Transactions on Fuzzy Sets and Systems (TFSS)
URL: https://tfss.journals.iau.ir/
Online ISSN: 2821-0131
Vol.2, No.1, (2023), 92-100
DOI: http://doi.org/10.30495/tfss.2022.1971064.1051

Article type: Original Research Article

On a New View of a Fuzzy Set

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Abstract. With this paper the authors try to newly reflect on Zadeh's concept of a fuzzy set. The departing point is the fact that not only fuzzy sets originate in Language, but that they are just 'linguistic entities' genetically different from the concept of 'crisp sets' whose origin is either in a physical collection of objects, or in a list of them.

Thus, a new definition of a fuzzy set is presented by means of two magnitudes: A qualitative one, a graph, the basic magnitude, and a quantitative one, a scalar magnitude. If the first reflects the language's relational ground of the fuzzy set, the second - and thanks to 'measuring the meaning of words' -, reflects the (numerical) extensional state in which it currently appears.

Since the second, the scalar magnitude, is essential for the applications, it is also introduced the concept of a 'working fuzzy set' by taking into account the numerical function, the meaning's measure or the membership function. The working fuzzy set, that enlarges the corresponding fuzzy set, allows us to see the 'same fuzzy set' with different membership functions, that is, the same graph appearing in different extensional states. Notice that a 'working crisp set' is but the same crisp set.

AMS Subject Classification 2020: 03B52; 68T27; 68T37
Keywords and Phrases: Fuzzy Set Theory, Fuzzy Logic, Meaning.

1 Introduction

1.1. From its inception in 1965 by the late Lotfi A. Zadeh [8], a fuzzy set $\mathbf{f_P}$ in a universe of discourse X is just viewed through its membership function or, more properly, as a triplet $(X, P, \mu_P) = \mathbf{f_P}$ where the universe of discourse X is supposed to be a set, P is the name of a predicate or property p shown by the elements in X, and μ_P is the membership function, indicating that $\mu_P(x)$ is the degree, between 0 and 1, up to which x verifies the property p, or that x is P.

When these values reduce to 0 and 1, because, respectively, either x does not at all verify P, or x verifies P completely, it is said that P or p is rigid, precise, or crisp on X; when, on the contrary, values strictly between 0 and 1 are taken by μ_P, it is said that P or p is vague, graduated, imprecise, or fuzzy on X.

With this definition, two fuzzy sets $\mathbf{f_P} = (X, P, \mu_P)$, and $\mathbf{f_Q} = (Y, Q, \mu_Q)$ are coincidental if and only if $X = Y$, $P = Q$, $\mu_P = \mu_Q$. Notwithstanding, this is something actually surprising since, as it is well known by everybody working with fuzzy sets, the same fuzzy set can exhibit different membership functions, like the same die can exhibit different probability distributions of obtaining its faces when throwing it.

In the same vein, the coincidence of the predicates P and Q is not necessary for the coincidence of the two fuzzy sets, it suffices that P and Q are synonyms, that have the same meaning on X.

Thus, there is a two face problem with Zadeh's definition of fuzzy set that concerns the identity of fuzzy sets, and what can mean the usually non unique membership function.

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Received: 19 October 2022; **Revised**: 29 November 2022; **Accepted**: 30 December 2022; **Published Online**: 7 May 2023.

How to cite: E. Trillas and A. R. de Soto, On a New View of a Fuzzy Set, *Trans. Fuzzy Sets Syst.*, 2(1), (2023), 92-100.

Part of the problem lies in the different background existing between the structure presumed for defining probabilities, that of a Boolean Algebra, and that of Language at which P and $\mathbf{f_P}$ belong to, and in which, for instance, the linguistic conjunction is not always commutative like it is the corresponding lattices meet in a Boolean Algebra. If the first is a very strong structure, language is endowed with a very weak one [5, 3, 6]. It could be said that the first is rigid, and the second is soft.

The properties of a Boolean Algebra are not universal in Language and Commonsense Reasoning, are but local, hold in just a part of language where it can be managed and mathematically represented when dealing with something requiring it.

1.2. Notice that a fuzzy set has, actually, an empirical linguistic counterpart that is an entity generated in language by people when speaking and with the finality of collectivizing what exhibits a property.

For instance, if the universe of discourse is the set of Londons inhabitants and P = young, people talk about the Young Londoners by referring to those Londoners that can be somehow recognized as young people; analogously, Old Londoners and Middle Age Londoners are also considered.

In each case it is supposed that a collective including such Londoners exists; that is, the collectives fuzzy set of Young Londoners, fuzzy set of Old Londoners, and fuzzy set of Middle Age Londoners, respectively.

Likes Human Brain to collectivize what enjoys a common property be it either rigid or graduated? Exists an innate human tendency to collectivize?

Where the answer to such questions is affirmative, it will still lack to answer the basic question concerning what can it serve, benefit, such (psychological) tendency. In any case it seems that a natural tendency to collectivization does actually exist. Notice that in language the idea behind what can be called a collective is wider than that of a simple collection (or set) of items.

For instance, the collective constituted by the weak Spanish vowels is the set {i, e}. The elements in such collectives can be closed between keys, as it happens with the collective of those Londoners that are less than 25 years old. Both are collections, sets; but neither the collective of the Young Londoners, nor that of Old Londoners can be shown between keys; it is not an atomic entity. For instance, if Spanish vowels are either week, or not week, Londoners are not only young, or not young, but young up to some degree.

1.3. What appears at the beginning of **1.1** shows that from the very beginning there is a conceptual gap in Zadeh's concept of a fuzzy set. A gap that requires to look at the empirical counterpart of a fuzzy set, to recognize that it is but a linguistic entity. An entity that is, of course, generated by people because commonsense reasoning, the everyday reasoning of everybody, is but language in action [6].

A first attempt to consider fuzzy sets from a language perspective was made, in 2003, by the first author in [4]. Nevertheless such a paper was not, in the end, fully satisfactory, since it contains confusion between the two basic concepts of maximality and measurability. Now the authors will try to finally jump over the gap by mathematically clarifying what is a fuzzy set, what a membership function represents, and which is the difference between theoretic fuzzy sets and those mathematical entities - functions - managed by practitioners.

2 Fuzzy Sets Newly Defined

2.1. To know how a predicate P linguistically acts on X, that is, how the elements in X can be distinguished by how they verify P, or how the property p varies along the universe of discourse, it should be known when, given two whatsoever elements x, y in X, which one of them shows p less than the other. That is, knowing if it is either x is less p than y, or y is less p than x.

Such variation that can be called the primary use on X of p, or of P, and when both possibilities happen, is said that x and y are equally P, are not distinguishable under P: The meaning of the statements x is P and y is P do coincide.

Shortening the statement x is less P than y by $x \prec_P y$, the (usually empirical) linguistic relation $\prec_P \subseteq X \times X$, facilitates the graph, or basic magnitude, $(X, \prec_P) = \mathbf{P}$, the fuzzy set in X with linguistic label P.

Notice that this new definition induces a sensible change: Two fuzzy sets with respective linguistic labels P and Q are coincidental, provided both are in the same universe of discourse and are primarily used in the same form. That is:

$$\mathbf{P} = \mathbf{Q} \Leftrightarrow (X, \prec_P) = (Y, \prec_Q) \Leftrightarrow X = Y \text{ and } \prec_P = \prec_Q .$$

The equality of two fuzzy sets means that their linguistic labels do have the same primary use or, in Wittgenstein words [7], (primary) meaning.

This solves the first problem with Zadeh's original definition of a fuzzy set, and offers a definition of **P** is contained in **Q**:

$$\mathbf{P} \subseteq \mathbf{Q} \Leftrightarrow \prec_P \subseteq \prec_Q,$$

supposed X= Y.

Notice that with this new definition of fuzzy set it is not presumed that for being $\mathbf{P} = \mathbf{Q}$ the predicates P and Q do be the same. To have the same meaning is to be synonyms.

Now lets try to solve the second: What is a membership function?

2.2. Since the idea behind membership functions — coming from supposing that meaning has extension — is to know up to which numerical level it can be said that x is P, that is, measuring the degree up to which x is P, its extension.

Lets introduce when a function $m_P : X \to [0, 1]$ is a measure in the graph $(X, \prec_P)$. Such a function measures (is a measure of) the meaning of P in X, whenever the following three properties are satisfied:

(*i*) $x \prec_P y \Rightarrow m_P(x) \leq m_P(y)$, that is, the measure grows along the relation $\prec_P$.

(*ii*) If z is minimal in the graph, that is, there is no t in X such that $t \prec_P z$, then $m_P(z) = 0$. Minimals do measure the minimum possible value.

Notice that if an element has zero measure, it does not imply that it is a minimal; condition (*ii*) is necessary but not sufficient. Minimal elements are also called anti-prototypes of P in X.

(*iii*) If w is maximal in the graph, that is, there is no v in X such that $w \prec_P v$, then $m_P(w) = 1$. Maximals do measure the maximum possible value.

Notice that if an element measures one, it does not imply that it will be maximal; condition (*iii*) is necessary but not sufficient. Maximal elements are also called prototypes of P in X.

2.3. It is interesting to note that, with the characteristic function:

$$R(x, y) = \begin{cases} 1, & \text{if } x \prec_P y \\ 0, & \text{if } x \not\prec_P y \end{cases}$$

condition (*i*) is equivalent to:

$$\min(m_P(x), R(x, y)) \leq m_P(y). \quad (1)$$

This condition generalized to any fuzzy relation R, and in particular to fuzzy preorders, any membership function μ and using a continuous t-norm T:

$$T(\mu(x), R(x, y)) \leq \mu(y),$$

gives rise to the definition of the fuzzy logic states of a fuzzy relation R, studied in [3] and [1]. Actually, condition given by (1) establishes that m_P and R satisfy the Modus Ponens Inequality with $T \leq \min$; so, condition (i) can also be seen as requiring that the measure m_P of the predicate P given by property p, is logically consistent with relation $\prec_P$.

The minimal and maximal elements of $\prec_P$ do not impose any restrictions using (1), however the minimum and maximum, if they exist, must have the smallest and largest value in m_P, respectively as it is imposed by laws (ii) and (iii) of m_P.

In addition, considering the relation $J_{\min}$ defined by residuation from min, (1) is equivalent to (see [3]):

$$R(x, y) < J_{\min}(m_P(x), m_P(y)) = 1, \text{ if } m_P(x) \prec m_P(y);$$

or

$$J_{\min}(m_P(x), m_P(y)) = m_P(y), \text{ in the contrary,}$$

showing that $J_{\min}$ is the maximum possible function R, that corresponds to the form and classical interpretation of $x \prec_P y$, as not x is P, or y is P.

It should be noticed that, usually, axioms (i) to (iii) are not sufficient to specify a single measure; they just characterize all measures, but, in general, to design a single one more conditions are necessary. This is not rare at all; remember how many probabilities can be associated to the six faces of the same die. At such respect, lets present in the next paragraph some very simple examples.

Lets still notice that measures m_P of P can be immediately identified with Zadeh's membership functions; in part, by clarifying what, concerning extensional meaning, appears in [8]. Thus, what is it a membership function is now mathematically clarified.

2.3. Examples

First.

Let it be $P =$ big in $X = [0, 10]$. Obviously, it is $\prec_{big} = \leq$, the total order of the Real Line ($4 \prec_{big} 6 \Leftrightarrow 4 \leq 6$). Hence the measures of the primary meaning of big in $[0, 10]$ are the mappings $m_{\text{big}} : [0, 10] \to [0, 1]$ that, non-decreasing, verify the two border conditions $m_{\text{big}}(0) = 0$ and $m_{\text{big}}(10) = 1$, since in $[0, 10]$ the only minimal is 0, the minimum, and the only maximal is 10, the maximum.

There is a great amount of these mappings and, without more conditions, it is not possible to specify one of them.

For instance, knowing that the measure it lineal, $m_{\text{big}}(x) = ax + b$, since the border conditions imply $b = 0$, and $10a = 1 \Leftrightarrow a = 1/10$, it is clear that, for big in the closed interval $[0, 10]$, there is just the unique lineal measure $m_{\text{big}}(x) = x/10$.

The situation is different when knowing that the measure is quadratic, $m_{\text{big}}(x) = ax^2 + bx + c$. In this case the border conditions imply $c = 0$ and $100a + 10b = 1$, or $b = \frac{(1-100a)}{10} = 0.110a$, with which it is $m_{\text{big}}(x) = ax^2 + (0.110a)x$, giving a one-parameter family of quadratic functions that, with $a = 0$, just recovers the lineal measure, and with $a = 0.01$ gives its square $m_{\text{big}}(x) = (\frac{x}{10})^2$.

Second.

a) If $X = [0, 10]$, and $P =$ "greater than four", it is obvious that $\mathbf{P} = \{x \in [0, 10]; 4 \prec x\} = (4, 10]$. In this case P is crisp and, consequently, $\mathbf{P}$ is a set, a collection of numbers, a semi-closed interval.

b) Returning to the first example for a while, lets consider that "big" is equated to "greater than eight". It is $\mathbf{P} = \{x \in [0, 10]; 8 \prec x\} = (8, 10]$, and their membership function is

$$m_P(x) = \begin{cases} 0, & \text{if } 0 \leq x \leq 8 \\ 1, & \text{if } 8 < x \leq 10, \end{cases}$$

a non-decreasing function verifying $m(0) = 0$ and $m(10) = 1$; clearly, is a particular case of "big".

Notice that in both the linear and the quadratic measures of "big", the membership functions or measures are continuous, but in this case there is a discontinuity of the measure at point $x = 8$. The crisp character of P in X causes the breaking of the measure.

2.4. Lets consider what happens when P is rigid in the universe X, a case in which the binary linguistic relation $\prec_P$ reduces to "x is equally P than y".

For instance, in the set $\mathbb{N}$ of Natural Numbers with P = "odd", it is clear that 5 is equally odd as 55 but not that 66; all odd numbers are equally, totally, odd, and the rest of the numbers, the pairs, are not odd at all. Something with a remarkable difference from the Londoners in, for instance, the "Middle Age" collective [2].

By writing $=_P = \prec_P \cap \prec_P^{-1}$, the fuzzy set of odd numbers results to be the graph $(X, =_{\text{odd}})$ that, being an algebraic equivalence, classifies perfectly X in two equivalence classes, that of odd numbers and that containing all the other numbers.

Consequently, since a number whatsoever n is either odd, or is not odd at all, there is just the following measure of the meaning of "odd":

$$m_{\text{odd}}(x) = \begin{cases} 1 & x \text{ is odd} \\ 0 & x \text{ is not odd} \end{cases}$$

and the set $m_{\text{odd}}^{-1}(1)$ is exactly the set rigid predicate odd specifies in $\mathbb{N}$, accordingly with the so-called "axiom of specification" in the Nave Theory of Sets.

2.5. As it was indicated, in general (i) does not give sufficient conditions to determine a membership function. Suppose, as an example, that we establish a similarity relationship among the elements of X with respect to the property p that defines the predicate P. For example, for the predicate P="big" in $X = [0, 10]$ we can establish the similarity relationship by means of the function,

$$E(x, y) = 1 - \frac{|\, x - y \,|}{10},$$

that is a fuzzy relation of indistinguishability (see [5, 3, 6]) and has a naturally associated preorder through its decomposition by means of next expression

$$E(x, y) = min(R(x, y), R(y, x)),$$

where the preorder R is defined by

$$R(x, y) = \begin{cases} 1 & \text{if } x \leq y \\ 1 - \frac{x}{10} + \frac{y}{10} & x > y, \end{cases}$$

The preorder R satisfies that $x \prec_P y$ iff $R(x, y) = 1$ and therefore satisfies condition (i). This preorder adds a graduation in the partial order relation between x and y given by $\prec_P$ when $x > y$.

The element 10 is a maximum of $\prec_P$, if the column of 10 is calculated, next expression is obtained

$$R(10, x) = \frac{x}{10},$$

which is a linear membership function between the points $(0, 0)$ and $(10, 1)$, which can be an extension of P="big".

If we take $x = 8$, the column gives rise to the extension

$$R(8, x) = m_p(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq 8 \\ 0.2 + \frac{x}{10} & \text{if } x > 8 \end{cases}$$

3 Working Fuzzy Sets

3.1. What we did is just passing from a basic magnitude, the graph, or fuzzy set $(X, \prec_P) = \mathbf{P}$, up to a scalar magnitude $(X, \prec_P, m_P)$ reflecting the state in which the predicate is currently managed and allowed to change of the relation $\prec_P$ by the (new) relation

$$x \prec_P^m y \Leftrightarrow m_P(x) \leq \ m_p(y),$$

a binary relation that not only is a new one, but is also different from $\prec_P$ since, for instance, the graph $(X, \prec_P^m)$ is a total or lineal one, because of all pair of points $(x, y) \in X \times X$, one of them will have smaller or equal measure than the other, that is, it will be necessarily either $x \prec_P^m y$, or $y \prec_P^m x$. Thus, under $\prec_P^m$ there are not incomparable, orthogonal, elements in X; but, instead, under $\prec_P$ such elements can exist, since it can be perceptively impossible capturing if one of both x and y is less P than the other.

3.2. The new graph $(X, \prec_P^m) = \mathbf{P^m}$, is called a "working fuzzy set associated to $\mathbf{P}$" since, usually, practitioners dont work with $\prec_P$ but with the total order relation $\leq$ of the Real Line inherited by the unit interval $[0, 1]$ that with the measure is the former total binary relation we just introduced.

Obviously, if P is rigid on X, the fuzzy set reduces to a set that, in addition, and since in this case there is just a unique measure, coincides with the corresponding working fuzzy set.

It should be noticed that the working fuzzy set $\mathbf{P^m}$ is, in general, larger than the fuzzy set $\mathbf{P}$. In fact:

$$x \prec_P y \Rightarrow m_P(x) \leq m_P(y) \Leftrightarrow x \prec_P^m y,$$

that is, $\prec_P \subseteq \prec_P^m$.

It can be said that the act of measuring the meaning enlarges the basic linguistic relation between the elements in the universe of discourse. Hence, the practitioner should be cautious at the respect since not all result valid in $\mathbf{P^m}$ will be, necessarily, neither valid in $\mathbf{P}$, nor in a different working fuzzy set, one endowed with a different measure.

Notice that in the first case of the former example 1, it is

$$x \prec_{\text{big}} y \Leftrightarrow x \leq y \Leftrightarrow \frac{x}{10} \leq \frac{y}{10} \Leftrightarrow m_{\text{big}}(x) \leq m_{\text{big}}(y) \Leftrightarrow x \prec_{\text{big}}^m \ y,$$

thus, the original linguistic relation $\prec_P$ and the new (measure dependant) one $\prec_P^m$ are coincidental: A coincidence that is not general.

4 Derivate Predicates

4.1. In natural language, graded concepts that are usually represented by fuzzy sets are characterized by admitting various related predicates such as the antonym or predicates intensified or weakened by modifier operators. The use of those predicates in the language is of course related to the use made of the main or primary predicate.

For example, the use of the antonym predicate ant$P$ could correspond to the inversion of the order that defines the use of P:

$$x \prec_{\text{ant}P} y \Leftrightarrow y \prec_P x,$$

and a measure $m_{\text{ant}P}$ should satisfy

$$x \prec_{\text{ant}P} y \Leftrightarrow y \prec_P x \Rightarrow m_{\text{ant}P}(x) \leq m_{\text{ant}P}(y) \wedge m_P(x) \geq m_P(y)$$

that is, it should invert the order associated with the extension of P. The minimal elements for P will become maximal and vice versa.

We can easily get an extension simply by defining

$$m_{\text{ant}P}(x) = 1 - m_P(x),$$

and the usual negation in fuzzy sets allows us to obtain a valid extension for the antonym that, nevertheless, makes it coincidental with the negation "not P". However, it is usual that in graded predicates the antonym does not coincide with the negation but rather that the antonym implies the negation of the predicate, that is

$$\text{If } x \text{ is ant}P\text{, then } x \text{ is not } P,$$

without always holding the reciprocal.

Another way to define the antonym is to take a bijection s on X that reverses the order defined by P

$$x \prec_P y \Leftrightarrow s(y) \prec_P s(x).$$

The membership function defined as

$$m_{\text{ant}P}(x) = m_P(s(x)),$$

is a valid extension for antP, since

$$x \prec_{\text{ant}P} y \Leftrightarrow y \prec_P x \Leftrightarrow s(x) \prec_P s(y) \text{ then } m_P(s(x)) \prec_P m_P(s(y)) \Leftrightarrow m_{\text{ant}P}(x) \prec m_{\text{ant}P}(y).$$

4.2. In example 2.5. the indistinguishability decomposes into a preorder R and its reciprocal $R^r(x,y) = R(y,x)$. The antonym predicate can be constructed using the reciprocal predicate by taking the minimum element column of the universe $X = [0,10]$, which is a maximal element for the reciprocal predicate,

$$R^r(0,y) = R(y,0) = 10 - y,$$

which is the linear function decreasing from the points $(0,1)$ to $(10,0)$. That membership function is also obtained by means of the inner bijection $s(x) = 10 - x$ on X from the extension of $P =$ "big" given by the column of the maximum 10 of the preorder R.

4.3. To derive a predicate iP that intensifies the meaning of P it suffices to define the order

$$x \prec_{iP} y \Leftrightarrow s(x) \prec_P s(y),$$

for a biyection s such as $s(x) \prec_P x$.

A membership function that is a valid extension of iP can be constructed by taking $m_{iP}(x) = m_P(s(x))$,

$$x \prec_{iP} y \Leftrightarrow s(x) \prec_P s(y) \text{ then } m_P(s(x)) \prec_P m_P(s(y)) \Leftrightarrow m_{iP}(x) \prec m_{iP}(y).$$

The membership m_{iP} represents an intensification of m_P because it is valid than $m_{iP}(x) \prec= m_P(x)$, given that $s(x) \prec_P x$.

Analogously, taking a biyection s such as $x \prec_P s(x)$ is possible to obtain a weakening predicate wP of P.

5 Conclusion

A fuzzy set **P** can be said to be "empty" when its linguistic label P refers to nothing in X. It happens when is either not known the binary linguistic relation $\prec_P$, or is known to be empty. In these cases it can be said that P is metaphysical in X and, on the contrary, P is said to be measurable in X, admits some measure of its meaning.

It should be noted that in the metaphysical case, no membership function can be known. But, what when a membership function is null? Can it be said that: If a membership function or measure is null, then the fuzzy set is empty? Knowing that the degree up to which all element in X is P is zero, does not mean the inexistence of graph **P**, the inexistence of relation $\prec_P$, the emptiness of the fuzzy set. It seems that the only that can be stated is: **P** is empty $\Leftrightarrow \prec_P$ is the empty set, in which case the existence of measures has no sense at all.

Nevertheless, if there is just a null measure, the fuzzy set cant have a single prototype, a single maximal element in the universe: No element in it belongs with a positive degree to the fuzzy set. Is it acceptable to state that in this case **P** is empty? Of course, what is admissible is to define that **P** is empty if and only if either $\prec_P$ is empty, or has a null measure.

This is a problem existing in the literature on fuzzy sets from its inception since it appears in [8], as a consequence of defining a fuzzy set by its membership function and by analogy with classical, crisp sets, viewed mathematically with independence of words even if the axiom of specification was added to the theory.

Nevertheless and In the same vein, the question of when a fuzzy set **P** can be identified with the universe X, has an obvious answer:

When for all x in X, the statement x is P is verified absolutely and, consequently, the only measure is $m_P(x) = 1$ for all x in X. The relation $\prec_P$ is $X \times X$, and all element x in X is a prototype of **P**.

Conflict of Interest: The authors declare that there are no conflict of interest.

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