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Buckling of Cylindrical Steel Shells with Random Imperfections due to Global Shear

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Abstract

This study aims to investigate the effects of geometric imperfections on buckling of thin cylindrical shells due to global shear. To this end, more than 320 finite element models of cylindrical shells with different diameter to thickness ratios were prepared. Random imperfections with different amplitudes were applied to numerical models. The results revealed that global buckling of cylindrical shells are susceptible to imperfection patterns. It was also shown that Yamaki's expression can be considered as upper band for plastic shear buckling of thin cylindrical shells.

Keywords: Cylindrical shell, Imperfection, Gobal shear buckling

1. Introduction

Cylindrical shells are common components of industrial plants such as oil refineries and petrochemical plants. Pressure vessels, liquid storage tanks, bins, and silos are some examples of shell structures. During the past decades, several cylindrical shells were damaged due to the extreme loads such as tornados, explosions and earthquakes [1-3]. Performance of shell structures during the past events showed that shell buckling is the most common failure mode of thin walled cylindrical shells. Although there is not a unique border between thin and thick cylindrical shells, thin shells usually have the radius to thickness ratio of 100 to 2000 [4]. Since 1900s, several analytical and experimental studies have been performed on buckling of cylindrical shells [5-10]. There is a wide scatter among the results of these studies. Buckling of thin shells is highly dependent on boundary conditions, shell imperfections, geometric specifications, etc. [4]. The wide scatter of the results of researches in the last decades is due to the fact that some of these parameters are random quantities. For this reason Arabocz [11] studied the buckling of shells with random imperfections due to axial compression and calculated the buckling stress of shells by means of reliability theory. Most of the research programs have focused on buckling of cylindrical shells due to axial compressive loads. In this study shear buckling of thin cylindrical shells with random imperfections have been taken into account.

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2. Shear Buckling of Cylindrical Shells

A horizontal load applied to a vertical cylindrical shell is termed as transverse shear or global shear. In thin cylindrical shells, global shear may cause buckling. Since prebuckling deformations are not axisymmetric, shear buckling is more complex than buckling due to the axial loads [4]. For the sake of simplicity, practical engineers usually link shear buckling to a simpler buckling mode torsional buckling. Results of previous studies showed that this simplification provides a reasonably accurate prediction [12, 13] (See Fig. 1).
In order to calculate the elastic critical shear stress the following formula can be used [14]:

$$\tau_e = \frac{4.82\sqrt{1+0.0239\omega^3}}{\omega^2}\frac{Et}{r} \qquad (1)$$

In which E, t, and r stand for Modulus of elasticity, shell thickness and radius respectively. The parameter ω can be calculated as follows:

$$\omega = \frac{L}{\sqrt{rt}} \quad (1a)$$

In the above relation L is the height of the cylinder. Hence global transverse shear load (Q_e) can be calculated as follows:

$$Q_e = \pi r t \tau_e \quad (2)$$

In order to calculate the global plastic buckling shear force (Q_p), Yamaki's expression can be used as follows [12]:

$$\frac{1}{{Q_p}^2} = \frac{1}{{Q_e}^2} + \frac{1}{{Q_y}^2} \quad (3)$$

In which the global shear force at yield can be calculated as follows:

$$Q_y = \pi r t \frac{\sigma_y}{\sqrt{3}} \quad (3a)$$

Where σ_y denotes the yield stress of the shell.

It is worth mentioning that the above mentioned expressions are related to perfect cylindrical shells.

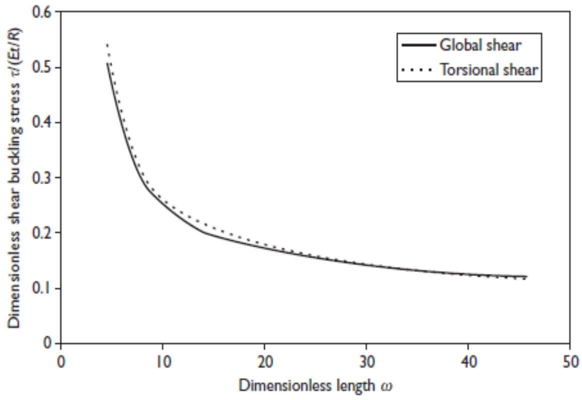

Fig. 1. Elastic global shear and torsional buckling [4].

3. Numerical Analysis

In order to study the global shear buckling of thin walled cylindrical shells nonlinear numerical analyses have been conducted. To this end, ANSYS multi-purpose FE code was used [15]. Four node Shell181 elements with three degrees of freedom at each node were used to model cylindrical shells (See Fig. 2). The elements were capable of considering material nonlinearity and large deformations. A bilinear elastic-plastic model was considered for modeling material properties of cylindrical shell. The yield stress of steel material was assumed to be 240 MPa.

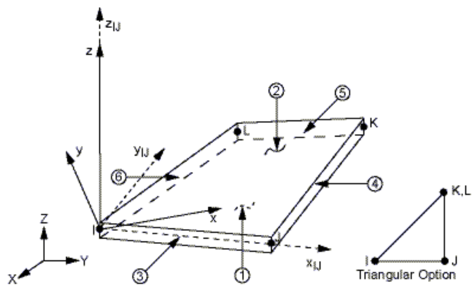

Fig 2. Specification of shell elements

3.1. *Specifications of Cylindrical Shells*

Three categories of cylindrical shells of the same height (H=14.75m) and different diameters (D=12, 15 and 17.5m) were modeled. For each category three different shell thicknesses (t=15, 20 and 25mm) were considered. For each cylindrical shell at least 15 different imperfection patterns with two imperfection ratios (w/t=1, 1.5) were randomly considered. Herein w is the maximum size of the geometric imperfection. For each model a perfect cylinder (w/t=0) was modeled too. In other words, 327 different cylindrical shells were modeled (See table 1). It deserves mentioning that the boundary conditions of the shells were fixed at the bottom and free at top of the shell.

Table 1. Specifications of the models

Model name	Diameter (m)	Thickness (mm)	Number of imperfection patterns
C1-1	10	15	40
C1-2	10	20	40
C1-3	10	25	40
C2-1	15	15	30
C2-2	15	20	30
C2-3	15	25	30
C3-1	17.5	15	36
C3-2	17.5	20	36
C3-3	17.5	25	36

3.2. Loading Pattern:

In order to observe post buckling path of the models, displacement control analyses were performed. To model global shear on cylindrical shells incremental displacements were horizontally applied to the top of the shells. It is worth mentioning that none of the cylindrical shells were pressurized.

4. Results

The global shear buckling loads of the cylindrical shells versus diameter to thickness ratio (D/t) are presented in Fig. 3. As indicated in this figure, there is no meaningful relationship between global shear buckling loads and the diameter to thickness ratio (D/t). On the other hand, there is a scatter among the results of numerical analyses for each cylinder which is related to the shell imperfection. The average buckling load and standard deviation of each set of models are presented in table 2. In order to obtain dimensionless shear buckling load, results of numerical analyses were normalized to plastic buckling force obtained from Yamaki's expression.

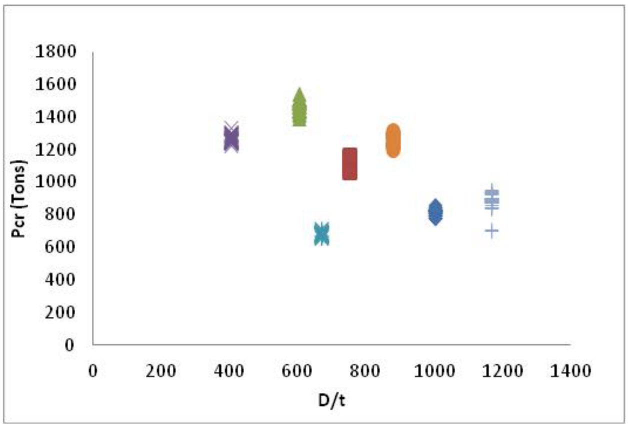

Fig. 3. Shear buckling loads of different cylindrical shells

Table 2. Average buckling loads of the cylindrical shells

D/t	Average buckling load, Q (kN)	Standard Deviation
400	124.9	21.74
500	94.9	14.50
600	154.3	35.47
666	67.1	12.21
700	163.9	56.35
750	113	28.68
875	126	33.71
1000	818	19.25
1167	886	51.40

Plots of dimensionless shear buckling loads for all the models are presented in Figures 4 to 6. As indicated in these figures, dimensionless buckling load for different models varies between o.3 and 0.8. In other words results of this study seem to indicate that Yamaki's relation is not conservative for imperfect shells. As shown in Figures 4 to 6, variation of dimensionless buckling load byimperfection pattern is much greater than that by imperfection amplitude (w/t). In other words, unlike axial buckling of cylindrical shells, global shear buckling is not very susceptible to imperfection amplitude.

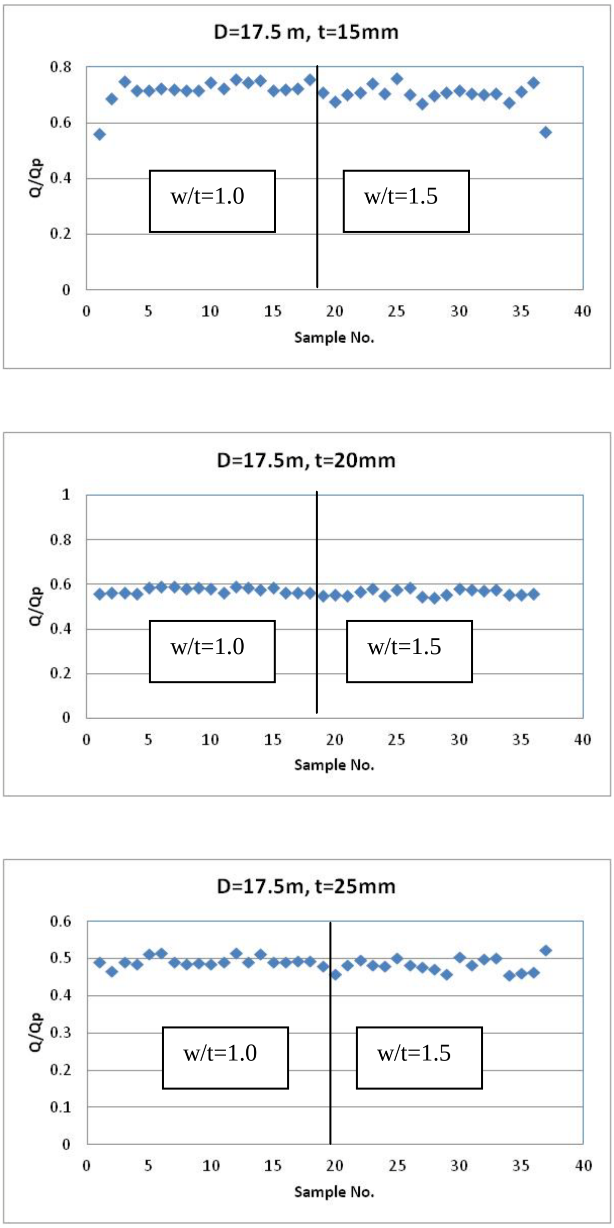

Fig 4. Dimensionless shear buckling loads for C1-1, C1-2 and C1-3

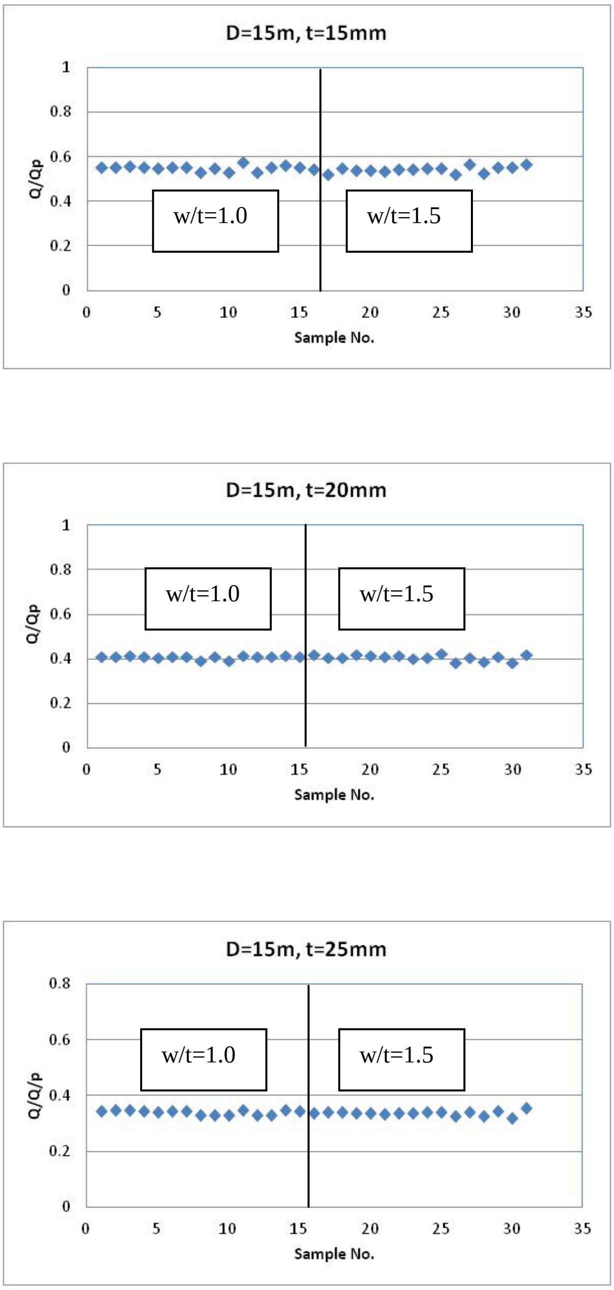

Fig 5. Dimensionless shear buckling loads for C2-1, C2-2 and C2-3

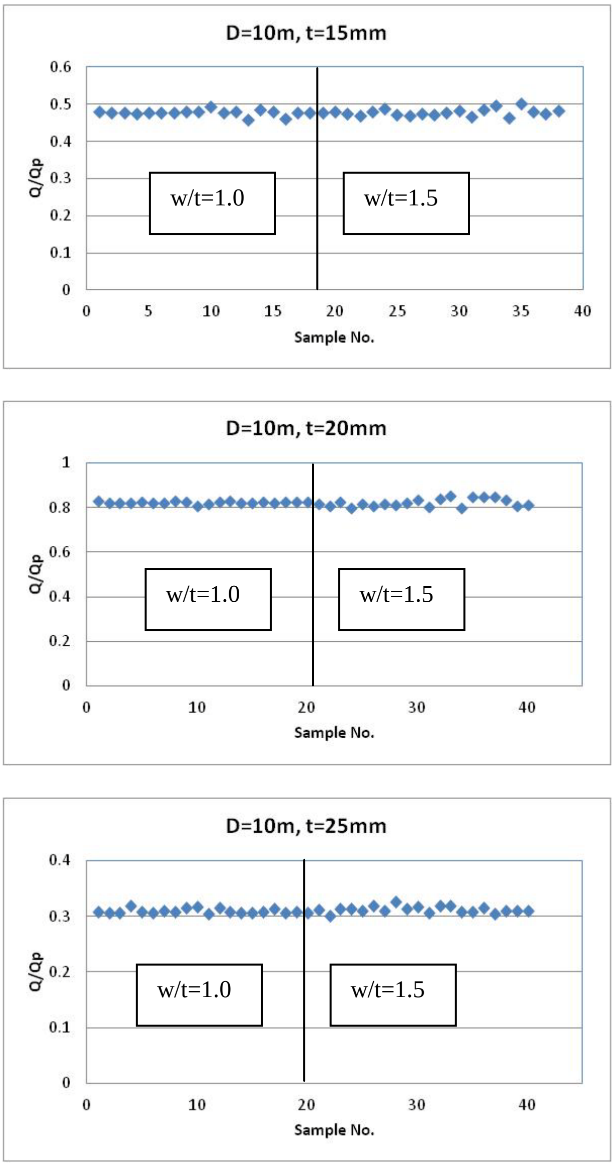

Fig 6. Dimensionless shear buckling loads for C3-1, C3-2 and C3-3

4.1. Stochastic Analysis

The imperfection patterns of shells are random in nature. Hence stochastic analysis is a suitable way for introducing the results of the analyses. To this end, reliability function $R(\lambda)$ can be introduced as follows:

$$R(\lambda) = P[\Lambda > \lambda] \quad (4)$$

In which λ is normalized load parameter (Q/Q_p) and Λ is the normalized random buckling load. Failure surface can be formulated as follows [11]:

$$Z(\lambda) = \Lambda_s - \lambda = \psi(x_1, x_2, \ldots x_n) - \lambda = 0 \quad (5)$$

The probability of failure (buckling) can be estimated as follows:

$$P_f(\lambda) = P[Z < 0] = F_z(0) = \int_{-\infty}^{0} F_z(t) df \tag{6}$$

where Fz(t)is probability distribution function and fz(t) is probability density function of Z. Assuming that Z(λ) is normally distributed:

$$f_z(t) = \frac{1}{\sigma_z \sqrt{2\pi}} exP\left[-\frac{1}{2}\left(\frac{t-a}{\sigma_z}\right)^2 \right] \tag{7}$$

in which $a = E[Z]$ and $\sigma_z = [Var(z)]^{0.5}$.

Base on the above calculation, the plot of R versus dimensionless buckling load is illustrated in Fig. 7. The allowable load can be defined as Q/Q_p in which the reliability function becomes less than 1. Hence in this study Q/Q_p=0.2. In other words, the shear buckling loads equal to $0.2Q_p$ should be considered to achieve reliable results in designing.

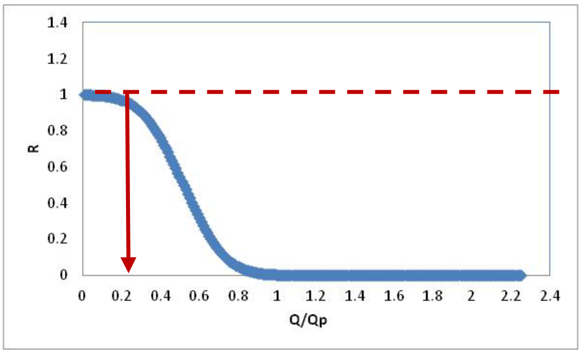

Fig. 7. Reliability function of global shear buckling

5. Conclusions:

Nonlinear static analyses were carried out to investigate the effects of geometric imperfections on global shear buckling of cylindrical shells. To this end, more than 320 FE models of different diameter to thickness ratios were prepared. Results of this study revealed that shear buckling capacity of cylindrical shells is susceptible to both imperfection amplitude and its pattern.

Furthermore results of this study seem to indicate that the plastic buckling capacity estimated by Yamaki's expression can be considered as an upper band of plastic shear buckling capacity of the imperfect cylindrical shells. However, for concrete conclusions more theoretical and experimental studies are required.

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