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Exact Mixed Integer Programming for Integrated Scheduling and Process Planning in Flexible Environment

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Abstracts

This paper presented a mixed integer programming for integrated scheduling and process planning. The presented process plan included some orders with precedence relations similar to Multiple Traveling Salesman Problem (MTSP), which was categorized as an NP-hard problem. These types of problems are also called advanced planning because of simultaneously determining the appropriate sequence and minimizing makespan in the process of scheduling. There are alternative machines for each operation and different sequences for each order, which create a flexible environment for production planning. In process planning ansd integrated scheduling, most mathematical models have two sets of ordered pairs with precedence or non-precedence relations between operations; therefore, these models cannot be solved using optimization software. Therefore, in this paper, this problem was modeled by a new approach and solved by GAMS software. The model was validated by the existing data in the literature.

Keywords: Integrated scheduling and process planning, Makespan, Flexible manufacturing.

1. Introduction

Process planning is looking for the best route for order manufacturing under system configuration conditions based on the existing resources in the total production time. Furthermore, process planning will be optimum if it is investigated according to time criteria. Process planning searches the best sequence for operations with precedence relations, determines scheduling, start time and completion time and selects suitable resources and machines for operation processing according to objective function of decision maker in time intervals. In other words, scheduling process tries to allocate operations to machines in the time intervals (saygin et al. (1999)). Based on this description, separated scheduling and process planning lead to interference and overlapping in production planning; i.e., in some cases, a proper process plan has not been appropriately selected and this causes deviation in objectives of the decision maker while integration between process planning and scheduling results in uniformity in solution space. Scheduling is connected to shop floor situation and causes some problems in the state of alternative process plans. In fixed process plan, first, one sequence is selected from the existing sequences; then, scheduling is done. Fixed process plan leads to scheduling with unbalanced loading, creation of additional bottlenecks and unbalanced utilization of machines and weakness in delivery times. In contrast, integrated process plan and scheduling are flexible in selecting operation sequence and scheduling. Figure 1 depicts fixed process plan and scheduling (process planning and scheduling are done in separate stages). Figure 2 shows integrated process planning and scheduling (sequence selecting and scheduling are simultaneously done).

Moon et al. (2008) integrated process plan and scheduling in a manufacturing supply chain; integration is one of the important tasks that results in high quality of products, low cost and high performance in supply chain. Besides, they demonstrated that utilization of alternative machines reduced lead time and improved overall use of resources. Nasr and Elsayed (1990) presented two heuristics to determine an efficient schedule for n orders and m resources problem with alternative routing allowance for each operation. Moon et al. (2002) presented a mathematical model for integrated process planning and scheduling in multi-plant supply chain that considered alternative machines and sequences, sequence dependent setup and distinct due date and was similar to multi traveling salesman problem. Chen and Ji (2002) presented advanced planning and scheduling that considered capacity constraint, operation sequence, lead times and delivery

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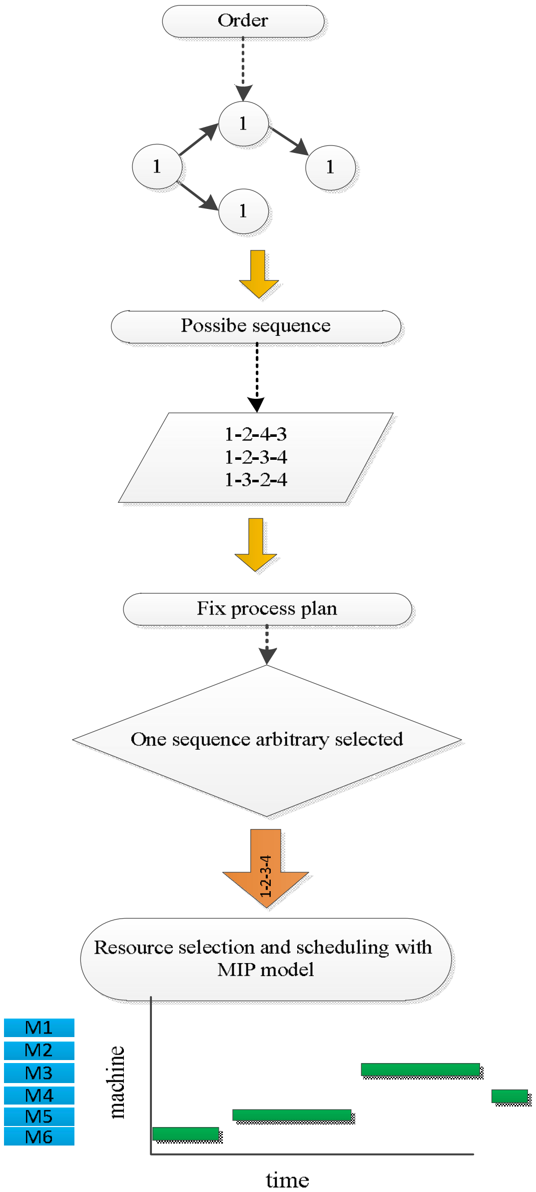

Fig.1. Fixed process plan and scheduling

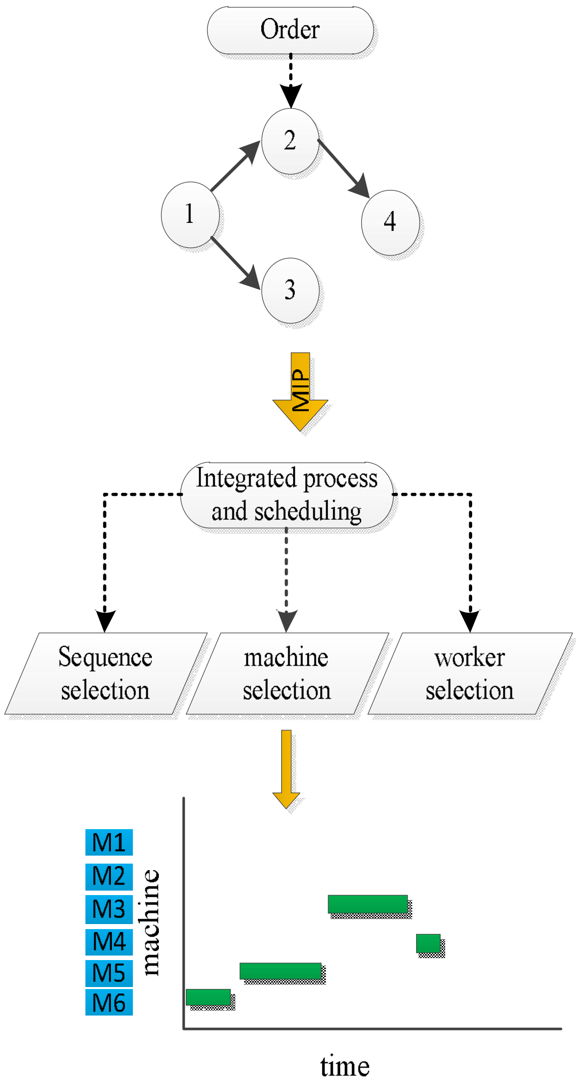

Fig. 2. Integrated process planning and scheduling

times in multi order environment. The model objective was to minimize cost of producing idle time and also tardiness and earliness penalty cost of orders. Chung et al. (2010) proposed advanced planning in multi-plant environment, each plant including several machines and each machine could process some operations of orders. Each factory's primary products may be the products for the assembly work of other companies and may produce some final products for final customers. Zhang et al. (2006) presented a multi objective function for process planning and scheduling and genetic algorithm was used for Pareto solution. Ey et al. (2000) utilized petri nets for job shop scheduling with routing and flexible sequence. Saygin and Kilic (1999) integrated scheduling and process plans in flexible environment and expressed that flexibility in process planning can include process flexibility, sequence flexibility and alternative machine tools. In integrated process planning and scheduling, most mathematical models have presented two sets of ordered pairs showing precedence or non-precedence relations between operations; therefore, they cannot be solved using optimization software and Table 1 illustrates this issue. In this paper, this problem was modeled by a new approach and solved by GAMS software. Of course, this problem was very similar to multiple traveling salesman with priority relations as one of NP-hard problems. Therefore, for large scale problems, meta-heuristic methods were proposed.

Table 1
Review works of study

Reference	problem type		modeling		solving method
	process plan	scheduling	conceptual	exact	
[1]	*	*		*	heuristic
[2]	*	*	*		GA
[3]	*	*	*		heuristic
[4]	*	*	*		GA
[5]	*	*		*	MIP
[6]	*	*	*		GA
[7]	*	*	*		moGA
[8]	*	*	*		Petri Nets
[9]	*	*		*	GA
[10]	*	*	-	-	GA

2. Prerequisites of the Model

In designing integrated process and scheduling, most mathematical models have presented two sets of ordered pairs that show precedence or non-precedence relations between operations, which is conceptually correct but is not solvable using optimization software. Therefore, in this paper, this problem was modeled by a new approach. For modeling precedence relations, two matrices called matrix A and matrix B were used. Matrix A guaranteed the defined precedence relations and matrix B was applied to develop acceptable sequences. Before forming the matrices, the number of operations of each order was increased to the maximum size of operations in the existing orders by adding virtual precedence relations and operations. Matrix A demonstrated precedence relations, which was essential for process plan, as shown with oriented vector. Matrix elements would be 1 if a relation existed between two operations. In matrix B, the whole operations that can have a relation in sequence were demonstrated in matrix elements. For better understanding, the following example can be observed in Figure (3). Line vectors depict infinitive and essential precedence relations and dash-line vectors depict relations that can exist in sequences. According to utilization of matrix B in the developed model, it is symmetric; i.e. all the relations between operations are covered in the sequences. Matrix A and B showed in Figure (4).

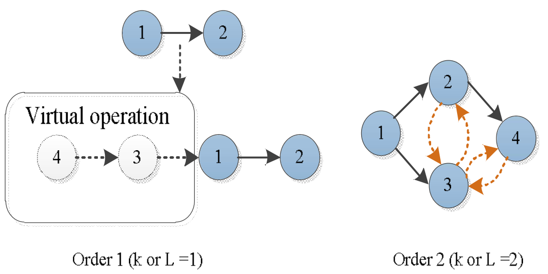

Fig. 3. Example of process plan

$$
\begin{array}{cc}
 & \begin{array}{cc} L=1 & \qquad L=2 \end{array} \\
\begin{array}{c} K=1 \\ \\ \\ K=2 \end{array} &
A = \left[\begin{array}{cccc|cccc}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
\end{array}
$$

$$
\begin{array}{cc}
 & \begin{array}{cc} L=1 & \qquad L=2 \end{array} \\
\begin{array}{c} K=1 \\ \\ \\ K=2 \end{array} &
B = \left[\begin{array}{cccc|cccc}
0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
\hline
1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\
1 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\
1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\
1 & 1 & 0 & 0 & 0 & 1 & 1 & 0
\end{array}\right]
\end{array}
$$

Fig. 4. Matrix A and B

3. Model Development

Advanced planning problem includes selecting operation sequences; a resource is selected for each operation and scheduling is done for all the operation. The presented model was highly flexible by integrating process plan and scheduling with a new approach and selecting alternative machines for operations. The following notations were used for describing the problem throughout this paper:

indexes

k, l	index for order, k, l= 1, 2, . . ., K where K is the number of orders
i, j	index for operation, i, j = 1, 2, . . ., $\max\{R_k\}$ where R_k is the number of operation of order k
m, n	index for machine, m, n = 1, 2, . . ., M where M is the number of machine
parameters	
P_{kim}	process time of operation i of order k on machine m
T_{mn}	Transportation time from machine m to n
q_k	Lot size for order k
G_{kim}	Determinant parameter of alternative machine for operation (if it be 1, operation i of order k can be process on machine m and otherwise zero)
A_{klij}	A is the precedence matrix and elements 1 illustrate relation between two operation
B_{klij}	B is the determinant matrix for precedence relation and used for sequencing between operations i of order k and operation j of order l and elements 1 illustrates possible relations between orders and operations.
U	an arbitrary large positive number
variables	
C_{kim}	Completion time of operation i order k on machine m
$X_{kim}\begin{cases}1\\0\end{cases}$	If operation i of order k process on machine m otherwise
$y_{kimljn}\begin{cases}1\\0\end{cases}$	If operation i of order k process on machine m precedes operation j of order l process on machine n otherwise

We introduced three decision variables. The first one determines completion time of each operation. The Second one is a binary variable for loading each operation on machines. The third one selects the operation sequence. In scheduling, the length of time required for completing orders for all customers is called Makespan. The Makespan is important when the number of customer orders is limited. It is denoted by Z and is defined as the time for the last order of a customer to leave the manufacturing system; i.e.

$$z = \max_{\forall k,i,m}\{C_{kim}\}$$

Minimizing Makespan forces the planner to balance workload on various machines. The mixed integer programming model for integrated process plan and scheduling (flexible planning) problem is presented below and linearization of the objective function is also done.

Minimize Z

Subject to

$$Z \ge c_{kim} \qquad \forall k,i,m \tag{1}$$

$$C_{ljn} - C_{kim} + M(1 - y_{kimljn}) \ge q_l . P_{ljn} \qquad \forall\, k,i,m,l,j,n \quad k=l \text{ or } m=n \tag{2}$$

$$C_{ljn} - C_{kim} + M(2 - X_{kimw} - X_{ljnv}) + M(1 - y_{kimljn}) \ge 0 \qquad \forall\, k,i,m,l,j,n \quad , k \ne l \tag{3}$$

$$\sum_{m=1}^{M}\sum_{n=1}^{M} y_{kjmwljnv} \ge A_{klij} \qquad \forall k,l,i,j \tag{4}$$

$$X_{kim} \le G_{kim} \qquad \forall k,i,m \tag{5}$$

$$\sum_{m=1}^{M} X_{kim} = 1 \qquad \forall k,i \tag{6}$$

$$X_{ljn} + X_{kim} \ge 2.\, y_{ljnkim} \qquad \forall\, k,i,m,l,j,n \tag{7}$$

$$S_{kilj} + S_{ljki} = \frac{B_{klij} + B_{lkji}}{2} \qquad \forall k,l,i,j \tag{8}$$

$$\sum_{m=1}^{M}\sum_{n=1}^{M} y_{kjmljn} \ge S_{kilj} \qquad \forall k,l,i,j \tag{9}$$

$$C_{kim} \ge q_k . P_{kim} \qquad \forall\, k,i,m \tag{10}$$

$$X_{kim}\ , y_{kimljn}\ , S_{kilj} \in \{0\ ,\ 1\} \quad \forall (k,i,m),(l,j,n) \tag{11}$$

Constraint (1) applied for linearization. Constraint (2) ensures that any two operations belonging to the same customer order or two operation loaded on same machine cannot be processed simultaneously. Constraint (3) prevents overlapping operations, and without this constraint interfering between operations of distinct orders occurs. Constraint (4) ensures that precedence relations between operations of each order are processed according to the determined precedence relation. Constraint (5) means that each operation processed on determined alternative machines. Constraint (6) ensures that only one resource for each operation should be selected. Constraint (7) guarantees that a precedence

relation from operation i of order k on resource m to operation j of order l on resource n is possible only when the resource m and n are assigned to the operations i and respectively.
Constraints (8) and (9) ensure that precedence relation preserved between operations of orders. Constraints (10) and (11) imply non-negativity of variables.

4. Numerical Results

For solving and demonstrating the validity of the proposed model, the data of the examples existing in the literature were used. This modeled example was first proposed by Lee-Jeong-Moon (2002) who solved it by the proposed Genetic Algorithm (GA). Process plan of this example is depicted in Figure 5 and processing time of operations on alternative machines is reported in Table 2. It should be noted that numbering of operation was continuous in the sample example; but, in this paper, numbering was separately done among the examples. Therefore, two numbers were written on operations of each 2, 3, 4 and 5 orders.
Then, Yan et al. (2007) improved solution of Lee-Jeong-Moon's model by applying local search in GA. Scheduling of Lee-Jeong-Moon and Yan et al. is shown in Figures 6 and 7.
By applying a different approach in this paper, this mixed integer programming (MIP) was solved by GAMS software. Scheduling of the proposed model is depicted in Figure 8 and comparison of Makespans is reported in Table 3.

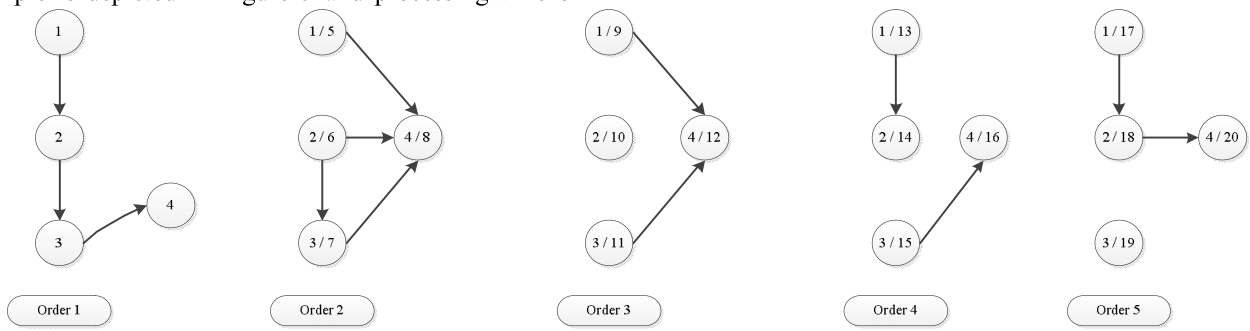

Fig. 5. Process plans of 5 orders

Table 2
Processing time for operations in alternative machines

		Order 1				Order 2				Order 3				Order 4				Order 5			
		1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4
Machine	1	5	-	-	-	7	-	-	-	4	-	-	4	-	-	-	-	3	-	-	6
	2	3	7	-	3	-	4	-	4	5	-	-	-	2	-	-	6	-	-	-	-
	3	-	-	6	-	-	6	7	-	8	-	-	-	6	8	3	-	5	7	-	-
	4	-	-	-	3	-	-	7	-	-	5	6	-	-	-	8	7	-	-	9	-
	5	-	-	-	4	-	-	-	10	-	-	5	4	-	-	-	4	-	-	6	3

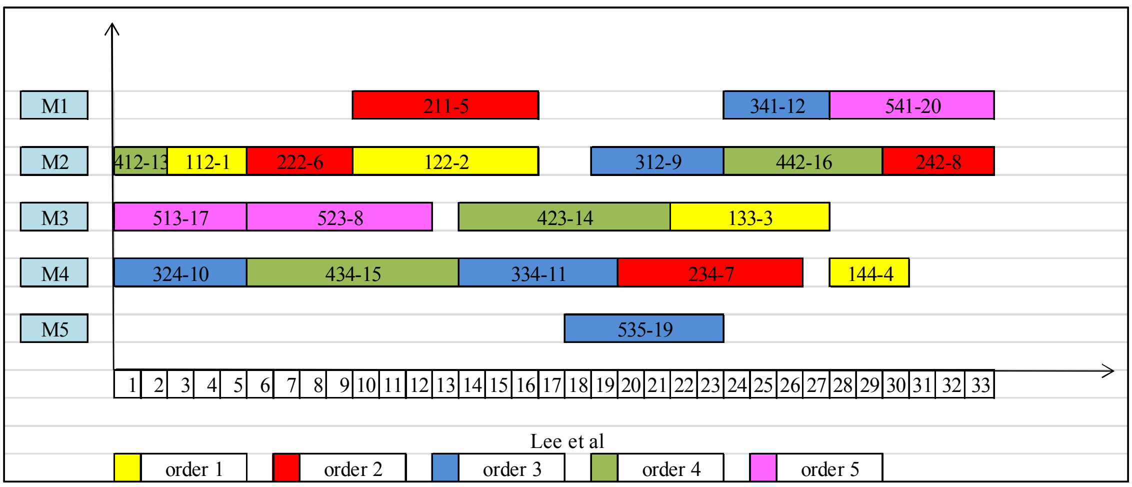

Fig. 6. Scheduling presented by Lee-Jeong-Moon

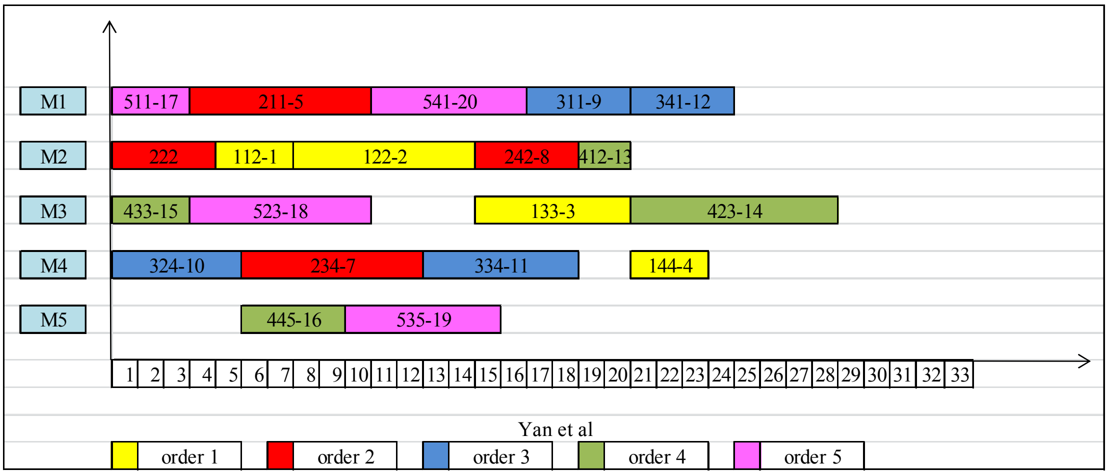

Fig. 7. Scheduling presented by yan et al

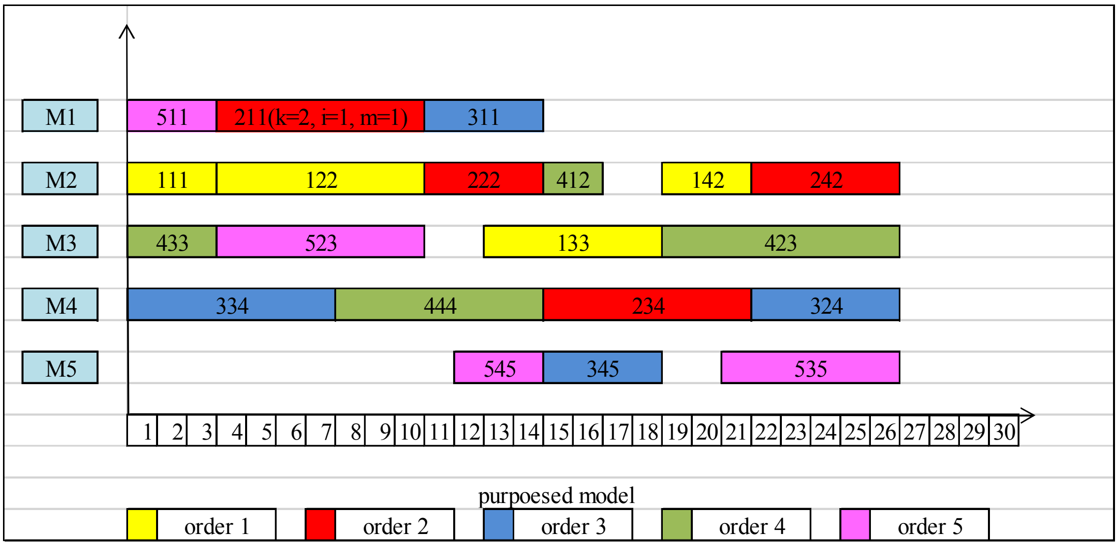

Fig. 8. purposed scheduling for presented model

Table 3
comparison of solution

Authors	solution method	makepan
Lee-Jeong-Moon	GA-based heuristic	33
Yan et al	GA with local search	28
Purposed model	MIP / GAMS software	26

5. Conclusion

This paper considered integrated process planning and scheduling, which is called advanced planning or flexible planning in manufacturing systems in the literature. Using a new method for modeling these problems, the Makespan, obtained by genetic algorithm in the literature, was improved. Also, the designed model utilized total floating time between operations, as one of the implicit results of the model. Start time of some operations can be visually changed without variation in the objective function. These problems are very similar to Multiple Traveling Salesman Problem (MTSP) that is categorized in NP-hard problem group. Therefore, for solving large scale problems, Meta heuristic methods are suggested.

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