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Inverse Data Envelopment Analysis to Estimate Inputs with Triangular Fuzzy Numbers

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Abstract

In the real world, all available data are not definitive and are considered based on quality. Estimating the values of the inputs when we change the values of the outputs as desired is one of the important applications of inverse data envelopment analysis. If we want to estimate the level of inputs (outputs) among a group of decision-making units (DMUs), when some or all of its outputs (inputs) are changed so that cost efficiency is maintained or improved, inverse data envelopment analysis is used. In this article, cost efficiency is investigated by increasing desired outputs along with triangular fuzzy data. The problem of inverse data envelopment analysis with fuzzy data is presented for the cost efficiency of the DMU under evaluation. Also, in this connection, the results of the proposed model will be examined in a numerical example.

Keywords: Inverse data envelopment analysis, improving cost efficiency, increasing outputs, membership function, triangular fuzzy numbers

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1. Introduction

Due to the nature of real-world problems, the data collected usually contain some degree of uncertainty. In fact, many data cannot be quantified due to their nature. Incomplete information or partial ignorance is another reason for applying fuzzy theory. Although in many cases accurate information can be obtained, some approximate data are judged to be good enough to avoid the high costs of accurate data collection. Therefore, many researchers prefer to include fuzzy data in their decision-making models to have more realistic results. In addition, according to some researchers, the data caused by human mental phenomena can be more realistically expressed with fuzzy numbers compared to clear or even random numbers.

Ebrahimnejad et al. presented the primal-dual method for LP problems with fuzzy variables [1]. Arenas proposed a method to solve an interactive fuzzy programming system [2]. Maleki et al. presented a concept to solve LP with fuzzy variables [3]. Rommelfanger presented a general concept for solving linear multicriteria programming problems with definite, fuzzy or random values [4]. Maleki proposed ranking functions and their applications for solving fuzzy linear programming [5]. Ramik has introduced some new concepts and results in secondary fuzzy linear programming [6]. Gansan et al. proposed a method for solving fuzzy linear programming problems with trapezoidal fuzzy numbers [7]. Naseri proposed a new method for solving fuzzy linear programming by solving linear programming [8]. Ezzati et al. presented a new algorithm for solving fully fuzzy programming problems using the MOLP problem [9]. Edalatpanah presented a direct model for triangular Neutrosophic linear programming [10]. Kumar et al proposed a mathematical model to solve fully fuzzy linear programming problem with trapezoidal fuzzy numbers [11]. Data envelopment analysis is a non-parametric method based on mathematical programming to measure the relative efficiency of decision-making units, which was presented by Charnes et al. in 1978, Data envelopment analysis is used to calculate the efficiency of each unit compared to other units [12]. Its main purpose is to compare and measure the efficiency of a number of similar decision-making units that have several inputs and several outputs. The purpose of comparing and measuring efficiency is how well a decision-making unit has used its resources in the direction of production compared to other decision-making units. Data envelopment analysis has many applications, which include the fields of education, economy, management, health and transportation, … [13]. In 1984, this technique was developed by Banker et al. [14]. And inverse data envelopment analysis is one of the significant topics in practical and theoretical sections where the relative efficiency of the DMU is evaluated and the goal is to determine the input and output levels of a DMU so that its relative efficiency remains constant or improves. Inverse data envelopment analysis was first studied and reviewed in 2000 by Zhang and Cui [15]. Wei et al. proposed a model to answer this question in 2000[16]. "If we increase one or more inputs in a particular unit among a group of DMUs, and assume that the DMU under evaluation maintains its efficiency relative to other units, by how much will the outputs of the DMU under evaluation increase?" In 2002, Yan et al. presented a linear programming problem for poorly performing units under evaluation, and also presented a MOLP model for inefficient units under evaluation [17]. In addition, Hadi-Vencheh et al. [18] in their studies developed the model presented by Wei et al. [16] to answer the following question. "If we increase several outputs in a particular unit among a group of DMUs, and assume that the DMU maintains its relative efficiency compared to the other units, by how much will the DMU's inputs be increased?" To answer the above question, Lertworasirikul et al, also proposed a model [19]. Data envelopment analysis is a non-parametric technique for evaluating the relative efficiency of decision-making units based on its nature. Maximizing revenue or profit, minimizing cost, are behavioral goals along with the price of inputs and outputs. Farrell first introduced the definition of cost effectiveness in 1957, which has played a significant role in the development of the concept of data envelopment analysis [20]. They obtain cost efficiency using linear programming technique. This linear programming model requires input prices of decision-making units in addition to input and output data. Cost efficiency (CE), as a DEA model, is a measure of the ability of a DMU to achieve current output at minimum cost. Fukuyama and his colleague in 2002 used the input ratio of direct and indirect pseudo-distance functions to measure output allocation efficiency [21]. Banihashem et al. evaluated the efficiency of profit, cost and revenue of multi-stage supply chains in three stages [22]. The fair allocation of the cost of shared fixed income was investigated by data envelopment analysis by Khodabakhshi et al [23]. income cost efficiency models in DEA_R were presented by Mozaffari et al. [24] A centralized method for re-allocating resources based on revenue efficiency among a set of decision-making units in a centralized environment was presented by Li Fang et al [25]. Amin et al. introduce a new inverse data envelopment analysis model based on a cost efficiency model to estimate the potential profit from mergers [26].

In this article, firstly, in part 2, the main definition and in part 3, data envelopment analysis, cost efficiency, improving cost efficiency by increasing outputs, and inverse data envelopment analysis with fuzzy data will be introduced. Then, in section 4, the inverse data envelopment analysis will be examined in order to estimate the inputs with triangular fuzzy numbers by presenting a numerical example, and in section 5, the article will be completed with the conclusions of the mentioned cases.

2. Basic definitions

Definition 1: The characteristic function μ_A of a fuzzy set $A \subseteq X$ assigns a value of 0 to 1 for each member in X. This function can be extended to a $\mu_A(x): X \to [0,1]$ membership function. The fuzzy set A defined by $\mu_A(x)$ for each $x \in X$ is described as follows:

$$A = \{(x, \mu_A(x)); x \in X\} \qquad (1)$$

Definition 2: The fuzzy number $\tilde{A} = (a,b,c)$ is called a triangular fuzzy number and is defined as follows:

$$\tilde{A}(x) = \begin{cases} \dfrac{x-a}{b-a} & a \le x \le b \\ \dfrac{c-x}{c-b} & b \le x \le c \\ 0 & otherwise \end{cases} \qquad (2)$$

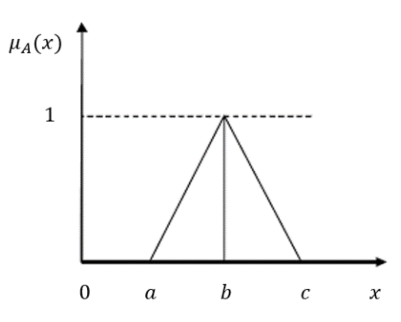

Figure 1: Triangular fuzzy number $\tilde{A} = (a,b,c)$

Definition 4: The fuzzy number $\tilde{A}=(a,b,c)$ is called a non-negative fuzzy number if and only if $a \geq 0$.

Definition 5: Unlike real numbers, there is no natural and specific way to compare fuzzy numbers. Ranking of fuzzy numbers is required to find the largest and smallest fuzzy numbers. Ranking function is an approach to sort fuzzy numbers. The ranking function is denoted by $M: F \rightarrow R$, where F is the set of fuzzy numbers defined on the real number axis R, which associates each fuzzy number with a real number where there is a natural order.

Definition 6: Suppose $\tilde{A}=(a_1,b_1,c_1)$ and $\tilde{B}=(a_2,b_2,c_2)$ are two triangular fuzzy numbers, then:

$$\begin{aligned} &i. \quad \tilde{A} \leq \tilde{B} \leftrightarrow M(\tilde{A}) \leq M(\tilde{B}) \\ &ii. \quad \tilde{A} \geq \tilde{B} \leftrightarrow M(\tilde{A}) \geq M(\tilde{B}) \\ &iii. \quad \tilde{A} \sim \tilde{B} \leftrightarrow M(\tilde{A}) = M(\tilde{B}) \end{aligned} \tag{3}$$

We consider the ranking functions of the triangular fuzzy number $\tilde{x}=(a,b,c)$ as follows[25] :

The first ranking formula:[27, 28]

$$M_1(x)=\frac{a+2b+c}{4} \tag{4}$$

The second ranking formula (ranking function of pentagonal fuzzy numbers) [29]:

$$M_2(x)=\frac{a+p+b+q+c}{5} \tag{5}$$

The third ranking formula[27]:

$$M_3(x)=\frac{a+4b+c}{6} \tag{6}$$

The fourth ranking formula[30]:

$$M_4(x)=\frac{a+7b+c}{9} \tag{7}$$

3. Data envelopment analysis, cost efficiency, improving cost efficiency by increasing outputs and inverse data envelopment analysis with fuzzy data

3.1. Data envelopment analysis

Data envelopment analysis is a linear programming model to evaluate the performance of a set of homogeneous decision-making units. One of the most important non-parametric methods for evaluating the performance of decision-making units based on linear programming is data envelopment analysis (DEA). Charnes et al. presented the CCR model in 1978 to evaluate the efficiency of the decision-making unit[12].

3.2. Cost efficiency

The concept of efficiency is the concept of not wasting resources and proper exploitation of resources and maximizing desired outputs. In other words, it means the efficiency of using the least inputs to get the most outputs. In addition to using the values of inputs and outputs, cost efficiency uses the price and value of inputs in calculating efficiency. Assume that $Y_{m\times n}, Y_{s\times n}$ and $C_{m\times n}$ are input, output and input cost matrices, respectively. The cost efficiency model to calculate the cost efficiency of DMU_o seeks to find the unit that spends the least cost to prepare inputs smaller than or equal to the inputs of the unit under evaluation, in order to produce outputs equal to the outputs of the unit under evaluation. Therefore, considering c_{io} as the cost corresponding to input i of DMU_o, the model is as follows:

$$\min z = \sum_{i=1}^{m} c_{io} x_i \tag{8}$$

$$s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij} \le x_i, \quad i = 1,\dots,m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro}, \quad r = 1,\dots,s$$

$$\lambda_j \ge 0, \qquad j = 1,\dots,n$$

If (λ^* and x^*) and z^* is the optimal solution to the above problem, the total cost efficiency of the jth unit under evaluation by dividing the minimum total cost $z^* = \sum_{i=1}^{m} c_{io} x_i^*$ is defined on the observed cost $z^* = \sum_{i=1}^{m} c_{io} x_{io}^*$.

$$E_{co} = \frac{\sum_{i=1}^{m} c_{io} x_i^*}{\sum_{i=1}^{m} c_{io} x_{io}} \tag{9}$$

Definition 7: DMU_o is called overall cost efficiency, if and only if DMU_o So that:

$$\frac{\sum_{i=1}^{m} c_{io} x_i^*}{\sum_{i=1}^{m} c_{io} x_{io}} \le 1 \tag{10}$$

3.3 Improving cost efficiency by increasing outputs

Now we present the problem of inverse data coverage analysis on unit cost efficiency under this type of evaluation. By perturbing the output vector, we calculate the perturbation of the input vector while maintaining the cost efficiency value. If the output of the unit under evaluation DMU_o changes from y_o to $y_o + \Delta y_o$ $(y_o + \Delta y_o > 0)$ so that its cost efficiency remains constant or improves, what is its input?

To answer the above question, we assume that for the unit under evaluation DMU_o, the output changes from y_o to $y_o + \Delta y_o \left(y_o + \Delta y_o > 0 \right)$ and the input changes from x_o to $x_o + \Delta x_o$ so that $\begin{pmatrix} x_o + \Delta x_o \\ y_o + \Delta y_o \end{pmatrix} > 0$. By replacing the vector $\begin{pmatrix} x_o + \Delta x_o \\ y_o + \Delta y_o \end{pmatrix}$ instead of $\begin{pmatrix} x_o \\ y_o \end{pmatrix}$ in the cost efficiency model (1) where the value of $y_o + \Delta y_o > 0$ is known, we calculate the cost efficiency. In this regard, we remove DMU_o from the set of observations and replace it with a new DMU_o whose input vector $x_o + \Delta x_o > 0$ is unknown and whose output vector is known value $y_o + \Delta y_o > 0$ let's do:

$$\min z = \sum_{i=1}^{m} c_{io} \left(x_{io} + \Delta x_{io} \right) \tag{11}$$

$$s.t. \sum_{\substack{j=1 \\ j \ne 0}}^{n} \lambda_j x_{ij} + \lambda_o \left(x_{io} + \Delta x_{io} \right) \le x_{io} + \Delta x_{io}, \quad i = 1,\dots,m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} + \lambda_o \left(y_{ro} + \Delta y_{ro} \right) \ge y_{ro} + \Delta y_{ro}, \ r = 1,\dots,s$$

$$\sum_{i=1}^{m} c_{io} \left(x_{io} + \Delta x_{io} \right) \ge z^*$$

$$\lambda^*{}_j \ge 0, \qquad j = 1,\dots,n$$

$$x_{io} + \Delta x_{io}, \qquad i = 1,\dots,m$$

where z^* is the optimal solution of model (8). c_{io} is the price or input cost of the i-th decision-making unit under the o-th evaluation. By simplifying and rewriting the model (11), we will have the following linear programming model:

$$\min z = \sum_{i=1}^{m} c_{io} x_i \qquad (12)$$

$$s.t. \sum_{\substack{j=1 \\ j\neq 0}}^{n} \lambda_j x_{ij} - x_i \leq 0, \qquad i = 1,...,m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro} + \Delta y_{ro}, \ r = 1,...,s$$

$$\sum_{i=1}^{m} c_{io} x_i \geq z^*$$

$$\lambda_j \geq 0, J \neq 0, \qquad j = 1,...,n$$

$$x_i \geq 0, \qquad i = 1,...,m$$

3.4 Inverse Data Envelopment Analysis with Fuzzy Data (FIDEA)

Now we present the problem of inverse data overlay analysis with fuzzy data on unit cost efficiency under evaluation as follows. If we consider a set of decision-making units DMU_j, $j = 1,...,n$ with input $\tilde{x}_{ij}$ and fuzzy output $\tilde{y}_{ij}$ that belong to the set of positive fuzzy numbers, suppose $\tilde{X}_{m\times n}$ and $\tilde{Y}_{s\times n}$ are the input and output matrices of non-negative pseudo-fuzzy numbers, respectively, and $C_{m\times n}$ is the input cost matrix with real numbers. The cost efficiency model to calculate the cost efficiency of DMU_o is as follows:

$$\min \tilde{z} = \sum_{i=1}^{m} c_{io} \tilde{x}_i \qquad (13)$$

$$s.t. \sum_{\substack{j=1 \\ j\neq 0}}^{n} \lambda_j \tilde{x}_{ij} \leq \tilde{x}_i, \qquad i = 1,...,m$$

$$\sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} \geq \tilde{y}_{ro}, \qquad r = 1,...,s$$

$$\lambda_j \geq 0, \qquad j = 1,...,n$$

If ($\lambda^*$and $\tilde{x}^*$) and $\tilde{z}^*$ is the optimal solution to the above problem, the total cost efficiency of the jth unit under evaluation by dividing the minimum total cost $z^* = \sum_{i=1}^{m} c_{io} \tilde{x}_{io}^*$ is defined on the observed cost $\sum_{i=1}^{m} c_{io} \tilde{x}_{io}^*$.

$$E_{co} = \frac{\sum_{i=1}^{m} c_{io} \tilde{x}_i^*}{\sum_{i=1}^{m} c_{io} \tilde{x}_{io}^*} \qquad (14)$$

where $0 < E_{co} < 1$ for $o = 1,...,n$.

Now we present the problem of inverse data envelopment analysis with fuzzy data to improve cost efficiency by increasing outputs as follows. And by disturbing the output vector, we calculate the amount of disturbance of the input vector while maintaining the cost efficiency value.

$$\min \tilde{z} = \sum_{i=1}^{m} c_{io} \tilde{x}_i \qquad (15)$$

$$s.t. \sum_{\substack{j=1 \\ j\neq 0}}^{n} \lambda_j \tilde{x}_{ij} - \tilde{x}_i \leq \tilde{x}_i, \qquad i = 1,...,m$$

$$\sum_{j=1}^{n} \lambda_j \tilde{y}_{rj} \geq \tilde{y}_{ro} + \Delta y_{ro}, \qquad r = 1,...,s$$

$$\sum_{i=1}^{m} c_{io} \tilde{x}_i \geq \tilde{z}^*$$

$$\lambda_j \geq 0, J \neq 0, \qquad j = 1,...,n$$

$$x_i \geq 0, \qquad i = 1,...,m$$

where $\tilde{z}^*$ is the optimal solution of model (13). c_{io} is the price or cost of the i-th input of the decision-making unit under the o-th evaluation. According to the provided ranking functions, a rank can be defined for each triangular fuzzy number. This helps us to transform the fuzzy data inverse envelope analysis ($FIDEA$) model presented in Equation (13) and (14) into a problem with deterministic data. To do this, we replace the rank of each triangular fuzzy number with the corresponding triangular fuzzy number in the considered FIDEA. Then all arithmetic operations are performed on real numbers. In other words, with the help of the ranking functions presented in relations (4), (5), (6) and (7), the fuzzy numbers of the problem are ranked and then with models (13), (14) and (15) of cost efficiency and we calculate the amount of new inputs of DMUs.

4. Numerical example

Consider ten DMUs with two inputs $\tilde{X}_{1_{old}}$ and $\tilde{X}_{2_{old}}$ and the prices (costs) of each input C_1 and C_1 and one output $\tilde{Y}$.

In Table No. 1, the data of ten DMUs with two inputs $\tilde{X}_{1_{old}}$ and $\tilde{X}_{2_{old}}$ and the prices (costs) of each input C_1 and C_1 and an output $\tilde{Y}$ are expressed.

First, with the help of the ranking functions presented in relations (4), (5), (6) and (7), the fuzzy numbers of the problem are ranked and then with the models (13), (14) and (15) of the cost efficiency of DMUs We calculate After increasing the output value of each of the DMUs by ٪10, we calculated the new inputs of DMUs and showed them in Table 3, and we obtained the cost efficiency of DMUs with new inputs and outputs and showed them in Table 2. In Table 2, cost efficiency can be seen before and after a ٪10increase in output with estimated inputs. which is ranked using the ranking function (4) of the triangular fuzzy numbers of the problem.

Table1: Data set for 10 decision making units

DMU_i	inputs		prices		output
	$\tilde{X}_{1_{old}}$	$\tilde{X}_{2_{old}}$	C_1	C_2	$\tilde{Y}$
DMU1	(2,6,7)	(1,3,5)	8	9	(0.7,1,1.1)
DMU2	(7,9,11)	(5,8,10)	3	5	(4,7,9)
DMU3	(3,4,5)	(6,7,9)	6	9	(2,3,5)
DMU4	(2,4,7)	(1,3,4)	7	3	(1.5,2,3)
DMU5	(4,5,7)	(5,6,7)	9	10	(0.8,1,1.3)
DMU6	(5,7,8)	(0.2,1,2)	10	4	(2,3,5)
DMU7	(6,9,11)	(2,4,5)	6	2	(1,2,3)
DMU8	(0.5,1,3)	(6,9,10)	2	3	(1.5,2,2.5)
DMU9	(7,8,10)	(1,2,3)	2	7	(0.6,1,2)
DMU10	(7,10,11)	(1,3,4)	3	5	(2.6,2.8,3.1)

Table 2: The value of cost efficiency ($E_{co}old$) and the value of cost efficiency after a ٪10increase in output ($E_{co}new$) with the ranking function with(4)

DMU10	DMU9	DMU8	DMU7	DMU6	DMU5	DMU4	DMU3	DMU2	DMU1	
0.52	0.24	0.36	0.34	0.81	0.21	0.71	0.56	0.81	0.27	$E_{co}old$
0.58	0.27	0.4	0.38	0.89	0.23	0.79	0.62	0.89	0.3	$E_{co}new$

Table 3: The value of the estimated inputs of DMUs after increasing the output by ٪10and the ranking function(4)

DMU10	DMU9	DMU8	DMU7	DMU6	DMU5	DMU4	DMU3	DMU2	DMU1	
6.45	2.63	4.57	2.93	4.77	2.34	3.12	7.42	15.42	2.17	x_1new
1	0.41	0.71	2.53	4.1	0.36	2.68	1.16	2.4	0.34	x_2new

Table 4: The value of cost efficiency ($E_{co}old$) and the value of cost efficiency after a %10increase in output ($E_{co}new$) with the ranking function with(5)

DMU10	DMU9	DMU8	DMU7	DMU6	DMU5	DMU4	DMU3	DMU2	DMU1	
0.52	0.24	0.36	0.35	0.84	0.21	0.72	0.55	0.79	0.27	$E_{co}old$
0.57	0.27	0.39	0.38	0.92	0.23	0.8	0.61	0.87	0.29	$E_{co}new$

Table 5: Estimating the input value of DMUs after increasing the output by %10and the ranking function(5)

DMU10	DMU9	DMU8	DMU7	DMU6	DMU5	DMU4	DMU3	DMU2	DMU1	
6.32	2.64	4.47	2.96	4.88	2.3	3.18	7.37	14.96	2.1	x_1new
1	0.42	0.71	2.53	4.17	0.36	2.72	1.17	2.37	0.33	x_2new

Table 6: The value of cost efficiency ($E_{co}old$) and the value of cost efficiency after a %10increase in output ($E_{co}new$) with the ranking function with(6)

DMU10	DMU9	DMU8	DMU7	DMU6	DMU5	DMU4	DMU3	DMU2	DMU1	
0.53	0.24	0.37	0.34	0.78	0.22	0.7	0.57	0.84	0.28	$E_{co}old$
0.58	0.26	0.41	0.37	0.85	0.24	0.77	0.63	0.92	0.3	$E_{co}new$

Table 7: Estimation of the input value of DMUs after increasing the output by %10and the ranking function(6)

DMU10	DMU9	DMU8	DMU7	DMU6	DMU5	DMU4	DMU3	DMU2	DMU1	
6.69	2.61	4.75	2.9	4.59	2.41	3.02	7.52	16.22	2.29	x_1new
1.01	0.39	0.72	2.52	3.99	0.36	2.63	1.14	2.45	0.35	x_2new

Table 8: The value of cost efficiency ($E_{co}old$) and the value of cost efficiency after a %10increase in output ($E_{co}new$) with the ranking function with(7)

DMU10	DMU9	DMU8	DMU7	DMU6	DMU5	DMU4	DMU3	DMU2	DMU1	
0.53	0.24	0.38	0.33	0.75	0.22	0.69	0.57	0.86	0.28	$E_{co}old$
0.59	0.26	0.42	0.36	0.83	0.24	0.76	0.63	0.94	0.31	$E_{co}new$

Table 9: Estimation of the input value of DMUs after increasing the output by %10and the ranking function(7)

DMU10	DMU9	DMU8	DMU7	DMU6	DMU5	DMU4	DMU3	DMU2	DMU1	
6.85	2.6	4.87	2.87	4.47	1.45	2.95	7.58	16.78	2.38	x_1new
1.02	0.39	0.72	2.52	3.92	1.27	2.59	1.12	2.49	0.35	x_2new

In Table 3, the estimated inputs of x_1new and x_2new are observed after a %10 increase in output. which is ranked using the ranking function (4) of the triangular fuzzy numbers of the problem. that the cost efficiency of all DMUs has been improved after increasing the outputs and calculating the inputs with the proposed method. For example, we can refer to DMU10, which with a %10increase in its output leads to a %64decrease in $X_2 = 2.75$ to $X^*_{2new} = 1$ and a %32decrease in $X_1 = 9.5$ to $X^*_{1new} = 6.45$ and an increase 9.6percent of cost efficiency from 0.52to 0.57.

In Table 5, the estimated inputs of x_1new and x_2new are observed after increasing the output by %10. which is ranked using the ranking function (5) of the triangular fuzzy numbers of the problem. that the cost efficiency of all DMUs has been improved after increasing the outputs and calculating the inputs with the proposed method. For example, we can refer to DMU7, which with a %10increase in its output leads to a%31.6 decrease in $X_2 = 3.7$ to $X^*_{2new} = 2.53$ and a %66 decrease in $X_1 = 8.7$ to $X^*_{1new} = 2.96$ and an increase 8.8percent of cost efficiency from 0.34to 0.37.

In Table 6, cost efficiency can be seen before and after a %10increase in output with estimated inputs. which is ranked using the ranking function (6) of the triangular fuzzy numbers of the problem.

In Table 7, the estimated inputs of x_1new and x_2new are observed after a %10 increase in output. which is ranked using the ranking function (6) of the triangular fuzzy numbers of the problem. that the cost efficiency of all DMUs has been improved after increasing the outputs and calculating the inputs with the proposed method. For example, we can refer to DMU4, which with a 10percent increase in its output leads to a 7percent decrease in $X_2 = 2.83$ to $X^*_{2new} = 2.63$ and a 27.4 percent decrease in $X_1 = 4.16$ to $X^*_{1new} = 3.02$ and an increase 10.1percent of cost efficiency from 0.69to 0.76.

In Table 8, cost efficiency can be seen before and after a %10increase in output with estimated inputs. which is ranked using the ranking function (7) of the triangular fuzzy numbers of the problem.

In Table 9, the estimated inputs of x_2new and x_1new are observed after a %10 increase in output. which is ranked using the ranking function (7) of the triangular fuzzy numbers of the problem. that the cost efficiency of all DMUs has been improved after increasing the outputs and calculating the inputs with the proposed method. For example, we can refer to DMU1, which with a %10increase in its output leads to an %88decrease in $X_2 = 3$ to $X^*_{2new} = 0.35$ and a %58decrease in $X_1 = 5.67$ to $X^*_{1new} = 2.38$ and an increase %10cost efficiency increased from 0.28to .0.31

5. Conclusion

The purpose of this research was to provide a perspective on the application of the envelope analysis method of inverse data with triangular fuzzy numbers. Estimation of inputs is very complex and sensitive with changes in the amount of outputs and improving the cost efficiency of DMUs. In this article, by changing the value of some or all outputs in a way that leads to maintaining or improving the cost efficiency, a new approach and method was presented in calculating the value of DMU inputs under evaluation by triangular fuzzy number ranking. According to the stated contents, by changing the amount of outputs of the unit under evaluation, in any case, the inputs can be estimated in such a way that its cost efficiency is maintained or improved. By using inverse data envelopment analysis with triangular fuzzy numbers, a new model was presented to estimate inputs while maintaining or improving cost efficiency, and we were able to answer this question: "Under maintaining or improving cost efficiency, when some or all outputs of a DMU increase What are the inputs of that DMU? To check the accuracy of the model, a numerical example with triangular fuzzy numbers was investigated. In future researches, the mentioned method should be extended for network DEA mode.

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