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Research Article

An Integrated Crew Scheduling Problem Considering a Reserve Crew in Air Transportation: Ant Colony Optimization Algorithm

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Abstract

A Crew Scheduling Problem (CSP) is a highly complex airline optimization problem, which includes two sub-problems, namely Crew Rostering Problem (CRP) and Crew Pairing Problem (CPP). Solving these problems sequentially may not lead to an optimal solution. To overcome this shortcoming, the present study introduces a new bi-objective formulation for the integrating CPP and CRP by considering the reserve crew with the objectives of crew cost minimization and crew reserve maximization. The integrated model generates and assigns pairings to a group of crew members by taking into account the rules and regulations about employing the manpower (i.e., crew member) and crew reservation in order to reduce flight delays or even cancellations due to the unexpected disruptions. An Ant Colony Optimization (ACO) algorithm is used to solve the considered problem. To justify the efficiency of this proposed algorithm in solving the presented model, different test problems are generated and solved by ACO and GAMS. The computational results indicate that solutions obtained by the proposed ACO algorithm have a 2.57% gap with the optimal solutions reported by GAMS as optimization software on average and significantly less CPU time for small-sized problems. Also, ACO obtains better solutions in significantly shorter CPU time for large-sized problems. The results indicate the efficient performance of the proposed algorithm in solving the given problems.

Keywords: Scheduling; Crew Planning; Multiple objective programming; Combinatorial optimization; Air transport; Meta-heuristics.
Introduction

1. Introduction

The scheduling problem includes various types of problems in production and service industries in which a timetable for resources is determined. In production systems, one of the main problems is determination of sequence and schedule of jobs operations on different machines. For the applications of scheduling problem in production systems, see Enayati et al., (2021) and Dehnavi-Arani et al. (2019). In service industries, the scheduling problem plays an important role in efficient usage of manpower and resources (Rashidi komijan et al., 2021a). This paper addresses crew scheduling problem which is a good example of a manpower scheduling problem. More specifically, a crew is a substantial airline resource that requires planning. A Crew Scheduling Problem (CSP) is the process of assigning flights to a set of crew members to minimize the cost. It is usually divided into two sub-problems known as the Crew Rostering Problem (CRP) and the Crew Pairing Problem (CPP). In the CPP, pairings starting from and ending at the same crew base(s) are generated in such a way that all the flights are covered with the minimum cost. Some rules about employing the manpower (i.e., crew member) should be considered during this procedure. It is worth noting that each pairing consists of consecutive duty days, and each duty day includes consecutive flights (Ahmadbeygi et al., 2009). In the CRP, the generated pairings as well as vacations and educational courses are assigned to a set of crew members based on some other rules (Kohl and Karisch, 2004).

Moreover, disruptions (e.g., crew unavailability, bad weather conditions, and air traffic) are inevitable on the operation day. These phenomena particularly affect crew scheduling and cause flight delays or even cancelations (Bazargan, 2016). As crew scheduling is performed one month before the operation day, unexpected disruptions may increase the operational crew cost up to 5%. This cost can be several million dollars in an airline, showing the importance of crew cost in airline expenditures (Shebalov et al., 2006). Fortunately, crew cost is controllable (Aggarwal et al., 2018), but requires very complex planning (Barnhart, 2008; Klabjan et al., 2001). The crew cost could be reduced by considering the reserve crew. In other words, in the case of crew unavailability, he/she can be replaced with an alternative crew member. Otherwise, flight delays or even cancelations will significantly increase, which may require expensive recovery actions.

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In this study, the integrated crew pairing and rostering problem is investigated by considering reserve crew teams. A few main contributions are as follows:

- This study integrates the CPP and the CRP in a bi-objective formulation. The integrated problem leads to better solutions compare to solving two separate problems sequentially. More specifically, the proposed model can simultaneously generate and assign pairings to several crew members based on some rules and regulations about the CPP and the CRP.
- In the integrated problem, the reserve crew and related constraints are considered. This can significantly reduce flight delays or even cancelations related to crew unavailability.

The crew is generally known as the cockpit and cabin crew members with different assignments. The focus of this research is only on cockpit crew scheduling.

The rest of the paper is structured as follows. In Section 2, the literature is reviewed. The problem definition and rules for integrating CPP and CRP by considering the reserve crew issue are presented in Section 3. Section 4 is dedicated to the mathematical model. In Section 5, the proposed meta-heuristic algorithm is introduced. Finally, Section 6 presents the computational results.

2. Literature Review

Usually, the CPP is formulated as a set covering or partitioning problem. Whereas each flight in the set covering problem is covered at least once, each flight in the set partitioning problem is covered by only one of the selected pairings. The CPP has received considerable research attention over the past years. Ahmadbeygi et al. (2009) introduced a set partitioning formulation of the CPP as an integer mathematical model and utilized the branch-and-bound (B&B) algorithm as their solution approach. Zeren and Ozkol (2012) applied a new operator (called perturbation) of a Genetic Algorithm (GA) to solve the CPP. Aydemir-Karadag et al. (2013) formulated the CPP as a set partitioning model and compared the results of three different algorithms (i.e., Knowledge Based Random Algorithm (KBRA), Hybrid Algorithm (HA), and Branch-and-Price (B&P)) as the solution approaches. In another study, Erdogan et al. (2015) formulated the CPP as a set partitioning problem and solved a large-scale problem using a Large Neighborhood Search (LNS) heuristic algorithm. Zeren and Ozkol (2016) addressed the CPP as a set partitioning model and developed a new column generation method with a pricing network design and a pricing elimination heuristic. Quesnel et al. (2017) considered the CPP as a set partitioning model with base constraints related to limiting the total working time at each crew base. They designed a novel branching method (i.e., retrospective branching) to solve the problem. Haouari et al. (2019) proposed a nonlinear model for optimization of the daily CPP, which could be solved in polynomial time by commercial MIP solvers. They linearized the formulation and compared the obtained results with non-linearized ones. Quesnel et al. (2020) proposed a new variant of the CPP, considered crew preferences to improve the CRP solutions in sequential order, and solved it using the column generation method.

Unlike the CPP, the CRP has drawn less attention from researchers. Different objective functions have been considered for the CRP in the previous studies. Maenhout and Vanhoucke (2010) formulated the CRP to generate a fair schedule for the crew as well as consider crew preferences and solved it using a Scatter Search (SS) algorithm. Santosa et al. (2010) addressed the CRP to set an equal number of flying days and off days among crew members and used Differential Evolution (DE) algorithm as the solution method. Hadianti et al. (2013) formulated a non-linear programming model for the CRP to minimize the deviation of a crew flying time from the ideal time and applied Simulated Annealing (SA) algorithm to solve the considered problem. De Armas et al. (2017) used a multi-start randomized heuristic algorithm as the solution approach for the CRP to balance crew workload. Doi et al. (2018) considered the CRP to set fair working time and solved it using a two-level decomposition-based algorithm.

A few studies have applied different solution methodologies to solve the integrated crew pairing and rostering problem (i.e., integrated CPP and CRP). Ozdemir and Mohan (2001) used a GA and specified its advantages in comparison to the previous methods in terms of cost function and CPU time. In another study, Deng and Lin (2011) applied the Ant Colony Optimization (ACO) algorithm to solve the integrated problem and showed its superiority to a GA. Moreover, Saddoune et al. (2012) developed a new algorithm by combining Column Generation and dynamic constraint aggregation algorithm to solve the integrated problem. Their findings indicated that the applied methodology could lead to cost savings in comparison to sequential order solving. Finally, Azadeh et al. (2013) solved the integrated crew pairing and rostering problem by Particle Swarm Optimization (PSO) algorithm and confirmed its better performance to ACO and GA.

Among studies focused on reserve crew scheduling, Sohoni et al. (2006) introduced a model, compared its results with those of another formulation, and showed its effectiveness in terms of decreasing the number of uncovered trips and reducing reserve availability. Bayliss et al. (2012) developed a probabilistic mathematical model for a reserve crew scheduling problem and suggested various solution approaches. The computational results showed a better performance of some algorithms (e.g., Variable Neighborhood Search (VNS) and Tabu Search (TS)) in terms of the objective functions and CPU time to the enumeration algorithm on some test problems. Finally, Bayliss et al. (2017) presented different simulation scenario-based approaches to schedule the reserve crew teams and compared the results in terms of the minimized overall disruptions on duties.

Recently, Shafipour-Omran et al. (2021) modeled the problem of crew pairing by considering the risk of COVID-19 in terms of minimizing daily flights as well as elapsed time. The results obtained by their solution approach called GA were better than GAMS, the exact method, for all real small and medium sized problems used in this study. Moreover, Rashidi Komijan et al. (2021b) presented an integrated model for the crew scheduling and fleet assignment problems and solved it by two meta-heuristic algorithm called Vibration Damping Optimization (VDO) and PSO. They indicated that VDO had better performance compared to PSO for all generated test problems. In another recent study, Saemi et al. (2021) presented a new mathematical model for the integrated problem by considering one or more off days in a pairing, which could provide the opportunities for the crew members to perform their preferred activities. They solved the problem by two meta-heuristic algorithms namely, GA and PSO. In the recent work, Saemi et al. (2022) presented a new mathematical formulation for the integrated problem and compered the results obtained by a meta-heuristic algorithm (i.e., ACO) for the integrated problem and sequential solving approach. Their findings indicated that solving the integrated problem cannot be much more difficult than solving the sequential approach and also it leads to solutions with significant lower cost and minimum number of assigned crew members.

In this study, a linear mathematical model for the integrated crew pairing and rostering problem by considering the reserve crew issue is presented. The integration of the CPP and the CRP can generate appropriate and accurate schedules for the crew in comparison to the sequentially solving two separate problems. On the other hand, considering the reserve crew can reduce flight delays or even cancelations, resulting in significant cost saving. A meta-heuristic algorithm based on ACO is applied to solve the considered problem. The results obtained by this proposed algorithm are compared with those results obtained by an exact method (i.e., GAMS) on some small-sized problems and its performance in solving the integrated problem is evaluated.

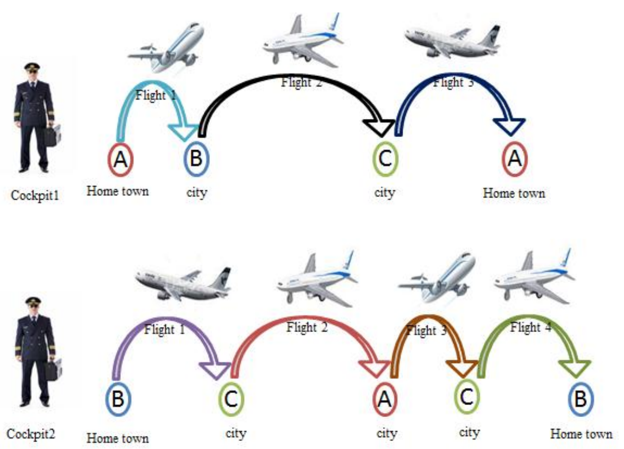

Fig. 1. Graphical Abstract

3. Problem Definition

The CSP includes two sub-problems, called CPP and CRP. The CPP is the process of generating a minimum cost set of pairings to cover all flights. The CRP is the task of assigning the previously generated pairings to a set of crew members considering some rules about employing the manpower (i.e., crew member). This study integrates these sub-problems into a single mathematical model. The main advantage of this integration is to provide accurate and appropriate work schedules for the existing crew members in comparison with sequential order models. More especially, the proposed formulation includes variables to generate and assign pairings to several crew members simultaneously. Furthermore, the suggested model is based on the following general rules about the CPP and the CRP:

- Each flight should be covered by only one of the existing crew members.
- Each pairing should begin/end from/to the same crew base.
- Each pairing includes at most two consecutive duty days.
- Each duty day consists of consecutive flights, in which the destination of each flight is the departure of the next one.
- A minimum sit time between two sequential flights on a duty day must be considered for the crew.
- A minimum layover time between two sequential duties in a pairing must be considered for the crew.
- Flying hours during each duty must be restricted by flying time and elapsed time, respectively. Also, a similar limitation should be considered for the total flying time duration in a pairing. It should be noted that the term "elapsed time" means working day time and includes not only the flying hours on a day but also the sit time between the consecutive daily flights assigned to a crew member.

The integrated problem is considered by several additional assumptions for the reserve crew teams based on related rules. These assumptions result in fewer flight delays or even cancelations due to the crew unavailability on the operation day. In other words, the crew scheduling is performed in such a way that when an assigned crew member cannot operate a flight, an alternative crew can be replaced. Moreover, this can also significantly result in more cost savings in the airline industry. Furthermore, general rules about the reserve crew policies are as follows:

A crew member can be reserved for a flight,

- If the minimum sit time period is considered between this flight and his/her previously assigned flight on the same day.
- If the minimum layover time period is considered between this flight and his/her previously assigned flight on the day before.
- If the total flying time is considered for both a duty day and a pairing.

- If both the reserve and the assigned crew members end their pairings on the similar day. This assumption guarantees that both assigned and reserve crew members can continue their original works' schedules (i.e., next pairings) without any interruptions.
- If both the reserve and the assigned crew members have the same home base.
- If the reserve crew's flight departure time starts earlier than his/her assigned flight according to schedule.

It should be noted that the CSP is developed several days before the operation day to inform the existing crew members about their duty and off days. Therefore, it is assumed that they can potentially be reserved for the flights not assigned to them on only their duty days.

The objective functions of the model tend to maximize the reserve crew and minimize the total cost, including the minimum guaranteed pay, additional pay for the extra flying hours on a duty, hoteling cost, and the deviation cost of the number of the duty days assigned to a crew member from an ideal one.

4. Problem Formulation

In this section, a novel mathematical model for the integrated crew pairing and rostering problem is formulated by considering the reserve crew issue. Sets, indices, parameters, and decision variables are as follows:

Sets and indices:

I	Set of all crew members (indexed by i).
F	Set of all flights (indexed by f).
D	Set of all days of planning horizon (indexed by d).
F_d	Set of all flights on day *d*
N	Set of flight rounds (indexed by n) considering that a crew member starts his/her duty day by flight *i* and continues it by flight *j*. For flights *i* and flight j, *n* equals 1 and 2, respectively.
CF_i	Set of all flights that originate from the home base of crew *i*.
CL_i	Set of all flights that terminate to the home base of crew *i*.
S	Set of all consecutive flight pairs, in which the destination of the first flight is the departure of the next one.
csb_i	Set of all crew members have the same home base as crew *i*

Parameters:

dep_{fd}	Departure time of flight f on day *d*
lan_{fd}	Arrival time of flight f on day *d*
$\min\_sit$	Minimum rest time required between two sequential flights on a single duty day
$\min\_rest$	Minimum rest time required between two sequential duty days in a pairing
$up1$	Maximum flying time in a duty
$up2$	Maximum elapsed time in a duty
$up3$	Maximum flying time in a pairing
GH	Minimum required flying time
e	Minimum guaranteed payment for a duty
r	Crew cost for each flying hour more than minimum required time
h	Hoteling cost
v	Penalty cost for the deviation of the number of duty days assigned to a crew member from an ideal one
q	Ideal number of duty days which can be assigned to a crew member
M	A big number

Decision variables:

x_{ifdn}	1 if flight *f* on day *d* is assigned to crew *i* in flight round *n*; 0, otherwise
y_{ifdn}	1 if crew *i* is reserved for flight *f* on day *d* in flight round n; 0, otherwise
u_{ifd}	1 if crew *i* ends his/her duty day *d* by the flight f; 0, otherwise
$z_{if'fdn}$	1 if flight *f* is handled after flight f' on day *d* by crew *i*; 0, otherwise
o_{id}	1 if duty day *d* is working day of crew *i*; 0, otherwise
g_{id}	Flying hours of crew *i* on day *d* exceeding the minimum required time (GH)
B_i	Number of duty days assigned to crew member i in the planning horizon

The objective function and constraints are as follows:

$$\boldsymbol{max} \sum_{f\in F_d}\sum_{d\in D}\sum_{i\in I}\Big(\sum_{n\in N} y_{ifdn}\Big) \tag{1}$$

$$\text{Min}\ \Big(\sum_{d\in D}\sum_{i\in I} o_{id}\,.\,e + \sum_{d\in D}\sum_{i\in I} g_{id}\,.\,r + \sum_{d\in D}\sum_{f\neq CL_i}\sum_{i\in I} u_{ifd}.\,h + \sum_{i\in I}|B_i - q|\,.\,v\Big) \tag{2}$$

Subject to:

$$o_{id} + o_{id+1} + o_{id+2} \le 2 \quad ;\forall i \in I, d \in D \tag{3}$$

$$\sum_{f\in F_d}\sum_{n\in N} x_{ifdn} \le M.(o_{id}) \quad ;\forall i \in I, d \in D \tag{4}$$

$$\sum_{n\in N} x_{ifdn} \le 1 \quad ;\forall i \in I, d \in D, f \in F_d \tag{5}$$

$$\sum_{f\in F_d} x_{ifdn} \le 1 \quad ;\forall i \in I, d \in D, n \in N \tag{6}$$

$$u_{ifd} \ge 1 - M.(1 - x_{ifdn}) \tag{7}$$

$$-M.\left(\sum_{f'\in F_d} x_{if'd,n+1}\right)$$

$$; \forall i \in I, d \in D, f \in F_d, n \in N$$

$$dep_{fd} - lan_{f'd} \geq \min\_sit$$
$$-M.\left(2 - x_{if'dn} - x_{ifd,n+1}\right) \quad (8)$$
$$; \forall i \in I, d \in D, f \in F_d, f' \in F_d, (f', f) \in s, n \in N$$

$$1440 + dep_{fd+1} - lan_{f'd} \geq \min\_rest$$
$$-M.\left(2 - u_{if'd} - x_{ifd+1,n}\right) \quad (9)$$
$$; \forall i \in I, d \in D, f \in F_{d+1}, f' \in F_d, (f', f) \in s, n = 1$$

$$\sum_{f\in F_d}\sum_{n\in N}\left(lan_{fd} - dep_{fd}\right).x_{ifdn} \leq up1 \quad (10)$$
$$; \forall i \in I, d \in D$$

$$2.z_{if'fdn} \leq x_{if'dn} + x_{ifd,n+1} \quad (11)$$
$$; \forall i \in I, d \in D, f \in F_d, f' \in F_d, (f', f) \in s, n \in N$$

$$z_{if'fdn} + 1 \geq x_{if'dn} + x_{ifd,n+1} \quad (12)$$
$$; \forall i \in I, d \in D, f \in F_d, f' \in F_d, (f', f) \in s, n \in N$$

$$\sum_{f\in F_d}\sum_{n\in N}\left(lan_{fd} - dep_{fd}\right).x_{ifdn}$$
$$+ \sum_{\substack{(f',f)\in s\\ f\in F_d\\ f'\in F_d}}\sum_{n\in N}\left(dep_{fd} - lan_{f'd}\right).z_{if'fdn} \leq up2 \quad (13)$$
$$; \forall i \in I, d \in D$$

$$\sum_{f\in F_{d'}}\sum_{\substack{d'\leq d+2\\ d'\geq d}}\sum_{n\in N}\left(lan_{fd'} - dep_{fd'}\right).x_{ifd'} \leq$$
$$up3 + M.\left(2 - (o_{id} + o_{id+1})\right) \quad (14)$$
$$; \forall i \in I, d \in D, (d \leq tot_{day} - 1)$$

$$\sum_{\substack{f\in CF_i\\ f\in F_d}} x_{ifdn} \geq 1 - M.(1 - o_{id}) - M.(o_{id-1}) \quad (15)$$
$$; \forall i \in I, d \in D, n = 1$$

$$\sum_{\substack{f\in Cl_i\\ f\in F_d}} u_{ifd} \geq 1 - M.(1 - o_{id}) - M.(o_{id+1}) \quad (16)$$
$$; \forall i \in I, d \in D$$

$$\sum_{f\in F_d} u_{ifd} \leq o_{id} \quad ; \forall i \in I, d \in D \quad (17)$$

$$\sum_{\substack{f'\in F_d\\ (f',f)\in s}} x_{if'dn} \geq 1 - M.\left(1 - x_{ifdn+1}\right) \quad (18)$$
$$; \forall i \in I, d \in D, f \in F_d, n \in N$$

$$\sum_{\substack{(f',f)\in s\\ f\in F_{d+1}}} x_{ifd+1n} \geq u_{if'd} - M.\left(2 - (o_{id+1} + o_{id})\right) \quad (19)$$
$$; \forall i \in I, d \in D, f' \in F_d, n = 1$$

$$\sum_{n\in N}\sum_{i\in I} x_{ifdn} = 1 \quad ; \forall d \in D, f \in F_d \quad (20)$$

$$\sum_{f\in F_d}\sum_{d\in D}\sum_{n\in N} x_{ifdn} \geq 1 \quad ; \forall i \in I \quad (21)$$

$$g_{id} \geq \sum_{f\in F_d}\sum_{n\in N}\left(lan_{fd} - dep_{fd}\right).x_{ifdn}$$
$$-(GH.o_{id}) \quad (22)$$
$$; \forall i \in I, d \in D$$

$$B_i \geq \sum_{p\in P} o_{id} \quad ; \forall i \quad (23)$$

$$\sum_{n\in N} y_{ifdn} \leq 1 - \sum_{n\in N} x_{ifdn} \quad (24)$$
$$; \forall i \in I, d \in D, f \in F_d$$

$$\sum_{\substack{f\in F_d\\ (f,f')\in s}} x_{ifdn} \geq 1 - M.\left(1 - y_{if'dn+1}\right) \quad (25)$$
$$; \forall i \in I, d \in D, f' \in F_d, n \in N$$

$$\sum_{\substack{f\in F_d\\ (f,f')\in s}} u_{ifd} \geq 1 - M.\left(1 - y_{if'd+1n}\right)$$
$$-M.(1 - o_{id}) \quad (26)$$
$$; \forall i \in I, d \in D, f' \in F_{d+1}, n \in N, n = 1$$

$$dep_{fd} - lan_{f'd} \geq \min\_sit$$
$$-M.\left(2 - x_{if'd,n} - y_{ifdn+1}\right) \quad (27)$$
$$; \forall i \in I, d \in D, f \in F_d, f' \in F_d, (f', f) \in s, n \in N$$

$$1440 + dep_{fd+1} - lan_{f'd} \geq \min\_rest$$
$$-M.\left(2 - u_{if'd} - y_{ifd+1,n}\right) \quad (28)$$
$$; \forall i \in I, d \in D, f \in F_{d+1}, f' \in F_d, (f', f) \in s, n = 1$$

$$\sum_{\substack{f\in F_d \\ f\notin CF_i}} y_{ifdn} \le M.(o_{id-1}) \quad ;\ \forall i\in I, d\in D, n\in N, n=1 \tag{29}$$

$$dep_{fd} \le dep_{f'd} + M.\left(2 - x_{if'dn} - y_{ifdn}\right) \quad ;\forall i, \in I, d\in D, f\in F_d, f'\in F_d, n\in N \tag{30}$$

$$o_{id+1} \ge o_{i'd+1} - M.\left(2 - x_{i'fdn'} - y_{ifdn}\right) \quad ;\forall i, i'\in I, i'\in csb_i, d\in D, f\in F_d, n, n'\in N \tag{31}$$

$$o_{id+1} \le o_{i'd+1} + M.\left(2 - x_{i'fdn'} - y_{ifdn}\right) \quad ;\forall i, i'\in I, i'\in csb_i, d\in D, f\in F_d, n, n'\in N \tag{32}$$

$$\sum_{f\in F_d}\sum_{n\in N} y_{ifdn} \le M.o_{id} \qquad ;\forall i\in I, d\in D \tag{33}$$

$$\sum_{f\in F_d{}'}\sum_{n''\ge n}(lan_{f'd} - dep_{f'd}).x_{if'dn''} + \sum_{f\in F_d{}'}\sum_{n''<n'}(lan_{f'd} - dep_{f'd}).x_{i'f'dn''} \le up1\ M.\left(2 - y_{i'fdn'} - x_{ifdn}\right) \quad ;\forall i, i'\in I, i'\in csb_i, d\in D, f\in F_d, n, n'\in N \tag{34}$$

$$\sum_{f\in F_{d'}}\sum_{\substack{d'\ge d-1\\ d'\le d+1}}\sum_{n''\ge n}(lan_{f'd'} - dep_{f'd'}).x_{if'd'n''} + \sum_{f\in F_{d'}}\sum_{\substack{d'\ge d-1\\ d'\le d+1}}\sum_{n''<n'}(lan_{f'd'} - dep_{f'd'}).x_{i'f'd'n''} \le up3 + M.\left(2 - y_{i'fdn'} - x_{ifdn}\right) \quad ;\forall i, i'\in I, i'\in csb_i, d\in D, f\in F_d, n, n'\in N \tag{35}$$

$$\sum_{\substack{i'\in I\\ i'\notin csb_i}}\sum_{n'\in N} y_{i'fdn'} \le M.(1 - \sum_{n\in N} x_{ifdn}) \quad ;\forall i\in I, d\in D, f\in F_d \tag{36}$$

$$\sum_{i\in I}|B_i - q| = \sum_{i\in I}(T_i + N_i) \tag{37}$$

$$B_i - q = T_i - N_i \quad ;\forall i \tag{38}$$

$$x_{ifdn}, z_{if'fdn}, u_{ifd}, y_{ifdn}, o_{id} = \{0,1\},\ B_i \ge 0\ and\ integer; g_{id}, T_i, N_i \ge 0 \quad ;\ \forall i\in I, d\in D, f, f'\in F_d, n\in N \tag{39}$$

The objective function (1) aims to maximize the number of reserved crew members. The objective function (2) includes four terms: crew cost, additional pay for flying times exceeding the required time for each crew member on duty days, hoteling cost and the deviation cost of the number of the duty days assigned to a crew member from an ideal one. Constraint (3) ensures that each pairing lasts for at most two consecutive duty days. Relation (4) states that no flight must be assigned to a crew member on his/her off day (*M*=number of flights on day *d*). Formula (5) ensures each flight can be assigned to any of the existing crew members. Based on Constraint (6), each crew member can cover at most one flight in the round. Constraint (7) demonstrates the last flight of each crew member on his/her duty day (*M*=1). Inequalities (8) and (9) are about the minimum sit time and minimum rest time, respectively ($M = \min\_sit + lan_{f'd}, M = \min\_rest + lan_{f'd}$). Constraint (10) restricts the total flying time on a duty day for each crew member. Constraint (13) is about elapsed time for each crew member. Constraint (14) limits each crew member's total flying time in his/her pairing (*M*= duration of all flights on days *d*, and *d*+1).

Inequalities (15) and (16) state that each pairing starts by a flight originating from the crew base and ends by another flight terminating to the same crew base (*M*=1). Constraint (17) ensures that a crew member should finish his/her duty day by one of his/her assigned flights. Constraints (18) and (19) ensure the integrity of flights assigned to a crew member (*M*=1). Equation (20) shows each flight must be assigned to only one of the existing crew members. Constraint (21) ensures that at least one flight must be assigned to each crew member. Constraint (22) shows the extra flying hour on a duty day, which is considered for the additional payment to the crew. Constraint (23) shows the number of duty days assigned to each crew member in the planning horizon. Constraint (24) implies that a crew member can be reserved for a flight only if the flight has not been assigned to him/her according to the schedule.

Constraints (25) and (26) state that a crew member can be reserved for a flight originating from city *j* while he/she has previously arrived there on either the same duty day or a day before, respectively (*M*=1). Constraint (27) indicates the minimum sit time for a crew member between two flights for which he/she is considered as a reserve crew and his/her previously assigned flight on the same day ($M = \min\_sit + lan_{f'd}$). Constraint (28) indicates the minimum rest time for a crew member between two flights for which he/she is considered as a reserve crew and his/her previously assigned flight on the day before ($M = \min\_rest + lan_{f'd}$). Constraint (29) ensures that a reserve crew should start his/her pairing by the flight originating from his/her home base. Constraint (30) ensures that a reserve crew's flight departure time should start earlier than his/her assigned flight according to the schedule ($M = dep_{fd}$). Constraints (31) and (32) ensure that both the reserve and the assigned crew to a flight should end their pairings on the same day (*M*=2). Constraint (33) implies that a crew member cannot be reserved for any flight on his/her off days. Constraints (34) and (35) show that the reserve crew's total flying time should be limited on his/her duty day and pairing, respectively (M= duration of all flights on days d, M= duration of all flights on days d *and* d*+1*). Finally, constraint (36) ensures both the reserve and the assigned crew to a flight must have the same home base. Since the proposed model includes a non-linearized absolute value term in the objective function, two non-negative variables (T_i, N_i) are defined and used for linearization of the model. Consequently, this term is replaced by Equation (37) and the additional related Constraint (38) is added to the model. Relation (39) shows the types of variables used in this study.

5. Solving Approach

Meta-heuristic algorithms can efficiently solve NP-hard problems by generating appropriate solutions in a reasonable time. In the current study, the ACO algorithm, as a major meta-heuristic algorithm, is used to solve the integrated crew pairing and rostering problem by considering the reserve crew teams. In this section, the proposed algorithm is introduced.

5.1. Ant colony optimization

ACO is swarm intelligence, a population-based, meta-heuristic algorithm designed by Dorigo et al. (1996) according to the behavior of the ants. Ants have a unique behavior in finding food sources outside their nests by choosing the shortest paths. They start this procedure by randomly depositing a chemical substance called pheromone trail while constructing their paths. The pheromone amount deposited on the path depends on the path quality; in other words, shorter paths are more likely to be selected in the next iterations. If an ant is positioned at node i, the following neighborhood node j has the following probability to be selected (equation (40)).

$$P_{ij} = \frac{[\tau_{ij}]^{\alpha}.[\eta_{ij}]^{\beta}}{\sum_{z \epsilon N_i}[\tau_{iz}]^{\alpha}.[\eta_{iz}]^{\beta}} \qquad ; \quad j \epsilon N_i \qquad (40)$$

where N_i is the set of neighborhood nodes connected to node i, α the relative importance of the trail, and β the relative importance of visibility. Moreover, τ_{ij} indicates the number of pheromone trails deposited on edge (i, j), and η_{ij} is the heuristic function obtained by subtracting the number of the uncovered flights between flights i and j from the number of flights in the planning horizon.

Next, the pheromone trails are updated so that a fraction of them on edge (i, j) is evaporated and the pheromone amount related to the path quality is added.

$$\tau_{ij}(t+1) = (1-\rho).\tau_{ij}(t) + \Delta\tau_{ij}(t) \qquad (41)$$

$$\Delta\tau_{ij}(t) = \frac{Q}{cost(T)} \;\; if \qquad (i,j) \in T \qquad (42)$$

where ρ is the evaporation rate, $\tau_{ij}(t)$ represents the number of pheromone trails on edge (i, j) in iteration t, and $\Delta\tau_{ij}(t)$ is the number of pheromone trails added to edge (i, j) in iteration t+1 (equation (41)). The additional pheromone trail is directly and inversely proportional to a fixed factor called Q and the quality of the tour ($cost(T)$), respectively (Eq. (42)).

At the end of each iteration, once the best solution is stored, the next iteration begins. This procedure continues until no better solution is achieved in the specified time period. The pseudo-code of the applied ACO algorithm is shown in Algorithm 1.

To represent the problem, the current study employs the flight graph representation, first introduced by Ozdemir and Mohan (2001), in which the nodes and arcs represent the flights and connections between them, respectively. Connecting arcs are assumed between all flight pairs, in which the destination of each flight is the departure point for the next one. A minimum time period between two sequential flights on the same day is required. Also, a minimum rest time period between two consecutive flights on sequential days is needed.

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t=0 and initialize all parameters (i.e., $n_k, \tau_O, \alpha, \beta, \rho, Q$).
Place all ants, k=1, …, n_k and initialize pheromone on each link;
Repeat
  For each ant k=1,… , n_k **do**
  Construct a path, $x_{(t)}{}^{k}$,
  End
  For each link (i, j) **do**
  Apply pheromone evaporation, and then update pheromone;
  $\tau_{ij}(t+1) = \tau_{ij}(t)$
  End
  t=t+1;
Until the stopping condition is true;

Algorithm 1. Pseudo code of ACO

In this study, the flight graph representation is revised based on the concept of inseparable flights first introduced by Azadeh et al. (2013). Inseparable flights are the sets of two flights, which should be assigned to a crew member sequentially. Moreover, two types of virtual nodes (i.e., initial and final nodes) are considered. The number of initial or final nodes is determined based on the number of the existing crew bases. If a flight starts from a specified crew base, a connecting edge should be considered between the flight and its relevant initial node. Also, if a flight ends at a specified crew base, a connecting edge is assumed between the flight and its relevant final node. On each day, flights are sorted based on their departure times, and each ant must take the sorted flights. In other words, each ant must start from an initial node and pass over some real nodes (i.e., flights) to reach a relevant final node. All the selected flights by the ant are then removed from the graph, and other ants must cover the remaining flights based on the aforementioned procedure. This procedure continues until other flights are covered.

5.2. Parameters setting

The algorithm efficiency depends on the parameters design, which was first introduced by Taguchi in the early 1960s. The ACO parameters are maximum iteration (t), number of ants (n_k), evaporation rate (ρ), a fixed value (Q), the importance of pheromone trails (α), and importance of visibility (β). Each parameter is valued at three levels. For example, t is valued as 30, 50, and 70. n_k is valued as 80, 100, and 120. Moreover, $1-\rho$ is valued as 0.9, 0.95, 0.99. Q is valued as 0.5, 0.7, and 0.9. Finally, α and β are valued as 2, 3, and 4. Based on the Design of Expert (DOE), 27 different scenarios are considered, each being run five times. Moreover, the mean *S/N* ratio plots are depicted for the ACO objective functions (Figure 2).

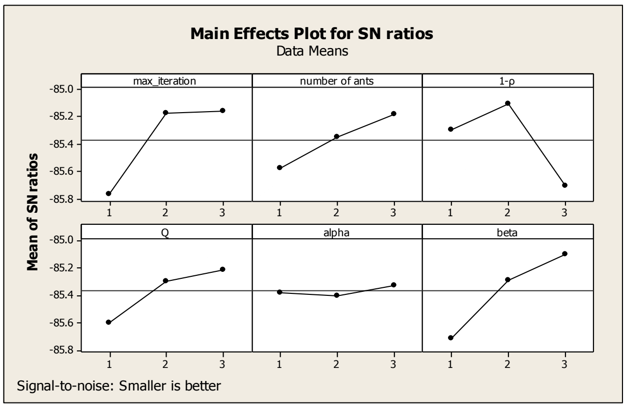

Fig. 2. Mean S/N ratio plot for each level of factors in ACO

The appropriate value for each parameter is obtained as follows: t =50, n_k =120, $1-\rho$ =0.95 , Q=0.9, $\alpha=4$ and $\beta=4$. Moreover, it should be noted that the initial pheromone is valued as 1 ($\tau_O=1$).

6. Computational Results

In this study, a set of problem instances extracted from the dataset (Kasirzadeh et al., 2017) is used to validate the integrated model and evaluate the performance of the applied algorithm. Table 1 shows these problems with their characteristics. As can be seen, seven test problems with different sizes are used. Columns 2-5 show the problem inputs including the planning horizon time, the number of flights, the number of crew members and the number of bases, respectively. The rest of the columns show the problem outputs. For example, Columns 6-9 show the computational results obtained by running the proposed model in GAMS software. It should be noted that the following single objective function (43) based on the LP-metric method is used for the problem.

$$\min z=\frac{f_1^*-f_1}{f_1^*}+\frac{f_2-f_2^*}{f_2^*} \tag{43}$$

Where, f_1 and f_2 refer to objective functions of the crew reserve and crew cost, respectively and the values f_1^* and f_2^* are their optimal amounts. Columns 10-12 demonstrate the ACO algorithm results run in MATLAB R2015a software. This algorithm is run 5 times for each problem, and the average cost function and CPU time are recorded in columns 10 and 12. Column 11 shows the best objective function of the ACO algorithm obtained in 5 runs. The last column (i.e., Column 13) represents the deviation of the ACO cost function from the optimal solution obtained by the exact method. This key parameter indicator (i.e., *GAP*) is one of the main indicators to compare the results and is measured by:

$$GAP=\frac{Min_{sol}-Gams_{sol}}{Gams_{sol}}*100 \tag{44}$$

Where $Gams_{sol}$ is the optimal solution obtained by the exact method and Min_{sol} is the minimum objective function obtained by running the ACO algorithm for five times.

As the mathematical model has the NP-hard complexity, the exact method is continued until sufficient CPU time (24 hours) for large-sized problems and the solutions are recorded as local solutions. As can be seen in Table 1, the ACO algorithm can obtain solutions with 2.57% average gaps in significantly shorter time in comparison to GAMS for small-sized problems.

In other words, based on Figures 3 and 4, the ACO algorithm can achieve appropriate solutions in significantly shorter CPU time, resulting in ACO outperformance in comparison to the exact method. Moreover, the ACO can obtain better solutions in a significantly less computational time compared to GAMS for large-sized problems. It should be noted the algorithm is run in a computer with specifications of 2.4GHz, Intel Corei7, 4GB of RAM, and Windows 7 (64bit).

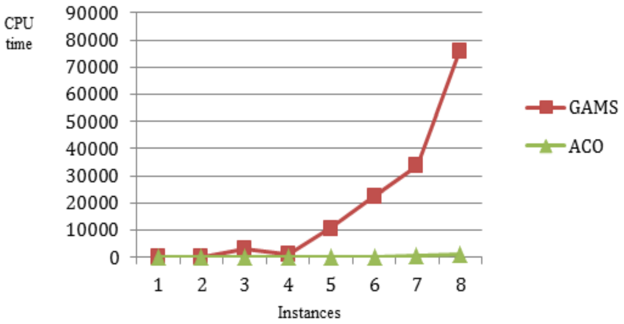

Fig. 3. Comparison of computational time between GAMS and ACO.

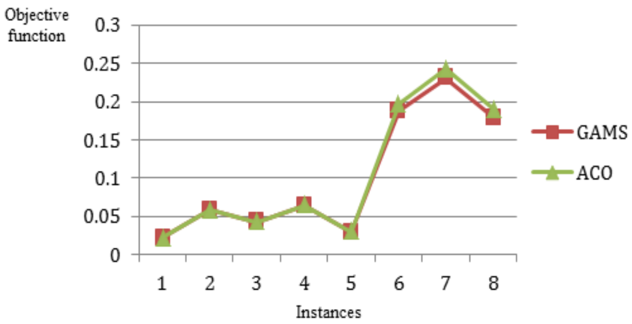

Fig. 4. Comparison of the obtained solutions between GAMS and ACO.

Table 1
Computational results

Instance	No. of days	No. of Flights	No. of crew members	No. of bases	GAMS				ACO algorithm			
					Objective function	CPU time (hr:min:sec)	No. of reserve crew teams	Status	Mean Objective function	Best Objective function	Mean CPU time (hr:min:sec)	Gap
1	2	20	6	2	0.021882	0:00:07	17	Opt.	0.021882	0.021882	0:00:1.41	0.00
2	2	24	8	2	0.05882	0:02:37	13	Opt.	0.05882	0.05882	0:00:2.24	0.00
3	2	32	9	1	0.04335	0:50:00	13	Opt.	0.04335	0.04335	0:00:7.79	0.00
4	3	30	8	2	0.06421	0:18:43	16	Opt.	0.06564	0.06507	0:00:20.8	1.34
5	3	54	11	2	0.02919	3:00:24	21	Opt.	0.03019	0.02996	0:00:59.5	2.64
6	4	58	12	2	0.1891	6:15:3	26	Opt.	0.1989	0.1973	0:01:38.2	4.36
7	5	52	9	3	0.2312	9:20:33	28	Opt.	0.24509	0.2436	0:06:07.5	5.37
8	4	118	17	3	0.1789	21:10:01	46	Opt.	0.1916	0.19102	00:16:4.4	6.78
9	5	394	24	3	0.2344	24:00:00	-	Local	0.2292432	0.225235	00:21:02.1	-
10	6	523	35	3	0.4519	24:00:00	-	Local	0.43829781	0.432513	00:39:43.4	-
11	5	721	46	3	0.8912	24:00:00	-	Local	0.86526608	0.845571	00:45:51.9	-
											Average GAP	2.57

7. Conclusion

Due to the complexity of the NP-hard problem, the Crew Scheduling Problem (CSP) was divided into two sub-problems, namely crew pairing problem (CPP) and crew rostering problem (CRP). The literature review showed that some studies separately explored these problems while only a few have used different algorithms to solve the integrated crew pairing and rostering problem. Therefore, this study presented a new bi-objective formulation for the integrated crew pairing and rostering problem by considering reserve crew with the objective functions of reserve crew maximization and crew cost minimization .It should be noted that the integrated problem could generate accurate and appropriate schedules for the crew members and, in turn, significantly reduce the costs. The variables in the formulation were defined in such a way that the pairings were simultaneously generated and assigned to crew members based on the rules about the CPP and the CRP. Also, the integrated problem was addressed by considering the reserve crew teams. It was noteworthy that the reserve crew teams could help to significantly decrease costs by reducing flight delays and cancelations on operation days, which might otherwise require the implementation of expensive recovery actions. The ACO algorithm was employed to solve the problem. As compared to the exact method, the computational results for small-sized problems in this study showed that the algorithm could achieve solutions with 2.57% average gaps in significantly less time. Furthermore, for large-sized problems, ACO could obtain better solutions in significantly less computational time as compared to GAMS. In other words, the ACO algorithm outperformed the exact method in terms of solutions and computational time. Future studies should focus on rescheduling crew members' work with minimum modifications on the operation day. Moreover, it is suggested that further research explores the integrated CSP by considering other regulations, like the 8-in-24 rule.

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