Dynamic Load Carrying Capacity of Flexible Manipulators Using Finite Element Method and Pontryagin's Minimum Principle

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Abstract

In this paper, finding Dynamic Load Carrying Capacity (DLCC) of flexible link manipulators in point to-point motion was formulated as an optimal control problem. The finite element method was employed for modelling and deriving the dynamic equations of the system. The study employed indirect solution of optimal control for system motion planning. Due to offline nature of the method, many difficulties such system nonlinearities and all types of constraints can be catered for and implemented easily. The application of Pontryagin's minimum principle to this problem was resulted in a standard two-point boundary value problem (TPBVP), solved numerically. Then, the formulation was developed to find the maximum payload and corresponding optimal path. The main advantage of the proposed method is that the various optimal trajectories can be obtained with different characteristics and different maximum payloads. Therefore, the designer can select a suitable path among the numerous optimal paths. In order to verify the effectiveness of the method, a simulation study considering a two-link flexible manipulator was presented in details.

Keywords: Flexible Manipulator; Finite Element Method; Pontryagin Minimum Principle.

1. Introduction

Most industrial robots in use today are composed of heavy and stiff links to satisfy the required repeatability and accuracy. These links, therefore, have inherently a large inertia, requiring in turn a long time to complete the motion and more power consumption in the actuators. To increase the productivity by fast motion and to complete a motion with small energy consumption, industrial robot manipulators are required to have light weight and flexible structures. On the other hand, the manipulators are typically used to repeat a prescribed task a large number of times, so even small improvements in their performance may result in large monetary saving. Sensitivity analysis of the geometric parameters such as length, thickness and width on the maximum deflection of the end effector and vibration energy of a single link flexible manipulator was investigated by Korayem et al. (2012).

Finding the full load motion for a point-to-point task can maximize the productivity and economic usage of the manipulators. Thomas et al. (1985) used the load capacity as a criterion for sizing the actuators of robotic manipulators at the design stage. In their study, they considered the maximum load in the neighbourhood of a robot configuration. The first formulation to obtain the maximum payload of a manipulator in point to point motion was presented by Wang and Ravani (1988). They used the iterative linear programming (ILP) method to solve the problem. Wang et al. (2001) solved the optimal control problem with the direct method in order to determine the maximum payload of a rigid manipulator. The basic idea of this work is to parameterize the joint trajectories by using B-spline functions and tuning the parameters in a nonlinear optimization until a local minimum that satisfies the constraints achieved. This method leaks from limiting the solution to a fixed-order polynomial as well as complexity issues arose in differentiating torques with respect to joint parameters and payload due to their constraints and discontinuity. Korayem and Ghariblu (2004) were presented a computational algorithm for maximum load determination

via linearizing the dynamic equation and constraints on the basis of Iterative Linear Programming (ILP) approach for flexible mobile manipulators. But because of some ILP approach's difficulties, in their work the link flexibility has not been considered either in the dynamic equation or simulation procedure.

In contrast, the optimal control method is known as an appropriate method in the cases where the system has a large number of degree of freedom, especially when nonlinear terms are large and fluctuating, e.g., in problems with consideration of flexibility in joints or links, gravity acceleration or having high speed motion. Furthermore, optimization of the various objectives is targeted by means of this approach. On the other hand, because of the nature of the optimal control problem, many difficulties like system nonlinearities and all types of constraints may be catered for and implemented easily. Thus, this method was widely used as a powerful and efficient tool in analysing the nonlinear system, such as path planning of the different types of rigid and mobile manipulators (Bertolazzi et al. (2005), Bessonnet and Chesse (2005), Callies and Rentrop (2008) and Korayem et al. (2011)).

On the other hand, in case of dynamic modelling flexible manipulators, the finite element method (FEM) has been used to solve very complex structural engineering problems during the past years. One of the main advantages of FEM over the most of other approximate solution methods is the fact that FEM can handle irregular geometries routinely. Another significant advantage of FEM, especially over analytical solution techniques is the ease with which nonlinear conditions can be handled. A comprehensive comparison of FEM with other available methods for dynamic robot analysing is addressed by Korayem and Rahimi (2011). In finite element modelling of dynamical manipulators, the elastic deformations are analysed by assuming a known rigid body motion and later superposing the elastic deformation with the rigid body motion (Usoro et al. (1986)). Dogan and Istefanopulos (2007) developed finite element models to describe the deflection of a planar two-link flexible robot manipulator. Zhang et al. (2004) proposed dynamic equation of planar cooperative manipulators with link flexibility in the absolute coordinates with the Timoshenko beam theory and the finite element method. Rashidifar et al. (2012) used finite element method to model a single link flexible manipulator by dividing the system into 10 elements. Then, they presented optimization of input shaping technique for vibration control of the system using genetic algorithms. Zebin (2012) presented theoretical investigation into the dynamic modelling and characterization of a constrained two-link flexible manipulator using finite element method. Then, the final derived model of the system was simulated to investigate the behaviour of the system. Mohamed and Tokhi (2004) derived the dynamic model of a single-link flexible manipulator using FEM and then studied the feed-forward control strategies for controlling the vibration. Yue et al.

(2001) used the finite element method for describing the dynamics of the system and computed the maximum payload of kinematically redundant flexible manipulators. Finally, they numerically simulated a planar flexible robot manipulator to validate their research work.

Nowadays the advantages of optimal control theory are well established and a host of issues related to this technique have been studying specially in the field of optimal motion planning of robots (Briot et al. (2012), Korayem (2013) and Bjorkenstam et al. (2013)). Rahimi et al. (2009) proposed indirect solution of open-loop optimal control method to trajectory optimization of flexible link/joint manipulator in the point-to-point motion. In the mentioned work, despite ILP based studies the complete form of the obtained nonlinear equation was used. Thus, unlike the previous ILP based works to solve the problem linearizing equations was not required. However, the paper employed assumed modes method to derive the robot dynamic moreover; fining the full load was not considered in this research study.

The main objective of the presenting paper is to provide a nonlinear dynamic modelling and optimal control of flexible manipulators in order to determine the dynamic load carrying capacity of such robots. The paper firstly deals with the nonlinear modelling of the general flexible links robot manipulators. Then, the optimal control problem that with employing of Pontryagin's minimum principle supports the execution of the optimization solution of model is expressed as a brief review; subsequently, an application example with the two-like flexible manipulator is presented and discussed to demonstrate the effective performance of the proposed approach. Lastly, the paper is concluded with highlighting the feature properties of the proposed model.

2. Dynamic Modeling

The finite element method is broadly used to derive dynamic equations of elastic robotic arms. Researcher usually used the Euler–Bernoulli beam element with multiple nodes and Lagrange shape function to achieve the reasonable finite element model. The node number can be selected according to requirement on precision. But, increasing the node number may enlarge the stiffness matrix and it cause to long and complex equations. Hence, choosing the proper node number is very important in the finite element analysing.

The overall finite element approach involves treating each link of the manipulator as an assemblage of n elements of length l_i. Consider link i to be divided into elements *'i1', 'i2', . . . , 'ij', . . . 'in_i'* of equal length, l_i, where n_i is the number of elements of the ith link. Let us define the following notation, where subscript ij refer to the jth element of link i. *OXY* is the inertia system of coordinates, $O_i X_i Y_i$ is the body-fixed system of coordinates attached

to link i. $u_{i,2j-1}$ is the flexural displacement at the common junction of elements *'i(j - 1)'* and *'ij'* of link *i*. $u_{i,2j}$ is the flexural slope at the tip of the common junction of elements *'i(j - 1)'* and *'ij'* of link *i*. This slope is measured with respect to axis $O_i X_i$. For each element the kinetic energy , T_{ij}, and potential energy, V_{ij}, are computed in terms of a selected system of generalized coordinate , q, and their rate of change with respect to time, $\dot{q}$. It is convenient to define r_i as the position vector of link i in the inertia reference frame in terms of the position of each point in the body-fixed coordinate system:

$$\vec{r}_1 = T_0^1 \begin{bmatrix} (j-1)l_1 + x_{1j} \\ y_{1j} \end{bmatrix},$$

$$\vec{r}_i = \left(T_0^1 \begin{bmatrix} L_1 \\ u_{2n_1+1} \end{bmatrix} + \cdots + T_0^1 T_1^2 \cdots T_{i-2}^{i-1} \begin{bmatrix} L_i \\ u_{2n_i+1} \end{bmatrix} \right) \\ + T_0^1 T_1^2 \cdots T_{i-1}^i \begin{bmatrix} (j-1)l_i + x_{2j} \\ y_{2j} \end{bmatrix} \tag{1}$$

for i = 2,3, $\cdots$ NL (Number of Links),

where T_{i-1}^i is the transformation matrix from $O_i X_i Y_i$ to its previous body-fixed coordinate system. It is obvious that $O_0 X_0 Y_0 = OXY$ is the inertia system of coordinates.

$$T_0^1 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) \\ \sin(\theta_1) & \cos(\theta_1) \end{bmatrix}$$

$$T_0^1 = \begin{bmatrix} \cos(\theta_i + u_{2n_{i-1}+2}) & -\sin(\theta_i + u_{2n_{i-1}+2}) \\ \sin(\theta_i + u_{2n_{i-1}+2}) & \cos(\theta_i + u_{2n_{i-1}+2}) \end{bmatrix} \tag{2}$$

for i = 2, 3, $\cdots$ NL,

where x_{ij} is the distance along $O_i X_i$ in a body-fixed coordinate system from node (*j-1*), l_i is the length of the elements in the ith link and θ_i is the joint angle between link i and *i-1*. Finally, y_{ij} is defined as the element displacement and expresses the deformation of each link due to its original shape:

$$y_{ij} = \sum_{k=1}^{4} \phi_k(x_{ij}) u_{i,2j-2+k}(t), \tag{3}$$

where u is flexural displacement at the common junction of elements *'i(j-1)'* and *'ij'* of link *i*. ϕ_k is the shape functions (Hermitian functions) of a beam element and obtain as:

$$\phi_1(x) = 1 - 3\left(\frac{x}{l}\right)^2 + 2\left(\frac{x}{l}\right)^3,$$

$$\phi_2(x) = x\left[1 - 2\left(\frac{x}{l}\right) + \left(\frac{x}{l}\right)^2\right],$$

$$\phi_3(x) = 3\left(\frac{x}{l}\right)^2 - 2\left(\frac{x}{l}\right)^3, \tag{4}$$

$$\phi_4(x) = x\left[-\left(\frac{x}{l}\right) + \left(\frac{x}{l}\right)^2\right].$$

Consequently, kinetic energy, T_{ij}, and potential energy, V_{ij}, for the jth element of link i can be computed by the following equation:

$$T_{ij} = \frac{1}{2} \int_0^{l_i} m_i \left[\frac{\partial r_i^T}{\partial t} \cdot \frac{\partial r_i}{\partial t} \right] dx_{ij} \tag{5}$$

And

$$V_{ij} = V_{gij} + V_{eij} =$$

$$\int_0^{l_i} m_i g [0 \quad 1] T_0^1 \begin{bmatrix} (j-1)l_i + x_{ij} \\ y_{ij} \end{bmatrix} dx_{ij} + \frac{1}{2} \int_0^{l_i} EI_i \left[\frac{\partial^2 y_{ij}}{\partial x_{ij}^2} \right]^2 dx_{ij} \tag{6}$$

In above equation, the potential energy is consisted of two parts. One part is due to gravity (V_{gij}) and another is related to elasticity of links (V_{eij}). m_i and EI_i are the mass and the flexural rigidity of ith element, respectively. After that, for each of these elements the kinetic energy T_{ij} and potential energy V_{ij} are computed in terms of a selected system of n generalized variables $q = (q_{1,} \quad q_{2,} \quad ..., \quad q_n)$ and their rate of change $\dot{q}$. These energies are then combined to obtain the total kinetic energy, T, and potential energy, V, for the entire system. Finally, using Lagrange equations the equations can be written in compact form as:

$$M(q)\ddot{q} + C(q,\dot{q}) + G(q) = U, \tag{7}$$

where M is the inertia matrix, C is the vector of Coriolis and centrifugal forces, G describes the gravity effects and U is the generalized force inserted into the actuator.

3. Formulation of the Optimal Control Problem

3.1. Statement of the optimal control problem

By defining $X = \{X_1, X_2\}^T \equiv \{q, \dot{q}\}^T$, Eq.(7) can be rewrite in state space form as:

$$\dot{X}(t) = f(X(t), U(t)) \tag{8}$$

For the maximum payload determination problem the state departing from the initial conditions $x(t_0) = x_0$ must reach the final conditions $x(t_f) = x_f$ during the overall time t_f in such a way that the maximum payload can be carried. Generating optimal movements can be achieved by minimizing a variety of quantities involving directly or not some dynamic capacities of the mechanical system as

$$J(u) = \int_{t_0}^{t_f} L(X(t), U(t))dt \qquad (9)$$

where the Lagrangian L may be specified in quite varied manners. In the presenting paper, the attention is restricting to define the performance measure as:

$$J_0(T) = 0.5\int_{t_0}^{t_f}(T^T R T + X_2^T W X_2)dt \qquad (10)$$

In the presented study, the objective function is defined as a function of the actuators velocities and torques to minimize energy consumption of the system Liu and Dong (2012). The objective function expressed by (10) is minimized over the entire duration of the motion. The first term in (10) is presented to minimize the total torque consumption of the system. The second and third terms are minimized the overall state variables during the motion. In the above equation, $X_2^T W X_2$ is the generalized squared norm of the state vector with respect to a symmetric, semi-definite weighting matrix W, $T^T R T$ is the generalized squared norm of the input vector regarding to a symmetric, definite weighting matrix R. This can combine, for instance, energy consumption, actuating torques, travelling time or bounding the velocity magnitude or maximum payload. By defining $\overline{U}$ as a set of admissible control torque over the time interval the imposed bound of torque for each motor can be expressed as:

$$\overline{U} = \{U^- \le U \le U^+\} \qquad (11)$$

If $\overline{U}$ be a set of admissible control torque over the time interval $t \in [t_0, t_f]$, for a specified payload, the optimal control problem is to obtain the $U^*(t) \in \overline{U}$ in such a manner that the objective criterion in Eq. (9) is minimized subject to the motion equations, boundary values and torque constraints.

3.2. Necessary condition for optimality

Now as the formulation of the optimal control problem has been completed, the solution of optimal problem should be formulated. In the presenting paper, an indirect solution of the optimal control is employed to solve the path planning problem. This technique provides an excellent tool to calculate optimal trajectory with high accuracy for robots that include, in this case, flexible arms. This method can overcome the high nonlinearity

nature of the optimization problem in spite of using complete nonlinear states. Accordingly, the method is a good candidate for the cases where the system has a large number of degrees of freedoms or high nonlinearities such as the flexible manipulators. By implementing Pontryagin's minimum principle for solving optimization problems the necessary conditions for optimality are obtained as stated on the basis of variational calculus. Defining the Hamiltonian function as:

$$H^*(X, U, Y, m_p, t) = Y^T f(X, U, t) + L(X, U, m_p, t) \qquad (12)$$

in addition to costate time vector-function $Y(t)$ that verifying the costate vector-equation (or adjoint system)

$$\dot{Y}^T = -\partial H^*/\partial X \qquad (13)$$

and the minimality condition for the Hamiltonian as:

$$\begin{cases} \partial H^*/\partial U = 0 , \\ \dot{X} = \partial H^*/\partial Y \end{cases} \qquad (14)$$

leads to transform the problem of optimal control into a non-linear multi-point boundary value problem, that there exist some numerical techniques for solving such problems.

Hence, the important task is to be achieving the explicit formulation of conditions (8), (13) and (14). Noticeably, these calculations need to compute the Jacobian matrices that require handling huge amounts of arithmetic operations when coping with complex dynamical systems. The fulfilment of such requirements with remaining all nonlinear state and control constraints is the main advantage of the presenting research study. There exist some numerical techniques for solving such problems, a number of which have been reported in associated literature such as those by Kirck (2009).

4. Simulation for a Two Link Flexible Manipulator

In this section, a flexible two link manipulator with the concentrated payload of mass m_p connected to the second link as depicted in Fig. 1 is considered to simulation.
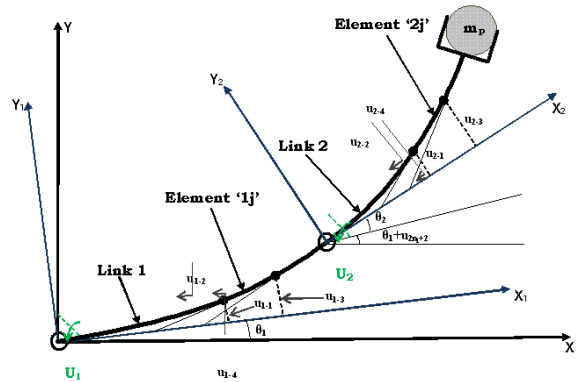

Fig. 1. A two-link manipulator with flexible links.

The physical parameters of the model used in these simulation studies were $E_1 I_1 = E_2 I_2 = 100 \, kg. \, m^6$, $L_1 = L_2 = 1 \, m$ and $m_1 = m_2 = 5 \, kg$.

By defining the state vectors as follows:

$$X_1 = Q^T = \begin{bmatrix} x_1 & x_3 & x_5 & x_7 & x_9 & x_{11} \end{bmatrix}^T, \qquad X_2 = \dot{Q}^T = \begin{bmatrix} x_2 & x_4 & x_6 & x_8 & x_{10} & x_{12} \end{bmatrix}^T. \tag{15}$$

The sate space equation of the system can be written as:

$$\dot{x}_{2i-1} = x_{2i} \,, \; \dot{x}_{2i} = F_2(i) \; ; i = 1 \ldots 6 \,, \tag{16}$$

where $F_2(i)$ can be obtained from Eq. (7). And the boundary condition can be expressed as:

$$\begin{aligned} &x_1(0) = \pi/2 \, \text{rad} \,, \; x_3(0) = 2 \times \pi/3 \, \text{rad} \\ &x_1(t_f) = \pi/6 \, \text{rad}, \qquad x_3(t_f) = \pi/3 \, \text{rad} \\ &x_{2i}(0) = x_{2i}(t_f) = 0, \quad i = 1 \ldots 6 \\ &x_5(0) = x_5(t_f) = x_7(0) = x_7(t_f) = 0 \end{aligned} \tag{17}$$

In order to derive the equations associated with optimality conditions, penalty matrices can be selected as:

$$\begin{aligned} &W = diag(w_1, w_2, w_3, w_4, w_5, w_6); \\ &R = diag(r_1, r_2). \end{aligned} \tag{18}$$

So the objective function is obtained by substituting Eq. (18) Into Eq. (10) as below

$$L = \frac{1}{2} \left(r_1 u_1^2 + r_2 u_2^2 + \sum_{i=1}^{6} w_i x_{2i}^2 \right) \tag{19}$$

Then, by considering the costate vector as $Y = \begin{bmatrix} y_1 & y_2 & \ldots & y_{12} \end{bmatrix}$, the Hamiltonian function can be expressed from as:

$$H = \frac{1}{2} \left(r_1 \tau_1^2 + r_2 \tau_2^2 + \sum_{i=1}^{6} w_i x_{2i}^2 \right) + \sum_{i=1}^{12} y_i \dot{x}_i \,, \tag{20}$$

where $\dot{x}_i, i = 1, \ldots, 12$ can be substituted from Eq. (16). Using Eq. (13) differentiating the Hamiltonian function with respect to the states, result in costate equations as follows:

$$\dot{y}_i = -\frac{\partial H}{\partial x_i} \,, \; i = 1, \cdots, 12 \tag{21}$$

The control function in the admissible interval can be computed using Eq. (14), by differentiating the Hamiltonian function with respect to the torques and setting the derivative equal to zero. Then, by applying motors torque limitation, the optimal control becomes:

$$PU_i = \begin{cases} U_i^+ & U_i > U^+ \\ U_i & otherwise \; ; i = 1, 2 \\ U_i^- & U_i < U^- \end{cases} \tag{22}$$

The actuators which are used for medium and small size manipulators are the permanent magnet D.C. motor. The torque speed characteristic of such D.C. motors may be represented by the following linear equation:

$$\begin{aligned} &u_1^+ = \tau_1 - S_1 \, x_{11} \; ; \; u_1^- = -\tau_1 - S_1 \, x_{11} \\ &u_2^+ = \tau_2 - S_2 \, x_{12} \; ; \; u_2^- = -\tau_2 - S_2 \, x_{12} \end{aligned} \tag{23}$$

where $S_i = (\tau_i / \omega_{mi})$, τ_i and ω_{mi} are the stall torque and maximum no-load speed of ith motor, respectively. In the presenting study, these parameters are considered as $w_{mi} = 3.5 \, \text{rad/s}$ and $\tau_{si} = 30 \, \text{N.m}$.

Finally, 24 nonlinear ordinary differential equations are obtained by substituting Eq.(23) into Eqs. (21) and (16), which with 24 boundary conditions given in Eq. (17) construct a two point boundary value problem (TPBVP). Nowadays there are numerous influential and efficient commands for solving such problems that are available in different software such as MATLAB®, MATEMATHICA or FORTRAN. These commands by employing competent methods such as shooting, collocation, and finite difference solve the problem. In this study, BVP4C command in MATLAB® which is based on the collocation method is used to solve the obtained problem. The details of this numerical technique are given in Shampine et al. (2000).

5. Results and Discussion

In this simulation, finding the maximum payload value carried between the initial and final point, during the overall time $t_f = 1.5$ second is presented. Using the obtained equations at section 2 and on the basis of the presented control method in section 3, the robot path planning problem is investigated by increasing the payload mass until the maximum allowable load is determined. The penalty matrices are considered to be $W =$ (20, 20, 1, 1, 1, 1) and $R =$ diag(0.1). The maximum payload for these values of penalty matrices is found to be 1 kg. The obtained angular velocities and torque curves graphs for a range of m_p are shown in Fig.s 2 and 3. It can be found that, increasing the m_p results in enlarging the velocity values. Also, as shown in the figures, increasing the payload increases the required torque until the maximum payload, so that for the last case the torque curves lay on their limits. Hence, it is the most possible values of the torques and increasing the payload that can lead to violating the boundary conditions.
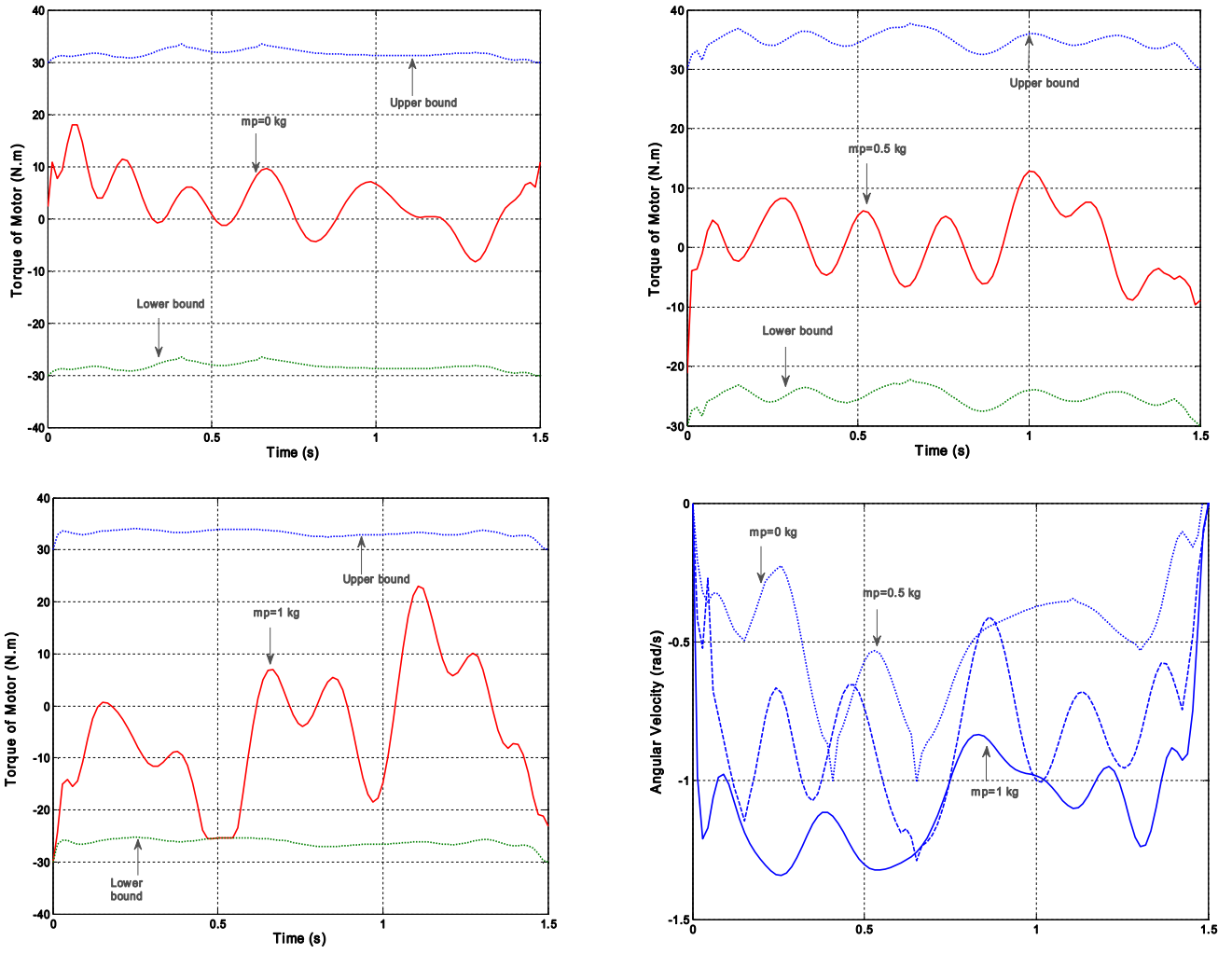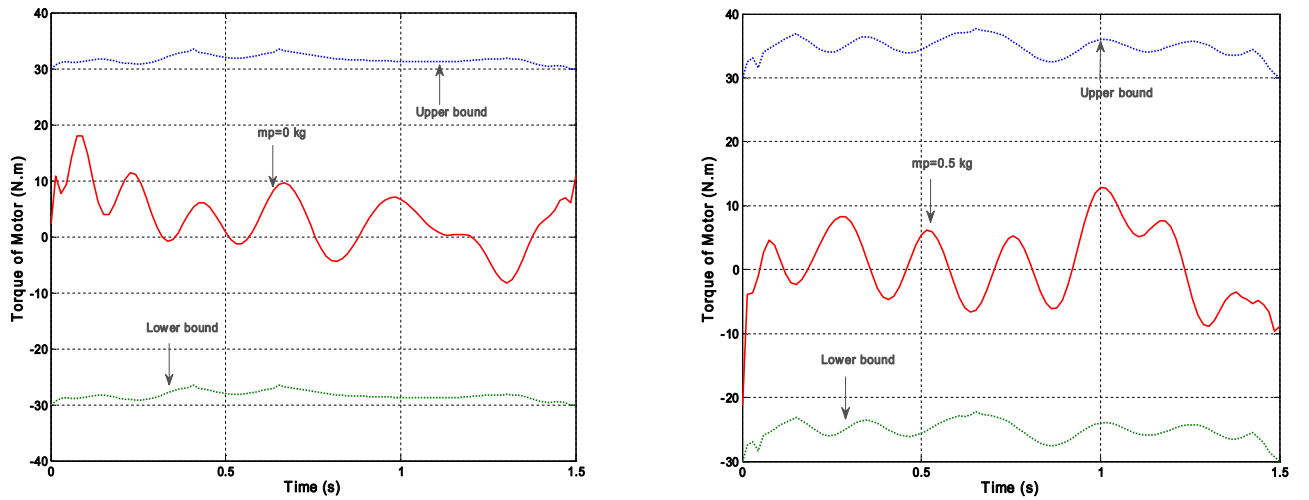
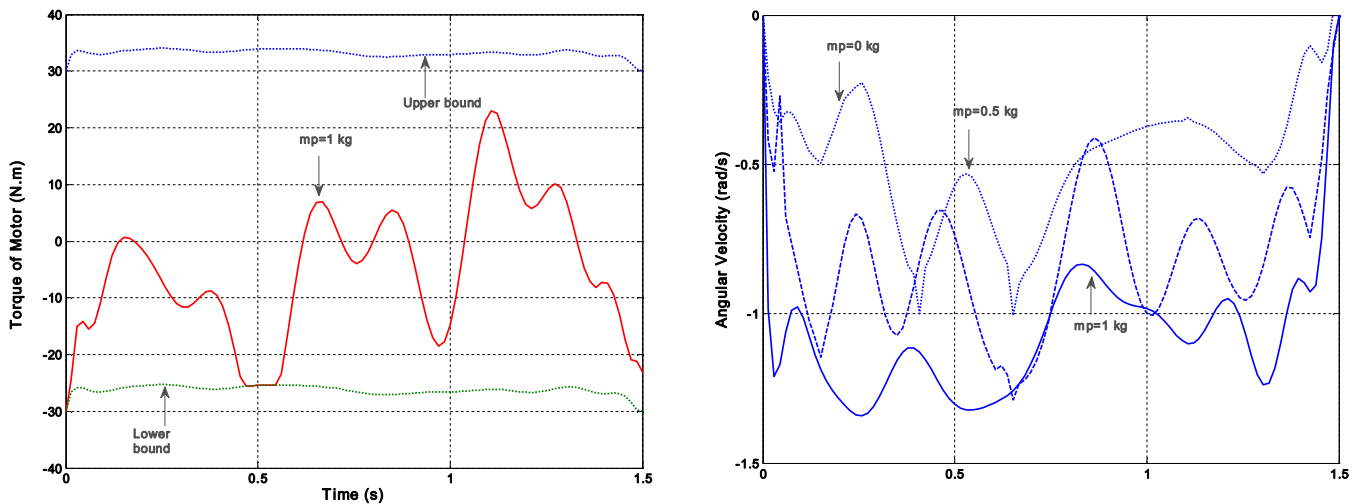Fig. 2- Angular velocities and torques of motors – First joint

Fig. 3. Angular velocities and torques of motors – Second joint

6. Conclusion

Full load motion planning of flexible manipulators for a given two-end-point task in point-to-point motion, based on indirect solution of optimal control problem has been addressed in this paper. We employed the finite element method to model and derive the nonlinear dynamic equations of flexible manipulator. It was found that in the presence of nonlinear and highly fluctuated terms in dynamic equations, open loop optimal control approach is a superior candidate for generating the full load motion path. The Hamiltonian function has been formed and the necessary conditions for optimality have been derived based on Pontryagin's minimum principle. The obtained equations established a two point boundary value problem which was solved by numerical techniques. Finally, simulations for a two-link planar manipulator with flexible links were carried out and the efficiency of the presented method was illustrated. The obtained results demonstrate the power and efficiency of the method to overcome the high nonlinearity nature of the optimization problem which with other methods may be very difficult or impossible. The optimal trajectory and corresponding input control obtained using this method can be used as a reference signal and feed forward command in control structure of flexible manipulators.

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