Journal of Industrial Strategic Management

A Hybrid Method of DEA and MODM in Grey Environment

Alireza Alinezhad[*a]

a Department of Industrial and Mechanical Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran.

CHRONICLE

Article history:
Received: 08/13/2016
Received in revised: 01/28/2017
Accepted: 04/30/2017

Keywords:
**Data Envelopment Analysis*
**Efficiency*
** Multi Objective Decision-Making*
** Grey Systems Theory.*

Abstract

DEA Classic models cannot be used for inaccurate and indeterminate data, and it is supposed that the data for all inputs and outputs are accurate and determinate. However, in real life situations uncertainty is more common. This article attempts to get the common weights for Decision-Making Units by developing DEA multi-objective models in the grey environment. First, we compute the privilege of DMUs efficiency to receive more accurate ranking. Finally, in order to assess the results, an example is presented to compare the results of the DMUs ranking between DEA Classic models and the presented model.

Introduction

Data Envelopment Analysis is a powerful managerial technique that provides managers with a device so that they could test the function of their companies against their competitors, and make decision for the better future based on the results (Jafarian-Moghaddam et al.,2011). DEA is an appropriate decision making device for assessing the relative function of a collection of under assessment units that have similar inputs and outputs. The first DEA model is CCR that is suggested by Charnes, copper and Roods in 1978 (Charnes et al.,1978). Generally, in DEA classic model to assess the efficiency of decision making units, the accurate and certain data is used (Charnes et al.,1978). In general, classical DEA problems are solved under the assumption that the values of parameters are specified precisely in a crisp environment, However, the observed values of the input and output data in real-world problems are often imprecise or vague (Khalili-Damghani et al., 2015). It is in case that, in real world the decision maker encounters circumstances in which variables and parameters are in uncertainty environment, and can`t determine the exact quantities for each of outputs and inputs and this criticizes the accuracy and correctness of model. In this circumstance, we need the model that could assess the efficiency of decision-making units with the uncertain data.

[*] Corresponding Author, email: alalinezhad@gmail.com

TELL:09123342560

Literature Review

In traditional DEA model, input and output values of decision-making groups are all real numbers and real variables. They are replaced by grey numbers and grey variables in grey DEA model respectively (Xu and Zhou, 2013).

MOLP and DEA combination could be used as a device in managing control and programming. The structure of these two models is nearly the same, but DEA assesses the past function as a part of control management function and MOLP is following planning to perform future programs (Cooper, 2004).

So far, many researchers have attempted studying and searching the Data Envelopment Analysis subject and its functions. Entani et al. (2002) and Wang et al. (2005) defined fuzzy data of the Data Envelopment Analysis models, in form of interval numbers, by applying the concept of alpha-cut.

Lozano and Villa (2004) presented tow models, radial DEA and un-radial DEA in which the decision maker simultaneously pays attention to either minimizing total consumer input or maximizing productive output by all units and maximizing the efficiency of every single units.

Cheng et al. (2007) did a research aiming to introduce procedure of Data Envelopment Analysis as another way for credit ranking of companies. Researchers at the first step explained the procedure and the approach of using it as an appropriate way for credit ranking and in the following, presenting a numerical example to show that the procedure of Data Envelopment Analysis had enough ability for credit ranking of trade units.

Ramanathan (2007) used DEA model to combine the results obtained from TCO and AHP approaches. He considered the weights obtained from AHP approach as output and the total expenses obtained from TCO approach as input of DEA model.

Hosseinzadeh et al. (2010) presented a balanced model between MOLP and DEA to show how the DEA subjects can be resolved by using MOLP, and they have used the method of Z-W to reflex the DM preferences in determining the efficiency.

Wu and Lee (2010) presented a probable DEA model in which, random variables, involuntary variables and sequence data are simultaneously taken into consideration. The function of the model they have presented is to value the multi-criteria subjects.

Hosseinzadeh et al. (2011) have presented a method to find efficient hyper plate with various return to scale technology in DEA by using multi objective linear model.

Moheb-Alizadeh et al. (2011) used multi objective Data Envelopment Analysis for location-allocation subjects in a fuzzy environment. They solved this model based on parametric planning manner, fuzzy and minimum deviation method.

Wu et al. (2012) presented a model of Probable Data Envelopment Analysis in their article that will be obtained by introducing the risk concept and the efficiency of decision-making units (DMU). Their model was used in environment efficiency assessment and its results were compared with classic models.

Wang and Liu (2012) used a CCR model to solve the DEA with grey interval data while the inputs/outputs have large interval length and found that lengths of efficiency intervals under the hypotheses are shorter, which produces more reliable and informative evaluation results and DMUs are dealt with more fairly.

Wang et al. (2016) proposed an effective approach based on grey theory and Data Envelopment Analysis (DEA) for selecting better partner for alliance among the world's 19 biggest automobile enterprises for Nissan Motor Co., Ltd.

Methodology

At first, we deliberate the BCC classic model and BCC model with interval grey numbers. If input and output of DMUs are definite, then the equation (1) will be introducer of classic BCC.

$$\max z_p = \frac{\sum_{r=1}^{s} u_r y_{rp} + a_p}{\sum_{i=1}^{m} v_i x_{ip}}$$

$$st. \quad \frac{\sum_{r=1}^{s} u_r y_{rj} + a_p}{\sum_{i=1}^{m} v_i x_{ij}} \le 1 \quad ; j = 1,2,\dots,n$$

$$u_r \ge \varepsilon > 0 \quad ; r = 1,2,\dots,s \qquad (1)$$

$$v_i \ge \varepsilon > 0 \quad ; i = 1,2,\dots,m$$

$$a_p \quad free$$

Now if the data of the model are sort of grey numbers, the equation (2) will be obtained.

$$\max z_p = \frac{\sum_{r=1}^{s} u_r \otimes y_{rp} + a_p}{\sum_{i=1}^{m} v_i \otimes x_{ip}}$$

$$st. \quad \frac{\sum_{r=1}^{s} u_r \otimes y_{rj} + a_p}{\sum_{i=1}^{m} v_i \otimes x_{ij}} \le 1 \quad ; j = 1,2,\dots,n$$

$$u_r \ge \varepsilon > 0 \quad ; r = 1,2,\dots,s \qquad (2)$$

$$v_i \ge \varepsilon > 0 \quad ; i = 1,2,\dots,m$$

$$a_p \quad free$$

If we want to compute the efficiency of DMUs based on equation (2), when the data of the subject is not definite and only their area is determined, many of DMUs may be introduced as efficient ones and their ranking will be inappropriate. Therefore we deliberate DEA model of multi objective in grey environment to resolve this problem and to obtain logical ranking.

At first, we take into consideration DEA multi objective in the image of model (3). The efficiency of each unit is taken into consideration as objective function in this model. In the other words, we will have as many quantities of objective function as DMUs in this model.

$$\max z_1 = \frac{\sum_{r=1}^{s} u_r y_{r1} + a}{\sum_{i=1}^{m} v_i x_{i1}}$$

$$\max z_2 = \frac{\sum_{r=1}^{s} u_r y_{r2} + a}{\sum_{i=1}^{m} v_i x_{i2}}$$

$$\vdots$$

$$\max z_n = \frac{\sum_{r=1}^{s} u_r y_{rn} + a}{\sum_{i=1}^{m} v_i x_{in}}$$

$$st. \quad \frac{\sum_{r=1}^{s} u_r y_{rj} + a}{\sum_{i=1}^{m} v_i x_{ij}} \le 1 \quad ; j = 1,2,\dots,n$$

$$u_r \ge \varepsilon > 0 \quad ; r = 1,2,\dots,s \qquad (3)$$

$$v_i \ge \varepsilon > 0 \quad ; i = 1,2,\dots,m$$

$$a \quad free \quad ; j = 1,2,\dots,n$$

Now if inputs and outputs of the model are not definite and are of sort of grey numbers, then model (3) will be changed into multi objective DEA model in grey environment in the image of model (4).

$$\max z_1 = \frac{\sum_{r=1}^{s} u_r \otimes y_{r1} + a}{\sum_{i=1}^{m} v_i \otimes x_{i1}}$$

$$\max z_2 = \frac{\sum_{r=1}^{s} u_r \otimes y_{r2} + a}{\sum_{i=1}^{m} v_i \otimes x_{i2}}$$

$$\vdots$$

$$\max z_n = \frac{\sum_{r=1}^{s} u_r \otimes y_{rn} + a}{\sum_{i=1}^{m} v_i \otimes x_{in}}$$

$$st. \quad \frac{\sum_{r=1}^{s} u_r \otimes y_{rj} + a}{\sum_{i=1}^{m} v_i \otimes x_{ij}} \le 1 \quad ; j = 1,2,\dots,n$$

$$u_r \ge \varepsilon > 0 \quad ; r = 1,2,\dots,s \qquad (4)$$

$$v_i \ge \varepsilon > 0 \quad ; i = 1,2,\dots,m$$

$$a \quad free \quad ; j = 1,2,\dots,n$$

Just as it is distinguished in the presented model, this model is multi objective and has (n) objective functions. One of the advantages of the model (4) is that the privilege of efficiency of all DMUs will be computable by solving only one model. There is also a possibility of applying decision maker`s interests and viewpoints in this model. In addition, considering that the model is indefinite, it can be used in the cases when the rate of data isn`t mooted in the image of definite and it has only definite limited area. Further, considering that the model is multi objective, less DMUs will be introduced as efficient units, and of course their efficiency will be determined more accurately, so we can make more logical and more appropriate ranking for DMUs.

We change the suggested model into a model with one objective function by using the fuzzy multi objectives linear programming approach.

For one objective function of max kind, the degree of membership linear function can be in the image of figure (1) in which z_j^L and z_j^R are quantities of z_j objective function that have the degree of membership of 1) and (2).

Supposing $\mu_j\left(z_j\right)=\alpha$ an objective function z_j can be obtained from the convex combination of z_j^L and z_j^R or in other words $z_j=\alpha z_j^R+(1-\alpha)z_j^L$ in which we have $0\le\alpha\le 1$.

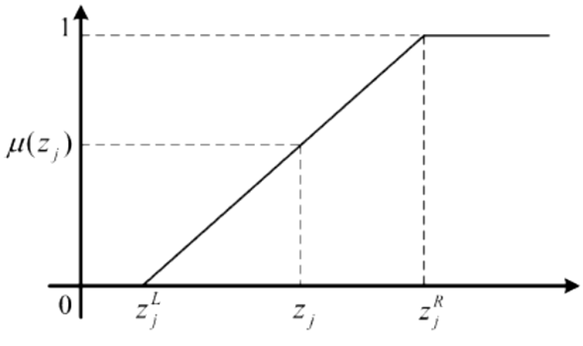

Figure 1. Max linear membership function (Jafarian and Ghoseiri., 2011)

Using theory of Zimmerman (1991) the equation (4) can be written in equation (5).

$$
\begin{aligned}
&\max\left\{\min_j \mu_j\left(z_j\right)\right\} \quad ;\ j=1,2,\dots,n\\
&st.\quad \frac{\sum_{r=1}^{s} u_r\otimes y_{rj}+a}{\sum_{i=1}^{m} v_i\otimes x_{ij}}\le 1 \quad ;\ j=1,2,\dots,n\\
&\quad u_r\ge\varepsilon>0 \quad ; r=1,2,\dots,s \qquad\qquad (5)\\
&\quad v_i\ge\varepsilon>0 \quad ; i=1,2,\dots,m\\
&\quad a \quad free \quad ; j=1,2,\dots,n
\end{aligned}
$$

Supposing

$\min\mu_j\left(z_j\right)=\alpha z_j^R+\left(1-\alpha\right)z_j^L\ ;\ j=1,2,\dots,n$

the model (5) will be changed into (6).

$$
\begin{aligned}
&\max\left\{\alpha z_j^R+\left(1-\alpha\right)z_j^L\right\} \quad ;\\
&\qquad\qquad j=1,2,\dots,n\\
&st.\quad \frac{\sum_{r=1}^{s} u_r\otimes y_{rj}+a}{\sum_{i=1}^{m} v_i\otimes x_{ij}}\le 1 \quad ;\\
&\qquad\qquad j=1,2,\dots,n \qquad\qquad (6)\\
&\quad \frac{\sum_{r=1}^{s} u_r\otimes y_{rj}+a}{\sum_{i=1}^{m} v_i\otimes x_{ij}}\ge \alpha z_j^R+\left(1-\alpha\right)z_j^L \quad ;\\
&\qquad\qquad j=1,2,\dots,n\\
&\quad u_r\ge\varepsilon>0 \quad ; r=1,2,\dots,s\\
&\quad v_i\ge\varepsilon>0 \quad ; i=1,2,\dots,m\\
&\quad a \quad free \quad ; j=1,2,\dots,n
\end{aligned}
$$

Model objective functions are advantages of the efficiency of decision making units and their quantities will be computed about [0,1]. Therefore we will have $z_j^L=0$ and $z_j^R=1$. Thus, based on this, model (7) will be obtained

$$\max \alpha$$
$$st. \quad \sum_{r=1}^{s} u_r \otimes y_{rj} - \sum_{i=1}^{m} v_i \otimes x_{ij} + a \le 0 \quad ; \quad j = 1,2,...,n$$
$$\sum_{r=1}^{s} u_r \otimes y_{rj} - \left(\alpha \bullet \left(\sum_{i=1}^{m} v_i \otimes x_{ij} \right) \right) + a \ge 0 \quad ; \quad j = 1,2,...,n \qquad (7)$$
$$u_r \ge \varepsilon > 0 \quad ; r = 1,2,...,s$$
$$v_i \ge \varepsilon > 0 \quad ; i = 1,2,...,m$$
$$a \quad free \quad ; j = 1,2,...,n$$
$$0 \le \alpha \le 1$$

Model (7) is the unique objective of model (4) that is obtained via the approach which is mooted by Zimmerman (1991). Model (7) is a unique objective model, but its parameters are of sort of grey uncertainty. By replacing grey interval numbers in the model (7), the model (8) will be obtained

$$\max \alpha$$
$$st. \sum_{r=1}^{s} u_r \left[c_{rj}, d_{rj} \right] - \sum_{i=1}^{m} v_i \left[a_{ij}, b_{ij} \right] + a \le 0 \; ; \quad j = 1,2,...,n$$
$$\sum_{r=1}^{s} u_r \left[c_{rj}, d_{rj} \right] - \left(\alpha \bullet \left(\sum_{i=1}^{m} v_i \left[a_{ij}, b_{ij} \right] \right) \right) + a \ge 0 \; ; \quad j = 1,2,...,n \qquad (8)$$
$$u_r \ge \varepsilon > 0 \quad ; r = 1,2,...,s$$
$$v_i \ge \varepsilon > 0 \quad ; i = 1,2,...,m$$
$$a \quad free \quad ; j = 1,2,...,n$$
$$0 \le \alpha \le 1$$

We convert models (9) and (10) to make certain model (8).

$$x_{ij} \in \left[a_{ij}, b_{ij} \right] \rightarrow x_{ij} = a_{ij} + \lambda_{ij} \left(b_{ij} - a_{ij} \right) \quad ; \quad 0 \le \lambda_{ij} \le 1$$
$$; i = 1,2,...,m \quad , \quad j = 1,2,...,n \qquad (9)$$

$$y_{rj} \in \left[c_{rj}, d_{rj} \right] \rightarrow y_{rj} = c_{rj} + \varphi_{rj} \left(d_{rj} - c_{rj} \right) \quad ; 0 \le \varphi_{rj} \le 1$$
$$; r = 1,2,...,s \quad , \quad j = 1,2,...,n \qquad (10)$$

As a result model (11) will be in the below.

$$\max \alpha$$
$$st. \quad \sum_{r=1}^{s} u_r c_{rj} + \sum_{r=1}^{s} u_r \varphi_{rj} \left(d_{rj} - c_{rj} \right) - \left(\sum_{i=1}^{m} v_i a_{ij} + \sum_{i=1}^{m} v_i \lambda_{ij} \left(b_{ij} - a_{ij} \right) \right) + a \le 0 \quad ; \quad j = 1,2,...,n$$
$$\sum_{r=1}^{s} u_r c_{rj} + \sum_{r=1}^{s} u_r \varphi_{rj} \left(d_{rj} - c_{rj} \right) - \left(\alpha \bullet \left(\sum_{i=1}^{m} v_i a_{ij} + \sum_{i=1}^{m} v_i \lambda_{ij} \left(b_{ij} - a_{ij} \right) \right) \right) + a \ge 0 \; ; \quad j = 1,2,...,n \qquad (11)$$
$$u_r \ge \varepsilon > 0 \quad ; r = 1,2,...,s$$
$$v_i \ge \varepsilon > 0 \quad ; i = 1,2,...,m$$
$$0 \le \lambda_{ij} \le 1 \quad ; i = 1,2,...,m \quad , \quad j = 1,2,...,n$$
$$0 \le \varphi_{rj} \le 1 \quad ; r = 1,2,...,s \quad , \quad j = 1,2,...,n$$
$$a \quad free \quad ; j = 1,2,...,n$$
$$0 \le \alpha \le 1$$

We settle:

$$v_i \lambda_{ij} = p_{ij} \rightarrow 0 \le p_{ij} \le v_i \; ; \; i = 1,2,...,m \quad , \quad j = 1,2,...,n \qquad (12)$$

$$u_r \varphi_{rj} = t_{rj} \rightarrow 0 \le t_{rj} \le u_r \; ; \; r = 1,2,...,s \quad , \quad j = 1,2,...,n \qquad (13)$$

By replacing the equations of (12) and (13) in the model (11), the model (14) will be obtained which is the certain model and unique objective model of the suggested model.

$$
\begin{aligned}
& \max \alpha \\
& st.\ \sum_{r=1}^{s} u_r c_{rj} + \sum_{r=1}^{s} t_{rj}\left(d_{rj} - c_{rj}\right) \\
& \quad -\left(\sum_{i=1}^{m} v_i a_{ij} + \sum_{i=1}^{m} p_{ij}\left(b_{ij} - a_{ij}\right)\right) + a \le 0 \quad ; \\
& \qquad\qquad\qquad\qquad j = 1,2,\dots,n \\
& \quad \sum_{r=1}^{s} u_r c_{rj} + \sum_{r=1}^{s} t_{rj}\left(d_{rj} - c_{rj}\right) \\
& \quad -\left(\alpha \bullet \left(\sum_{i=1}^{m} v_i a_{ij} + \sum_{i=1}^{m} p_{ij}\left(b_{ij} - a_{ij}\right)\right)\right) + a \ge 0 \ ; \\
& \qquad\qquad\qquad\qquad j = 1,2,\dots,n \\
& \quad u_r \ge \varepsilon > 0 \quad ; r = 1,2,\dots,s \\
& \quad v_i \ge \varepsilon > 0 \quad ; i = 1,2,\dots,m \\
& \quad 0 \le p_{ij} \le v_i \quad ; \begin{matrix} i = 1,2,\dots,m \\ j = 1,2,\dots,n \end{matrix} \ , \\
& \quad 0 \le t_{rj} \le u_r \quad ; \begin{matrix} r = 1,2,\dots,s \\ j = 1,2,\dots,n \end{matrix} \ , \qquad (14) \\
& \quad a \quad free \quad ; j = 1,2,\dots,n \\
& \quad 0 \le \alpha \le 1
\end{aligned}
$$

Two Numerical Examples

Numerical Example (1)

In this part to assess the function of suggested model that was mooted in the image of equation (4) and to determine the advantages of this model as related to the main grey DEA model that is presented in the equation (2), we have profited from an example including cement companies of Fars Province comprising nine factories named DMU_1 to DMU_9 and 4 inputs and 3 outputs that are in the interval of grey recommended numbers.

This example data is received from Kaviani and Abbasi (2014). Table 1 and Table 2 present the data related to the example.

Table 1. Input parameters of numerical example

	Quality	Cost	Dependability	Flexibility	Speed
DMU_1	[0.9,1]	[0.6,0.9]	[0.5,0.6]	[0.9,1]	[0.3,0.4]
DMU_2	[0.6,0.9]	[0.9,1]	[0.6,0.9]	[0.5,0.6]	[0.6,0.9]
DMU_3	[0.5,0.6]	[0.6,0.9]	[0.1,0.3]	[0.3,0.4]	[0.5,0.6]
DMU_4	[0.9,1]	[0.3,0.4]	[0.6,0.9]	[0.9,1]	[0.5,0.6]
DMU_5	[0.5,0.6]	[0.4,0.5]	[0.9,1]	[0.5,0.6]	[0.1,0.3]
DMU_6	[0.6,0.9]	[0.4,0.5]	[0.3,0.4]	[0.6,0.9]	[0.6,0.9]
DMU_7	[0.5,0.6]	[0.9,1]	[0.5,0.6]	[0.5,0.6]	[0.1,0.3]
DMU_8	[0.6,0.9]	[0.4,0.5]	[0.6,0.9]	[0.4,0.5]	[0.9,1]
DMU_9	[0.6,0.9]	[0.3,0.4]	[0.6,0.9]	[0.4,0.5]	[0.5,0.6]

Table 2. Output parameters of numerical example

	Return on investment	Return on assets	Market share
DMU_1	[0.5,0.6]	[0.4,0.5]	[0.5,0.6]
DMU_2	[0.6,0.6]	[0.3,0.4]	[0.4,0.5]
DMU_3	[0.4,0.5]	[0.5,0.6]	[0.4,0.5]
DMU_4	[0.6,0.9]	[0.5,0.6]	[0.5,0.6]
DMU_5	[0.4,0.5]	[0.3,0.4]	[0.3,0.4]
DMU_6	[0.5,0.6]	[0.4,0.5]	[0.4,0.5]
DMU_7	[0.3,0.4]	[0.4,0.5]	[0.5,0.6]
DMU_8	[0.1,0.3]	[0.3,0.4]	[0.5,0.6]
DMU_9	[0.4,0.5]	[0.4,0.5]	[0.3,0.4]

Table 3. The results of suggested model performance and BCC model

	The efficiency advantage obtained of performing the suggested model	The efficiency advantage obtained of performing the BCC grey classic model
DMU_1	0.998	1
DMU_2	0.996	1
DMU_3	1	1
DMU_4	0.988	1
DMU_5	0.999	1
DMU_6	1	1
DMU_7	0.997	1
DMU_8	0.998	1
DMU_9	0.998	1

The results of example performance via suggested model –equation (4), and also the solution results via CCR classic grey model – equation (2), are presented in Table 3. It is necessary to say that the Lingo software has been used to solve the models.

Numerical Example (2)

In this part to assess the function of suggested model that was mooted in the image of equation (4) and to determine the advantages of this model as related to the main grey DEA model that is presented in the image of equation (2), we have profited from an example including 4 decision making units and 3 inputs and 4 outputs that are in the interval of grey recommended numbers.

This example data is received from Jahanshahloo et al. (2011). Table 4 and Table 5 present the data related to the example.

The results of example performance via suggested model –equation (4), and also the solution results via CCR classic grey model – equation (2), are presented in Table 6.

Table 4. Input parameters of numerical example

	$\otimes x_1$	$\otimes x_2$	$\otimes x_3$
DMU_1	[106,112]	[84,92]	[1600,1600]
DMU_2	[102,102]	[107,111]	[2500,2500]
DMU_3	[82,88]	[92,94]	[2800,2800]
DMU_4	[77,82]	[92,94]	[1630,1630]

Table 5. Output parameters of numerical example

	$\otimes y_1$	$\otimes y_2$	$\otimes y_3$	$\otimes y_4$
DMU_1	[85399,87220]	[43,97]	[10206,10775]	[179,289]
DMU_2	[56144,58816]	[30,57]	[7380,7936]	[185,430]
DMU_3	[87716,90250]	[28,43]	[630,660]	[51,167]
DMU_4	[50210,50593]	[6,16]	[10247,10256]	[28,295]

Table 6. The results of suggested model performance and BCC model

	The efficiency advantage obtained of performing the suggested model	The efficiency advantage obtained of performing the BCC grey classic model
DMU_1	0.985	1
DMU_2	0.955	1
DMU_3	1	1
DMU_4	0.952	1

As shown in Tables 3 and 6, the units which are recommended in efficient suggested model are also efficient in DEA classical model.It shows the correct function of suggested model. Besides, the suggested model has recommended less efficient units than DEA classic model. This shows the preference of this model and it means the suggested model adopts a more severe approach to recommend a decision making unit to be efficient. In addition, the model multi-objective nature of the model, has made it possible to make DM view point effects on the model and obtain common weights for DMUs. Another preference of this model is that we spent less time to obtain the advantage of efficiency of decision making units.

Conclusion

The model presented in this article can be used to obtain the exact quantity of the efficiency of a decision-making unit. The model is preferred over other models as it is a multi-objective and can consider the DM view points. Then solving one model instead of solving (n) models can reduce the time needed to obtain the efficiency of decision-making units. In addition, it introduces fewer DMUs as efficient units in comparison with DEA classic model and provides more logical and accurate ranking.

References

1. Charnes, A., Cooper, W.W. and Rhodes, E. (1978). Measuring the efficiency of decision making units. *Eur.J.Oper*, 2, 429-444.
2. Cheng, E.W.L., Chiang, Y. H. and Tang, B.S. (2007). Alternative approach to credit scoring by DEA: Evaluating borrowers with respect to PFI projects. Building and Environment, 42, 1752– 1760.
3. Cooper, W. W. (2004). Origins, uses, and equation between goal programming and Data Envelopment Analysis, in: MCDM 2004, Whistler, BC. Canada, 6-11.
4. Entani, T., Maeda, Y. and Tanaka, H. (2002). Dual models of interval DEA and its extension to interval data. European Journal of Operational Research, 136, 32– 45.
5. Hosseinzadeh Lotfi, F., Jahanshahloo, G.R., Mozaffari, M.R. and Geramic, J. (2011). Finding DEA-efficient hyperplanes using MOLP efficient faces. Journal of Computational and Applied Mathematics, 235, 1227–1231.
6. Hosseinzadeh Lotfi, F., Jahanshahloo, G.R., Soltanifar, M., Ebrahimnejad, A. and Mansourzadeh, S.M. (2010). Equationship between MOLP and DEA based on output-orientated CCR dual model. Expert Systems with Applications, 37, 4331–4336.
7. Jafarian-Moghaddam, A. R. and Ghoseiri, K. (2011). Fuzzy dynamic multi-objective Data Envelopment Analysis model. Expert Systems with Applications, 38, 850–855.
8. Jahanshahloo, G.R., Hosseinzadeh Lotfi, F., Rezaie, V. and Khanmohammadi, M. (2011). Ranking DMUs by ideal points with interval data in DEA. Applied Mathematical Modelling, 35, 218–229.
9. Kaviani, M. A. and Abbasi, M. (2014). Analyzing the operations strategies of manufacturing firms using a hybrid Grey DEA approach–A case of Fars

Cement Companies in Iran. International Journal of Supply and Operations Management, 1(3), 371-391.

10. Khalili-Damghani, K., Tavana, M. and Haji-Saami, E. (2015). A Data Envelopment Analysis model with interval data and undesirable output for combined cycle power plant performance assessment. Expert Systems with Applications, 42(2), 760-773.
11. Lozano, S. and Villa, G. (2004). Centralized resource allocation using Data Envelopment Analysis. Journal of Productivity Analysis, 22, 143–161.
12. Moheb-Alizadeh, H., Rasouli, S. M. and Tavakkoli-Moghaddam, R. (2011). The use of Multi criteria Data Envelopment Analysis for Location-allocation problems in a Fuzzy Environment. Expert systems with application, 38, 5687-5695.
13. Ramanathan, R. (2007). Supplier selection problem: Integrating DEA with the approaches of total cost of ownership an AHP. Supply Chain Management, 12, 258-261.
14. Wang, C. N., Nguyen, X. T. and Wang, Y. H. (2016). Automobile industry strategic alliance partner selection: The application of a hybrid DEA and grey theory model. Sustainability, 8(2), 173.
15. Wang, J. and Liu, S. (2012). Efficiency measures in DEA with grey interval data under the hypotheses of data consistency. Grey Systems: Theory and Application, 2(1), 63-69.
16. Wang, Y. M., Greatbanks, R. and Yang, J. B. (2005). Interval efficiency assessment using Data Envelopment Analysis. Fuzzy Sets and Systems, 153, 347–370.
17. Wu, C., Li, Y., Liu, Q. and Wang, K. (2012). A stochastic DEA model considering undesirable outputs with weak disposability. Mathematical and Computer Modelling.
18. Wu, D (Dash). and Lee, C.G. (2010). Stochastic DEA with ordinal data applied to a multi-attribute pricing problem. European Journal of Operational Research, 207, 1679–1688.
19. Xu, H. and Zhou, S. (2013). A GREY DATA ENVELOPMENT ANALYSIS MODEL. Journal of Theoretical & Applied Information Technology, 48(3), 1552-1557.
20. Zimmerman, H. J. (1991). Fuzzy set theory and its applications (2nd ed.). Boston: Kluwer Academic Publishers.