Developing a Method for Increasing Accuracy and Precision in Measurement System Analysis: A Fuzzy Approach

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Abstract

Measurement systems analysis (MSA) has been applied in different aspect of industrial assessments to evaluate various types of quantitative and qualitative measures. Qualification of a measurement system depends on two important features: accuracy and precision. Since the capability of each quality system is severely related to the capability of its measurement system, the weakness of the two mentioned features can reduce the reliance on the qualitative decisions. Consequently, since in the literature fuzzy MSA is not considered as an independent study, in this paper, a fuzzy method is developed for increasing method accuracy and precision by encountering the impreciseness of some measures of MSA. To do so, bias, capability, and gauge repeatability and reproducibility (GR&R) indices are considered as triangular fuzzy numbers. The application of the proposed method is illustrated through a case study taken from an automotive parts industry.

Keywords: MSA; Fuzzy numbers; GR&R; Quality techniques.

1. Introduction

Currently, there are many quality control techniques for recognizing causes of errors and preventing their occurrence. Measurement system analysis (MSA) is considered as an important one. Measurement is defined as "the assignment of numbers or values to material things to represent the relationship among them with respect to particular properties" (C.Eisenhart [5]). It is the process of evaluating an unknown quantity and expressing it into numbers that is usually considered as precedence of any statistical process control. Moreover, MSA has been recognized as one of the key requirements in the old QS9000 Quality Standard, Six Sigma technique, and even new standards such as ISO TS16949:2009. These extensive applications are due to advantages of MSA including promoting the compatibility of the measurement system for the given process and reducing the contamination of measurement variation in the total process variation.

MSA is based on an important philosophy which argues that measurement errors creep into any process easurement method. Therefore, it should be considered as the precedent process of any quality measurement system (Harry & Lawson [10]). MSA quantifies a measurement error via the examination of multiple sources of variation in a process, including the variation resulting from the measurement system, from the operators, and from the parts [2].

Since statistical measures are estimated by data which are obtained through sampling, they are usually unreliable [7]. In this case, it is helpful to think of a measured value as the sum of two variables; the quantity of measured value and its error as Eq.(1).

$$Y_i(Measured\_value) = X_i(True\_value) + e_i \qquad (1)$$

The measurement system increases the total observed variability (σ^2_{obs}) of the measured parts. In any measuring, some of the observed variability is due to variability in the

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process (σ_p^2), whereas the rest variability is due to the measurement error or gauge variability (σ_{msa}^2). The variance of the total observed measurements can be expressed as Eq. (2) [17]. It means that total variability equals to the sum of process variability and measurement variability.

$$\sigma_{obs}^2 = \sigma_p^2 + \sigma_{msa}^2 \quad (2)$$

In this equation σ_{msa}^2 includes two major types of error by itself which are called repeatability and reproducibility. Repeatability ($\sigma_{\text{Repeatability}}^2$) which can be determined by measuring a part for several times, quantifies the variability in a measurement system, resulted from its gauge, [17], [19], [24]. Reproducibility ($\sigma_{\text{Reproducibility}}^2$) which is determined from the variability created by several operators measuring a part for several times, quantifies the variation in a measurement system resulted from the operators of the gauge and environmental factors, [2], [19], [3], [25]. Square root of σ_{msa}^2 is called gauge repeatability and reproducibility (GR&R) that as it mentioned models the all error related to the gauge. It can be shown as Eq. (3).

$$\sigma_{msa}^2 = \sigma_{repeatability}^2 + \sigma_{reproducibility}^2 \quad (3)$$

Foster [6] proposed some procedures for calculating different indices of MSA in order to calculate GR&R as major output of MSA. In order to distinguish product variance from device variance, Grubbs [7], Juran and Gyrna [9] carried out MSA studies on two or more measurement devices and proposed a procedure for estimating the sensitivity of the measurement devices. Senol [22] statistically evaluated MSA method by the means of designed experiments to minimize α-β risks and n (sample size). A GR&R study which was introduced in [19] estimates the repeatability and reproducibility components of measurement system variation with the primary objective of assessing whether or not the gauge is appropriate for the intended applications. Evaluating measurement and process capabilities by GR&R with four quality measures presented by Al-Refaie and Bata [1].

As it mentioned, one reason that causes indices calculated by sample data are unreliable is the uncertainty of the data. Therefore, statistical calculations such as standard deviation, point and interval estimation, hypothesis testing and other similar one are used. Besides, unreliability of indices has another reason which is resulted from the impreciseness of data. In the literature to deal with impreciseness usually fuzzy concept (introduced by Zadeh [26]) is used.

However, in the literature of the quality issues any work with the legend of fuzzy MSA (FMSA) was not found and we have just reviewed the most relevant work such as the application of fuzzy modelling in different quality indices and quality control charts. Lee [14] and Hong [11] proposed C_{pk} index estimation using fuzzy numbers. Parchami et al. [20] using fuzzy specification limits rather than precise ones proposed a new fuzzy number for all types process capability indices. Parchami et al. [21] introduced a new method that uses confidence interval of capability indices to produce fuzzy number for them. Faraz and Bameni Moghadam [4] and Gulbay and Kahraman [8] proposed efficient methods for creating fuzzy quality control charts.

According to what was mentioned, MSA help to judge about compatibility of the measurement system with the given measurement process and provides conditions for more reliable decision. This study makes important contribution to the MSA literature and develops fuzzy concept on MSA method to create indices of MSA much more accurate.

The rest of the paper is organized as follows. The next section illustrates classical MSA and its different indices. Section 3 develops fuzzy concept to create FMSA method. Section 4 describes a case study in automotive parts industry in Kachiran Company. Finally, in Section 5 some notes about concluding remarks and future research are presented.

2. Measurement System Analysis (MSA)

Measurement system is the collection of instruments or gauges, standards, operations, methods, fixtures, software, personnel, environment and assumption used to quantify a unit of measure or fix assessment [5]. Correspondingly, MSA is a collection of statistical methods for the analysis of measurement system capability (Automotive Industry Action Group (AIAG) [2]; Smith et al. [24]). It seeks to describe, categorize, evaluate the quality of measurements; improve the usefulness, accuracy, precision, meaningfulness of measurements; and propose methods for developing better measurement instruments by Montgomery, Runger [16]. Some stated goals of MSA are estimated components of measurement error, estimate the contribution of measurement error to the total variability of a process or equipment parameter, determine stability of a metrology tool over time, and to compare and correlate multiple metrology tools. Measurement process is a kind of production process that its output is number. Fig.1 presents measurement process with its inputs and outputs, [5].

According to the type of data, MSA has two categories of measurements; quantitative measurement and qualitative one. In this paper, quantitative measurements are discussed. In the rest of the section, we illustrate the steps required to execute MSA.

2.1. Bias

There are the difference between the observed average of measurements and the master average of the same parts using precision instruments [5]. Actually, bias is a measure that represents the difference between the averages value of the measurement and certified value of a specific part.

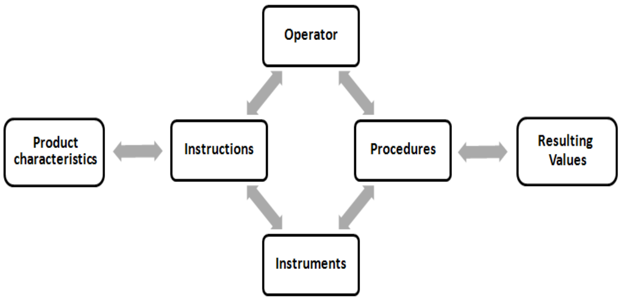

Fig.1. Measurement system analysis process [5]

Fig.2 represents bias concept schematically, [5]. In order to compute this index, we measure a part with an instrument for at least ten times. Then we should acquire the average of these observations and compare it with the true value of the part. We can obtain the value of bias by Eq. (3).

$$B = \bar{x}_g - x_m \qquad (3)$$

Where B is bias value, $\bar{x}_g$ is the average of measured data, x_m is a traceable standard. If a traceable standard C_g is not available, you can measure the part ten times in a controlled environment and then the average of values is used to determine the reference value. This sample will be considered as the Master Sample. Decision making based on only bias value is unusual. Actually, after obtaining, we can recognize appropriate level of bias.

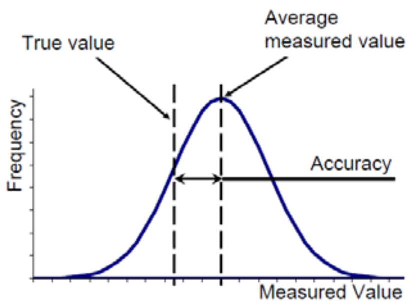

Fig.2. Scheme of bias [5]

2.2. Capability

Capability is a measure of process ability to consistently produce a result that meets the specification requirements. The term process capability was synonymous with process variation measures such as standard deviation or range of the observed data. However these measures do not consider customer requirements and it is not suitable for general comparisons among processes Leung PK, Spiring F, [15]. Capability indices C_p, C_{pk}, C_{nm}, and C_{nmk} have been proposed in the manufacturing and service industries, providing numerical measures on whether or not a process is capable of reproducing items within the specification limits, Shishebori, Hamadani [22]. Similarly, C_g and C_{gk} are used to show the capability of the measurement gauge. The indices and C_g and C_{gk} are defined by Eq.(4), (5).

$$C_g = \frac{0.2T}{6S_g} \qquad (4)$$

$$C_{gk} = \frac{0.1T - \left|\bar{x}_g - x_m\right|}{3S_g} \qquad (5)$$

Where T is part tolerance and S_g shows standard deviation of observed values using measurement instrument. The minimum acceptance criteria for C_g and C_{gk} are equal to 1.33 [12].

2.3. Gauge Repeatability and Reproducibility (GR&R)

As it was mentioned earlier, the total measurement variation is the sum of variation due to repeatability and reproducibility Eq. (3). Repeatability and reproducibility can influence the precision and accuracy respectively.

2.4. Repeatability

The same characteristic of the product should be measured repeatedly in order to determine the sensitivity of the measurement process [6]. When an inspector uses the same gauge to measure a product several times under the same conditions, several different values of measurement may occur. This error, called repeatability, comes from the gauge itself [18]. Repeatability is computed as Eq.(6).

$$EV = 5.15\frac{\bar{R}}{d_2^*} \qquad (6)$$

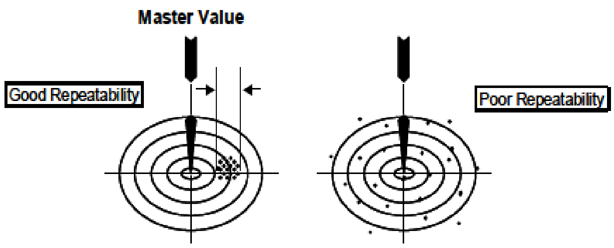

Fig.3. Precision of Repeatability [5]

Where $\bar{R}$ is the average of variation range, d_2^* is obtained from a specific table, and EV is tool variation.

Also 5.15σ interval involves 99 percents of data in normal distribution. Fig.3 represents the variation among successive measurements of the same characteristic, by the same person using the same instrument [5].

2.5. Reproducibility

This error occurs when different inspectors measure a product under the same condition. Practically, it is due to deficient trained inspectors or out of standard measuring methods [18]. It is computed as Eq. (7).

$$AV = \sqrt{(5.15\frac{\bar{X}_{DIF}}{d_2^*})^2 - \frac{(EV)^2}{n.r}} \quad (7)$$

Where $\bar{X}_{DIF} = \max(\bar{X}_i) - \min(\bar{X}_i)$; i=1,2,…,numbers of operator, d_2^* obtained from same table within previous subsection with *g=1* , *m* is number of operators, $\frac{\bar{x}_{DIF}}{d_2^*}$ is standard deviation of reproducibility, *EV* is repeatability value, *AV* is appraiser variation, *n* is number of used parts and *r* is number of trials that each piece is measured. If $5.15\frac{\bar{X}_{DIF}}{d_2^*} >> \frac{(EV)^2}{n.r}$ then $AV = \frac{5.15\bar{X}_{DIF}}{d_2^*}$ and otherwise reproducibility value is equal to zero.

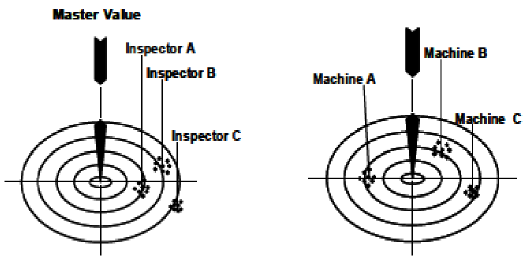

Fig.4. Precision of Reducibility [5]

Fig.4 represents the standard deviation of the averages of the measurements made by different persons, machines, and tools [5] when measure the identical characteristic on the same part.

2.5.1. Gauge R&R

A GR&R study is a method of determining the suitability of a gauge system for measuring a particular process. Every measurement has some associated error, and if this error is large compared to the allowable range of values (the tolerance band), the measuring device will frequently accept bad parts and reject good ones [5].

Total GR&R is the estimation of the combined estimated variation from repeatability and reproducibility [1]. In a GR&R study, we try to quantify measurement variation as a percent of process variation. GR&R index is computed as (8).

$$R\,\&\,R = \sqrt{EV^2 + AV^2} \quad (8)$$

An ideal measurement system should not have any variation. However, this is impossible and we have to be satisfied with a measurement system that has variation less than 10% of the process variation. As the portion of variation due to measurement system increases, the value of measurement system reduces. If this proportion is more than 30%, the measurement system is unacceptable. Table1 summarizes system status for obtained %GR&R [1].

Table 1
System status after computing %GR&R

%GR&R	Decision Guideline
<%10	Acceptable measurement system
%10 to %30	This needs to be agreed with the customer
>%30	Unacceptable measurement system

3. Fuzzy Measurement System Analysis (FMSA)

In this section, some basic concept of fuzzy sets and fuzzy numbers are reviewed. Fuzzy set theory which was introduced by Professor Lofti Zadeh in 1965 [26] is a typical method for encountering with ambiguity and imprecision. Since most practical and industrial methods and problems are encountered with imprecise data or lack of data considering fuzzy techniques help us to make our methods much more accurate.

Nowadays we deal with different quality problems that all of them are preceded by MSA. In this situation, if the quality of measurement system is low, it can be expected that process analysis will not be valid. Therefore, we consider MSA with fuzzy numbers to have a more precise and accurate measurement system and data analysis. This precise data analysis will lead to a more accurate decision making and quality system. Experimental results of a case study will represent performance of this new type of MSA in an industrial real world instance and show how MSA executed with fuzzy calculations in detail. Meanwhile, first some mathematical operations in fuzzy concept is reviewed, then they are expanded on different indices in MSA in the rest of this section.

Fuzzy calculations are handled with fuzzy numbers. A fuzzy number is a convex fuzzy subset of the real line *R* and is completely defined by its membership function. Different type of membership functions are considered in the literature of fuzzy numbers. This paper uses triangular membership function [13] for different indices of MSA and shows detailed calculations of MSA in the environment of fuzzy concept. The rest of the section illustrates all of these calculations.

Denoting the triangular fuzzy number $\widetilde{M}$ by a triplet (a, b, c) and $\widetilde{N}$ by a triplet (d, e, f) [26], the addition, subtraction, multiple, and division of the two triangular fuzzy numbers can be shown as Eq.(9) to Eq.(12) respectively.

$$\tilde{M}+\tilde{N}=(a,b,c)+(d,e,f)=(a+d,b+e,c+f)$$
$$\tilde{M}-\tilde{N}=(a,b,c)-(d,e,f)=(a-f,b-e,c-d)$$
$$\tilde{M}\times\tilde{N}=(a,b,c)\times(d,e,f)=(a\times d,b\times e,c\times f)$$
$$\tilde{M}\div\tilde{N}=(a,b,c)\div(d,e,f)=(\frac{a}{f},\frac{b}{e},\frac{c}{d}) \qquad (9)$$

Our method for making a fuzzy triangular number is defined in Eq.(10) [13].

$$\tilde{X}_b=(\tilde{X}_b\sigma_1,\tilde{X}_b,\tilde{X}_b\sigma_2); \qquad \sigma_1=0.999,\sigma_2=1.001 \qquad (10)$$

The corresponding shape of the fuzzy number is plotted as Fig.5.

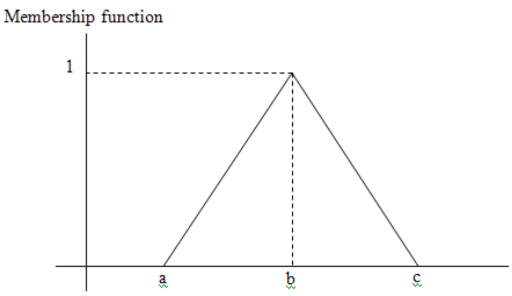

Fig. 5. Membership function of *b* fuzzy number

It should be mentioned that in this study for ranking two fuzzy numbers like (a,b,c) and (d,e,f) the third member of each set is considered as the criteria of comparison. For instance among fuzzy M and N (defined at Eq.(14)) , since $\max(11,12)=12$, fuzzy M is selected.

$$\widetilde{N}=(6,8,11)\ \&\ \widetilde{M}=(5,9,12) \qquad (11)$$

The indices of fuzzy measurement system are presented below. Equations (12)-(16) are fuzzy indices of (3), (6), (7), and (8), respectively. For the sake of completeness, we have provided the complete set of equations.

$$\begin{aligned}\tilde{B}&=\tilde{\bar{x}}_g-x_m\\&=(\tilde{\bar{x}}_a-x_m,\tilde{\bar{x}}_b-x_m,\tilde{\bar{x}}_c-x_m)\\&=(\tilde{B}_a,\tilde{B}_b,\tilde{B}_c)\end{aligned} \qquad (12)$$

$$\begin{aligned}E\tilde{V}&=5.15\frac{\tilde{\bar{R}}}{d_2^*}\\&=5.15\frac{(\tilde{\bar{R}}_a,\tilde{\bar{R}}_b,\tilde{\bar{R}}_c)}{d_2^*}\\&=(\frac{5.15}{d_2^*}\tilde{\bar{R}}_a,\frac{5.15}{d_2^*}\tilde{\bar{R}}_b,\frac{5.15}{d_2^*}\tilde{\bar{R}}_c)\\&=(E\tilde{V}_a,E\tilde{V}_b,E\tilde{V}_c)\end{aligned} \qquad (13)$$

Then, according to the corresponding ranking approach the results are as follows. We suppose that i=1 is the largest member of a fuzzy number and i=2 is the smallest member of a fuzzy number.

$$\begin{aligned}\tilde{\bar{X}}_{DIFF}&=max(\tilde{\bar{X}}_i)-min(\tilde{\bar{X}}_i)\\&=max(\tilde{\bar{X}}_i^{\,a},\tilde{\bar{X}}_i^{\,b},\tilde{\bar{X}}_i^{\,c})-min(\tilde{\bar{X}}_i^{\,a},\tilde{\bar{X}}_i^{\,b},\tilde{\bar{X}}_i^{\,c})\\&=(\tilde{\bar{X}}_1^{\,a},\tilde{\bar{X}}_1^{\,b},\tilde{\bar{X}}_1^{\,c})-(\tilde{\bar{X}}_2^{\,a},\tilde{\bar{X}}_2^{\,b},\tilde{\bar{X}}_2^{\,c})\\&=(\tilde{\bar{X}}_1^{\,a}-\tilde{\bar{X}}_2^{\,c},\tilde{\bar{X}}_1^{\,b}-\tilde{\bar{X}}_2^{\,b},\tilde{\bar{X}}_1^{\,c}-\tilde{\bar{X}}_2^{\,a})\\&=(\tilde{\bar{X}}_{DIFF}^{\,a},\tilde{\bar{X}}_{DIFF}^{\,b},\tilde{\bar{X}}_{DIFF}^{\,c})\end{aligned} \qquad (14)$$

$$\begin{aligned}A\tilde{V}&=\sqrt{(5.15\frac{\bar{X}_{DIF}}{d_2^*})^2-\frac{(EV)^2}{n.r}}\\&=\sqrt{(5.15\frac{(\tilde{\bar{X}}_{DIFF}^{\,a},\tilde{\bar{X}}_{DIFF}^{\,b},\tilde{\bar{X}}_{DIFF}^{\,c})}{d_2^*})^2-\frac{(E\tilde{V}_a,E\tilde{V}_b,E\tilde{V}_c)^2}{n.r}}\\&=(\sqrt{\frac{5.15}{d_2^*}\tilde{\bar{X}}^2_{DIFF}{}^{a}-\frac{5.15}{n.r.d_2^*}\tilde{\bar{R}}^2},\\&\quad\sqrt{\frac{5.15}{d_2^*}\tilde{\bar{X}}^2_{DIFF}{}^{b}-\frac{5.15}{n.r.d_2^*}\tilde{\bar{R}}_b^{\,2}},\\&\quad\sqrt{\frac{5.15}{d_2^*}\tilde{\bar{X}}^2_{DIFF}{}^{c}-\frac{5.15}{n.r.d_2^*}\tilde{\bar{R}}_a^{\,2}})\\&=(A\tilde{V}_a,A\tilde{V}_b,A\tilde{V}_c)\end{aligned} \qquad (15)$$

Finally, fuzzy GR&R is defined as (18). All of the membership functions of these indices are presented in next section in a case study.

$$\begin{aligned}\tilde{R}\,\&\,\tilde{R}&=\sqrt{E\tilde{V}^2+A\tilde{V}^2}\\&=\sqrt{(E\tilde{V}_a,E\tilde{V}_b,E\tilde{V})^2+(A\tilde{V}_a,A\tilde{V}_b,A\tilde{V}_c)}\\&=(\tilde{R}\,\&\,\tilde{R}^a,\tilde{R}\,\&\,\tilde{R}^b,\tilde{R}\,\&\,\tilde{R}^c)\end{aligned} \qquad (16)$$

Table 2
Case study data

Part Number	Operator 1 Measurements (*mm*)		Operator 2 Measurements (*mm*)		Operator 3 Measurements (*mm*)	
	M1	M2	M1	M2	M1	M2
1	(62.14,62.2,62.26)	(62.1,62.16,62.22)	(62.09,62.15,62.21)	(62.08,62.14,62.2)	(62.09,62.15,62.21)	(62.1,62.16,62.22)
2	(62.13,62.19,62.25)	(62.13,62.19,62.25)	(62.13,62.19,62.25)	(62.13,62.19,62.25)	(62.13,62.19,62.25)	(62.14,62.2,62.26)
3	(62.05,62.11,62.17)	(62.06,62.12,62.18)	(62.05,62.11,62.17)	(62.04,62.10,62.16)	(62.04,62.1,62.16)	(62.05,62.11,62.17)
4	(62.11,62.17,62.23)	(62.11,62.17,62.23)	(62.11,62.17,62.23)	(62.11,62.17,62.23)	(62.11,62.17,62.23)	(62.11,62.17,62.23)
5	(62.19,62.25,62.31)	(62.19,62.25,62.31)	(62.19,62.25,62.31)	(62.20,62.26,62.32)	(62.19,62.25,62.31)	(62.19,62.25,62.31)
6	(62.06,62.12,62.18)	(62.06,62.12,62.18)	(62.06,62.12,62.18)	(62.06,62.12,62.18)	(62.06,62.12,62.18)	(62.06,62.12,62.18)
7	(62.07,62.13,62.19)	(62.08,62.14,62.2)	(62.08,62.14,62.20)	(62.07,62.13,62.19)	(62.07,62.13,62.19)	(62.07,62.13,62.19)
8	(62.14,62.2,62.26)	(62.14,62.2,62.26)	(62.14,62.2,62.26)	(62.14,62.2,62.26)	(62.14,62.2,62.26)	(62.14,62.2,62.26)
9	(62.24,62.3,62.36)	(62.24,62.3,62.36)	(62.24,62.3,62.36)	(62.23,62.29,62.35)	(62.23,62.29,62.35)	(62.24,62.3,62.36)
10	(62.22,62.28,62.34)	(62.22,62.28,62.34)	(62.22,62.28,62.34)	(62.22,62.28,62.34)	(62.22,62.28,62.34)	(62.21,62.27,62.33)

4. Numerical Example

In this section a case study which investigates housing clutch in automotive parts industry in Kachiran Company in Asia within crisp environment is considered. Then, to extract more accurate indices and make a more reliable decision making we bring the crisp case study into the fuzzy environment and propose FMSA. In our case study, we have 10 parts, 3 operators, 2 trials and the tolerance of corresponding part is 62.2 ± 0.1. Therefore, X_m is 62.2 and T is 0.2. Also, the coefficient for right hand side and left hand side corresponding fuzzy number is 0.001. Table (2) shows case study data and Table (3), (4) represent results of MSA indices with fuzzy number. It should be mentioned that stability and linearity of the data had tested in an exact environment before the data become fuzzy. It means that our non fuzzy data had the both basic features which are stability and linearity. Finally, Fig. 6 to Fig. 10 plots the membership function of MSA indices.

Table 3
Results within fuzzy environment

Index	Operator 1	Operator 2	Operator 3
$\tilde{\tilde{X}}_1$	(62.13,62.19,62.26)	(62.13,62.19,62.25)	(62.13,62.19,62.25)
$\tilde{B}_1$	(-0.07,-0.01,0.06)	(-0.07,-0.01,0.05)	(-0.07,-0.01,0.05)
$\tilde{\tilde{R}}_1$	(-0.12,0.01,0.13)	(-0.12,0,0.13)	(-0.12,0.01,0.13)

As it can be seen in table 2, for each operator, integer parts of all observations are the same and decimal part of the data has little variation. This achievement shows that the repeatability of the gauges is appropriate. On the other hand, according to the table 3, results of the different operators do not have any differences, that shows reproductively of the measurement system is in a high level. Therefore, having appropriate reproductively and repeatability causes a good GR&R. Table 4 proves our mentioned expectations are right. Our graphical outputs in

Table 4
MSA indices with fuzzy numbers

Indices	Fuzzy Number
$\tilde{\tilde{R}}$	(-0.12,0.01,0.13)
$E\tilde{V}$	(-0.53,0.03,0.6)
$\tilde{\tilde{X}}_{DIFF}$	(-0.13,0,0.12)
$A\tilde{V}$	(0,0.01,0.29)
$\tilde{R}$ & $\tilde{R}$	(0,0.04,0.67)

Fig.6 to Fig.10 illustrates indices schematically. Through this figures or fuzzy numbers, vagueness of the indices is modeled. Figure 10 as an example models and removes vagueness of the GR&R criterion. This figure not only includes crisp GR&R in the point 0.04, but also is consisted of any other number around this value that is in the doubtful area. Therefore, considering MSA indices as fuzzy numbers causes to improve quality of measurement system and corresponding decision will make with more accurate information.

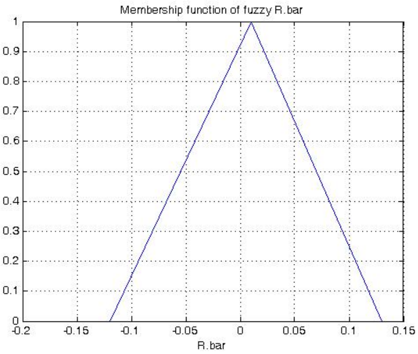

Fig.6. Membership function of $\tilde{\tilde{R}}$

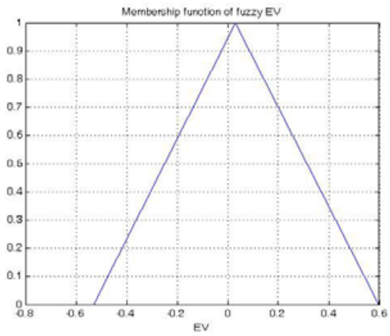

Fig.7. Membership function of $E\tilde{V}$

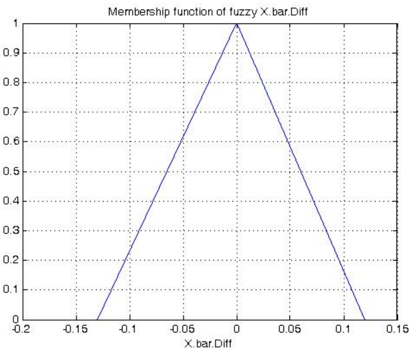

Fig.8. Membership function of $\tilde{\tilde{X}}_{DIFF}$

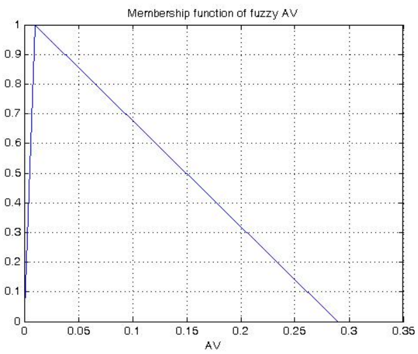

Fig.9. Membership function of $A\tilde{V}$

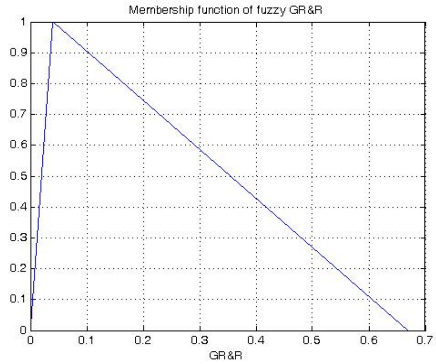

Fig.10. Membership function of $\tilde{R}$ & $\tilde{R}$

5. Conclusion and Future Works

MSA is a collection of statistical methods for analyzing measurement systems. Since most practical systems are encountered with imprecise data, considering fuzzy concept help us to make MSA much more accurate. It means that, the underlying data are usually assumed to be precise numbers, but it is much more realistic to consider them through fuzzy values. However, although FMSA can be a considerable issue, in the literature any corresponding paper has not been found. Thus, in this paper MSA was modeled in fuzzy environment and triangular fuzzy numbers were proposed for MSA indices. The output of fuzzy indices were illustrated through different tables and figures and shown that all the crisp concepts and definitions of MSA can be developed in fuzzy environment.

Finally, a real-world example taken from a housing clutch manufacturing process was investigated to explain efficient performance of FMSA more explicitly. Interpretations of the FMSA were also done according to tables just like interpretation of the crisp MSA to explain relationship among these two areas. For future works, following options can be suggested:

- Using other types of membership functions that can leads to better results.
- Expanding this method for qualitative measurement data
- Considering some variable together in the problem and developing a fuzzy control system with various rules
- Developing a new ranking approach other than the ranking approach of this paper
- Considering other aspects of MSA such as fuzzy stability or fuzzy linearity

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