Transactions on Fuzzy Sets and Systems
URL: https://tfss.journals.iau.ir/
Online ISSN: 2821-0131
Vol.1, No.1, (2022), 106-113
DOI: http://doi.org/10.30495/tfss.2022.690594

Article Type: Original Research Article (Invited Article)

Use of Soft Sets and the Blooms Taxonomy for Assessing Learning Skills

Michael Gr. Voskoglou

Abstract. Learning, a universal process that all individuals experience, is a fundamental component of human cognition. It combines cognitive, emotional and environmental influences for acquiring or enhancing ones knowledge and skills. Volumes of research have been written about learning and many theories have been developed for the description of its mechanisms. The goal was to understand objectively how people learn and then develop teaching approaches accordingly. In this paper soft sets, a generalization of fuzzy sets introduced in 1999 by D. Molodstov as a new mathematical tool for dealing with the uncertainty in a parametric manner, are used for assessing student learning skills with the help of the Blooms taxonomy. Blooms taxonomy has been applied and is still applied by generations of teachers as a teaching tool to help balance assessment by ensuring that all orders of thinking are exercised in student learning. The innovative assessment method introduced in this paper is very useful when the assessment has qualitative rather than quantitative characteristics. A classroom application is also presented illustrating its applicability under real conditions.

AMS Subject Classification 2020: 03399; 03E72; 97C30
Keywords and Phrases: Fuzzy sets, Soft sets, Learning, Blooms taxonomy, Assessment methods.

1 Introduction

Learning, a universal process that all individuals experience, is a fundamental component of human cognition. It combines cognitive, emotional and environmental influences for acquiring or enhancing ones knowledge or skills.

Curiosity about how humans learn dates back to the ancient Greek philosophers Socrates, Plato and Aristotle, who explored whether knowledge and truth mostly come from intellectual reasoning, i.e. they could be found within oneself (*rationalism*) or through external observation (*empiricism*). Thousands of years later, during the 17th and 18th century, the same question was the reason for a historical confrontation of two academic schools of European philosophy: The rationalists Descartes, Spinoza, Leibniz (European continent), versus the U.K. empirists Bacon, Locke, Hume.

By the 19th century, psychologists began to answer this question with systematic scientific studies. Volumes of research have been written about learning and many theories have been developed for the description of its mechanisms. The goal was to understand objectively how people learn and then develop teaching approaches accordingly.

In 20th century, the debate among the educational specialists centred on whether people learn by responding to external stimuli (*behaviorism* [3]) or by using their brains to construct knowledge from external data (*cognitivism* [19]).

Corresponding author: Michael Gr. Voskoglou, Email: mvoskoglou@gmail.com, ORCID: 0000-0002-4727-0089
Received: 17 February 2022; Revised: 17 April 2022; Accepted: 17 April 2022; Published Online: 7 May 2022.

How to cite: M. Gr. Voskoglou, Use of Soft Sets and the Blooms Taxonomy for Assessing Learning Skills, *Trans. Fuzzy Sets Syst.*, 1(1) (2022), 106-113.

Constructivism, a philosophical framework based on Piagets theory for learning and formally introduced by von Clasersfeld during the 1970s, suggests that knowledge is not passively received from the environment, but is actively constructed by the learner through a process of adaptation based on and constantly modified by the learners experience of the world [13]. This is usually referred as *cognitive constructivism.*

The synthesis of the ideas of constructivism with Vygoskys social development theory [4] created the issue of *social constructivism* [9]. According to Vygosky learning takes place within some socio-cultural setting. Shared meanings are formed through negotiation in the learning environment, leading to the development of common knowledge. The basic difference between cognitive and social constructivism is that the former argues that thinking precedes language, whereas the latter supports the exactly inverse approach.

In addition to the primary learning theories outlined above, i.e. behaviorism, cognitivism, constructivism and social constructivism, there are still more options [6]. *Humanism,* for example, focuses on creating an environment leading to self-actualization, where learners are free to determine their own goals while the teacher assists in meeting those goals. The *experiential* theory suggests to combine both learning about something and experiencing it, so that learners be able to apply the new knowledge to real-world situations. Also, the *transformative* theory, which is particularly relevant to adult learners, considers that the new information can change our world views when paired with critical reflection, etc.

The increasing use of technology as an educational tool has changed during the last years the learning landscape. Strongly influenced by technology, *connectivism,* focuses on a learners ability to frequently source and update accurate information. Knowing how and where to find the best information is as important as the information itself [5].

The target of the present paper is to use the Blooms taxonomy for teaching and learning and soft sets as tools for obtaining an assessment method of student learning skills in a parametric manner.

The motivation for writing this paper came from the fact that frequently the student assessment is attempted using not numerical, but linguistic grades, like A, B, C, D, E, F and sometimes $B-$, $B+$, etc. Also, it is important and useful to assess the student learning skills at each level of the learning process, as those levels are described by the Blooms taxonomy (see next section).

The rest of the paper is formulated as follows: A brief account of the Blooms taxonomy is exposed in the next section. The definition of soft set and its connection to fuzzy sets are presented in the third section. The assessment method is developed in fourth section with a classroom application and the main text closes with the final conclusion and some hints for future research contained in fifth section. An Appendix is presented also at the end of the paper, after the list of references, containing the questionnaire used in the classroom application.

2 The Blooms Taxonomy for Teaching and Learning

In 1956 Benjamin Bloom with collaborators Max Englehart, Edward Furst, Walter Hill, and David Krathwohl published a framework for categorizing educational goals, the *Taxonomy of Educational Objectives* [2]. The publication of the taxonomy followed a series of conferences from 1949 to 1953, which took place in order to improve communication between educators on the design of curricula and examinations. A revised version of the Blooms taxonomy was created in 2000 by Lorin Anderson [1], former student of Bloom. The six major levels of the revised taxonomy, moving through the lowest order processes to the highest, can be described as follows:

• L_1: *Knowing-Remembering:* Retrieving, recognizing, and recalling relevant knowledge from long-term memory.

• L_2: *Organizing-Understanding:* Constructing meaning from oral, written, and graphic messages through interpreting, exemplifying, classifying, summarizing, inferring, comparing, and explaining. Understand uses and implications of terms, facts, methods, procedures, concepts.

• L_3: *Applying:* Make use of theory, solve problems and use information in new situations.

• L_4: *Analyzing:* Breaking material into constituent parts, determining how the parts relate to one another and to an overall structure or purpose through differentiating, organizing, and attributing.

• L_5: *Generating-Evaluating:* Making judgements based on criteria and standards through checking and critiquing. Accept or reject on basis of criteria.

• L_6: *Integrating-Creating:* Put things together, bring together various parts, write theme, present speech, plan experiments and put information together in a new and creative way.

Most researchers and educators consider the last three levels L_4, L_5 and L_6 as being parallel, i.e. as happening simultaneously. For teaching a topic, the instructor should arrange his/her class work in the order to synchronize it with the six levels of Blooms taxonomy. The typical questions for evaluating the student achievement at the corresponding level must focus:

For Knowing-Remembering, on clarifying, recalling, naming, and listing. For Organizing-Understanding, on arranging information, comparing similarities and differences, classifying, and sequencing. For Applying, on prior knowledge to solve a problem. For Analyzing, on examining parts, identifying attributes, relationships, patterns, and main idea. For Generating-Evaluating, on producing new information, inferring, predicting, and elaborating with details. For Integrating-Creating, on connecting, combining, summarizing information and restructuring existing information to incorporate new information. For Evaluating, on reasonableness and quality of ideas, criteria for making judgments and confirming accuracy of claims.

Blooms taxonomy has been used and is still used by generations of teachers as a teaching tool to help balance assessment by ensuring that all orders of thinking are exercised in student's learning.

3 Fuzzy and soft sets

Probability theory used to be until the middle of the 1960's the unique tool in hands of the experts for dealing with the existing in real life and science situations of uncertainty. Probability, however, based on the principles of the bivalent logic, has been proved sufficient for tackling only problems of uncertainty connected to randomness, but not those connected to imprecision or incomplete information of the given data.

The *fuzzy* set theory, introduced by Zadeh in 1965 [20], and the connected to it infinite-valued in the interval $[0, 1]$ *fuzzy logic* [8] gave to scientists the opportunity to model under conditions of uncertainty which are vague or not precisely defined, thus succeeding to mathematically solve problems whose statements are expressed in the natural language. Through fuzzy logic the fuzzy terminology is translated by algorithmic procedures into numerical values, operations are performed upon those values and the outcomes are returned into natural language statements in a reliable manner.

Fuzzy systems are considered to be part of the wider class of *Soft Computing,* also including *probabilistic reasoning* and *neural networks,* which are based on the function of biological networks [11]. One may say that neural networks and fuzzy systems try to emulate the operation of the human brain. The former concentrate on the structure of the human mind, i.e. the hardware, and the latter concentrate on the software emulating human reasoning.

Let U be the universal set of the discourse. It is recalled that a fuzzy set on U is defined with the help of its *membership function* $m : U \to [0, 1]$ as the set of the ordered pairs

$$A = \{(x, m(x)) \ : \ x \in U\}. \tag{1}$$

The real number $m(x)$ is called the *membership degree* of x in . The greater is $m(x)$, the more x satisfies the characteristic property of . Many authors, for reasons of simplicity, identify a fuzzy set with its membership function.

A crisp subset A of U is a fuzzy set on U with membership function taking the values $m(x) = 1$ if x belongs to A and 0 otherwise. In other words, the concept of fuzzy set is an extension of the concept of the ordinary sets.

It is of worth noting that there is not any exact rule for defining the membership function of a fuzzy set. The methods used for this purpose are usually empirical or statistical and the definition is not unique depending on the personal goals of the observer. The only restriction about it is to be compatible to the common logic; otherwise the resulting fuzzy set does not give a reliable description of the corresponding real situation.

For example, defining the fuzzy set of the young people of a country one could consider as young all those being less than 30 years old and another all those being less than 40 years old. As a result they assign different membership degrees to people with ages below those two upper bounds.

For general facts on fuzzy sets, fuzzy logic and the connected to them uncertainty we refer to the chapters $4-7$ of the book [15].

A lot of research has been carried out during the last 60 years for improving and extending the fuzzy set theory on the purpose of tackling more effectively the existing uncertainty in problems of science, technology and everyday life. Various generalizations of the concept of fuzzy set and relative theories have been developed like the type-2 fuzzy set, the intuitionistic fuzzy set, the neutrosophic set, the rough set, the grey system theory, etc. [17]. In 1999, Dmtri Molodstov, Professor of the Computing Center of the Russian Academy of Sciences in Moscow, proposed the notion of *soft set* as a new mathematical tool for dealing with the uncertainty in a parametric manner [10].

Let E be a set of parameters, let A be a subset of E and let f be a mapping of A into the set $\Delta(U)$ of all subsets of U. Then the soft set on U connected to A, denoted by (f, A), is defined as the set of the ordered pairs

$$(f, A) = \{(e, f(e) \ : \ e \in A\}. \tag{2}$$

In other words, a soft set is a parametrized family of subsets of U. Intuitively, it is "soft" because the boundary of the set depends on the parameters.

For example, let $U = \{H_1, H_2, H_3\}$ be a set of houses and let $E = \{e_1, e_2, e_3\}$ be the set of the parameters $e_1 = cheap$, $e_2 = expensive$ and $e_3 = beautiful$. Let us further assume that H_1, H_2 are the cheap and H_2, H_3 are the beautiful houses. Set $A = \{e_1, e_3\}$, then a mapping $f : A \to \Delta(U)$ is defined by $f(e_1) = \{H_1, H_2\}$, $f(e_3) = \{H_2, H_3\}$. Therefore, the soft set (f, A) representing the cheap and beautiful houses of U is the set of the ordered pairs

$$(f, A) = \{(e_1, \{H_1, H_2\}), (e_3, \{H_2, H_3\})\}. \tag{3}$$

A fuzzy set on U with membership function $y = m(x)$ is a soft set on U of the form $(f, [0, 1])$, where $f(\alpha) = \{x \in U \ : \ m(x) \geq \alpha\}$ is the corresponding α-cut of the fuzzy set, for each α in $[0, 1]$. The concept of soft set is, therefore, a generalization of the concept of fuzzy set.

An important advantage of soft sets is that, by using the set of parameters E, they pass through the existing difficulty of defining properly the membership function of a fuzzy set.

The theory of soft sets has found many and important applications to several sectors of the human activity like decision making, parameter reduction, data clustering and data dealing with incompleteness, etc. [14]. One of the most important steps for the theory of soft sets was to define mappings on soft sets, which was achieved by A. Kharal and B. Ahmad and was applied to the problem of medical diagnosis in medical expert systems [7]. But fuzzy mathematics has also significantly developed at the theoretical level providing important insights even into branches of classical mathematics like algebra, analysis, geometry, topology etc. For example, one can extend the concept of topological space, the most general category of mathematical space, to fuzzy structures and in particular can define soft topological spaces and generalize the concepts of convergence, continuity and compactness within such kind of spaces [12].

4 The Soft Set Assessment Method

In earlier works the present author has developed various methods for assessing human-machine performance under fuzzy conditions, including the measurement of uncertainty in fuzzy systems, the use of the Center of Gravity (COG) defuzzification technique, the use of fuzzy or grey numbers, etc. [16]. Recently he also constructed a soft set model for assessment in a parametric manner and provided examples to illustrate its applicability to real situations [18].

In this model the set of the discourse U is the set of all objects which are under assessment. Consider the set $E = \{e_1, e_2, e_3, e_4, e_5\}$ of the parameters $e_1 = excellent$, $e_2 = verygood$, $e_3 = good$, $e_4 = mediocre$ and $e_5 = failed$ and the mapping $f : E \to \Delta(U)$ assigning to each parameter of E the subset of U consisting of all elements whose performance is described by this parameter. Then the soft set

$$(f, U) = \{(e_i, f(e_i)), i = 1, 2, 3, 4, 5\}, \tag{4}$$

represents mathematically a qualitative assessment of the elements of U.

Here this model will be adapted for assessing student learning skills in terms of the Blooms taxonomy.

The student assessment will be materialized through the following classroom application, which was performed with subjects 30 students of the School of Technological Applications (prospective engineers) of the Graduate Technological Educational Institute (T. E. I.) of Western Greece attending the course Mathematics I of their first term of studies.

This course involved an introductory module repeating and extending the students knowledge from secondary education about real numbers. After the module was taught, the instructor wanted to investigate the students progress according to the principles of the Blooms taxonomy. For this, he asked them to answer in the classroom the written test given in the Appendix at the end of this paper, which is divided to six different parts, one for each level of the taxonomy.

The students answers were assessed separately for each level with respect to the parameters of the set E outlined above. The tests results are depicted in the following table, where L_i, $i = 1, 2, 3, 4, 5, 6$ denote the levels of the Blooms taxonomy and P denotes the student overall performance.

Table 1: The results of the test

Parameter	L_1	L_2	L_3	L_4	L_5	L_6	P
e_1	8	6	5	3	2	3	3
e_2	9	11	10	8	7	8	8
e_3	10	9	10	12	10	8	12
e_4	3	3	3	5	7	8	5
e_5	0	1	2	2	4	3	2

The instructor numbered the students with respect to their overall performance in the test moving from the best one to the worst by $S_1, S_2, \ldots, S_{30}$.

Let $U = \{S_1, S_2, \ldots, S_{30}\}$ be the set of the discourse and let $f : E \to \Delta(U)$ be the mapping assigning to each parameter of E the subset of U consisting of the students whose overall performance was assessed by this parameter. Then the soft set

$$\begin{aligned}(f, U) = \{&(e_1, \{S_1, S_2, S_3\}), (e_2, \{S_4, S_5, \ldots, S_{11}\}), (e_3, \{S_{12}, S_{13}, \ldots, S_{23}\}), \\ &(e_4, \{S_{24}, S_{25}, \ldots, S_{28}\}), (e_5, \{S_{29}, S_{30}\})\}.\end{aligned} \tag{5}$$

represents mathematically the student overall performance in the test.

In an analogous way one can represent by a soft set the student performance at each level of the Blooms taxonomy. In those cases, however, an additional search is required, because the data of Table 1 is not enough for finding the students whose performance was assessed by the corresponding parameter.

For example, for level L_5 the instructor found that the student performance can be represented by the soft set

$$(f,U) = \{(e_1, \{S_1, S_3\}), (e_2, \{S_2, S_4, S_5, S_6, S_8, S_9, S_{12}\}), (e_3, \{S_7, S_{10}, S_{11}, S_{13}, S_{14}, S_{15}, S_{16}, S_{18}, S_{19}, S_{22}\}), (e_4, \{S_{17}, S_{20}, S_{21}, S_{23}, S_{24}, S_{25}, S_{28}\}), (e_5, \{S_{26}, S_{27}, S_{29}, S_{30}\})\}, \quad (6)$$

etc.

This method gives also the opportunity to represent with a soft set each students individual profile with respect to his/her performance at the levels of the Blooms taxonomy. For this, consider $U = \{L_1, L_2, L_3, L_4, L_5, L_6\}$ as the set of the discourse and let $g : E \to \Delta(V)$ be the mapping assigning to each parameter of E the subset of V consisting of the levels of the Blooms taxonomy in which the corresponding students performance was assessed by this parameter. For example the soft set

$$(g,V) = \{(e_1, \{L_1, L_2\}), (e_2, \{L_3\}), (e_3, \{L_4\}, (e_4, \{L_5, L_6\}), (e_5, \varnothing)\}, \quad (7)$$

corresponds to the profile of a student who demonstrated excellent performance at levels L_1 and L_2, very good at level L_3, good at level L_4 and mediocre performance at levels L_5 and L_6.

5 Conclusion

The discussion performed in this study leads to the conclusion that soft sets offer a potential tool for a qualitative assessment of student learning skills in a parametric manner with the help of the Blooms taxonomy.

Due to the generality of the assessment method used, a promising area for future research is the application of this method for assessing other student skills, like problem solving, mathematical modelling, analogical reasoning, etc. It could be also an interesting idea the development of alternative assessment methods under fuzzy conditions by using other types of generalizations of fuzzy sets or related theories, as they have been mentioned in the third section of this work.

Conflict of Interest: The author declares no conflict of interest.

References

[1] L. W. Anderson and D. R. Krathwohl, A taxonomy for learning, teaching, and assessing: A revision of Bloom's taxonomy of educational objectives, *Boston, Allyn and Bacon*, (2000).

[2] B. S. Bloom, M. D. Engelhart, E. J. Furst, W. H. Hill and D. R. Krathwohl, Taxonomy of educational objectives: The classification of educational goals, *Handbook I: Cognitive domain, New York, David McKay Company*, (1956).

[3] K. Cherry, History and Key Concepts of Behavioral Psychology, (2019). Available online: https://www.verywellmind.com/behavioral-psychology-4157183.

[4] K. Crawford, Vygotskian approaches in human development in the information era, *Educ. Stud. Math.* 31(1996), 43-62.

[5] G. Siemens, Connectivism: A learning theory for the digital age, *International Journal of Instructional Technology and Distance Learning*, 2(1) (2005). Available online: http://www.itdl.org/Journal/Jan05/article01.htm.

[6] B. Fairbanks, 5 educational learning theories and how to apply them, (2021). Available online: https://www.phoenix.edu/blog/educational-learning-theories.

[7] A. Kharal and B. Ahmad, Mappings on Soft Classes, *New Mathematics and Natural Computation*, 7(3) (2011), 471-481.

[8] B. Kosko, Fuzzy Thinking: The New Science of Fuzzy Logic, *NY, Hyperion*, (1993).

[9] J. McKinley, Critical argument and writer identity: Social constructivism as a theoretical framework for EFL academic writing, *Crit. Inq. Lang. Stud* 12 (2015), 184-207.

[10] D. Molodtsov, Soft Set Theory-First Results, *Computers and Mathematics with Applications*, 37(4-5) (1999), 19-31.

[11] A. P. Paplinski, Neuro-Fuzzy Computing, *Lecture Notes, Monash University, Australia*, (2005).

[12] M. Shabir and M. Naz, On Soft Topological Spaces, *Computers and Mathematics with Applications*, 61 (2011), 1786-1799.

[13] K. S. Taber, Constructivism as educational theory: Contingency in learning and optimally guided instruction, in Hassaskhah, J. (Ed.), Educational Theory, Hauppauge, *NY, Nova Science Publishers*, (2011), 39-61.

[14] B. K. Tripathy and K. R. Arun, Soft Sets and Its Applications, in J. S. Jacob (Ed.), Handbook of Research on Generalized and Hybrid Set Structures and Applications for Soft Computing, *Hersey, PA, IGI Global*, (2016), 65-85.

[15] M. Gr. Voskoglou, Finite Markov Chain and Fuzzy Logic Assessment Models: Emerging Research and Opportunities, *Columbia, SC, Createspace.com Amazon*, (2017).

[16] M. Gr. Voskoglou, Assessing Human-Machine Performance under Fuzzy Conditions, *Mathematics*, 7 (2019).

[17] M. Gr. Voskoglou, Generalizations of Fuzzy Sets and Related Theories, in M. Voskoglou (Ed.), An Essential Guide to Fuzzy Systems, *N. Y., Nova Publishers*, (2019), 345-352.

[18] M. Gr. Voskoglou, Application of Soft Sets to Assessment Processes, *American Journal of Applied Mathematics and Statistics*, 10(1) (2022), 1-3.

[19] B. Wallace, A. Ross, J. B. Davies and T. Anderson, The Mind, the Body and the World: Psychology after Cognitivism, *Upton Pyne, UK, Imprint Academic*, (2007).

[20] L. A. Zadeh, Fuzzy Sets, *Information and Control*, 8 (1965), 338-353.

Appendix

The questionnaire used in our classroom application (Topic: Real numbers)

1. *Knowing-Remembering*

• Give the definitions and examples of a periodic decimal and of an irrational number (in the form of an infinite decimal).

2. *Organizing*

• Compare the set of all fractions with the set of periodic decimals. Compare the set of irrational numbers with the set of all roots (of any order) that have no exact values.

3. *Applying*

• Which of the following numbers are natural, integers, rational, irrational and real numbers?

$$2, \quad -\frac{5}{3}, \quad 0, \quad 9\cdot 08, \quad 5, \quad 7\cdot 333\ldots, \quad \pi = 3\cdot 14159\ldots, \quad -\sqrt{4}, \quad \frac{22}{11}, \quad 5\sqrt{3},$$

$$-\frac{\sqrt{5}}{\sqrt{20}}, \quad (\sqrt{3}+2)(\sqrt{3}-2), \quad -\frac{\sqrt{5}}{2}, \quad \sqrt{7}-2, \quad \sqrt{\left(\frac{5}{3}\right)^2}.$$

• Write the number $0\cdot 345345345\ldots$ in its fractional form.

4. *Analyzing*

• Find the digit which is in the 1005th place of the decimal $2\cdot 825342342\ldots$.

• Compare the numbers 5 and $4\cdot 9999\ldots$.

• Construct the line segment of length $\sqrt{3}$ with the help of the Pythagorean Theorem. Give a geometric interpretation.

5. *Generating - Evaluating*

• Justify why the decimals $2\cdot 001313113111311111\ldots$ and $0\cdot 1234567891011\ldots$ are irrational numbers.

• Construct the line segment of length $\sqrt[3]{2}$ by using the graph of the function $f(x) = \sqrt[3]{x}$.

6. *Integrating - Creating*

• Define the set of the real numbers in terms of their decimal representations (this definition was not given by the instructor to the class before the test).

Michael Gr. Voskoglou
Department of Mathematical Sciences,
Graduate Technological Educational Institute of Western Greece,
Meg. Alexandrou 1, 263 34 Patras, Greece.
E-mail: voskoglou@teiwest.gr; mvoskoglou@gmail.com