Journal of Optimization in Industrial Engineering
Vol.15, Issue 2, Summer & Autumn 2022, 191- 200
DOI: 10.22094/joie.2022.1891205.1721

Research Article

A Mathematical Programming Model for Single Round-Robin Tournament Problem: a Case Study of Volleyball Nations League

Hamed Jafari [a,*], Morteza Rajabzadeh [b]

[a] *Department of Industrial Engineering,, Golpayegan College of Engineering, Isfahan University of Technology, Golpayegan 87717-67498, Iran*
[b] *Faculty of Engineering, Mahallat Institute of Higher Education, Mahallat, Iran*

Received 18 January 2020; Revised 07 June 2022; Accepted 12 June 2022

Abstract

In this study, a mathematical programming model is developed for a single round-robin tournament problem to provide a schedule for the preliminary round of the Volleyball Nations League. In this setting, the aim is to assign the teams to the pools at each week as well as to specify the host teams of the pools. This schedule is obtained by minimizing the sum of the differences between the total distance traveled by every team and the average of the total distances traveled by all teams. Then, to evaluate the performance of the developed model, it is applied to obtain the optimal schedule for the preliminary round of the Volleyball Men's Nations League in year 2018. The results indicate that the sum of the travel distance deviations from the average of the total travel distances of all teams obtained from the schedule provided by the mathematical model is significantly lower than that calculated from the schedule presented by the International Volleyball Federation. Moreover, the schedule presented by the International Volleyball Federation leads to a percentage gap of 449.92% in comparison with the optimal schedule provided by the developed model.

Keywords: Sport scheduling; Single round-robin tournament; Total travel distance; Volleyball Nations League; Mathematical programming model.

1. Introduction

Nowadays, the sports leagues are considered as a major economic activity around the world (Kostuk and Willoughby 2012). In this point of view, teams' managers do not want to waste their investments because of providing poor schedules of matches (Ichim and Moyano-Fernández 2017). In this setting, the issue of the *sport scheduling* has received considerable attentions in recent years (Ribeiro and Urrutia 2012; Westphal 2014). Scheduling is planning of the activities by achieving to the goals in the available time (Jafari 2021; Behnamian 2020; Jafari and Haleh 2021; Hassani et al. 2021; Jafari 2020a; Yavari et al. 2020; Jafari 2020b). Providing a sports league schedule is a troublous task due to variety of requirements that have to be met (Saur et al. 2012). This research area is considered as an interesting topic in Operations Research (Alarcón et al. 2017; Cocchi et al. 2018; Durán et al. 2007; Durán et al. 2012; Januario and Urrutia 2015; Kendall et al. 2010).
The *traveling tournament problem* is considered as one of the famous problems in the sport scheduling (Bonomo et al. 2012; Trick et al. 2012). In this problem, the aim is to provide a schedule for the home and away matches in the tournament by meeting some feasibility requirements as well as by minimizing the total distance traveled by all teams (Bhattacharyya 2016). There are several studies by investigating the issue of the traveling tournament problem that most of them are addressed as follows:

*Corresponding author Email address: hamed.jafari@iut.ac.ir

Carvalho and Lorena (2012) developed an integer programming model with dynamic constraints for the traveling tournament problem. They used the data benchmarks from a baseball tournament to evaluate the developed model. Guajardo and Jornsten (2017) proposed an integer programming model to provide a stable schedule with respect to the teams' preferences. In each round of this schedule, the teams play against preferable opponents. Urrutia and Ribeiro (2004, 2006) provided a lower bound to the problem by minimizing the number of the breaks as well as the traveled distances. Cheung (2009) and Rasmussen and Trick (2007) proposed a Benders decomposition algorithm to the problem, whereas Irnich (2010) applied a branch-and-price approach to solve the problem. Furthermore, Toffolo et al. (2016) and De Oliveira et al. (2016) presented a branch-and-bound approach for the problem.
Thielen and Westphal (2011) investigated the complexity of the traveling tournament problem. They proved that the problem is NP-Hard when the upper bound of the maximum number of the consecutive away matches is equal to three. Khelifa and Boughaci (2015) proposed a variable neighborhood search algorithm for the problem. In this algorithm, first, a feasible solution is provided for the problem. Then, a search process is applied to find a good solution by minimizing the total traveled distances. Moreover, Lim et al. (2006) used the simulated annealing algorithm to provide a good quality solution to the
A *round-robin* is a tournament in which a team plays against other teams at least once. As a matter of fact, it

contrasts with an elimination tournament (Erzurumluoğlu 2018; Horbach 2010; Knust 2008; Rasmussen 2008).

A *single round-robin tournament* is a well-known format of the sporting events (Briskorn and Drexl 2009; Briskorn and Horbach 2012; Januario et al. 2016). In this problem, the matches are scheduled such that in each period each team plays against other teams exactly once (Briskorn 2009; Briskorn and Knust 2010). Examples with the single round-robin tournament format are the preliminary rounds of the FIFA World Cup and UEFA European Football Championship. Below, some of these studies are presented:

Knust and Von Thaden (2006) developed a two-stage approach for a single round robin tournament problem. At the first stage, the home and away modes are determined for each match, while the matches are scheduled at the second stage. Moreover, Januario and Urrutia (2016) presented a new neighborhood structure for the problem based on the graph theory.

A *double round-robin tournament* is also considered as one of the most common formats of the sports leagues schedules. In this problem, each team plays against all other teams exactly twice: once at home as the host and once at away as the guest (Rasmussen and Trick 2008). Around the world, the football leagues are scheduled based on the double round-robin tournament format, mostly. Several researchers have studied the double round-robin tournament problem. Below, most of these studies are discussed:

Atan and Cavdaroglu (2018) developed two integer programming models and one constraint programming formulation for the double round-robin tournament problem. They evaluated the performance of their models by comparing the obtained schedules with those provided for several instances. Cocchi et al. (2018) proposed a mathematical model to provide an optimal schedule for the Italian Volleyball League. Lewis and Thompson (2011) investigated the double round-robin format by considering it as one of the graph coloring problems. Moreover, Duran et al. (2014) analyzed the problem by maximizing the total revenue generated by all teams participated in the tournament.

Zeng and Mizuno (2012) applied the constraint programming model to present a schedule by minimizing the number of the breaks for the problem. Knust and Lucking (2009) found a feasible schedule for the problem by minimizing the number of the breaks as well as the total costs to hold the matches. Furthermore, Elf et al. (2003) used a branch-and-cut algorithm to solve a break minimization problem.

Miyashiro and Matsui (2005) proposed a polynomial-time algorithm to obtain the equitable home and away assignments for a given timetable of the problem, whereas Briskorn (2008) presented a necessary condition for the problem that i checked in a polynomial time.

Hof et al. (2010) incorporated a fairness constraint into the problem, while Suksompong (2016) considered the issues of the quality and fairness of a tournament. Furthermore, Della Croce and Oliveri (2006) investigated the Italian Football League as one of the double round-robin tournament formats.

The current study investigates the single round-robin tournament problem to provide an efficient schedule for the Volleyball Nations League as a new annual international volleyball tournament.

In the Volleyball Nations League tournament, sixteen teams are qualified to compete together during five weeks. At each week, the teams are divided into four pools of four teams and all teams belonged to each pool compete together on three consecutive days in a single round-robin format.

In the last two decades, the operational research approaches have been increasingly applied to find good solutions for various problems in real world (Jafari 2019; Rasti-Barzoki et al. 2017; Jafari et al. 2020; Jafari 2022). In this study, a novel mathematical programing model is developed to provide an optimal schedule for the matches at the preliminary round of the tournament by minimizing the sum of the differences between the total distance traveled by every team and the average of the total distances traveled by all teams. Moreover, applying the developed model, the teams are assigned to the pools at each week and the host team of each pool is specified.

The remainder of the paper is organized as follows: In Section 2, the considered research problem is descripted. The mathematical model is presented in Section 3. In Section 4, the developed model is evaluated. Furthermore, Section 5 deals with the conclusions.

2. Problem Description

In this section, the detailed descriptions of the considered research problem are provided.

2.1. Problem definition

The Volleyball Nations League is a new annual international volleyball tournament that has been replaced with the former Volleyball World League since 2018. In this tournament, sixteen teams are qualified to compete together. Twelve of them are qualified as the core teams which cannot be relegated from the tournament, whereas other four teams are selected as the challenger teams which can face the relegation at the next year.

At the preliminary round, the sixteen teams compete together in a single round-robin format. The matches are held during five weeks. At each week, the teams are divided into four pools of four teams and all teams belonged to each pool compete together on three consecutive days in a single round-robin format. After the preliminary round, the top five teams join the host of the final round to compete at the final round. At the final round, the six teams are divided into two pools of three teams and the teams belonged to each pool compete together in a single round-robin format. The top two teams of each pool are qualified for the semifinals. In this round, the winner of each pool plays against the runner-up of another pool. The semifinals winners advance to compete at the final. The winner of these two teams is known as the champion of the Volleyball Nations League. Note there is no priority between the core and challenger teams to provide the schedule of the matches. Moreover, the relegation takes into account only among the four challenger teams. The last ranked challenger team will be excluded from the next tournament. And, the winner of the Challenger Cup will be qualified for the next tournament as a challenger team.

2.2. Constraints

Below, the considered constraints are addressed:

- Each pool consists of four teams.
- At each pool, one team is exactly considered to be as the host.
- The host team of each pool should belong to that pool.
- Each team should be considered as the host at least at one pool and at most at two pools during the planning horizon.
- At each week, each team exactly belongs to one of the pools related to that week.
- If two specific teams belong to one specific pool, then they cannot belong to one of the other pools, simultaneously.
- During the planning horizon, each team plays against the other teams exactly once (Obviously, this constraint is redundant regarding the previous constraint).
- A specific team cannot be considered as the host at two consecutive weeks (This constraint has been considered to balance the total travel distance of the teams).

2.3. Objective

In this study, the attempts will be made to provide the schedule of the matches at the preliminary round during the planning horizon in order to minimize the sum of the differences between the total travel distance for every team and the average of the total travel distances for all teams.

Regarding the considered objective, the aim is to assign the teams to the pools at each week at the preliminary round. As a matter of fact, the matches concerning the teams belonged to each pool are held in the host country of that pool. As a result, it is sufficient to assign the teams to the pools and specify the host team of the pools to calculate the total distance traveled by every team during the planning horizon.

3. Mathematical Programming Model

In this section, a mathematical programming model is presented to assign the teams to the pools at the preliminary round and specify the host team of the pools, simultaneously.

3.1. Sets

The sets are defined as follows:

$Pool$ Set of the pools ($Pool = \{1, 2, \dots, 20\}$)

$Pool^1$ Set of the pools related to the first week ($Pool^1 = \{1, 2, 3, 4\}$)

$Pool^2$ Set of the pools related to the second week ($Pool^2 = \{5, 6, 7, 8\}$)

$Pool^3$ Set of the pools related to the third week ($Pool^3 = \{9, 10, 11, 12\}$)

$Pool^4$ Set of the pools related to the fourth week ($Pool^4 = \{13, 14, 15, 16\}$)

$Pool^5$ Set of the pools related to the fifth week ($Pool^5 = \{17, 18, 19, 20\}$)

$Week$ Set of the weeks ($Week = \{1, 2, 3, 4, 5\}$)

$Week^1$ Set of the weeks except the first week ($Week^1 = \{2, 3, 4, 5\}$)

$Team$ Set of the teams ($Team = \{1, 2, \dots, 16\}$)

3.2. Indices

The indices are as follows:

p Index of the pools

w Index of the weeks

t Index of the teams

3.3. Parameters

The considered parameters are denoted as follows:

$D_{tt'}$ Distance between the cities that have been introduced as the host of teams t and t'

3.4. Decision variables

Below, the decision variables are defined:

x_{tp} = 1 if team t belongs to pool p, and = 0 otherwise.

y_{tp} = 1 if team t is considered as the host of pool p, and = 0 otherwise.

$z_{tt'p}$ = 1 if team t belongs to pool p and team t' is the host of this pool, and = 0 otherwise.

d_{tw} Travel distance for team t from week $w-1$ to week w.

dI_t Travel distance for team t from its country to its host country at the first week.

dF_t Travel distance for team t from its host country at the last week to its country.

dT_t Total travel distance for team t during the planning horizon.

dA Average of the total distances traveled by all teams during the planning horizon.

3.5. Mathematical programming model

The mathematical programming model is formulated as follows:

$$\text{Minimize} \sum_{t \in Team} |dT_t - dA| \tag{1}$$

Subject to:

$$\sum_{t \in Team} x_{tp} = 4 \qquad \forall\, p \in Pool \tag{2}$$

$$\sum_{t \in Team} y_{tp} = 1 \qquad \forall\, p \in Pool \tag{3}$$

$$x_{tp} \geq y_{tp} \qquad \forall\, t \in Team,\ \forall\, p \in Pool \tag{4}$$

$$1 \leq \sum_{p \in Pool} y_{tp} \leq 2 \qquad \forall\, t \in Team \tag{5}$$

$$\sum_{p \in Pool^{w}} x_{tp} = 1 \qquad \forall\, t \in Team,\ \forall\, w \in Week \tag{6}$$

$$x_{tp} + x_{t'p} \leq 3 - x_{tp'} - x_{t'p'} \qquad \forall\, t, t'(t' \neq t) \in Team,\ \forall\, p, p'(p' \neq p) \in Pool \tag{7}$$

$$\sum_{p \in Pool^{w-1}} y_{tp} + \sum_{p \in Pool^{w}} y_{tp} \leq 1 \qquad \forall\, t \in Team,\ \forall\, w \in Week^{1} \tag{8}$$

$$z_{tt'p} \geq x_{tp} + y_{t'p} - 1 \qquad \forall\, t, t' \in Team,\ \forall\, p \in Pool \tag{9}$$

$$2\, z_{tt'p} \leq x_{tp} + y_{t'p} \qquad \forall\, t, t' \in Team,\ \forall\, p \in Pool \tag{10}$$

$$d_{tw} \leq D_{t't''} + M\left(2 - \sum_{p \in Pool^{w-1}} z_{tt'p} - \sum_{p \in Pool^{w}} z_{tt''p}\right) \qquad \forall\, t, t', t'' \in Team, \forall w \in Week^{1} \tag{11}$$

$$d_{tw} \geq D_{t't''} - M\left(2 - \sum_{p \in Pool^{w-1}} z_{tt'p} - \sum_{p \in Pool^{w}} z_{tt''p}\right) \qquad \forall\, t, t', t'' \in Team, \forall w \in Week^{1} \tag{12}$$

$$dI_{t} = \sum_{t' \in Team} \sum_{p \in Pool^{1}} D_{tt'}\, z_{tt'p} \qquad \forall\, t \in Team \tag{13}$$

$$dF_{t} = \sum_{t' \in Team} \sum_{p \in Pool^{5}} D_{t't}\, z_{tt'p} \qquad \forall\, t \in Team \tag{14}$$

$$dT_{t} = dI_{t} + \sum_{w \in Week^{1}} d_{tw} + dF_{t} \qquad \forall\, t \in Team \tag{15}$$

$$dA = \frac{1}{16} \sum_{t \in Team} dT_{t} \tag{16}$$

$$x_{tp}, y_{tp}, z_{tt'p} \in \{0, 1\} \ \ \forall\, t, t' \in Team, \forall\, p \in Pool \tag{17}$$

$$d_{tw},\ dI_{t},\ dF_{t},\ dT_{t},\ dA \geq 0 \qquad \forall\, t \in Team,\ \forall\, w \in Week \tag{18}$$

Objective function (1) minimizes the sum of the differences between the total travel distance for every team and the average of the total travel distances for all teams. Constraint (2) ensures that each pool consists of four teams. Regarding constraint (3), each pool exactly has one host team. Incorporating constraint (4) into the model, the host team of each pool belongs to that pool. Considering constraint (5), each team is as the host at least at one pool and at most at two pools during the planning horizon. Regarding constraint (6), at each week, each team exactly belongs to one of the pools of that week. If two teams belong to one pool, then they cannot belong to one of the other pools, simultaneously. Constraint (7) meets this fact. Furthermore, constraint (8) is incorporated into the model to ensure that each team cannot be considered as the host at two consecutive weeks.

Regarding constraints (9) and (10), if team t belongs to pool p and team t' is considered as the host of this pool, then the value of decision variable $z_{tt'p}$ is equal to one and vice versa. Travel distance for each team between each two consecutive weeks is determined using constraints (11) and (12). In these relations, symbol M denotes a sufficiently big number that can be considered as the maximum possible distance between each couple of the host cities. Moreover, the distance travelled by each team from its country to its host country at the first week, the distance travelled by each team from its host country to its country at the last week, the total distance travelled by each team over the planning horizon, and the average of the total distances travelled by all teams during the planning horizon are calculated by incorporating constraints (13)-(16) into the model, respectively.

The developed mathematical model can be easily converted to a linear optimization problem as follows:

$$\text{Minimize} \quad \sum_{t \in Team} s_{t} \tag{19}$$

Subject to:

$$dT_{t} - dA \leq s_{t} \qquad \forall\, t \in Team \tag{20}$$

$$dA - dT_{t} \leq s_{t} \qquad \forall\, t \in Team \tag{21}$$

$$s_{t} \geq 0 \qquad \forall\, t \in Team \tag{22}$$

And other constraints (2)-(18)

Obviously, decision variable s_t denotes the difference between the total travel distance for team t and the average of the total travel distances for all teams.

4. Model Evaluation

In this section, the performance of the developed mathematical model is evaluated. For this reason, the proposed model is applied to obtain the optimal schedule at the preliminary round of the Volleyball Men's Nations League in year 2018. Then, the schedule obtained from the developed model is compared with the schedule provided by the International Volleyball Federation (FIVB) in this year.

Note that IBM ILOG CPLEX optimization Studio version 12.2 was applied to solve the problem using the mathematical model. Furthermore, a Core i3-6006U CPU 2.00 GHz PC with 4.00 GB RAM was used for this reason.

The core teams qualified for the tournament are Argentina (ARG), Brazil (BRA), China (CHN), France (FRA), Germany (GER), Iran (IRN), Italy (ITA), Japan (JPN), Poland (POL), Russia (RUS), Serbia (SRB), and United States (USA), whereas the challenger teams are Australia (AUS), Bulgaria (BUL), Canada (CAN), and South Korea (KOR).

The pools composition at the preliminary round provided by the FIVB has been presented in Table 1. Note that the information has been gathered from the International Volleyball Federation website.

Table 1
Pools composition presented by FIVB

Week	The first week			
Pool	Pool 1	Pool 2	Pool 3	Pool 4
Host team	France	China	Poland	Serbia
Host city	Rouen	Ningbo	Katowice	Kraljevo
Teams	France Australia Iran Japan	China Argentina Bulgaria United States	Poland Canada Russia South Korea	Serbia Brazil Germany Italy
Week	The second week			
Pool	Pool 5	Pool 6	Pool 7	Pool 8
Host team	Bulgaria	Brazil	Argentina	Poland
Host city	Sofia	Goiania	San Juan	Katowice
Teams	Bulgaria Australia Russia Serbia	Brazil Japan South Korea United States	Argentina Canada Iran Italy	Poland China France Germany
Week	The third week			
Pool	Pool 9	Pool 10	Pool 11	Pool 12
Host team	Canada	Japan	Russia	France
Host city	Ottawa	Osaka	Ufa	Rouen
Teams	Canada Australia Germany United States	Japan Bulgaria Italy Poland	Russia Brazil China Iran	France Argentina Serbia South Korea
Week	The fourth week			
Pool	Pool 13	Pool 14	Pool 15	Pool 16
Host team	South Korea	Germany	United States	Bulgaria
Host city	Seoul	Ludwigsburg	Hoffman Estates	Sofia
Teams	South Korea Australia China Italy	Germany Argentina Japan Russia	United States Iran Poland Serbia	Bulgaria Brazil Canada France
Week	The fifth week			
Pool	Pool 17	Pool 18	Pool 19	Pool 20
Host team	Australia	China	Iran	Italy
Host city	Melbourne	Ningbo	Tehran	Modena
Teams	Australia Argentina Brazil Poland	China Canada Japan Serbia	Iran Bulgaria Germany South Korea	Italy France Russia United States

Regarding Table 1, Poland has been considered as the host team of pools 3 (at week 1) and 8 (at week 2), and this violates the constraint that does not permit the teams to be as the host at two consecutive weeks. As it is previously stated, this constraint has been incorporated to model to balance the total distance traveled by the teams.

The travel distances between the host cities have been provided in Table 2. Note these distances have been considered as the air distance (straight-line distance) between each couple of the host cities. The distances have been obtained from the Google Maps website.

Table 2
Travel distances between the host cities

Air distance (Km)	Rouen	Ningbo	Katowice	Kraljevo	Sofia	Goiania	San Juan	Ottawa	Osaka	Ufa	Seoul	Ludwigsburg	Hoffman Estates	Melbourne	Tehran	Modena
Rouen	0	9415	1285	1620	1862	8859	11319	5537	9636	3696	8975	591	6556	16869	4300	914
Ningbo	9415	0	8312	8553	8419	18275	19032	11463	1417	5797	992	8993	11457	7909	6451	9159
Katowice	1285	8312	0	738	903	9970	12483	6591	8703	2518	7981	724	7556	15584	3055	871
Kraljevo	1620	8553	738	0	242	9759	12306	7133	9087	2800	8326	1051	8132	15364	2755	784
Sofia	1862	8419	903	242	0	9905	12456	7370	8998	2708	8226	1290	8365	15131	2527	1019
Goiania	8859	18275	9970	9759	9905	0	2551	7402	17956	12488	17663	9285	7651	13774	12024	9131
San Juan	11319	19032	12483	12306	12456	2551	0	8587	17751	14999	18445	11782	8425	11556	14518	11660
Ottawa	5537	11463	6591	7133	7370	7402	8587	0	10569	7984	10511	6082	1064	16569	9549	6448
Osaka	9636	1417	8703	9087	8998	17956	17751	10569	0	6290	830	9314	10392	8119	7383	9575
Ufa	3696	5797	2518	2800	2708	12488	14999	7984	6290	0	5526	3217	8725	13288	2147	3361
Seoul	8975	992	7981	8326	8226	17663	18445	10511	830	5526	0	8616	10472	8582	6553	8849
Ludwigsburg	591	8993	724	1051	1290	9285	11782	6082	9314	3217	8616	0	7085	16291	3709	490
Hoffman Estates	6556	11457	7556	8132	8365	7651	8425	1064	10392	8725	10472	7085	0	15536	10439	7470
Melbourne	16869	7909	15584	15364	15131	13774	11556	16569	8119	13288	8582	16291	15536	0	12612	16149
Tehran	4300	6451	3055	2755	2527	12024	14518	9549	7383	2147	6553	3709	10439	12612	0	3536
Modena	914	9159	871	784	1019	9131	11660	6448	9575	3361	8849	490	7470	16149	3536	0

The optimal pools composition given by the proposed mathematical model has been provided in Table 3.

Table 3
Optimal pools composition provided by mathematical model

Week	The first week			
Pool	Pool 1	Pool 2	Pool 3	Pool 4
Host team	Bulgaria	South Korea	Brazil	Argentina
Host city	Sofia	Seoul	Goiania	San Juan
Teams	Bulgaria Iran Serbia Russia	South Korea France Japan Australia	Brazil China Canada Germany	Argentina Poland United States Italy
Week	The second week			
Pool	Pool 5	Pool 6	Pool 7	Pool 8
Host team	Italy	Japan	Serbia	Russia
Host city	Modena	Osaka	Kraljevo	Ufa
Teams	Italy Bulgaria Brazil South Korea	Japan Argentina Germany Iran	Serbia China France United States	Russia Canada Poland Australia
Week	The third week			
Pool	Pool 9	Pool 10	Pool 11	Pool 12
Host team	Canada	France	South Korea	United States
Host city	Ottawa	Rouen	Seoul	Hoffman Estates
Teams	Canada Serbia Japan Italy	France Poland Brazil Iran	South Korea China Argentina Russia	United States Bulgaria Germany Australia
Week	The fourth week			
Pool	Pool 13	Pool 14	Pool 15	Pool 16
Host team	Poland	Brazil	Iran	Argentina
Host city	Katowice	Goiania	Tehran	San Juan
Teams	Poland Germany Serbia South Korea	Brazil United States Japan Russia	Iran China Australia Italy	Argentina France Bulgaria Canada
Week	The fifth week			
Pool	Pool 17	Pool 18	Pool 19	Pool 20
Host team	United States	Australia	China	Germany
Host city	Hoffman Estates	Melbourne	Ningbo	Ludwigsburg
Teams	United States Canada South Korea Iran	Australia Serbia Brazil Argentina	China Japan Poland Bulgaria	Germany Russia France Italy

The results related to the schedule presented by the FIVB and the developed model have been also summarized in Tables 4 and 5, respectively.

The total travel distances for the teams calculated using the schedules provided by the FIVB and the mathematical model have been shown in Fig. 1. Regarding the figure, the total travel distances obtained from the schedule presented by the mathematical model for Poland, Brazil, Argentina, Canada, Japan, South Korea, United States, and Iran decrease in comparison with those obtained from the schedule provided by the FIVB, while this consequence is reversed for the other teams. Moreover, the variance of the total travel distances for the teams calculated using the schedule provided by the mathematical model is lower than that presented by the FIVB, obviously.

Table 4
Results related to schedule presented by FIVB

Objective value	Decision variables value									Teams
	dA	s_t	dT_t	dF_t	d_{t5}	d_{t4}	d_{t3}	d_{t2}	dI_t	
248428	41726	35361	6365	914	1019	1862	1285	1285	0	France
		24378	17348	0	992	5526	2518	8312	0	China
		8489	50215	15584	15536	10392	8703	0	0	Poland
		13056	28670	8553	11457	6556	1862	242	0	Serbia
		1838	39888	2527	2527	8998	8998	8419	8419	Bulgaria
		21893	63619	13774	15131	2708	12488	9759	9759	Brazil
		36095	77821	11556	16291	591	11319	19032	19032	Argentina
		13187	54913	11463	8419	7370	8587	12483	6591	Canada
		14449	56175	1417	8993	9314	17956	8859	9636	Japan
		28529	13197	3361	490	3217	2708	903	2518	Russia
		7165	48891	6553	6553	8975	8859	9970	7981	South Korea
		19846	21880	3709	3709	6082	6591	738	1051	Germany
		11412	53138	7470	7470	1064	7402	18275	11457	United States
		3468	45194	0	8582	10511	7370	1862	16869	Australia
		8056	49782	0	10439	8725	14999	11319	4300	Iran
		1206	40520	0	8849	830	17751	12306	784	Italy

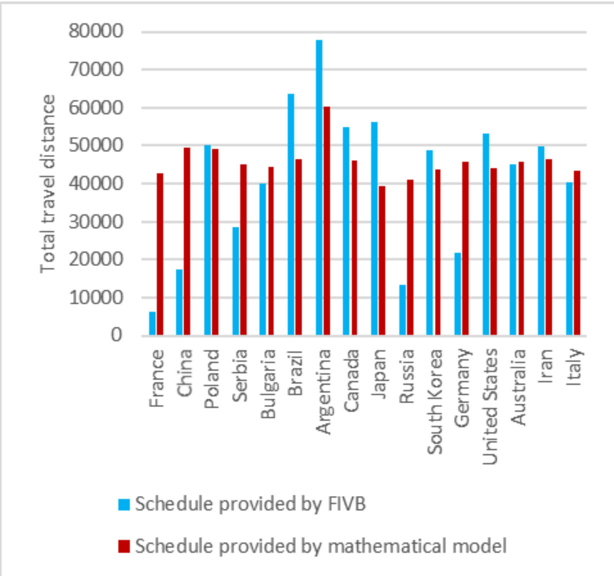

Fig. 1. Total travel distances

Table 5
Results obtained from proposed mathematical model

Objective value	Decision variables value									Teams
	dA	s_t	dT_t	dF_t	d_{t5}	d_{t4}	d_{t3}	d_{t2}	dI_t	
45175.24	45817.44	3204.40	42613	591	11782	11319	1620	8326	8975	France
		3546.60	49364	0	6451	6553	8326	9759	18275	China
		3269.60	49087	8312	8312	1285	3696	14999	12483	Poland
		661.44	45156	15364	15584	6591	7133	242	242	Serbia
		1452.40	44365	8419	19032	8425	7470	1019	0	Bulgaria
		634.56	46452	13774	13774	8859	914	9131	0	Brazil
		14321.00	60138	11556	11556	18445	830	17751	0	Argentina
		132.56	45950	1064	8425	8587	7984	12488	7402	Canada
		6494.40	39323	1417	18275	7402	10569	830	830	Japan
		4710.40	41107	3217	9285	17663	5526	2708	2708	Russia
		2110.40	43707	10472	7556	7981	8849	8849	0	South Korea
		95.56	45913	0	724	7556	10392	17956	9285	Germany
		1652.40	44165	0	7651	7651	8132	12306	8425	United States
		66.56	45884	0	12612	10439	8725	5526	8582	Australia
		521.56	46339	10439	10439	4300	9636	8998	2527	Iran
		301.40	43516	490	3709	9549	6448	11660	11660	Italy

The difference between the total distance traveled by each team and the average of the total distances traveled by all teams has been also indicated in Fig. 2. Clearly, the objective value obtained from the schedule provided by the mathematical model is considerably better than that calculated by the schedule presented by the FIVB.

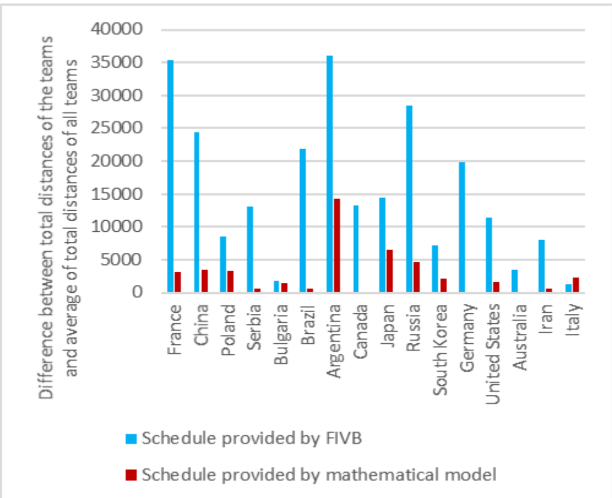

Fig. 2. Difference between total travel distances of the teams and average of total travel distances of all teams

The average of the total travel distances for all teams calculated from the schedules presented by the FIVB and the mathematical model has been exhibited in Fig. 3. Moreover, the sum of the differences between the total travel distance for every team and the average of the total travel distances for all teams obtained from the schedules provided by the FIVB and the mathematical model has been indicated in Fig. 4.

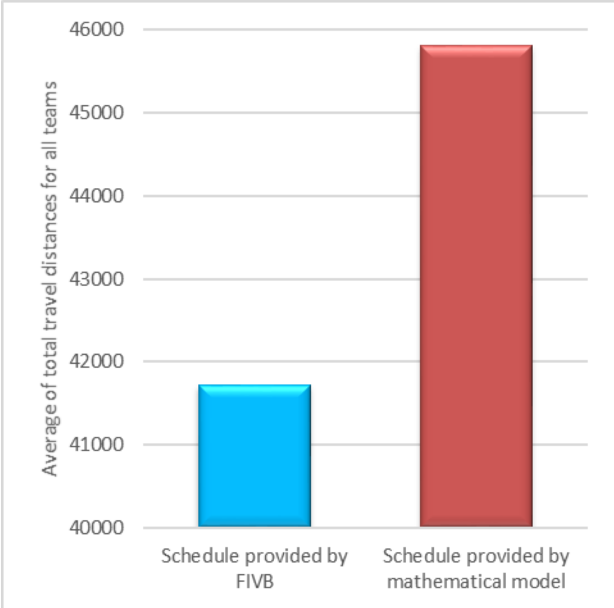

Fig. 3. Average of total travel distances

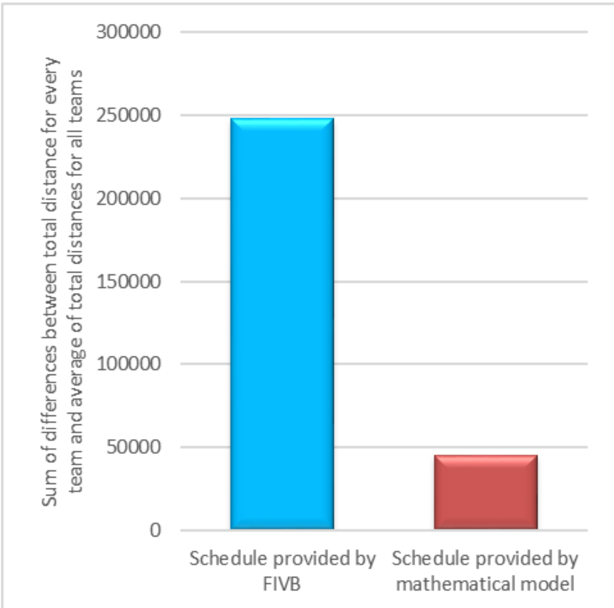

Fig. 4. Sum of differences between total distance for every team and average of total distances for all teams

Regarding Fig. 3 and Fig. 4, the average of the total travel distances for all teams in the schedule presented by the mathematical model is greater than by the FIVB. While, the sum of the travel distance deviations from the average of the total travel distances of all teams obtained from the schedule provided by the mathematical model is significantly lower than that calculated from the schedule presented by the FIVB.

In the current study, the aim is to provide a schedule by minimizing the sum of the differences between the total travel distance for every team and the average of the total travel distances for all teams. As a result, the schedule provided by the mathematical model is obviously obtained from this objective's point of view. Note that the schedule presented by the FIVB leads to a percentage gap of $(248428 - 45175.24)/45175.24 \times 100 = 449.92\%$ compared to the optimal schedule provided by the proposed mathematical model.

5. Summary and Conclusion

In this study, a single round-robin tournament problem was discussed. As a case study, Volleyball Nations League was considered as a new annual international volleyball tournament. In this tournament, sixteen teams are qualified to compete together. The matches are held during five weeks. At each week, the teams are divided into four pools of four teams and all teams belonged to each pool compete together on three consecutive days in a single round-robin format.

In this setting, the attempts were made to provide the schedule of the matches at the preliminary round of the tournament by minimizing the sum of the differences between the total travel distance for every team and the average of the total travel distances for all teams. For this reason, a mathematical programming model was proposed to assign the teams to the pools at each week in addition to specify the host team of the pools.

Then, the performance of the developed model was evaluated. For this reason, the model was used to find the optimal schedule at the preliminary round of the Volleyball Men's Nations League in year 2018.

Regarding the obtained results, the sum of the travel distance deviations from the average of the total travel distances of all teams obtained from the schedule provided by the mathematical model is significantly lower than from the schedule presented by the International Volleyball Federation. Furthermore, the schedule obtained from the International Volleyball Federation leads to a percentage gap of 449.92 % compared to the optimal schedule provided by the mathematical model.

References

Alarcón, F., Durán, G., Guajardo, M., Miranda, J., Muñoz, H., & Ramírez, L. (2017). Operations research transforms the scheduling of chilean soccer leagues and south american world cup qualifiers. *Interfaces*, 47(1), 52-69.

Atan, T., & Çavdarolu., B. (2018). Minimization of Rest Mismatches in Round Robin Tournaments. *Computers & Operations Research*, 99, 78-89.

Behnamian, J. (2020). Parallel Jobs Scheduling with a Specific Due Date: Asemi-definite Relaxation-based Algorithm. *Journal of Optimization in Industrial Engineering*, 13(2), 199-210.

Bhattacharyya, R. (2016). Complexity of the Unconstrained Traveling Tournament Problem. *Operations Research Letters*, 44(5), 649-654.

Bonomo, F., Cardemil, A., Durán, G., Marenco, J., & Sabán, D. (2012). An application of the traveling tournament problem: The Argentine volleyball league. *Interfaces*, 42(3), 245-259.

Briskorn, D. (2008). Feasibility of home–away-pattern sets for round robin tournaments. *Operations Research Letters*, 36(3), 283-284.

Briskorn, D. (2009). Combinatorial properties of strength groups in round robin tournaments. *European Journal of Operational Research*, 192(3), 744-754.

Briskorn, D., & Drexl, A. (2009). A branching scheme for finding cost-minimal round robin tournaments. *European Journal of Operational Research*, 197(1), 68-76.

Briskorn, D., & Horbach, A. (2012). A Lagrangian approach for minimum cost single round robin tournaments. *Computers & Operations Research*, 39(3), 718-727.

Briskorn, D., & Knust, S. (2010). Constructing fair sports league schedules with regard to strength groups. *Discrete Applied Mathematics*, 158(2), 123-135.

Cheung, K.K. (2009). A Benders approach for computing lower bounds for the mirrored traveling tournament problem. *Discrete Optimization*, 6(2), 189-196.

Cocchi, G., Galligari, A., Nicolino, F.P., Piccialli, V., Schoen, F., & Sciandrone, M. (2018). Scheduling the Italian National Volleyball Tournament. *Interfaces*, 48(3), 271-284.

De Carvalho, M.A.M., & Lorena, L.A.N. (2012). New models for the mirrored traveling tournament problem. *Computers & Industrial Engineering*, 63(4), 1089-1095.

De Oliveira, L., & De Souza, C.C., & Yunes, T. (2016). Lower bounds for large traveling umpire instances: New valid inequalities and a branch-and-cut algorithm. *Computers & Operations Research*, 72, 147-159.

Della Croce, F., & Oliveri, D. (2006). Scheduling the Italian football league: An ILP-based approach. *Computers & Operations Research*, 33(7), 1963-1974.

Durán, G., Guajardo, M., Miranda, J., Sauré, D., Souyris, S., Weintraub, A., & Wolf, R. (2007). Scheduling the Chilean soccer league by integer programming. *Interfaces*, 37(6), 539-552.

Durán, G., Guajardo, M., & Wolf-Yadlin, R. (2012). Operations research techniques for scheduling Chile's second division soccer league. *Interfaces*, 42(3), 273-285.

Duran, S., Özener, O.Ö., & Yaki, E. (2014). League scheduling and game bundling in sports industry. *Computers & Industrial Engineering*, 74, 92-101.

Elf, M., Jünger, M., & Rinaldi, G. (2003). Minimizing breaks by maximizing cuts. *Operations Research Letters*, 31(5), 343-349.

Erzurumluoglu, A. (2018). Constructing day-balanced round-robin tournaments with partitions. *Discrete Applied Mathematics*, 235, 81-91.

Guajardo, M., & Jörnsten, K. (2017). The Stable Tournament Problem: Matching sports schedules with preferences. *Operations Research Letters*, 45(5), 461-466.

Hassani, A., Hosseini, S.M., & Behroozi, F. (2021). Minimizing the operational costs in a flexible flow shop scheduling problem with unrelated parallel machines. *Journal of Optimization in Industrial Engineering*, 14(1), 169-184.

Horbach, A. (2010). A combinatorial property of the maximum round robin tournament problem. *Operations Research Letters*, 38(2), 121-122.

Ichim, B., & Moyano-Fernández, J.J. (2017). On the score sheets of a round-robin football tournament. *Advances in Applied Mathematics*, 91, 24-43.

Irnich, S. (2010). A new branch-and-price algorithm for the traveling tournament problem. *European Journal of Operational Research*, 204(2), 218-228.

Jafari, H. (2019). Sustainable development by reusing of recyclables in a textile industry including two collectors and three firms: A game-theoretic approach for pricing decisions. *Journal of Cleaner Production*, 229, 598-610.

Jafari, H. (2020a). Developing a fuzzy model for the nurse scheduling problem. *Journal of Operational Research in Its Applications*, 17(2), 93-107.

Jafari, H. (2020b). Investigating uncertainties in nurse sch eduling problem using fuzzy mathematical modeling approach. *Sharif Journal of Industrial Engineering & Management*, 36(1), 77-85.

Jafari, H. (2021). Nurse scheduling problem by considering total number of required nurses as well as nurses' preferences for working shifts: An algorithmic game-theoretic approach. *Scientia Iranica*, Inpress.

Jafari, H. (2022). Investigating environmental and economic aspects of sustainability by recycling PET plastic bottles: A game-theoretic approach. *Clean Technologies and Environmental Policy*, 24(3), 829-842.

Jafari, H., & Haleh, H. (2021). Nurse scheduling problem by considering fuzzy modeling approach to treat uncertainty on nurses' preferences for working shifts and weekends off. *Journal of Optimization in Industrial Engineering*, 14(1), 69-78.

Jafari, H., Hejazi, S.R., & Rasti-Barzoki, M. (2020). Game theoretical approach to price a product under two-echelon supply chain containing e-tail selling channel. International Journal of Services and Operations Management, 36(2), 131-160.

Januario, T., & Urrutia, S. (2015). An analytical study in connectivity of neighborhoods for single round robin tournaments. Operations Research and Computing: *Algorithms and Software for Analytics*, 188-199.

Januario, T., & Urrutia, S. (2016). A new neighborhood structure for round robin scheduling problems. *Computers & Operations Research*, 70, 127-139.

Januario, T., Urrutia, S., Ribeiro, C.C., & De Werra, D. (2016). Edge coloring: A natural model for sports scheduling. *European Journal of Operational Research*, 254(1), 1-8.

Kendall, G., Knust, S., Ribeiro, C.C., & Urrutia, S. (2010). Scheduling in sports: An annotated bibliography. *Computers & Operations Research*, 37(1), 1-19.

Khelifa, M., & Boughaci, D. (2015). A variable neighborhood search method for solving the traveling tournaments problem. *Electronic Notes in Discrete Mathematics*, 47, 157-164.

Knust, S. (2008). Scheduling sports tournaments on a single court minimizing waiting times. *Operations Research Letters*, 36(4), 471-476.

Knust, S., & Lücking, D. (2009). Minimizing costs in round robin tournaments with place constraints. *Computers & Operations Research*, 36(11), 2937-2943.

Knust, S., & Von Thaden, M. (2006). Balanced home–away assignments. *Discrete Optimization*, 3(4), 354-365.

Kostuk, K.J., & Willoughby, K.A. (2012). A decision support system for scheduling the Canadian Football League. *Interfaces*, 42(3), 286-295.

Lewis, R., & Thompson, J. (2011). On the application of graph colouring techniques in round-robin sports scheduling. *Computers & Operations Research*, 38(1), 190-204.

Lim, A., Rodrigues, B., & Zhang, X. (2006). A simulated annealing and hill-climbing algorithm for the traveling tournament problem. *European Journal of Operational Research*, 174(3), 1459-1478.

Miyashiro, R., & Matsui, T. (2005). A polynomial-time algorithm to find an equitable home–away assignment. *Operations Research Letters*, 33(3), 235-241.

Rasmussen, R.V. (2008). Scheduling a triple round robin tournament for the best Danish soccer league. *European Journal of Operational Research*, 185(2), 795-810.

Rasmussen, R.V., & Trick, M.A. (2007). A Benders approach for the constrained minimum break problem. *European Journal of Operational Research*, 177(1), 198-213.

Rasmussen, R.V., & Trick, M.A. (2008). Round robin scheduling–a survey. *European Journal of Operational Research*, 188(3), 617-636.

Ribeiro, C.C., & Urrutia, S. (2012). Scheduling the Brazilian soccer tournament: Solution approach and practice. *Interfaces*, 42(3), 260-272.

Saur, M.C., Starr, K., Husted, M., & Newman, A.M. (2012). Scheduling softball series in the rocky mountain athletic conference. *Interfaces*, 42(3), 296-309.

Suksompong, W. (2016). Scheduling asynchronous round-robin tournaments. *Operations Research Letters*, 44(1), 96-100.

Thielen, C., & Westphal, S. (2011). Complexity of the traveling tournament problem. *Theoretical Computer Science*, 412(4-5), 345-351.

Toffolo, T.A., Wauters, T., Van Malderen, S., & Berghe, G.V. (2016). Branch-and-bound with decomposition-based lower bounds for the Traveling Umpire

Problem. *European Journal of Operational Research*, 250(3), 737-744.

Trick, M.A., Yildiz, H., & Yunes, T. (2012). Scheduling major league baseball umpires and the traveling umpire problem. *Interfaces*, 42(3), 232-244.

Urrutia, S., & Ribeiro, C.C. (2004). Minimizing travels by maximizing breaks in round robin tournament schedules. *Electronic Notes in Discrete Mathematics*, 18, 227-233.

Urrutia, S., & Ribeiro, C.C. (2006). Maximizing breaks and bounding solutions to the mirrored traveling tournament problem. *Discrete Applied Mathematics*, 154(13), 1932-1938.

Van't Hof, P., Post, G., & Briskorn, D. (2010). Constructing fair round robin tournaments with a minimum number of breaks. *Operations Research Letters*, 38(6), 592-596.

Westphal, S. (2014). Scheduling the German basketball league. *Interfaces*, 44(5), 498-508.

Yavari, S., Azab, A., Fazle Baki, M., Alcelay, M., & Britt, J. (2020). Machine scheduling for multitask machining. *Journal of Optimization in Industrial Engineering*, 13(2), 1-15.

Zeng, L., & Mizuno, S. (2012). On the separation in 2-period double round robin tournaments with minimum breaks. *Computers & Operations Research*, 39(7), 1692-1700.

This article can be cited: Jafari, H., & Rajabzadeh, M. (2022). A Mathematical Programming Model for Single Round-Robin Tournament Problem: A case study of Volleyball Nations League. *Journal of Optimization in Industrial Engineering*, *15*(2), 191-200.
Doi: 10.22094/joie.2022.1891205.1721