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NeutroAlgebra & AntiAlgebra vs. Classical Algebra

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Abstract. NeutroAlgebra & AntiAlgebra vs. Classical Algebra is a like Realism vs. Idealism. Classical Algebra does not leave room for partially true axioms nor partially well-defined operations. Our world is full of indeterminate (unclear, conflicting, unknown, etc.) data.

This paper is a review of the emerging, development, and applications of the NeutroAlgebra and AntiAlgebra [2019-2022] as generalizations and alternatives of classical algebras.

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1 Introduction

The Classical Algebraic Structures were generalized in 2019 by Smarandache [16] to NeutroAlgebraic Structures (or NeutroAlgebras) {whose operations and axioms are partially true, partially indeterminate, and partially false} as extensions of Partial Algebra, and to AntiAlgebraic Structures (or AntiAlgebras) {whose operations and axioms are totally false} and on 2020 he continued to develop them [18, 20, 17].

The NeutroAlgebras & AntiAlgebras form a *new field of research*, which is inspired by our real world. Many researchers from various countries around the world have contributed to this new field, such as F. Smarandache, A.A.A. Agboola, A. Rezaei, M. Hamidi, M.A. Ibrahim, E.O. Adeleke, H.S. Kim, E. Mohammadzadeh, P.K. Singh, D.S. Jimenez, J.A. Valenzuela Mayorga, M.E. Roja Ubilla, N.B. Hernandez, A. Salama, M. Al-Tahan, B. Davvaz, Y.B. Jun, R.A. Borzooei, S. Broumi, M. Akram, A. Broumand Saeid, S. Mirvakilii, O. Anis, S. Mirvakili, etc (See [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24]).

2 Distinctions between Classical Algebraic Structures vs. NeutroAlgebras & AntiAlgebras

In classical algebraic structures, all operations are 100% well-defined, and all axioms are 100% true, but in real life, in many cases, these restrictions are too harsh since in our world we have things that only partially verify some operations or some laws.

Using the process of *NeutroSophication* of a classical algebraic structure we produce a NeutroAlgebra, while the process of *AntiSophication* of a classical algebraic structure produces an AntiAlgebra.

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3 The neutrosophic triplet (Operation, NeutroOperation, AntiOperation)

When we define an operation on a given set, it does not automatically mean that the operation is well-defined. There are three possibilities:

(i) The operation is well-defined (also called inner-defined) for all set's elements [degree of truth $T = 1$] (as in classical algebraic structures; this is a classical **Operation**). Neutrosophically we write: Operation$(1, 0, 0)$.

(ii) The operation if well-defined for some elements [degree of truth T], indeterminate for other elements [degree of indeterminacy I], and outer-defined for the other elements [degree of falsehood F], where (T, I, F) is different from $(1, 0, 0)$ and from $(0, 0, 1)$ (this is a **NeutroOperation**). Neutrosophically we write: NeutroOperation(T, I, F).

(iii) The operation is outer-defined for all set's elements [degree of falsehood $F = 1$] (this is an **AntiOperation**). Neutrosophically we write: AntiOperation$(0, 0, 1)$.

4 The neutrosophic triplet (Axiom, NeutroAxiom, AntiAxiom)

Similarly for an axiom, defined on a given set, endowed with some operation(s). When we define an axiom on a given set, it does not automatically mean that the axiom is true for all set elements. We have three possibilities again:

(i) The axiom is true for all set's elements (totally true) [degree of truth $T = 1$] (as in classical algebraic structures; this is a classical **Axiom**). Neutrosophically we write: Axiom$(1, 0, 0)$.

(ii) The axiom is true for some elements [degree of truth T], indeterminate for other elements [degree of indeterminacy I], and false for other elements [degree of falsehood F], where (T, I, F) is different from $(1, 0, 0)$ and from $(0, 0, 1)$ (this is **NeutroAxiom**). Neutrosophically we write NeutroAxiom(T, I, F).

(iii) The axiom is false for all set's elements [degree of falsehood $F = 1$](this is **AntiAxiom**). Neutrosophically we write AntiAxiom$(0, 0, 1)$.

5 The neutrosophic triplet (Theorem, NeutroTheorem, AntiTheorem)

In any science, a classical Theorem, defined on a given space, is a statement that is 100% true (i.e. true for all elements of the space). To prove that a classical theorem is false, it is sufficient to get a single counter-example where the statement is false.

Therefore, the classical sciences do not leave room for the *partial truth* of a theorem (or a statement). But, in our world and our everyday life, we have many more examples of statements that are only partially true, than statements that are totally true. The NeutroTheorem and AntiTheorem are generalizations and alternatives of the classical Theorem in any science.

Let's consider a theorem, stated on a given set, endowed with some operation(s). When we construct the theorem on a given set, it does not automatically mean that the theorem is true for all set elements. We have three possibilities again:

(i) The theorem is true for all set's elements [totally true] (as in classical algebraic structures; this is a classical **Theorem**). Neutrosophically we write Theorem$(1, 0, 0)$.

(ii) The theorem is true for some elements [degree of truth T], indeterminate for other elements [degree of indeterminacy I], and false for the other elements [degree of falsehood F], where (T, I, F) is different from $(1, 0, 0)$ and from $(0, 0, 1)$ (this is a **NeutroTheorem**). Neutrosophically we write NeutroTheorem(T, I, F).

(iii) The theorem is false for all set's elements (this is an **AntiTheorem**). Neutrosophically we write AntiTheorem$(0, 0, 1)$.

And similarly, for (Lemma, NeutroLemma, AntiLemma), (Consequence, NeutroConsequence, AntiConsequence), (Algorithm, NeutroAlgorithm, AntiAlgorithm), (Property, NeutroProperty, AntiProperty), etc.

6 The neutrosophic triplet (Algebra, NeutroAlgebra, AntiAlgebra)

(i) An algebraic structure whose all operations are well-defined and all axioms are totally true is called a classical Algebraic Structure (or **Algebra**).

(ii) An algebraic structure that has at least one NeutroOperation or one NeutroAxiom (and no AntiOperation and no AntiAxiom) is called a NeutroAlgebraic Structure (or **NeutroAlgebra**).

(iii) An algebraic structure that has at least one AntiOperation or one Anti Axiom is called an AntiAlgebraic Structure (or **AntiAlgebra**).

Therefore, a neutrosophic triplet is formed: ¡Algebra, NeutroAlgebra, AntiAlgebra¿, where Algebra can be any classical algebraic structure, such as a groupoid, semigroup, monoid, group, commutative group, ring, field, vector space, BCK-Algebra, BCI-Algebra, etc.

7 Theorems and Examples

Theorem 7.1. *If a Classical Statement (theorem, lemma, congruence, property, proposition, equality, inequality, formula, algorithm, etc.) is totally true in a classical Algebra, then the same Statement in a NeutroAlgebra maybe be:*

- *totally true (degree of truth $T = 1$, degree of indeterminacy $I = 0$, and degree of falsehood $F = 0$);*
- *partially true (degree of truth T), if partial indeterminate (degree of indeterminacy I), and partial falsehood (degree of falsehood F), where $(T, I, F) \notin \{(1, 0, 0), (0, 0, 1)\}$.*
- *totally false (degree of falsehood $F = 1$, degree of truth $T = 0$, and degree of indeterminacy $I = 0$).*

Example 7.2. (**Examples of Classical Algebra, NeutroAlgebra, and AntiAlgebra**)

Let $S = \{a, b, c\}$ be a set, and a binary law (operation) $*$ defined on S:

$$* : S^2 \to S.$$

As in the below Cayley Table:

$*$	a	b	c
a	a	c	a
b	a	b	a
c	b	c	a

Then:

1. $(S, *)$ is a Classical Grupoid since the law $*$ is totally (100%) well-defined (classical law), or $\forall\ x, y \in S,\ x * y \in S$.

2. $(S, *)$ is a NeutroSemigroup, since:

(i) the law $*$ is totally well-defined (classical law);

(ii) the associativity law is a NeutroAssociativity, i.e.

- partially true, because $\exists\ a, b, c \in S$ such that
$$(a * b) * c = c * c = a = a * (b * c) = a * a = a,$$
the degree of truth $T > 0$,
- degree of indeterminacy $I = 0$ since no indeterminacy exists;
- and partially false, because $\exists\ c, c, c \in S$ such that
$$(c * c) * c = a * c = a \neq c * (c * c) = a * a = b,$$
so degree of falsehood $F > 0$.

3. $(S, *)$ is an AntiCommutative NeutroSemigroup, since:
(i) the law $*$ is totally well-defined (classical law);
(ii) the associativity is a NeutroAssociativity (as proven above);
(iii) the commutativity is an AntiCommutativity, since:
$$\forall\ x, y \in S, \quad x * y \neq y * x.$$

Proof.
$$a * b = c \neq a = b * a,$$
$$a * c = a \neq b = c * a,$$
$$b * c = a \neq c = c * b.$$

□

Theorem 7.3. *If a Classical Statement is false in a classical Algebra, then in a NeutroAlgebra it may be:*
(i) *either a NeutroStatement, i.e. true (T) for some elements, indeterminate (I) for other elements, and false (F) for the others, where (T, I, F) is different from $(1, 0, 0)$ and from $(0, 0, 1)$;*
(ii) *or an AntiStatement, i.e. false for the elements.*

Theorem 7.4. *A Classical Group can be:*
(i) *either Commutative (the commutative law is true for all elements);*
(ii) *or NeutroCommutative (the commutative law is true (T) for some elements, indeterminate (I) for others, and false (F) for the other elements where (T, I, F) is different from $(1, 0, 0)$ and from $(0, 0, 1)$;*
(iii) *or AntiCommutative (the commutative law is false for all the elements).*

Corollary 7.5. *The Classical Non-Commutative Group is either NeutroCommutative or AntiCommutative.*

Corollary 7.6. *The Classical Non-Associative Groupoid is either NeutroAssociative or AntiAssociative.*

8 Conclusion

The Classical Structures in science mostly exist in theoretical, abstract, perfect, homogeneous, idealistic spaces - because in our everyday life almost all structures are NeutroStructures, since they are neither perfect nor applying to the whole population, and not all elements of the space have the same relations and same attributes in the same degree (not all elements behave in the same way).

The indeterminacy and partiality, with respect to the space, to their elements, to their relations or their attributes are not taken into consideration in the Classical Structures. But our Real World is full of structures with indeterminate (vague, unclear, conflicting, unknown, etc.) data and partialities.

There are exceptions to almost all laws, and the laws are perceived in different degrees by different people in our every-day life.

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