Extension of Portfolio Selection Problem with Fuzzy Goal Programming: A Fuzzy Allocated Portfolio Approach

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Abstract

Recently, the economic crisis has resulted in instability in stock exchange market and this has caused high volatilities in stock value of exchanged firms. Under these conditions, considering uncertainty for a favorite investment is more serious than before. Multi-objective Portfolio selection (Return, Liquidity, Risk and Initial cost of Investment objectives) using MINMAX fuzzy goal programming for a Fuzzy Allocated Portfolio is considered in this research and all the main sectors of investment are assumed under uncertainty. A numerical example on stock exchange is presented to demonstrate the validity and strengths of the proposed approach.
Keywords: Portfolio selection; Fuzzy Allocated Portfolio (FAP); Fuzzy goal programming; MINMAX Approach.

1. Introduction

In many corporations and industries, decision makers face many important problems including scheduling problem, logistics, data mining and asset allocation problem. In these problems, it is important that they predict the total future return and decide an optimal asset allocation maximizing them under some constraints, particularly a budget constraint. Furthermore, in recent investment fields, not only big companies and institutional investors but also individual investors called Day-Traders invest in stock, currency land and property. Therefore, the role of investment theory called portfolio theory becomes more and more important. Of course, they easily decide the most suitable allocation provided that they know future returns a priori. Furthermore, in the real world, there may be probabilitistic and possibilitistic factors derived from the lack of efficient information and an ambiguous prediction of decision maker. So the concept of Portfolio selection is an interesting concept for scientists.

So far, various studies with respect to portfolio selection problems have been done. Portfolio selection, as originally introduced by Markowitz (1952) was one of the most important fields of research in theory of finance and his mean-variance model has been challenged and modified by many studies that examines the trade-offs between risk and return objectives in the "mean-variance" context. Commonly, portfolio selection models assume that the future condition of stock market can be accurately predicted by historical data without paying attention to the accuracy of the previous data (Chen and Huang, 2009). As far as most of the real world problems take place in an imprecise environment, this is not an appropriate assumption for the real financial markets due to the high volatility of market environments. Therefore, fuzzy set theory, proposed by Zadeh et al. (1987), has become a helpful tool in handling the imprecise conditions and attributes of portfolio selection. A brief literature on this subject in the previous years with focus on fuzzy approach follows. Two portfolio selection models based on fuzzy probabilities and possibility distributions were proposed by Tanaka et al. 2000. Inuiguchi and Tanino 2000 proposed a new possibilistic programming approach based on the worst regret to the portfolio selection.

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Tiryaki 2001 used DEA to analyze more complex portfolio systems. A fuzzy goal programming with fuzzy goals and fuzzy constraints was formulated by Parra et al. [15] assuming three criteria: return, risk and liquidity. Ong et al. (2005) proposed a method that incorporates the grey and possibilistic regression models. A multistage stochastic fuzzy program with soft constraints and recourse in order to capture both uncertainty and imprecision was developed by Lacagnina and Pecorella 2006. Huang et al. (2006) revised the conventional mean–variance method to determine the optimal portfolio selection under conditions of uncertainty. Terol et al. 2006 formulated a fuzzy compromise programming problem and Zhang et al. 2007 proposed two kinds of portfolio selection models based on lower and upper possibilistic means and possibilistic variance. Huang 2007 dealt with the problem of portfolio selection when security returns contain both randomness and fuzziness. Gupta et al. 2008 applied multi-criteria decision making via fuzzy mathematical programming to develop comprehensive models of asset portfolio optimization.

In this research, an FAP problem will be introduced, where; all the main sectors of investment are considered under uncertainty. We extend this concept in the framework of a multi-objective fuzzy portfolio selection problem whose objectives are return, risk, liquidity and initial cost of investment. Then, we use the MINMAX Approach model (Yaghoobi and Tamiz, 2007) to optimize this multi-objective fuzzy problem.

The remainder of this paper is organized as follows. Section 2 briefly discusses the theoretical background and fundamental concepts of fuzzy goal programming. Modeling an optimization of a multi-objective problem to the Iran stock exchange market and solving them with two famous models will be illustrated in Section 3. In section 4, FAP and "linguistic" constraints are used to help investors to find the efficient portfolio under uncertainty. Finally, conclusion and further research will be considered in Section 5.

2. The Fuzzy Goal Programming (FGP) Model

An objective with an imprecise aspiration level can be treated as a fuzzy goal. The fuzzy goals can be identified as fuzzy sets defined over the feasible set with the membership functions. Mostly, linear membership functions, such as follows, are used in the literature (Narasimhan, 1980; Hannan, 1981):

$$\mu_i(AX) = \begin{cases} 0, & (AX)_i \le b_i - \Delta_{iL}, \quad i = j_0+1,\dots,K \\ 1 - \dfrac{b_i - (AX)_i}{\Delta_{iL}}, & b_i - \Delta_{iL} \le (AX)_i \le b_i, \quad i = j_0+1,\dots,K \\ 1 - \dfrac{(AX)_i - b_i}{\Delta_{iR}}, & b_i \le (AX)_i \le b_i + \Delta_{iR}, \quad i = j_0+1,\dots,K \\ 0, & (AX)_i \ge b_i + \Delta_{iR}, \quad i = j_0+1,\dots,K \end{cases} \tag{1}$$

where Δ_{iL} and Δ_{iR} are the maximum admissible violations from the aspiration level b_i (for i = 1,…, K). They are either subjectively chosen by DM (Narasimhan, 1980; Hannan, 1981) or tolerances in a technical process (Kim and Whang, 1998. The above membership function is depicted respectively in Figure 1.

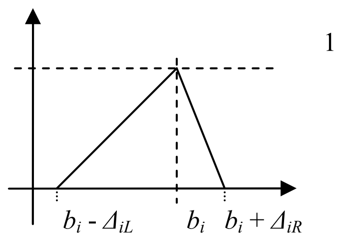

Fig. 1. Linear membership functions

Now, consider multi-objective fuzzy model for portfolio selection problem as follows:

$$\begin{aligned} & \max \tilde{f}_h(x), \quad h = 1,\dots,H \\ & \min \tilde{f}_l(x), \quad l = 1,\dots,L \\ & \text{s.t.} \\ & \textstyle\sum_{j=1}^{n} x_j = 1, \\ & x \epsilon S, \end{aligned} \tag{2}$$

Where $\tilde{f}_h(x)$ and $\tilde{f}_l(x)$ respectively are fuzzy objectives, and x_j (for j = 1,…, n) is the invested proportion of security j in optimal portfolio. Finding optimal solution x^* is equivalent to solve the following crisp model (Zimmermann, 1978):

$$\begin{aligned} & \max \lambda \\ & \text{s.t.} \\ & \lambda \le \mu_{f_h}(x), \quad h = 1,\dots,H \\ & \lambda \le \mu_{f_l}(x), \quad l = 1,\dots,L \\ & \sum_{j=1}^{n} x_j = 1, \\ & x \epsilon S, \end{aligned} \tag{3}$$

Where $\mu_{f_h}(x)$ and $\mu_{f_l}(x)$ represent the membership functions of objectives, respectively, and $0 \le \lambda \le 1$ is the achievement degree of the membership functions.

Yang et al. (1991) proposed a model to solve FGP problems with triangular linear membership functions. In fact, they extended the well-known Zimmerman's (1978) approach to transform the problem into a conventional single LP model. Yaghoobi and Tamiz (2007) developed Yang et al. (1991) and presented the following model for solving FGP problems.

$$\begin{aligned} & \max \lambda \\ & \text{s.t.} \\ & (AX)_i - P_i \le b_i, \quad i = 1,\dots,i_0 \\ & (AX)_i + n_i \ge b_i, \quad i = i_0+1,\dots,j_0 \\ & (AX)_i + n_i - P_i = b_i, \quad i = j_o+1,\dots,K \\ & \lambda + \frac{1}{\Delta_{iR}} P_i \le 1, \quad i = 1,\dots,i_0 \end{aligned}$$

$$\lambda + \frac{1}{\Delta_{iL}} n_i \le 1, \quad i = i_0 + 1, \ldots, j_0$$
$$\lambda + \frac{1}{\Delta_{iL}} n_i + \frac{1}{\Delta_{iR}} P_i \le 1, \quad i = j_o + 1, \ldots, K$$
$$x \epsilon S,$$
$$\lambda \ge 0;\ n_i \ge 0;\ P_i \ge 0, \quad i = 1, \ldots, K \tag{4}$$

where b_i (for i = 1,…, K) is the precise aspiration level for goal ith, n_i and p_i (for i = 1,…, K) are respectively the negative and positive deviations from aspiration value of goal ith, Δ_{iL} and Δ_{iR} indicate left and right admissible violations for fuzzy goal ith, respectively. In this model, weights are considered equally for the fuzzy goals and the fuzzy decision is symmetrical.

3. Portfolio Selection in Iran Stock Exchange Market

In this section, two different fuzzy approaches in the portfolio selection will be compared in a real sample of 15 main stocks from the Iran stock exchange market during 2006-2008. In order to study this problem, we consider four selected objectives as follows:

- Return: Instead of the crisp representations used in this paper, rate of return is represented as fuzzy numbers to reflect the uncertainty at the evaluation stage. The fuzzy rate of return ($\tilde{r}_j = (\tilde{P}_{j,t} - P_{j,t-1} + \tilde{D}_{j,t})/P_{j,t}$) measures the profitability of the stock where $\tilde{P}_{j,t}$ is the fuzzy price of the stock j at time t and $\tilde{D}_{j,t}$ is the fuzzy dividend received during the period [t –1, t].
- Beta risk: $\tilde{\beta}_j = Cov(\tilde{r}_j, \tilde{r}_m)/Var(\tilde{r}_m)$, where $\tilde{r}_j$, j=1, 2, …,15 is the fuzzy rate of return of stock j and $\tilde{r}_m$ is the fuzzy rate of market return. This objective indicates the performance on its own rather than by the movements of the market. The aim is to choose a diversified portfolio with small β.
- Initial cost of investment: In real world, many people do not have enough money for secure investments. Thus, the aim is to enable people to spend less money while they will obtain their favorite results from other objectives. $\tilde{P}_j$ is the fuzzy price of stock j (with known formal currency) in the last under study day. Let N be the total number of existent securities (stocks) in the optimum portfolio. Therefore, the initial cost of investment objective function can be obtained without considering the value N as follows:

$$\begin{aligned} &Z = \tilde{P}_1(Nx_1) + \tilde{P}_2(Nx_2) + \ldots + \tilde{P}_{15}(Nx_{15}) \\ &\Rightarrow Z = N(\tilde{P}_1 x_1 + \tilde{P}_2 x_2 + \ldots + \tilde{P}_{15} x_{15}) \\ &\Rightarrow Z = N(\textstyle\sum_{j=1}^{15} \tilde{P}_j x_j) \Rightarrow Z/N = N(\textstyle\sum_{j=1}^{15} \tilde{P}_j x_j)/N \\ &\Rightarrow Z/N = f_3 = \textstyle\sum_{j=1}^{15} \tilde{P}_j x_j \end{aligned} \tag{5}$$

Finally, optimum value of cost for selection and allocation of optimum portfolio is equal to $Z^* = f_3^* N$. We consider the price of the last day ($\tilde{P}_j$) to purchase stock j.

- Liquidity: Liquidity is measured as the possibility of converting an investment into cash without any significant loss in its value. Other things being equal, the investors prefer greater liquidity (Parra et al, 2001). The exchange flow ratio ($\tilde{EF}_j = \tilde{N}_{j,s}/\tilde{N}_m$), with $\tilde{N}_{j,s}$ being the fuzzy number of days when the stock j has been traded and $\tilde{N}_m$ being the fuzzy number of days that the market has been opened.

Furthermore, our aim is to include into our framework linguistic labels, such as “little rate of return”, “sufficient initial cost of investment” and “near absolutely liquid”. These natural expressions have a fit representation through fuzzy numbers used in the work. However, the main portfolio selection problem can be formulated as follows:

$$\begin{aligned} \max \quad & f_1 = \textstyle\sum_{j=1}^{15} \tilde{r}_j x_j, \quad j = 1, \ldots, 15 \\ \min \quad & f_2 = \textstyle\sum_{j=1}^{15} \tilde{\beta}_j x_j, \quad j = 1, \ldots, 15 \\ \min \quad & f_3 = \textstyle\sum_{j=1}^{15} \tilde{P}_j x_j, \quad j = 1, \ldots, 15 \\ \max \quad & f_4 = \textstyle\sum_{j=1}^{15} \tilde{EF}_j x_j, \quad j = 1, \ldots, 15 \\ \text{s.t.} \quad & \\ & x_1 + x_2 + x_3 + x_{15} = 0.25, \\ & x_5 + x_6 + x_7 + x_8 = 0.25, \\ & x_4 + x_{13} + x_{14} = 0.25, \\ & x_9 + x_{10} + x_{11} + x_{12} = 0.25, \\ & 0 \le x_j \le 0.1, \quad j = 1, \ldots, 15. \end{aligned} \tag{6}$$

Where, in order to diversify the selected portfolios and maximum utilization of the all existent capacities of investment, DM proposes to invest 25% in automotive industry (for stocks j = 1, 2, 3, 15), banking and leasing (for stocks j = 5, 6, 7, 8), investment sectors (for stocks j = 4, 13, 14) and another sectors (for stocks j = 9, 10, 11, 12). Moreover, we set a lower and an upper bound for each stock in order to diversify the portfolio, $0 \le x_j \le 0.1$, for j = 1, 2, …, 15 where the x_j is the proportion to be invested in the stock j.

Model (11) is transformed to an MA model (Yaghoobi and Tamiz, 2007) as follows:

$$
\begin{aligned}
&\max \quad \lambda \\
&\text{s.t.} \\
&\sum_{j=1}^{15} \tilde{r}_j x_j + n_1 \ge b_1, \qquad j = 1,\dots,15 \\
&\sum_{j=1}^{15} \tilde{\beta}_j x_j - p_2 \le b_2, \qquad j = 1,\dots,15 \\
&\sum_{j=1}^{15} \tilde{P}_j x_j - p_3 \le b_3, \qquad j = 1,\dots,15 \\
&\sum_{j=1}^{15} E\tilde{F}_j x_j + n_4 \ge b_4, \qquad j = 1,\dots,15 \\
&\lambda + \frac{1}{\Delta_{1L}} n_1 \le 1, \\
&\lambda + \frac{1}{\Delta_{2R}} p_2 \le 1, \\
&\lambda + \frac{1}{\Delta_{3R}} p_3 \le 1, \\
&\lambda + \frac{1}{\Delta_{4L}} n_4 \le 1, \\
&x_1 + x_2 + x_3 + x_{15} = 0.25, \\
&x_5 + x_6 + x_7 + x_8 = 0.25, \\
&x_4 + x_{13} + x_{14} = 0.25, \\
&x_9 + x_{10} + x_{11} + x_{12} = 0.25, \\
&0 \le x_j \le 0.1, \qquad j = 1,\dots,15 \\
&\lambda \ge 0; n_1, n_4 \ge 0; p_2, p_3 \ge 0.
\end{aligned} \tag{7}
$$

Furthermore, the Yang et al. (1991) model for solving model (11) is as follows:

$$
\begin{aligned}
&\max \quad \lambda \\
&\text{s.t.} \\
&\lambda \le 1 - \frac{b_1 - \sum_{j=1}^{15} \tilde{r}_j x_j}{\Delta_{1L}}, \quad j = 1,\dots,15 \\
&\lambda \le 1 - \frac{\sum_{j=1}^{15} \tilde{\beta}_j x_j - b_2}{\Delta_{2R}}, \quad j = 1,\dots,15 \\
&\lambda \le 1 - \frac{\sum_{j=1}^{15} \tilde{P}_j x_j - b_3}{\Delta_{3R}}, \quad j = 1,\dots,15 \\
&\lambda \le 1 - \frac{b_4 - \sum_{j=1}^{15} E\tilde{F}_j x_j}{\Delta_{4L}}, \quad j = 1,\dots,15 \\
&x_1 + x_2 + x_3 + x_{15} = 0.25, \\
&x_5 + x_6 + x_7 + x_8 = 0.25, \\
&x_4 + x_{13} + x_{14} = 0.25, \\
&x_9 + x_{10} + x_{11} + x_{12} = 0.25, \\
&0 \le x_j \le 0.1, \quad j = 1,\ \dots,\ 15 \\
&\lambda \ge 0.
\end{aligned} \tag{8}
$$

The model involves expressions of set of fuzzy decision goals $\tilde{b} = (\tilde{b}_1, \tilde{b}_2, \tilde{b}_3, \tilde{b}_4)$, which is associated with a set of fuzzy objectives $f(x) = (f_1(x), f_2(x), f_3(x), f_4(x))$. The problem formulation allows the objectives to be under-or over-achieved enabling the DM to be relatively imprecise about initial design goals. Table 1 presents the de-fuzzified goal values of objectives: return, risk, initial cost of investment and liquidity. The goal value of Beta objective is equal to 1 (Lee and Chesser, 1980).

Table 1
De-fuzzified goal values of fuzzy objectives (for i = 1, 2, 3, 4)

Objective (f_i)	Δ_{iL}	b_i	Δ_{iR}
f_1	0.0008	0.002	–
f_2	–	1	0.05
f_3	–	1300	100
f_4	0.002	0.126	–

Table 2 presents data concerning the 15 main stocks of the Iran stock exchange market during 2006-2008. We considered de-fuzzified numbers instead of fuzzy numbers and applied fuzzy decision goals in the FAP problem. The five columns of Table 2 are the stocks, the stock price in the last exchanged day, the risk β, the expected rate of return of each security and the exchange flow ratio of each security, respectively.

Table 2
De-fuzzified data under study

Stocks (j)	Stock price in the last exchanged day (P_j)	Beta risk (β_j)	Expected rate of return (μ_j)	Exchange flow ratio (EF_j)
PARS AUTO	926	0.59815	0.0012654	0.1292097
MEH IRAN AUTO	700	1.15065	−0.0006437	0.1301761
SAIPA	926	0.17812	0.0015994	0.1214500
PAY SAIPA INV	2392	2.60025	0.0027148	0.1067670
PERSIAN BANK	2337	1.05606	0.0021114	0.1140610
KAR AFR BANK	1435	2.00207	0.0019685	0.1304640
IRAN LEAS	2115	−0.02369	0.0027717	0.1304920
IND & MIN LEAS	967	1.23007	0.0022249	0.1399780
PARS ALU	948	2.14956	−0.0001838	0.1289632
ALUMTAK	1385	−0.82301	0.0016264	0.1100235
IRAN BEHNUSH	2373	−0.00125	0.0009780	0.1224789
PARS MINOO	2477	3.67891	−0.0021901	0.1263525
OIL IND INV	1180	1.67921	0.0011433	0.1324790
SEPAH INV	1180	2.12003	0.0011433	0.1240011
SAIPA DIESEL	920	0.89782	−0.0004956	0.1203698

Models (12) and (13) were solved by Lingo software package and Table 3 presents optimal portfolios and optimal values of each objective:

Table 3
Result of solving problem (18) with different models

Optimal solution	Yang et al. (1991) model	MA model
x_1	0.1	0.1
x_2	0.05	0.05
x_3	0.1	0.1
x_4	0.0996	0.0996
x_5	0	0
x_6	0.0549	0.0549
x_7	0.0951	0.0951
x_8	0.1	0.1
x_9	0.1	0.1
x_{10}	0.1	0.1
x_{11}	0.05	0.05
x_{12}	0	0
x_{13}	0.1	0.1
x_{14}	0.0504	0.0504
x_{15}	0	0
f_1	0.0014839	0.0014839
f_2	1	1
f_3	1365	1365
f_4	0.1253	0.1253
Optimal values of deviation variables	–	$n_1 = 0.000516$
	–	$p_2 = 0.033256$
	–	$p_3 = 64.51238$
	–	$n_4 = 0.00129$

Results of Table 3 show that MA model (Yaghoobi and Tamiz, 2007) is in general equivalent to Yang et al. (1991) model and can optimize the FGP problems.

4. Fuzzy Allocated Portfolio (FAP)

In this section, FAP model as a novel approach to portfolio selection problem will be discussed. To diversify the selected portfolios and maximum utilization of the all existent capacities of investment, FAP allocates a percentage of total selected portfolios to any investment sector under uncertainty. By this definition, allocated constraints of FAP are defined as "linguistic" constraints. We propose this kind of portfolio selection for decreasing the above-mentioned current problems concerning investment in Iran stock exchange market. Hence, with regard to the above advantages, we will develop our portfolio selection problem and use Yaghoobi and Tamiz (2007) model to solve it.

The membership function related to fuzzy allocated constraint t-th (t = 1, 2, 3, 4) of the main portfolio selection problem may be presented as follows:

$$\mu_t(AX) = \begin{cases} \dfrac{(AX)_t - 0.27}{0.03}, & 0.27 \le (AX)_t \le 0.3, \quad t = 1,2,3,4 \\ \dfrac{0.32 - (AX)_t}{0.02}, & 0.3 \le (AX)_t \le 0.32, \quad t = 1,2,3,4 \\ 0, & (AX)_t \prec 0.27 \text{ and } (AX)_t \succ 0.32, \quad t = 1,2,3,4 \end{cases} \tag{9}$$

Then, the main FAP problem can be formulated as follows:

$$\begin{aligned} &\max\ \tilde{f}_1 = \sum\nolimits_{j=1}^{15} \tilde{r}_j x_j \qquad j = 1,\ ...,\ 15 \\ &\min\ \tilde{f}_2 = \sum\nolimits_{j=1}^{15} \tilde{\beta}_j x_j \qquad j = 1,\ ...,\ 15 \\ &\min\ \tilde{f}_3 = \sum\nolimits_{j=1}^{15} \tilde{P}_j x_j \qquad j = 1,\ ...,\ 15 \\ &\max\ \tilde{f}_4 = \sum\nolimits_{j=1}^{15} E\tilde{F}_j x_j \qquad j = 1,\ ...,\ 15 \\ &\text{s.t.} \\ &x_1 + x_2 + x_3 + x_{15} \cong 0.3 \\ &x_5 + x_6 + x_7 + x_8 \cong 0.3 \\ &x_4 + x_{13} + x_{14} \cong 0.3 \\ &x_9 + x_{10} + x_{11} + x_{12} \lesssim 0.3 \\ &\sum_{j=1}^{15} x_j = 1, \\ &0 \le x_j \le 0.1, \quad j = 1,\ ...,\ 15. \end{aligned} \tag{10}$$

Based on the MA model of Yaghoobi and Tamiz (2007), model (15) is transformed as follows:

$$\begin{aligned} &\max \quad \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 + \varphi + \tau + \omega + \psi \\ &\text{s.t.} \\ &\sum_{j=1}^{15} \tilde{r}_j x_j + n_1 \ge b_1, \quad j = 1,\dots,15 \\ &\sum_{j=1}^{15} \tilde{\beta}_j x_j - p_2 \le b_2, \quad j = 1,\dots,15 \\ &\sum_{j=1}^{15} \tilde{P}_j x_j - p_3 \le b_3, \quad j = 1,\dots,15 \\ &\sum_{j=1}^{15} E\tilde{F}_j x_j + n_4 \ge b_4, \quad j = 1,\dots,15 \\ &\lambda_1 + \frac{1}{\Delta_{1L}} n_1 \le 1, \end{aligned} \tag{11}$$

Model (16) is solved by Lingo software package and the optimal solution is obtained as follows:
$x_1^* = 0.1$, $x_2^* = 0.1$, $x_3^* = 0.1$, $x_4^* = 0.0726$, $x_5^* = 0$, $x_6^* = 0.1$, $x_7^* = 0.1$, $x_8^* = 0.1$, $x_9^* = 0$, $x_{10}^* = 0.1$, $x_{11}^* = 0.0273$, $x_{12}^* = 0$, $x_{13}^* = 0.1$, $x_{14}^* = 0.1$, $x_{15}^* = 0$, $f_1^* = 0.0015336$, $f_2^* = 1$, $f_3^* = 1320$, and $f_4^* = 0.12592$.

It seems that by interpreting constraints as "linguistic" the feasible solution space gets bigger than before and the results obtained will not worsen. Therefore, the comparison of the results for models (12) and (16) reveals that the optimal solution obtained after interpreting constraints as "linguistic" improve.

$$\lambda_2 + \frac{1}{\Delta_{2R}} p_2 \le 1,$$
$$\lambda_3 + \frac{1}{\Delta_{3R}} p_3 \le 1,$$
$$\lambda_4 + \frac{1}{\Delta_{4L}} n_4 \le 1,$$
$$\varphi \le \frac{0.32 - (x_1 + x_2 + x_3 + x_{15})}{0.02},$$
$$\varphi \le \frac{(x_1 + x_2 + x_3 + x_{15}) - 0.27}{0.03},$$
$$\tau \le \frac{0.32 - (x_5 + x_6 + x_7 + x_8)}{0.02},$$
$$\tau \le \frac{(x_5 + x_6 + x_7 + x_8) - 0.27}{0.03},$$
$$\omega \le \frac{0.32 - (x_4 + x_{13} + x_{14})}{0.02},$$
$$\omega \le \frac{(x_4 + x_{13} + x_{14}) - 0.27}{0.03},$$
$$\psi \le \frac{0.32 - (x_9 + x_{10} + x_{11} + x_{12})}{0.02}, \tag{12}$$
$$\sum_{j=1}^{15} x_j = 1,$$
$$\varphi, \tau, \omega, \psi \ge 0; n_1, n_4 \ge 0; p_2, p_3 \ge 0; \lambda_i \ge 0, \qquad i = 1, \ldots, 4$$
$$0 \le x_j \le 0.1, \qquad j = 1, \ldots, 15.$$

It is realistic in most cases that poor performance on one criterion cannot easily be balanced with good performance on other criteria. In this case, we can reformulate the model so that the achievement level of membership functions should not be less than the allowed value. The α-cut approach can be utilized to ensure that the degree of achievements for any goals and fuzzy constraints should not be less than a minimum allowed value α. In this case, the model (16) should be reformulated by adding new constraints of λ_i (for i = 1, 2, 3, 4), φ, τ, ω, ψ ≥ α, $\alpha \in [\alpha^-, \alpha^+]$ to other system constraints. This approach requires that DM have to choose reasonable values for α to avoid getting infeasible solutions (Chen, [1]).

In this example, α^- is assumed to be 0.0878 and α^+ can be obtained from Zimmermann's (1978) approach in which all objective functions and constraints are equally important. In fact, α^+ is the maximum achievement degree of membership functions of fuzzy objectives and constraints. In this example, α^+ is calculated at 0.4976982 and then α can vary from 0.0878 to a maximum level of 0.4976982. To change α from α^- to α^+, causes the problem solutions to vary from asymmetric to fully symmetric decision making. In this case, α is changing in steps 0.045, from 0.0878 to 0.4976982. Table 4 (Appendix 1.) presents all optimal solutions S1 to S11 related to these α-cut levels. Fig 2 represents achievement level variations of membership functions according to α-cut level approach.

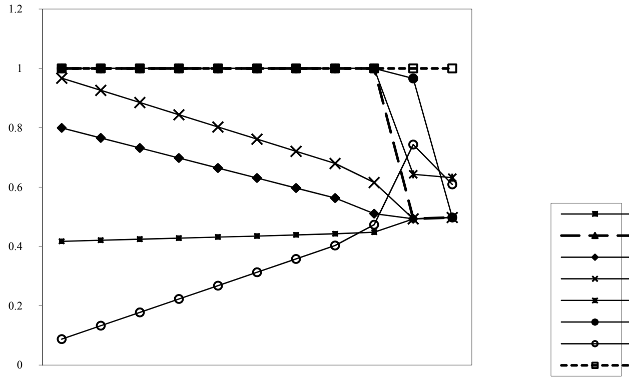

Fig. 2. Degree of achievement objective functions and constraints(α-cut level from 0.0878 to 0.4976982)

5. Conclusion

To deal with the nature of uncertainty in the portfolio selection problem, a multi-objective problem with four objectives was introduced and applied to selecting optimal portfolio in Iran stock exchange market. The coefficients and goal value of objectives were considered based on fuzzy set theory as unbalanced triangular fuzzy numbers. Then, the multi-objective fuzzy problem was converted to a model of FGP and, in order to solve it, we considered two approaches: the MA model (Yaghoobi and Tamiz, 2007) and Yang et al. (1991) model. Both models were solved according to FAP approach. The α-cut approach was used for the obtained results to insure that the achievement level of objective functions should not be less than the minimum level α. It was shown that by increasing α level, objectives improvement of problem will decrease unless about expected rate of return. This matter represented trade-offs between the objectives under uncertainty environment.

Further research may address using group decision making, stochastic fuzzy constraints and changing the objectives.

6. References

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Appendix 1.

Table 4

Optimal solution S1 to S11 related to α-cut level

Solutions	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11
α-cut	0.0878	0.133	0.178	0.223	0.268	0.313	0.358	0.403	0.448	0.493	0.4976982
x_1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
x_2	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.0331	0.0356
x_3	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
x_4	0.0726	0.0740	0.0753	0.0767	0.0780	0.0794	0.0807	0.0821	0.0842	0.0923	0.0883
x_5	0	0.0037	0.0074	0.0111	0.0148	0.0186	0.0223	0.0260	0.0318	0.0007	0.01
x_6	0.1	0.0963	0.0925	0.0888	0.0851	0.0814	0.0777	0.0740	0.0682	0.1	0.1
x_7	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
x_8	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
x_9	0	0	0	0	0	0	0	0	0	0	0
x_{10}	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
x_{11}	0.0273	0.0260	0.0246	0.0233	0.0219	0.0206	0.0192	0.0179	0.0158	0.0177	0.0127
x_{12}	0	0	0	0	0	0	0	0	0	0	0
x_{13}	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
x_{14}	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
x_{15}	0	0	0	0	0	0	0	0	0	0.0562	0.0533
f_1	0.0015336	0.0015366	0.0015392	0.0015422	0.0015449	0.0015481	0.0015508	0.0015539	0.0015583	0.0015944	0.0015981
f_2	1	1	1	1	1	1	1	1	1	1.025	1.025
f_3	1320	1323	1326	1330	1333	1337	1340	1344	1349	1351	1350
f_4	0.12592	0.12585	0.12575	0.12567	0.12558	0.12552	0.12543	0.12536	0.12523	0.12498	0.12498
λ_1	0.4172	0.4208	0.4244	0.4280	0.4316	0.4352	0.4388	0.4424	0.4480	0.5	0.5
λ_2	1	1	1	1	1	1	1	1	1	0.5	0.5
λ_3	0.7992	0.7654	0.7316	0.6979	0.6641	0.6304	0.5967	0.5629	0.5103	0.5	0.5
λ_4	0.9670	0.9258	0.8848	0.8437	0.8027	0.7616	0.7206	0.6795	0.6155	0.5	0.5
φ	1	1	1	1	1	1	1	1	1	0.6431	0.6318
τ	1	1	1	1	1	1	1	1	1	0.9659	0.4977
ω	0.0878	0.1330	0.1780	0.2230	0.2680	0.3130	0.3580	0.4030	0.4731	0.7435	0.6094
ψ	1	1	1	1	1	1	1	1	1	1	1