	2
	r=2	4	7	6			
i=4	r=1	11	8	13	3	2	3
	r=2	8	8	10			
i=5	r=1	11	10	12	4	2	4
	r=2	6	9	10			
i=6	r=1	7	9	10	1	5	1
	r=2	0	0	0			
i=7	r=1	12	10	0	3	4	2
	r=2	10	10	8			
i=8	r=1	11	11	11	3	2	3
	r=2	8	9	9			

Table 12. Total cost and profit function.

Results check	Cost and o2 calculated by allocating the proposed algorithm before NSGA-II		Cost and o2 calculated by allocating the proposed algorithm after NSGA-II	
	Costfunction	Amount o2	Costfunction	Amount o2
The total cost of the system, if retailers provide services to probabilistic customers	14806589	5082	14385401	4449
The total cost of the system, if probabilistic customers go to retailers for service	14572321	4913	14260093	4311
Conclusion	Customers go to retailers for services		Customers go to retailers for services	

Table 13. Retailer coordinates before and after solving by NSGA-II.

Number Retailer	Coordinates of the retailers before solving with NSGA-II	Coordinates of the retailers after solving with NSGA-II
1	[3200, 5000]	[4489.6, 4288.2]
2	[5500, 6000]	[5457.6, 4939.4]
3	[7000, 5000]	[4715.9, 3572.7]

In table 13, Retailer coordinates before and after solving by NSGA-II are showed. In Table 14, the allocation of each customer to retailers are showed.

5. Computational result

The GCLSC issue, as a multi-objective issue, is one of the most prominent branches in SC issues. Due to the entry of pollutants into the environment, much attention is paid to this issue nowadays. One of the primary measures in such issues is to decrease the service distance of retailers or lessen the time for customers to reach the service centers. In this study, which is a special case of CLSC issues, the problem is addressed by presenting heuristic allocation algorithms and focusing on retailers with known coordinates and the level of their coverage distance to send services. Moreover, customers have probabilistic coordinates and the coverage time of visiting the retail centers. In this model, for the first time, the optimal allocation is done by simultaneously comparing the distances and expected coverage of retailers and probabilistic customers. Also, using the NSGA-II algorithm, the best places of retailers are discovered. The distance coverage radius between retailers and the time coverage radius of the customers considering the amount of standard radius, upper and lower bounds of each of the retailers and customers is calculated. To block further dispersal in solving this example, we held the potential location search range for probabilistic customers within [1000, 7000] and the optimal search location for retail centers within [1000, 8000] spans. Thus, the optimal coordinates of retailers are calculated in the same span. Figure 3 shows simultaneously the Random points, Algorithm first point and the best point of the total cost and profit until reaching the optimal point. In Figure 4, in addition to the initial coordinates of retailers and probabilistic customers, using the results of Table 12, the calculated optimal coordinates that retailers can offer to these customers are also depicted.

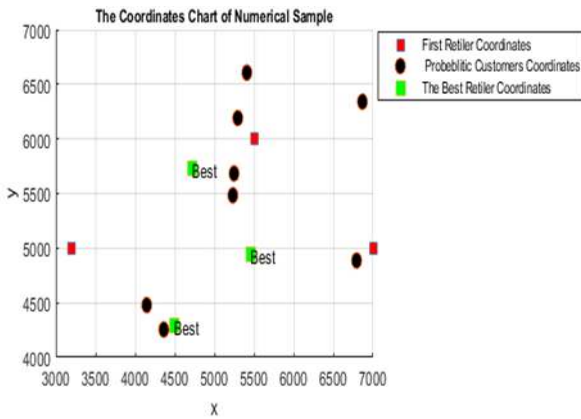


Figure 3. Optimal coordinates of retailers

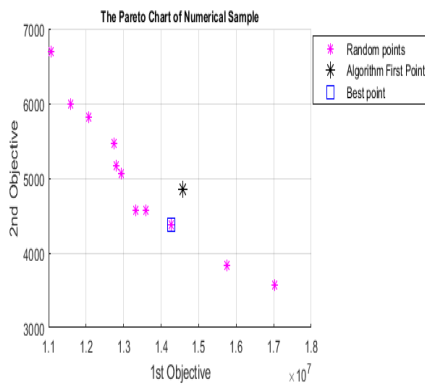


Figure 4. The Pareto front chart of Numerical Sample

Table 14. Probabilistic customers assigned to retailers

		Before solving by Nsga2			After solving by Nsga2		
		ζ_1	ζ_2	ζ_3	ζ_1	ζ_2	ζ_3
i=1	r=1	✓				✓	
	r=2		✓			✓	
i=2	r=1			✓			✓
	r=2		✓				✓
i=3	r=1		✓			✓	
	r=2	✓				✓	
i=4	r=1		✓				✓
	r=2	✓					✓
i=5	r=1		✓			✓	
	r=2	✓				✓	
i=6	r=1	✓					✓
	r=2			✓			✓
i=7	r=1			✓		✓	
	r=2			✓		✓	
i=8	r=1		✓		✓		
	r=2	✓			✓		

6. Conclusion

In this study, the structure of which is GCLSC, in the first stage, considering the probabilistic place of customers and the fixed location of retailers, the expected distance is calculated at the first step according to the type of movement (Rectangular, Euclidean, Ecclesiastical Square, Chebyshev). At the second step, based on the coverage radius, the distance between retailers and the time of probabilistic customers is calculated probabilistically by integral calculations. Additionally, how to allocate customers to retailers or vice versa is achieved by presenting algorithms 1 and 2. In the second stage, which is the general solution to the problem, the NSGA-II algorithm is applied. The results of applying model to the studied example indicate that with regard to costs and O₂ emissions, customers should move to retailers for receiving services. Moreover, considering the calculated expected coverage time, the type of motion is also hinted. Furthermore, new coordinates are calculated for retailers, which come with the lowest cost for customers and enable the optimal allocation of retailers to customers. Using various scenarios in a time window, probabilistic demand and relocating time can be possible themes for future research.

References

- [1]L.Wang, X.Wang, and W. Yang,"Optimal design of electric vehicle battery recycling network – From the perspective of electric vehicle manufacturers", Applied Energy, 2020,275: p. 115328.
- [2]C.K.Chen and M.A. Ulya,"Analyses of the reward-penalty mechanism in green closed-loop supply chains with product remanufacturing", International Journal of Production Economics, 2019, 210: p. 211-223.
- [3]J.Gharemani Nahr, S.H.R. Pasandideh, and S.T.A.Niaki, "A robust optimization approach for multi-objective, multi-product, multi-period,closed-loopgreen supply chain network designs under uncertainty and discount", Journal of Industrial and Production Engineering, 2020, 37(1): p. 1-22.
- [4]L.Huang, L. Murong, and W. Wang, "Green closed-loop supply chain network design considering cost control and CO₂ emission", Modern Supply Chain Research and Applications, 2020, 2(1): p. 42-59.
- [5]C.Mondal and B.C. Giri, "Pricing and used product collection strategies in a two-period closed-loop supply chain under greening level and effort dependent demand",

Journal of Cleaner Production, 2020, 265: p. 121335.

[6]B.M. Tosarkani and S.H. Amin, "A possibilistic solution to configure a battery closed-loop supply chain: Multi-objective approach", Expert Systems with Applications, 2018, 92: p. 12-26.

[7]K.Tehrani, "A smart cyber physical multi-source energy system for an electric vehicle prototype", Journal of Systems Architecture, 2020,111: p. 101804.

[8]G.Aldemir,T.Beldek, and D.Celebi,"AClosed-Loop Sustainable Supply Chain Network Design with System Dynamics for Waste Electrical and Electronic Equipment", in Industrial Engineering in the Industry 4.0 Era. 2018. Cham: Springer International Publishing.

[9]P.Georgiadis and M.Besiou,"Environmental and economical sustainability of WEEE closed-loop supply chains with recycling: a system dynamics analysis", The International Journal of Advanced Manufacturing Technology, 2010,47(5): p. 475-493.

[10]Y.A. Alamerew and D. Brissaud, "Modelling reverse supply chain through system dynamics for realizing the transition towards the circular economy", A case study on electric vehicle batteries, Journal of Cleaner Production, 2020,254: p. 120025.

[11]M.T.Islam andN. Huda, "Reverse logistics and closed-loop supply chain of Waste Electrical and Electronic Equipment (WEEE)/E-waste", A comprehensive literature review. Resources, Conservation and Recycling, 2018, 137: p. 48-75.

[12]P.Georgiadis and M. Besiou,"Sustainability in electrical and electronic equipment closed-loop supply chains: A System Dynamics approach", Journal of Cleaner Production, 2008, 16(15): p. 1665-1678.

[13]D.Qiao,W. Gaoshang, G. Tianming, W. Bojie,and D. Tao,"Potential impact of the end-of-life batteries recycling of electric vehicles on lithium demand in China 2010–2050", Science of The Total Environment, 2021, 764: p. 142835.

[14]X.Chen and A.Chuluunsukh," Closed-Loop Supply Chain Model with Efficient Operation Strategy", 2018, Cham: Springer International Publishing.

[15]S.H.R.Pasandideh, S.T.A. Niaki, and K. Asadi, "Bi-objective optimization of a multi-product multi-period three-echelon supply chain problem under uncertain environments: NSGA-II and NREGA", Information Sciences, 2015, 292: p. 57-74.

[16]M.P.Garg, A. Jain, and G. Bhushan, "Modelling and multi-objective optimization of process parameters of wire electrical discharge machining using non-dominated sorting genetic algorithm-II. Proceedings of the Institution of Mechanical Engineers", Part B: Journal of Engineering

Manufacture, 2012, 226(12): p. 1986-2001.

[17]İ.Kuban.Altinel,E.Durmaz, N.Aras, and K. CanÖzkisacık,"A location–allocation heuristic for the capacitated multi-facility Weber problem with probabilistic customer locations", European Journal of Operational Research, 2009,198(3): p. 790-799.

[18]T.Drezner, Z. Drezner, and Z. Goldstein, "A Stochastic Gradual Cover Location Problem", Naval Research Logistics (NRL), 2010, 57: p. 367-372.

Appendix:

$$\begin{aligned}
 f_{1(i,i)} &= \min (E[dic_{i,i}]) \\
 &= \min(E[(dicR_{j,i}), (dicED_{j,i}), (dicEDS_{j,i}), (dicCH_{j,i})]) \\
 &= \min \left[\left(\sum_{j=1}^J \sum_{i=1}^I (|d_{j1} - \mu_{i1}| \right. \right. \\
 &\quad \left. \left. + |d_{j2} - \mu_{i2}| \right) \cdot \left(\sum_{j=1}^J \sum_{i=1}^I \left(\sqrt{(d_{j1} - \mu_{i1})^2 + (d_{j2} - \mu_{i2})^2} \right) \right. \right. \\
 &\quad \left. \left. + \frac{1}{2} \left(\frac{\sigma_{i1}^2 + \sigma_{i2}^2}{\sqrt{(d_{j1} - \mu_{i1})^2 + (d_{j2} - \mu_{i2})^2}} \right) \right) \right] \cdot \left(\sum_{j=1}^J \sum_{i=1}^I ((d_{j1} - \mu_{i1})^2 + \sigma_{i1}^2 \right. \\
 &\quad \left. + (d_{j2} - \mu_{i2})^2 + \sigma_{i2}^2) \right) \cdot \left(\max \left(\sum_{j=1}^J \sum_{i=1}^I |d_{j1} - \mu_{i1}|, |d_{j2} - \mu_{i2}| \right) \right) \Big]
 \end{aligned}$$

$$\begin{aligned}
 f_{2(i,j)} &= \min (E[dic_{i,j}]) \\
 &= \min(E[(dicR_{i,j}), (dicED_{i,j}), (dicEDS_{i,j}), (dicCH_{i,j})]) \\
 &= \min \left[\left(\sum_{i=1}^I \sum_{j=1}^J (|\mu_{i1} - d_{j1}| \right. \right. \\
 &\quad \left. \left. + |\mu_{i2} - d_{j2}| \right) \cdot \left(\sum_{i=1}^I \sum_{j=1}^J \left(\sqrt{(\mu_{i1} - d_{j1})^2 + (\mu_{i2} - d_{j2})^2} \right) \right. \right. \\
 &\quad \left. \left. + \frac{1}{2} \left(\frac{\sigma_{i1}^2 + \sigma_{i2}^2}{\sqrt{(\mu_{i1} - d_{j1})^2 + (\mu_{i2} - d_{j2})^2}} \right) \right) \right] \cdot \left(\sum_{i=1}^I \sum_{j=1}^J ((\mu_{i1} - d_{j1})^2 + \sigma_{i1}^2 \right. \\
 &\quad \left. + (\mu_{i2} - d_{j2})^2 + \sigma_{i2}^2) \right) \cdot \left(\max \left(\sum_{i=1}^I \sum_{j=1}^J |\mu_{i1} - d_{j1}|, |\mu_{i2} - d_{j2}| \right) \right) \Big]
 \end{aligned}$$

Lemma1. The following Equations are always confirmed.

$$[dicR_{j,i}] = \sum_{j=1}^J \sum_{i=1}^I (|d_{j1} - \mu_{i1}| + |d_{j2} - \mu_{i2}|)$$

$$[dicR_{i,j}] = \sum_{i=1}^I \sum_{j=1}^J (|\mu_{i1} - d_{j1}| + |\mu_{i2} - d_{j2}|)$$

Proof. The Parameters a_{i1}, a_{i2} Are independent of each other. As a result, we have the independence of customers of this model.

$$E [dic_{R_{j,i}}] = E[l(d_j, a_i)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} l(d_j, a_i) f_i(a_i) la_{i1} la_{i2}$$

$$\begin{aligned} & \int_{-\infty}^{+\infty} (|d_{j1} - a_{i1}| f_i(a_i) la_{i1} la_{i2} + \int_{-\infty}^{+\infty} |d_{j2} - a_{i2}| f_i(a_i) la_{i1} la_{i2}) = \\ & \int_{-\infty}^{+\infty} (|d_{j1} - a_{i1}| f_i(a_i) la_{i1}) f_i(a_i) la_{i2} + \\ & \int_{-\infty}^{+\infty} (|d_{j2} - a_{i2}| f_i(a_i) la_{i1}) f_i(a_i) la_{i2} = \\ & [(\int_{-\infty}^{+\infty} (d_{j1} f_i(a_i) la_{i1}) f_i(a_i) la_{i2}) - [(a_{j1} f_i(a_i) la_{i1}) f_i(a_i) la_{i2}]] + \\ & [(\int_{-\infty}^{+\infty} (d_{j2} f_i(a_i) la_{i1}) f_i(a_i) la_{i2}) - [(a_{j2} f_i(a_i) la_{i1}) f_i(a_i) la_{i2}]] = [|d_{j1} - \mu_{i1}| + |d_{j2} - \mu_{i2}|] \end{aligned}$$

Lemma2. The following Equations are always confirmed [17].

$$E[dicED_{j,i}] = \sum_{j=1}^J \sum_{i=1}^I \left[\left(\sqrt{(d_{j1} - \mu_{i1})^2 + (d_{j2} - \mu_{i2})^2} \right) + \frac{1}{2} \left(\frac{\sigma_{i1}^2 + \sigma_{i2}^2}{\sqrt{(d_{j1} - \mu_{i1})^2 + (d_{j2} - \mu_{i2})^2}} \right) \right]$$

$$E[dicED_{i,j}] = \sum_{i=1}^I \sum_{j=1}^J \left[\left(\sqrt{(\mu_{i1} - d_{j1})^2 + (\mu_{i2} - d_{j2})^2} \right) + \frac{1}{2} \left(\frac{\sigma_{i1}^2 + \sigma_{i2}^2}{\sqrt{(\mu_{i1} - d_{j1})^2 + (\mu_{i2} - d_{j2})^2}} \right) \right]$$

Lemma 3. The following Equations are always confirmed.

$$E[dicEDS_{j,i}] = \sum_{j=1}^J \sum_{i=1}^I ((d_{j1} - \mu_{i1})^2 + \sigma_{i1}^2 + (d_{j2} - \mu_{i2})^2 + \sigma_{i2}^2)$$

$$E[dicEDS_{i,j}] = \sum_{i=1}^I \sum_{j=1}^J ((\mu_{i1} - d_{j1})^2 + \sigma_{i1}^2 + (\mu_{i2} - d_{j2})^2 + \sigma_{i2}^2)$$

Proof.

$$E[l(d_j, a_i)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} l(d_j, a_i) f_i(a_i) la_{i1} la_{i2}$$

The Parameters a_{i1}, a_{i2} Are independent of each other. As a result, we have the independence of customers of this model.

$$f_j = (a_{i1} \times a_{i2}) = f_{i1}(a_{i1}) \times f_{i2}(a_{i2})$$

$$\begin{aligned} E[l(\mathbf{d}_j, \mathbf{a}_i)] &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} [(d_{j1} - a_{i1})^2 + (d_{j2} - a_{i2})^2] f_i(a_i) \\ & \quad \times f_i(a_i) la_{i1} la_{i2} \\ &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} [(d_{j1} - a_{i1})^2 f_i(a_{i1}) \times f_i(a_{i2}) la_{i1} la_{i2} \\ & \quad + (d_{j2} - a_{i2})^2 f_i(a_{i1}) \times f_i(a_{i2}) la_{i1} la_{i2}] \end{aligned}$$

$$\begin{aligned} &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \left[(d_{j1}^2 + a_{i1}^2 - 2d_{j1}a_{i1}) \times f_i(a_{i1}) \times f_i(a_{i2}) la_{i1} la_{i2} \right. \\ & \quad + (d_{j2}^2 + a_{i2}^2 - 2d_{j2}a_{i2}) \\ & \quad \times f_i \left(\int_{-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} d_{j1}^2 \times f_i(a_{i1}) la_{i1} \times f_i(a_{i2}) la_{i2} \right. \right. \\ & \quad \left. \left. + \int_{-\infty}^{+\infty} a_{i1}^2 \times f_i(a_{i1}) la_{i1} \times f_i(a_{i2}) la_{i2} \right. \right. \\ & \quad \left. \left. - \int_{-\infty}^{+\infty} (2d_{j1}a_{i1}) \times f_i(a_{i1}) la_{i1} \times f_i(a_{i2}) la_{i2} \right. \right. \\ & \quad \left. \left. + \int_{-\infty}^{+\infty} \left[\int_{-\infty}^{+\infty} d_{j2}^2 \times f_i(a_{i1}) la_{i1} \times f_i(a_{i2}) la_{i2} + \int_{-\infty}^{+\infty} a_{i2}^2 \right. \right. \right. \\ & \quad \left. \left. \times f_i(a_{i1}) la_{i1} \times f_i(a_{i2}) la_{i2} - \int_{-\infty}^{+\infty} 2d_{j2}a_{i2} \times f_i(a_{i1}) la_{i1} \right. \right. \\ & \quad \left. \left. \times f_i(a_{i2}) la_{i2} \right] a_{i1}) \times f_i(a_{i2}) \right] la_{i1} la_{i2} \Big] = \end{aligned}$$

On the other hand, the following statements are always valid:

$$\int_{-\infty}^{+\infty} a_{ik} f_{ik}(a_{ik}) la_{ik} = \mu_{ik}$$

$$\int_{-\infty}^{+\infty} f_{ik}(a_{ik}) la_{ik} = 1$$

$$\int_{-\infty}^{+\infty} a_{ik}^2 f_{ik}(a_{ik}) la_{ik} = \sigma_{ik}^2 + \mu_{ik}^2$$

$$[d_{j1}^2 + (\sigma_{i1}^2 + \mu_{i1}^2) - 2d_{j1}\mu_{i1}] + [d_{j2}^2 + (\sigma_{i2}^2 + \mu_{i2}^2) - 2d_{j2}\mu_{i2}]$$

Lemma 4. The following Equations are always confirmed.

$$E[dicCH_{j,i}] = \max \left(\sum_{j=1}^J \sum_{i=1}^I |d_{j1} - \mu_{i1}|, |d_{j2} - \mu_{i2}| \right)$$

$$E[dicCH_{i,j}] = \max \left(\sum_{i=1}^I \sum_{j=1}^J |\mu_{i1} - d_{j1}|, |\mu_{i2} - d_{j2}| \right)$$

Proof:

$$\begin{aligned} E[l(\mathbf{d}_j, \mathbf{a}_i)] &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} (\text{MAX}(|d_{j1} - a_{i1}|, |d_{j2} - a_{i2}|) f_i(a_i) la_{i1} la_{i2}) \\ &= \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \text{MAX} \left((|d_{j1} - a_{i1}| f_i(a_i) la_{i1}, |d_{j2} \right. \\ & \quad \left. - a_{i2}|) f_i(a_i) la_{i2} \right) = \end{aligned}$$

$$\begin{aligned} & \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \text{MAX} \left(\sum_{j=1}^J |d_{j1} f_i(a_i) la_{i1}| - \int_{-\infty}^{+\infty} a_{i1} f_i(a_i) la_{i1} \right), \left(\int_{-\infty}^{+\infty} d_{j2} f_i(a_i) la_{i2} \right) - \\ & \int_{-\infty}^{+\infty} a_{i2} f_i(a_i) la_{i2} \Big] = \text{MAX} \left(\sum_{j=1}^J \sum_{i=1}^I |d_{j1} - \mu_{i1}|, |d_{j2} - \mu_{i2}| \right) \end{aligned}$$

Lemma 5. the following Equations always confirmed [18].

$$f_{3(j,i,r)} = \text{Max} E(\text{COVER}^1(l_{j,i,r})) =$$

$$\begin{cases}
 1 & l_{jr} \leq L_j - e_j \\
 \left(\frac{1 + \left(\frac{l_{jr} - L_j}{e_j} \right)}{2} \right) + 2 \left(\frac{L_j - l_{jr}}{e_j} \right) \times \ln 2 - \\
 \frac{1}{2} \left(\left(\frac{L_j - l_{jr}}{e_j} + 1 \right)^2 \times \ln \left(1 + \left(\frac{L_j - l_{jr}}{e_j} \right) \right) \right) & L_j - e_j < l_{jr} \leq L_j \\
 - \left(\left(\frac{L_j - l_{jr}}{e_j} \right)^2 \times \ln \left(\frac{L_j - l_{jr}}{e_j} \right) \right) \\
 \left(\frac{1 - \left(\frac{l_{jr} - L_j}{e_j} \right)}{2} \right) - 2 \left(\frac{l_{jr} - L_j}{e_j} \right) \times \ln 2 + \\
 \frac{1}{2} \left(\left(\frac{l_{jr} - L_j}{e_j} + 1 \right)^2 \times \ln \left(1 + \left(\frac{l_{jr} - L_j}{e_j} \right) \right) \right) & L_j < l_{jr} \leq L_j + e_j \\
 - \left(\left(\frac{l_{jr} - L_j}{e_j} \right)^2 \times \ln \left(\frac{l_{jr} - L_j}{e_j} \right) \right) \\
 0 & l_{jr} > L_j + e_j
 \end{cases}$$

Lemma 6. the following Equations always confirmed [18].

$$f_{4(i,j,r)} = \text{Max } E(\text{COVER}^2(t_{i,j,r})) =
 \begin{cases}
 1 & t_{i,j,r} \leq T_i - \theta_i \\
 \left(\frac{1 + \left(\frac{T_i - t_{i,j,r}}{\theta_i} \right)}{2} \right) + 2 \left(\frac{T_i - t_{i,j,r}}{\theta_i} \right) \times \ln 2 - \\
 \frac{1}{2} \left(\left(\frac{T_i - t_{i,j,r}}{\theta_i} + 1 \right)^2 \times \ln \left(1 + \left(\frac{T_i - t_{i,j,r}}{\theta_i} \right) \right) \right) & T_i - \theta_i < t_{i,j,r} \leq T_i \\
 - \left(\left(\frac{T_i - t_{i,j,r}}{\theta_i} \right)^2 \times \ln \left(\frac{T_i - t_{i,j,r}}{\theta_i} \right) \right) \\
 \left(\frac{1 - \left(\frac{t_{i,j,r} - T_i}{\theta_i} \right)}{2} \right) - 2 \left(\frac{t_{i,j,r} - T_i}{\theta_i} \right) \times \ln 2 + \\
 \frac{1}{2} \left(\left(\frac{t_{i,j,r} - T_i}{\theta_i} + 1 \right)^2 \times \ln \left(1 + \left(\frac{t_{i,j,r} - T_i}{\theta_i} \right) \right) \right) & T_i < t_{i,j,r} \leq T_i + \theta_i \\
 - \left(\left(\frac{t_{i,j,r} - T_i}{\theta_i} \right)^2 \times \ln \left(\frac{t_{i,j,r} - T_i}{\theta_i} \right) \right) \\
 0 & t_{i,j,r} > T_i + \theta_i
 \end{cases}$$