Journal of Optimization in Industrial Engineering
Vol. 16, Issue 2, Summer & Autumn 2023, 49-62
DOI: 10.22094/JOIE.2023.1984123.2057

Research Article

Forecasting Seasonal and Trend-Driven Data: A Comparative Analysis of Classical Techniques

Zahira Marzak*, Rajaa Benabbou, Salma Mouatassim, Jamal Benhra

Laboratory of Advanced Research in Industrial and Logistic Engineering (LARILE), OSIL Team, Department of Industrial and Logistic Engineering (GIL), ENSEM, Hassan II University of Casablanca, Casablanca, Morocco

Received 17 April 2023; Revised19 June 2023; Accepted 09 July 2023

Abstract

Making future predictions based on past and present data is known as forecasting. In the face of uncertainty, organizations rely on this valuable tool to make informed decisions, develop better strategies, and become more proactive. This study presents a comprehensive comparison of the performance of several classical quantitative forecasting methods, namely, Moving Average, Single Exponential Smoothing, Holt's Double Exponential Smoothing with a trend, Holt-Winter's Triple Exponential Smoothing with a trend and seasonality, ARIMA, ARIMAX, SARIMA, SARIMAX, and Multiple Linear Regression method. This research's aim is to identify the most effective technique for predicting weekly sales of a product, a critical aspect of supply chain management, with the emphasis being placed on the capability of each technique to capture the trend and seasonality components of the dataset. For this, an out-of-sample validation procedure was used; the evaluation of the performance of each technique's model was conducted using three accuracy metrics: Mean Absolute Percentage Error (MAPE), Mean Absolute Error (MAE), and Root Mean Square Error (RMSE). The results revealed that the SARIMAX model outperformed the other techniques, providing the most accurate forecasts for the product's weekly sales. This paper contributes to the field of industrial engineering by offering insights into the application of these classical quantitative forecasting methods in real-world scenarios, particularly in sales forecasting. The findings of this study can assist businesses and organizations in making up-to-date decisions and developing more effective and successful strategies.

Keywords: Forecasting; Moving Average; Exponential Smoothing; Holt-Winter, ARIMA; Regression.

1. Introduction

Forecasting is a fundamental practice in industrial engineering, particularly in the realm of supply chain management. The ability to predict future trends based on historical and current data allows organizations to make informed decisions, develop effective strategies, and create sustainable systems(You et al., 2009).
Accurate forecasting of trends and seasonality is vital for managing inventory, optimizing production, and improving customer satisfaction (Dallasega & Rauch, 2017). However, it also presents significant challenges due to factors such as market volatility, changing consumer behaviours, and external events that can disrupt trends(Anderson et al., 2000).
In recent years, the integration of classical forecasting methods with modern machine learning techniques has shown promise in improving forecasting performance (Dash et al., 2019).
Despite the significance and intricacy of this practice, there is a gap in the literature surrounding the comparative analysis of traditional quantitative forecasting approaches.
By providing a thorough comparison of several traditional quantitative forecasting techniques, such as Moving Average, Single Exponential Smoothing, Holt's Double Exponential Smoothing with a Trend, Holt-Winter's Triple Exponential Smoothing with a Trend and Seasonality, ARIMA, ARIMAX, SARIMA, SARIMAX, and Multiple Linear Regression, this study seeks to close this gap.
The comparison is based on an out-of-sample validation procedure, which is a common approach in comparative analyses of forecasting techniques across various disciplines.
This paper emphasizes the importance of synchronized production planning and control in achieving sustainable supply chains. It suggests that effective forecasting is crucial for managing inventory, optimizing production, and improving customer satisfaction.
The results of this study will provide valuable insights into the strengths and weaknesses of these methods when applied to real-world sales data, guiding practitioners in selecting the most suitable forecasting method for their specific needs.
Finding the best method for estimating a product's weekly sales, a crucial component of supply chain management, is the aim of this study.
The paper is structured as follows:

- Section 1 being this introduction,
- Section 2 provides an overview of forecasting methods and describes the traditional methods used in this study,
- Section 3 presents a brief synopsis of related material,
- Section 4 presents the chosen forecasting accuracy measures,
- Section 5 details the methodology employed in this study and introduces the data we used.
- Section 6 presents the empirical results and discusses the performance of all forecasting methods.
- Section 7 concludes the paper's key findings.

* Corresponding author Email address: zahira.marzak.doc21@ensem.ac.ma

- Finally, suggestions for future research are included in section 8.

2. Forecasting Methods

2.1 Classification of Forecasting Methods

Organizations have been trying to achieve sustainable competitive advantages in recent years by establishing effective and efficient supply chain systems. These systems rely on several metrics to minimize resources such as product quality, delivery performance, backorders and inventory levels while optimizing distribution channels through service quality and the service necessities of crucial companies (Mohamad et al., 2018).

Forecasting or predicting future events such as demand for a product emerged as a key element in supply chain management, enabling organizations to create sustainable systems and achieve a level of efficiency and effectiveness required to succeed on a global scale.

Nowadays, organizations rely on forecasting methods to make critical decisions and improve their business; these methods are primarily classified as qualitative and quantitative. An example of the categorization of the most commonly used forecasting techniques is shown in Fig. 1.

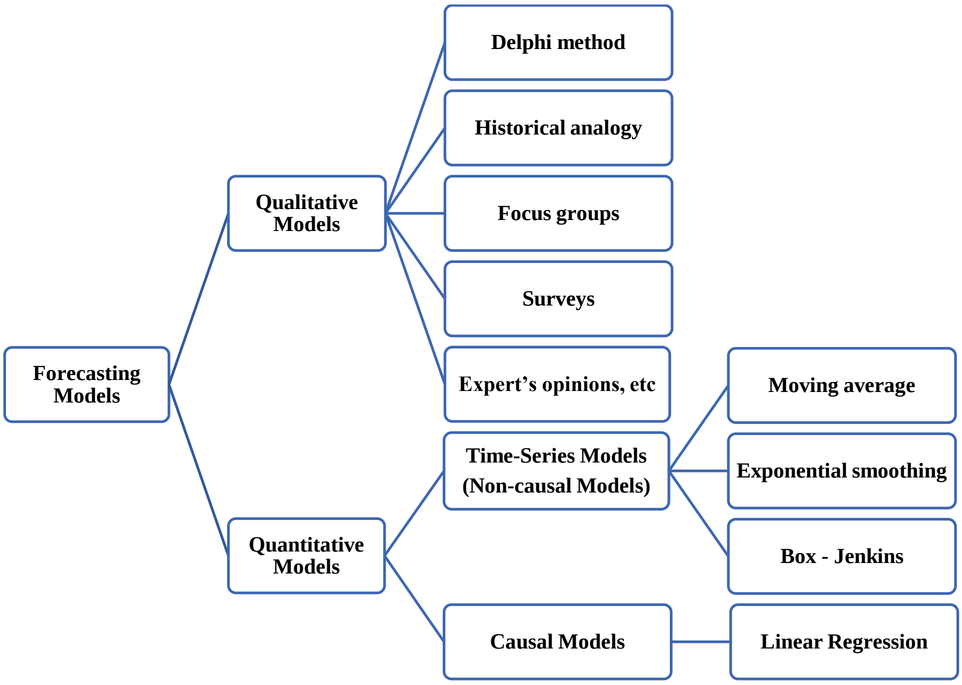

Fig. 1. Categorization of well-known Forecasting Models

Qualitative methods:

Qualitative methods are an interpretation of intuitive information used in the absence of historical data. Subjective in nature, they are often based on opinions, beliefs, experiences, perceptions, and judgment. They exhibit no analytic structure, nor rely on mathematical models. They are often used to make moderate or long-term decisions. Some examples of qualitative forecasting techniques are the Delphi method, historical analogy, focus groups, surveys, expert's opinions, etc.

Quantitative methods:

In contrast, quantitative methods are objective in nature and rely on the accessible data to construct models and make predictions, supposing that some characteristics of the past pattern will remain into the future (Makridakis et al., 1998). These models are usually used to make short or intermediate-range decisions, and fall into two major categories: non-causal and causal models (Wang & Chaovalitwongse, 2011).

- **Time-series models** are another name for non-causal models, they are used when patterns such as trends and seasonality are observed in past data, the predictions of future values are then generated using the extrapolation of the detected patterns, assuming a certain continuality of the patterns. These methods are usually used for single-variable datasets.

The analysis of time series consists of describing four basic properties:

- **Trend**: which describes the movement and tendency exhibited in the data.
- **Seasonality**: which represent patterns repeated at a fixed interval.
- **Cycles**: which correspond to periodical changes or patterns repeated at varying intervals.
- **Irregular variations**: which are other non-random sources of variations of series.

Different time series methods have been used to analyse these four properties, the most common of which are: Exponential Smoothing, Moving average, and Box-Jenkins models.

- **Causal methods**, on the other hand, assume a relationship between the forecasted variable and
- other factors that might explain and influence its behaviour. Regression models are the most popular causal models.

In a standard regression model, the relationship between the forecasted variable also called the response or the dependent variable, and its related independent predictor or explanatory factor, is investigated.

Table 1 summarizes the applicable situations for different types of forecasting models, as described by (Wang & Chaovalitwongse, 2011).

Table 1
Forecasting Models: Types, Applications, and Data Requirements

Model Category		Specific Situations	Forecasting period	Required Data
Qualitative Models		▪ Past data is unavailable ▪ There is no clear understanding of causality ▪ The data is too expensive to collect ▪ Short-term accuracy is not necessary	Can be used for both long-term and short-term forecasting	Background information and survey data
Quantitative Models	Time Series Models	▪ Historical time series data is available ▪ Presence of a stable patterns ▪ Short-term accuracy is necessary	Primarily used for short-term forecasting	Time series data
	Causal Models	▪ Past data available ▪ Clear and stable causal relationship ▪ Explanatory variables are controllable	Primarily used for short-term forecasting	Response and explanatory data

In this paper, the focus will be on the quantitative forecasting methods, namely time series, and causal methods, also known as the classical or traditional forecasting methods. Precisely, the following methods will be explored:

1. Multiple linear regression (MLR)
2. Moving average (MA),
3. Single Exponential Smoothing (SES),
4. Holt's Double Exponential Smoothing with trend (DES),
5. Holt-Winter's triple exponential smoothing with trend and seasonality (HW),
6. ARIMA and ARIMAX models,
7. SARIMA and SARIMAX models.

The aim is to determine the best forecasting method for a dataset describing the weekly sales of a product. The evaluation of the models performance was conducted using three accuracy metrics.

2.2 Overview of Classical Forecasting Models

1.1.1 Regression Models

Regression models has been in use since the early 20th century. These methods assumes a linear relationship between explanatory variables X_{1t}, X_{2t}, ... X_{nt} and the dependent variable Y_t:

A general linear regression model is given by (Dinesh Kumar, 2017):

$$F_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \cdots + \beta_n X_{nt} + \varepsilon_t \quad (1)$$

F_t is the forecasted value of Y_t, X_{1t} through X_{nt} are the explanatory variables or the predictors, β_i through β_n are the linear regression coefficients to be estimated, and ε_t is the error term.

1.1.2 Moving Average (MA)

The moving average, one of the earliest forecasting techniques which forecasts the future value of a time series by averaging previous data (Dinesh Kumar, 2017), is a simple yet effective technique for stable data, but struggles with trends and seasonality.

This technique was used decades before R.H. Hooker introduced it in 1901 using the term "**instantaneous averages"** which was later coined by G. U. Yule as "**moving-averages**" In 1909.

In the absence of trends and with the most recent N observations given the same weights, the forecast for period t+1 is given as follows:

$$F_{t+1} = \frac{1}{N}\sum_{k=t+1-N}^{t} Y_k \quad (2)$$

1.1.3 Simple Exponential Smoothing (SES)

Single or Simple exponential smoothing (SES) was first introduced by Robert Goodell Brown in 1956. Another common forecasting method that combines both the actual value with the predicted value of the latest period to generate a forecast.

A smoothing factor Alpha, with a value varying from 0 to 1, is used to calculate the weight assigned to the predicted value.

The equation that makes up the SES model at time (t + 1) is given by (Dinesh Kumar, 2017):

$$F_{t+1} = \alpha\, Y_t + (1 - \alpha)\, F_t \quad (3)$$

The SES method improved upon the Moving Average by giving more weight to recent data, but still didn't support trends and seasonality.

1.1.4 Holt's Double Exponential Smoothing (DES)

Holt's Double Exponential Smoothing, first introduced by Charles Holt in 1957, improved upon Simple Exponential Smoothing by handling trends in the data.
In the presence of a pattern or a linear trend, a Double Exponential Smoothing forecast is generated based on two equations: the level or the short-term average value and the trend:
Level (or Intercept) equation (L_t):

$$L_t = \alpha Y_t + (1 - \alpha)[L_{t-1} + T_{t-1}] \quad (4)$$

Trend equation (T_t):

$$T_t = \beta(L_t - L_{t-1}) + (1 - \beta)T_{t-1} \quad (5)$$

The forecast F_{t+1} using DES method is given by:

$$F_{t+1} = L_t + T_t \quad (6)$$

The forecast F_{t+n} for n periods ahead is given by:

$$F_{t+n} = L_t + nT_t \quad (7)$$

The smoothing factors for the level and trend are respectively, alpha and beta. Values of both vary between 0 and 1. Where n is the number of time periods into the future (Dinesh Kumar, 2017).

1.1.5 Holt-Winter's Model (HW) or ***Triple Exponential Smoothing (TES)***

Charles Holt and Peter Winters introduced the Triple Exponential Smoothing, commonly known as Holt-Winter's model in 1960. It is an extension of Holt's method used to capture the trend as well as the seasonality. In this case, the forecast is generated based on three equations (Dinesh Kumar, 2017):

For a multiplicative model:

Level (or Intercept) equation (L_t):

$$L_t = \alpha \frac{Y_t}{S_{t-c}} + (1 - \alpha)[L_{t-1} + T_{t-1}] \quad (8)$$

Trend equation (T_t):

$$T_t = \beta(L_t - L_{t-1}) + (1 - \beta) T_{t-1} \quad (9)$$

Seasonal equation (S_t):

$$S_t = \gamma \frac{Y_t}{L_t} + (1 - \gamma) S_{t-c} \quad (10)$$

The forecast F_{t+1} using HW method is given by:

$$F_{t+1} = [L_t + T_t] S_{t+1-c} \quad (11)$$

The forecast Ft + n for n periods ahead is given by:

$$F_{t+n} = [L_t + nT_t] S_{t+n-c} \quad (12)$$

For an additive model:

Level (or Intercept) equation (L_t):

$$L_t = \alpha (Y_t - S_{t-c}) + (1 - \alpha)[L_{t-1} + T_{t-1}] \quad (13)$$

Trend equation (T_t):

$$T_t = \beta(L_t - L_{t-1}) + (1 - \beta) T_{t-1} \quad (14)$$

Seasonal equation (S_t):

$$S_t = \gamma (Y_t - L_t) + (1 - \gamma) S_{t-c} \quad (15)$$

The forecast F_{t+1} using HW method is given by:

$$F_{t+1} = L_t + T_t + S_{t+1-c} \quad (16)$$

The forecast F_{t+n} for n periods ahead is given by:

$$F_{t+n} = L_t + nT_t + S_{t+n-c} \quad (17)$$

The smoothing factors for the level, trend and seasonality are respectively, alpha, beta and omega. Their values vary between 0 and 1. Where c is the length of seasonality.
The key to achieving an accurate forecast is selecting the optimal smoothing factors α, β and γ. Generally, this step is carried out by solving a non-linear optimization problem which consists of minimizing the Root Mean Square Error (RMSE) (Dinesh Kumar, 2017) or the average of squared forecasts error of the data set (M.S.E)(Rublíková, 1998).
Nearly all statistical programs offer an optimization option as part of their standard packages.

1.1.6 Autoregressive Integrated Moving Average (ARIMA)

Autoregressive Integrated Moving Average (ARIMA) also known as Box-Jenkins models was initially presented by George Box and Gwilym Jenkins in 1970, and since became the most commonly used linear model in forecasting non-stationary time series.
ARIMA models describe the autocorrelations in the data. It is a more complex technique that can handle trends. Only historical values of the forecast variable are used in the model which is an Auto Regressive Moving Average (ARMA) model combining the (AR) and (MA) components with an Integration component (I) to render the time series stationary.
The abbreviation ARIMA (p, d, q) is used to represent the model, where p is the order of AR part, d is the degree of differencing, while q is the order of the MA part.
AR (p) model equation:

$$Y_t = \sum_{i=1}^{p} \alpha_i Y_{t-i} + \varepsilon_t \quad (18)$$

MA (q) model equation:

$$Y_t = \sum_{i=1}^{q} \beta_i \varepsilon_{t-i} + \varepsilon_t \quad (19)$$

ARMA (p,q) model equation is given by:

$$Y_t = \sum_{i=1}^{p} \alpha_i Y_{t-i} + \sum_{i=1}^{q} \beta_i \varepsilon_{t-i} + \varepsilon_t \quad (20)$$

Where α_i,.., α_p, β_i,… β_q are the parameters of the models, ε_t are white noise error terms.
An ARIMA (p, d, q) model equation is given by:

$$\Delta^d Y_t = \sum_{i=1}^{p} \alpha_i \Delta^d Y_{t-i} + \sum_{i=1}^{q} \beta_i \varepsilon_{t-i} + \varepsilon_t \quad (21)$$

The first difference (d=1) is the difference between consecutive values of the time series (Y_t and Y_{t-1}) and is given by (Dinesh Kumar, 2017):

$$\Delta Y_t = Y_t - Y_{t-1} \quad (22)$$

The second difference (d=2) is the difference of the first difference and is given by(Dinesh Kumar, 2017):

$$\Delta^2 Y_t = \Delta(\Delta Y_t) = Y_t - 2Y_{t-1} + Y_{t-2} \quad (23)$$

1.1.7 Autoregressive Integrated Moving Average with Explanatory Variable (ARIMAX)

The ARIMAX model, also known as the dynamic regression model or vector ARIMA was introduced later as an extended version of the ARIMA model with contributions from various researchers.
While ARIMA is suitable for univariate datasets, ARIMAX is used for multivariate dataset where explanatory variables X_t are available and can be used to enhance the predictive power of model.
ARIMAX could be described simply as an ARIMA model with exogenous covariates, the model is represented as ARIMAX (p, d, q), and is given by:

$$\Delta^d Y_t = \sum_{i=1}^{p} \alpha_i \, \Delta^d Y_{t-i} + \sum_{i=1}^{q} \beta_i \, \varepsilon_{t-i} + \sum_{i=1}^{k} \gamma_i \, X_{i,t} + \varepsilon_t \quad (24)$$

1.1.8 Seasonal Autoregressive Integrated Moving Average (SARIMA)

The Seasonal Autoregressive Integrated Moving Average (SARIMA), also introduced in 1970, is a version of ARIMA that supports the modelling of seasonal element of univariate time series.
This variation of ARIMA adds new parameters to describe the seasonal component of the series.
SARIMA is represented as SARIMA (p, d, q) (P, D, Q) m , where p, d , and q are non-seasonal factors representing the order of the autoregressive component, degree of the first differencing, and order of the moving average component, while P, D , and Q are seasonal factors. In addition, the seasonal parameter m refers to the period of seasonality (Hyndman & Athanasopoulos, 2013).
The SARIMA model is similar to the ARIMA model, with an added set of autoregressive and moving average elements, offset by the frequency of seasonality.

SARIMA model is given by:

$$\Delta^d Y_t = \sum_{i=1}^{p} \alpha_i \, \Delta^d Y_{t-i} + \sum_{i=1}^{q} \beta_i \, \varepsilon_{t-i} + \sum_{i=1}^{P} \varphi_i \, \Delta^d Y_{t-\mathrm{m}i} + \sum_{i=1}^{Q} \eta_i \, \varepsilon_{t-\mathrm{m}i} + \varepsilon_t \quad (25)$$

A seasonal difference is the difference between an observation and the previous observation from the same season, a seasonally differenced series is given by (Hyndman & Athanasopoulos, 2013).

$$\Delta Y_t = Y_t - Y_{t-m} \quad (26)$$

While a twice differenced series is given by (Hyndman & Athanasopoulos, 2013) :

$$\Delta^2 Y_t = \Delta Y_t - \Delta Y_{t-1} \quad (27)$$

$$\Delta^2 Y_t = (Y_t - Y_{t-m}) - (Y_{t-1} - Y_{t-m-1}) \quad (25)$$

$$\Delta^2 Y_t = Y_t - Y_{t-1} - Y_{t-m} + Y_{t-m-1}) \quad (26)$$

1.1.9 Seasonal Autoregressive Integrated Moving Average with Explanatory Variable (SARIMAX)

The SARIMAX model is another version of the SARIMA model that was later introduced to enhance its predictive power. It is simply a Seasonal Auto-Regressive Integrated Moving Average with exogenous factors X_t ,
SARIMAX model, represented as SARIMAX (p, d, q) (P, D, Q)m and is given by:

$$\Delta^d Y_t = \sum_{i=1}^{p} \alpha_i \, \Delta^d Y_{t-i} + \sum_{i=1}^{q} \beta_i \, \varepsilon_{t-i} + \sum_{i=1}^{k} \gamma_i \, X_{i,t} + \sum_{i=1}^{P} \varphi_i \, \Delta^d Y_{t-\mathrm{m}i} + \sum_{i=1}^{Q} \eta_i \, \varepsilon_{t-\mathrm{m}i} + \varepsilon_t \quad (28)$$

Where $\alpha_i, .., \alpha_p, \beta_i, \dots \beta_q$, $\varphi_i, \dots \varphi_P$, $\eta_i, \dots \eta_Q$ are the parameters of the models, ε_t are white noise error terms, while the seasonal parameter m represents the length of the cycle.
Error! Reference source not found. provides a historical context for our review and shows the evolution of the classical forecasting method over time.

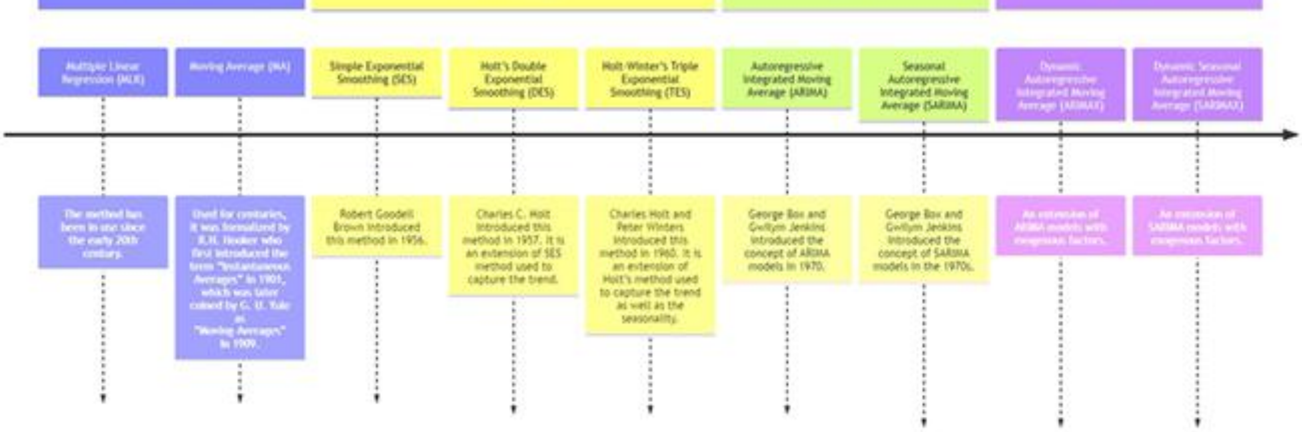

Fig. 2. Classical Forecasting Methods TimeLine

3. Literature review

3.1 Forecasting with classical methods

This section summarizes earlier studies that are relevant to our paper and illustrates the importance of classical forecasting methods in contemporary research.

These papers cover a range of applications for the classical forecasting methods we are comparing in our study.

Our literature review begins by exploring the importance of forecasting in supply chain management. Several studies have highlighted the role of accurate forecasting in boosting customer satisfaction, production optimization, and inventory management (Rahbari et al., 2023; Tang et al., 2023). However, forecasting trends and seasonality presents significant challenges due to factors such as market volatility, changing consumer behaviors, and external events that can disrupt trends (Widowati et al., 2023; Xu & Tan, 2023).

In an effort to increase the accuracy of forecasts for these datasets, numerous studies and research conducted by both academia and organizations employed traditional methods such as multiple linear regression models, moving average, and different types of exponential smoothing (simple, double, and triple) as well as the ARIMA and SARIMA models.

Some of the major features of these classical models, which made them very popular, are their simplicity and efficiency. Forecasts using these models could be made on the available computers using statistical software tools like R studio, Minitab, SPSS, Statgraphics Centurion, SAS or even Excel.

In this section, we review some of the work that has been done to compare the performance of these methods; the evaluation is usually based on the following accuracy metrics: Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE), and Root Mean Square Error (RMSE).

The early studies conducted on forecasting sales focused on the Moving Average (MA) and Single Exponential Smoothing techniques, both simple methods that can provide accurate forecasts when data is stable but may not perform well when there are trends or seasonal patterns (Saxena, 2020).

Researchers then moved on to more complex models, exploring the potential of different type of Exponential Smoothing techniques: from using Holt's model also known as the Double Exponential Smoothing (DES) to capture trends in data, and then Holt-Winters model (HW) or Triple Exponential Smoothing to capture trends as well as seasonality. While the most recent work focused on the ARIMA models (Ayyagari Venkata, 2018) . all of which can handle trends and seasonality but may be more complex and computationally intensive (Sharma & Nigam, 2020; Syaharuddin et al., 2020).

(Frank et al., 2003) conducted a comparative analysis to forecast women's clothing sales using HW and SES. The results showed that when it came to capturing seasonal trends, the HW model outperformed the SES model.

In another study to forecast Ford Mustang Sales Data (Ayyagari Venkata, 2018; RAWAL, 2021) used traditional methods, including MA, ES, and the ARIMA methods, the results showed that the DES method was superior in forecasting the sales yearly data.

(Ullah et al., 2016) came with the same conclusion while comparing the ARIMA models with exponential smoothing models in order to forecast the unemployment rate. The DES method was found to be the best model.

Recent studies have also emphasized the importance of accurate sales forecasting for balancing demand and supply. For instance, (Saxena, 2020) used ARIMA and SARIMAX models to predict the monthly sales of champagne for 10 years, highlighting the use of time series forecasting in various organizations and marketing research.

(Jaramillo & Carrión, 2022) outlines an approach for creating an adaptable model for sudden changes in electricity consumption by combining optimization with SARIMA time series.

(Ayad, 2022) used a seasonal ARIMA model to predict the long-term air temperature in the city of Tetouan.

(Zhang et al., 2022) combined SARIMA with the Gravitational Search Algorithm (GSA) to tackle the non-linear and seasonal characteristics of runoff data.

(Arshad & Tahir, 2022) used Deep learning (Long-Short Term Memory) with ARIMA to predict the upcoming demand of cardiac products.

(Syaharuddin et al., 2020) used the Holt Exponential Smoothing method in 2020 to forecast the rise in poverty in Indonesia during the following ten years.

(Sharma & Nigam, 2020) examined the Covid-19 pandemic's growing pattern in India using regression analysis, ARIMA model, and Exponential Smoothing and Holt-Winters models.

Table 2 provides a comparison of the key features of each forecasting method discussed in this review.

Table 2

Comparison Matrix of the different classical forecasting techniques

Forecasting Method	Handles Trend	Handles Seasonality	Multivariate	Complexity
Moving Average	No	No	No	Low
Simple or Single Exponential Smoothing	No	No	No	Low
Holt's Exponential Smoothing	Yes	No	No	Moderate
Holt-Winter's Exponential Smoothing	Yes	Yes	No	High
ARIMA	Yes	No	No	High
ARIMAX	Yes	No	Yes	High
SARIMA	Yes	Yes	No	High
SARIMAX	Yes	Yes	Yes	High

Multiple Linear Regression	Yes	No	Yes	Moderate

While

Table 3 summarizes their key characteristics, strengths and weaknesses.

Table 3
Key characteristics, Strengths and Weaknesses of classical forecasting techniques

Forecasting Method	Key Characteristics	Strengths	Weaknesses
Moving Average	Averages past observations good for stable data	Simple Easy to understand and implement	Does not handle trends or seasonality
Simple Exponential Smoothing	Gives more weight to recent observations Good for stable data	Simple Easy to understand and implement	Does not handle trends or seasonality
Holt's Exponential Smoothing	Handles trends	Can model linear trends	Does not handle seasonality More complex and computationally intensive
Holt-Winter's Exponential Smoothing	Handles trends and seasonality	Can model linear trends and seasonality	More complex and computationally intensive than other methods
ARIMA	Univariate Can handle trends	Flexible, Can model a variety of data patterns Can handle a wide range of time series data, as long as they are univariate	Can be complex to implement and interpret Not suitable for multivariate time series It involves extensive data preprocessing and tuning, as you need to check the stationarity, autocorrelation, and partial autocorrelation of the data, and find the optimal values of the parameters using trial and error or grid search.
ARIMAX	Extension of ARIMA that includes additional explanatory variables	Can model complex data patterns and include additional variables	Can be complex to implement and interpret
SARIMA	Extension of ARIMA that handles seasonal patterns	Can model complex data patterns including seasonality	Can be complex to implement and interpret
SARIMAX	Extension of SARIMA that includes additional explanatory variables	Can model complex data patterns including seasonality and include additional variables	Can be complex to implement and interpret
Multiple Linear Regression	Uses multiple explanatory variables to predict the outcome	Can model complex relationships between variables	Assumes a linear relationship between variables

In our previous work, we conducted a comparative analysis of seasonal and trend-driven data using the following classical techniques:

- Multiple linear regression (MLR)
- Moving average (MA),
- Single Exponential Smoothing (SES),
- Holt's Double Exponential Smoothing with trend (DES),
- Holt-Winter's triple exponential smoothing with trend and seasonality (HW),
- ARIMA and SARIMA models.

For this paper, we are going to try to improve our forecasts by using these same techniques in addition to the ARIMAX and SARIMAX models, making use of the available explanatory variables.

3.2 Research gap

Despite the depth of research on these forecasting techniques, there aren't many studies that thoroughly compare how well they predict weekly product sales. Since most research only employ one or two methods and frequently use distinct datasets, it is challenging to directly compare the findings of different studies. Few researches have also looked at how well these strategies operate throughout a range of time frames and seasonality types.
By contrasting a number of conventional quantitative forecasting techniques, this study seeks to close this gap by determining which technique is most suitable for this particular application. We compared the performance of various methods directly in order to gain a deeper knowledge of their advantages and disadvantages. To do this, we used a single dataset of weekly sales data.

4. Forecast Accuracy

The accuracy of the forecast is a crucial factor when comparing different forecasting methods; it consists of comparing the data forecasted by a predictive model with the actual data on an existing time period. The difference between the two values is known as the prediction error. The smaller the error, the more precise the model.
The most frequently used metrics to evaluate the performance of forecasting techniques are: Mean Absolute Percentage Error (MAPE), Mean Absolute Error (MAE), and Root Mean Square Error (RMSE).
A good forecasting ability is achieved by minimizing these three metrics (Wang & Chaovalitwongse, 2011). The smaller their value, the higher the prediction accuracy of the forecast.
For a validation data with n observations, the formulas for these metrics are given as follows (Dinesh Kumar, 2017):

$$\text{MAPE} = \frac{1}{n}\sum_{t=1}^{n}\frac{|Yt-Ft|}{Yt} \times 100\% \quad (29)$$

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(Yt-Ft)^2} \quad (30)$$

$$\text{MAE} = \sum_{t=1}^{n}\frac{|Yt-Ft|}{n} \quad (31)$$

Y_t is the real value of Y at time t, F_t is the predicted value and n is the number of predictions.
For the analysis, the MAPE is going to be our primary metric to evaluate the forecasting accuracy of our models.

Table 4 presents an interpretation of typical MAPE, as described by (Jafari-Samimi et al., 2007).

Table 4
Interpretation of typical MAPE values

MAPE	Forecasting ability
<10%	Excellent
10-20%	Good
20-50%	Reasonable
>50%	Bad

5. Methodology and dataset

5.1 Methodology

The methodology employed in this study involved an out-of-sample validation procedure, which is a common approach in comparative analyses of forecasting techniques across various disciplines.
The technique refers to using only a subset of the available data to build and train the model while withholding some of the sample data to evaluate its performance later on.
The procedure involves a number of steps:

Step 1: Data collection

The dataset used in this study comprised weekly sales data for a specific product collected from February 2010 to October 2012. The data contained 143 observations describing the total units sold weekly, along with information about holidays and promotion periods.

Step 2: Data splitting

Splitting the available data into two periods: a training period for estimating parameters and building models, and a test period for assessing the performance of the models, usually, it corresponds to the most recent observations.
For each period, the data size should be carefully chosen in order to achieve accurate forecasts.
As a rule, the use of 20% to 25% of the existing data for validation is recommended with a minimum of 40 observations (Poole et al., 2002), or 50 observations for accurate estimation (McCleary & Hay, 1980).
Overall, the sample size should be sufficient to detect the desired pattern and take into account the type of information being collected (Jebb et al., 2015).
For particular techniques like the ARIMA model, at least 30 observations are needed to produce a forecast that is both accurate and dependable (Box, G.E., Jenkins, G.M., Reinsel, G.C., Ljung, G.M., n.d.)..
Regression models on the other hand require using a minimum of 10 observations per variable.

Step 3: Model Building

The training data was used to estimate the parameters of the forecasting methods and then build the models.

Step 4: Forecast Generation

Forecasts for the test period were then produced using the adjusted models.

Step 5: Performance Evaluation

The performance of the models was evaluated based on the Root Mean Square Error (RMSE), the Mean Absolute Error (MAE), and the Mean Absolute Percentage Error (MAPE).

Step 6: Best Model Selection

The models with the best performance (lowest RMSE, MAE, and MAPE) was selected.
The out-of-sample validation procedure was carried out with the use of STATGRAPHICS Centurion a software with a statistically significant level of $p<0.05$.
This software has been used as a forecasting tool by many researchers using classical methods. For example, a study used STATGRAPHICS to build a Holt-Winters’ model predicting the number of employed and unemployed individuals in the Slovak Republic’s economy (Rublíková, 1998).
In another study, it was used to compare five forecasting techniques (linear and quadratic trend models, saturation curve models, Holt’s model, and ARIMA models to generate forecast for revenue and passenger traffic of the Sheremetyevo International Airport (Kochkina et al., 2021).
Recently, it was used to construct ARIMA models predicting the COVID-19 prevalence of Italy, France, and Spain (Ceylan, 2020).
The flow chart in Fig. 2. Methodology flowchart illustrate the steps of the methodology, from data collection to model selection.

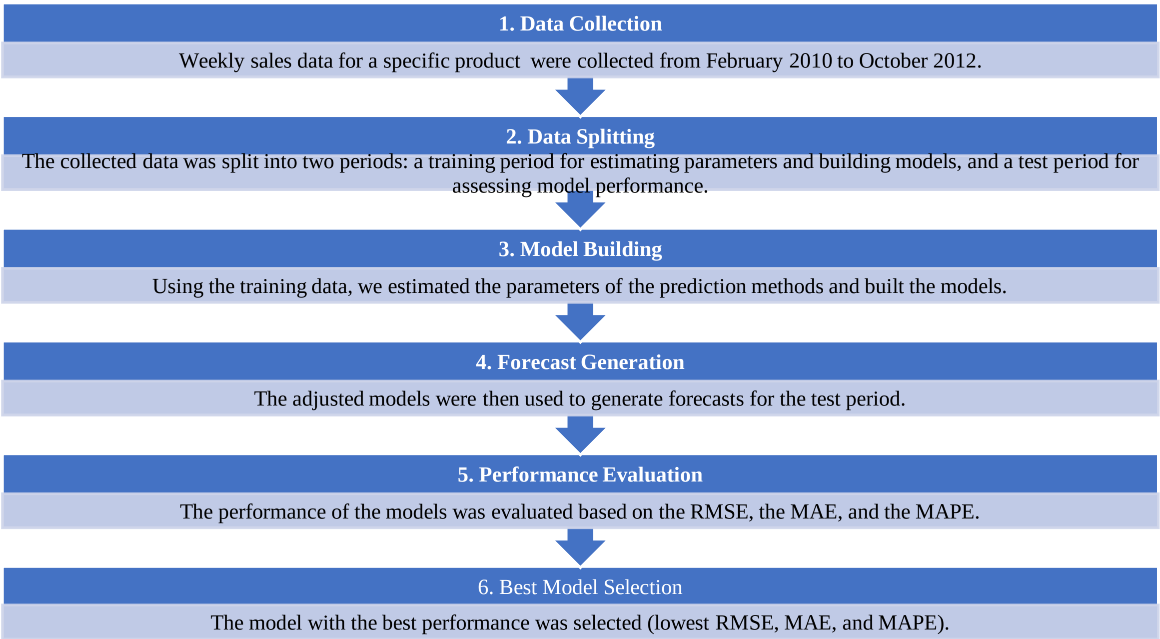

Fig. 2. Methodology flowchart

5.2 *Dataset: Weekly sales*

From February 2010 to October 2012, weekly sales data for a certain product was collected. The data contains 143 observations describing the total unit sold weekly as well as information about holidays and promotion periods.

Time series plot, Autocorrelation Function (ACF), and Partial Autocorrelation Function (PACF) graphs were plotted to examine trends, stationarity and seasonality of the time series and are presented, respectively, in Fig. 3, Fig. 4, and Fig. 5.

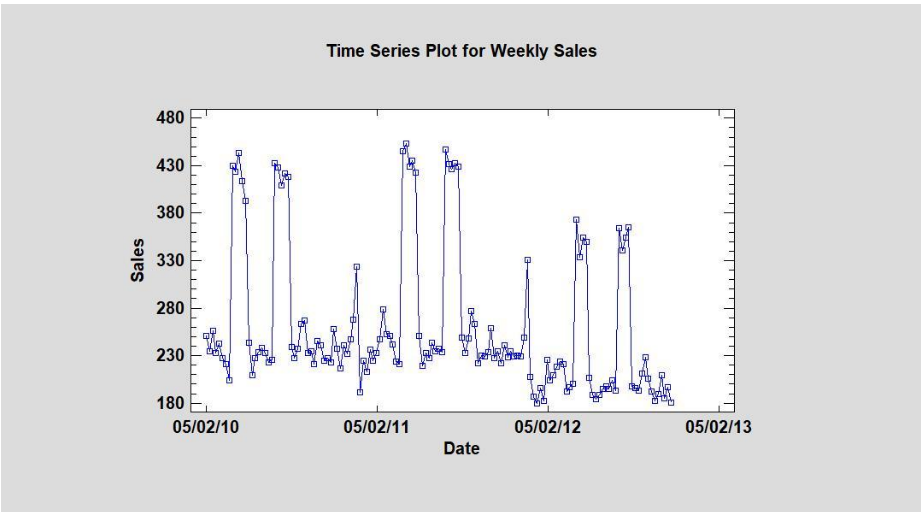

Fig. 3. Time Sequence Plot of Weekly Sales from 2010 to 2012

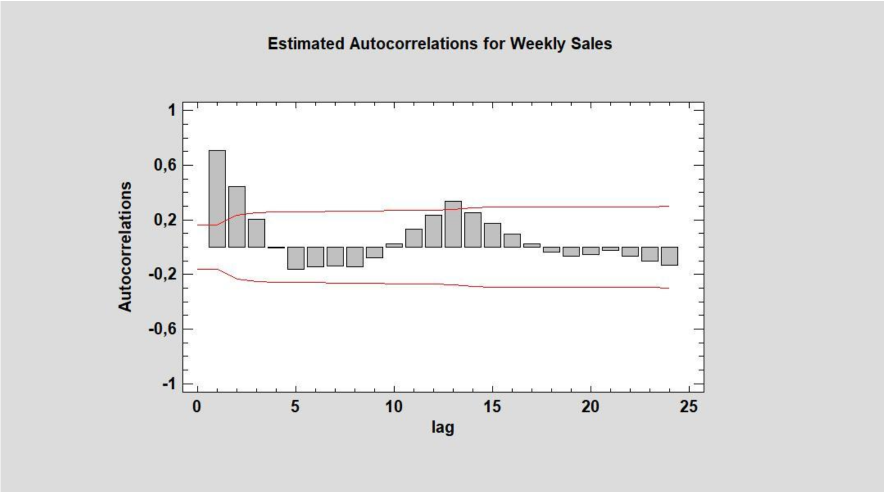

Fig. 4. Autocorrelations plot for the Weekly sales

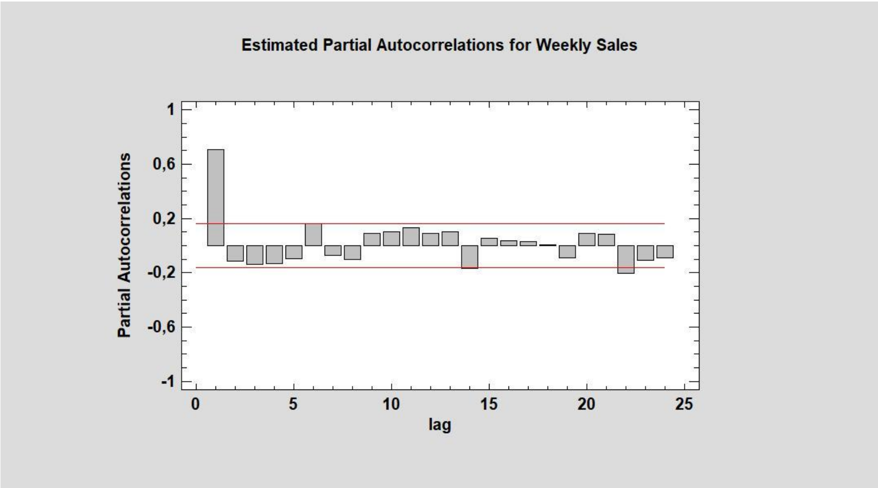

Fig. 5. Partial autocorrelations plot for the Weekly sales

The time series plot shows that the datasets is characterized by a trend and a seasonality.

Based on the visual inspection of the ACF graph, we can see that the ACF is slowly decaying, indicating a trend as well as non-stationarity, while the significant spike at lag 1 that decreases after a few lags, indicates an autoregressive term in the data.

Based on the plot of partial autocorrelations, a significant spike at lag 1 with much lower spikes for the following lags, suggests a first-order difference should be considered, while an AR(1) model is expected to be more suitable for this time series.

6. Results and Discussion

After dividing our data into training and test data, the training dataset was used to estimate and then optimize the parameters of the mathematical model for each method following the methodology presented in section 5.

Evaluation of the performance of the forecasting models was made based on comparing the prediction generated by the models with the actual data for both the training and the test periods. . The accuracy metrics used are MAPE, MAE, and RMSE. The empirical results of the analysis are shown in

Table 5.

Table 5
Error calculation

Model	Training Data Analysis			Test Data Analysis		
	RMSE	MAE	MAPE	RMSE	MAE	MAPE
Moving Average						
N=2	65,8961	36,8491	13,0613%	61,2388	34,6286	14,3686%
N=3	72,4646	44,2889	15,6926%	67,4921	43,5238	18,0089%
N=4	78,0937	50,2019	17,5912%	73,5689	51,3071	21,0818%

Exponential Smoothing						
SES	59,3737	30,6408	11,0890%	57,0533	29,4427	11,9167%
DES	60,0298	30,8051	11,0614%	57,2879	29,5347	**11,8756%**
Holt-Winter	22,6385	16,4578	5,9736%	41,3721	24,3637	**10,3296%**
ARIMA						
(1,0,0)	55,3193	33,4464	11,7244%	53,1820	35,4442	15,1095%
(1,0,1)	55,2119	33,1598	11,6484%	53,3831	36,8031	15,6030%
(1,0,2)	55,3052	33,0137	11,6024%	53,1540	36,9743	15,6796%
(1,1,0)	59,9406	30,9148	11,1788%	57,0418	29,4079	11,8952%
(1,1,1)	60,2235	30,9099	11,1788%	57,0451	29,4294	11,9043%
(1,1,2)	60,3731	30,7061	11,1238%	57,1231	30,3916	12,2641%
ARIMAX						
(1,0,0)	20,8711	14,4765	5,79338%	35,5891	31,3157	13,4213%
(1,0,1)	20,9598	14,5231	5,81469%	36,2635	32,1376	13,8043%
(1,0,2)	20,6801	14,1782	5,64792%	35,0379	31,2524	13,4393%
(1,1,0)	24,3778	16,6849	6,68189%	19,7276	14,6112	**6,3397%**
(1,1,1)	21,465	14,606	5,76013%	27,2683	22,4487	9,3305%
(1,1,2)	21,3603	14,7058	5,85553%	27,665	22,8682	9,4954%
SARIMA						
$(1,1,1)x(1,0,0)_{52}$	11,3084	6,65176	2,74076%	54,1281	25,5382	12,1837%
$(1,1,1)x(1,1,1)_{52}$	2,80908	0,7603	0,3299%	69,0473	45,1523	18,8866%
$(1,1,0)x(1,0,0)_{52}$	12,55610	6,8508	2,7800%	59,2801	30,8260	13,4902%
$(1,1,0)x(1,0,1)_{52}$	11,9997	8,22613	3,25544%	60,4789	31,5203	13,9096%
$(0,0,0)x(1,1,1)_{52}$	3,19127	0,7687	0,3447%	53,7891	27,3874	12,1330%
$(1,1,1)x(1,1,0)_{52}$	2,78380	0,6429	0,2966%	68,6640	44,7578	18,6923%
$(0,1,1)x(1,1,0)_{52}$	6,19611	1,4212	0,6517%	49,9107	27,3481	13,1267%
$(0,1,0)x(1,1,0)_{52}$	19,0524	14,2790	5,7353%	69,1614	33,9636	14,3923%
SARIMAX						
$(1,1,1)x(1,0,0)_{52}$	11,3257	6,71786	2,76681%	20,1695	14,6425	**5,8665**%
$(1,1,1)x(1,1,1)_{52}$	17,5368	12,2805	5,1120%	57,7974	33,4046	14,8570%
$(1,1,0)x(1,0,0)_{52}$	12,5634	7,1149	2,8802%	20,4227	15,5380	6,4276%
$(1,1,0)x(1,0,1)_{52}$	12,7149	8,02933	3,23894%	19,1032	15,0565	6,4830%
$(0,0,0)x(1,1,1)_{52}$	32,0253	27,8146	10,5370%	62,3006	37,2316	16,1490%
$(1,1,1)x(1,1,0)_{52}$	17,4580	12,3015	5,1293%	58,1314	33,7569	15,0312%
$(0,1,1)x(1,1,0)_{52}$	16,6585	12,3864	5,0439%	56,8565	28,7568	12,8994%
$(0,1,0)x(1,1,0)_{52}$	19,4376	14,2956	5,7420%	69,1193	33,9231	14,3713%
Regression						
MLR	21,9257	14,4955	5,79824%	46,5411	42,1784	18,0423%

From the

Table 5, we can see that all traditional methods performed well with MAPE values of less than 20%, their forecasting ability is classified as good to excellent in the training period.

As for the test period, two methods performed well producing MAPE values of less than 10%, classifying their forecasting ability as excellent.

- The first best model is SARIMAX, which proved to be the best fit for our dataset. The chosen model SARIMAX$(1,1,1)x(1,0,0)_{52}$ registered the lowest values of our three metrics RMSE, MAE and MAPE compared to the other methods for this period with values of 20,1695, 14,6425 and 5,8665% respectively, reflecting its excellent forecasting ability.

- The next best model is ARIMAX, the model ARIMAX (1,1,0) with a MAPE value of 6,3397% registered the lowest values of RMSE and MAE, 19,7276 and 14,6112 respectively.
- The ES techniques came in third place with a good forecasting ability: HW with a MAPE of 10, 3296%, and DES with a MAPE of 11,8756%.

Fig. 6 presents a bar graph summarizing the performance of our classical forecasting methods based on RMSE, MAE, and MAPE.
Based on these results, the best forecasting method for data exhibiting trends and seasonality is found to be the SARIMAX model.

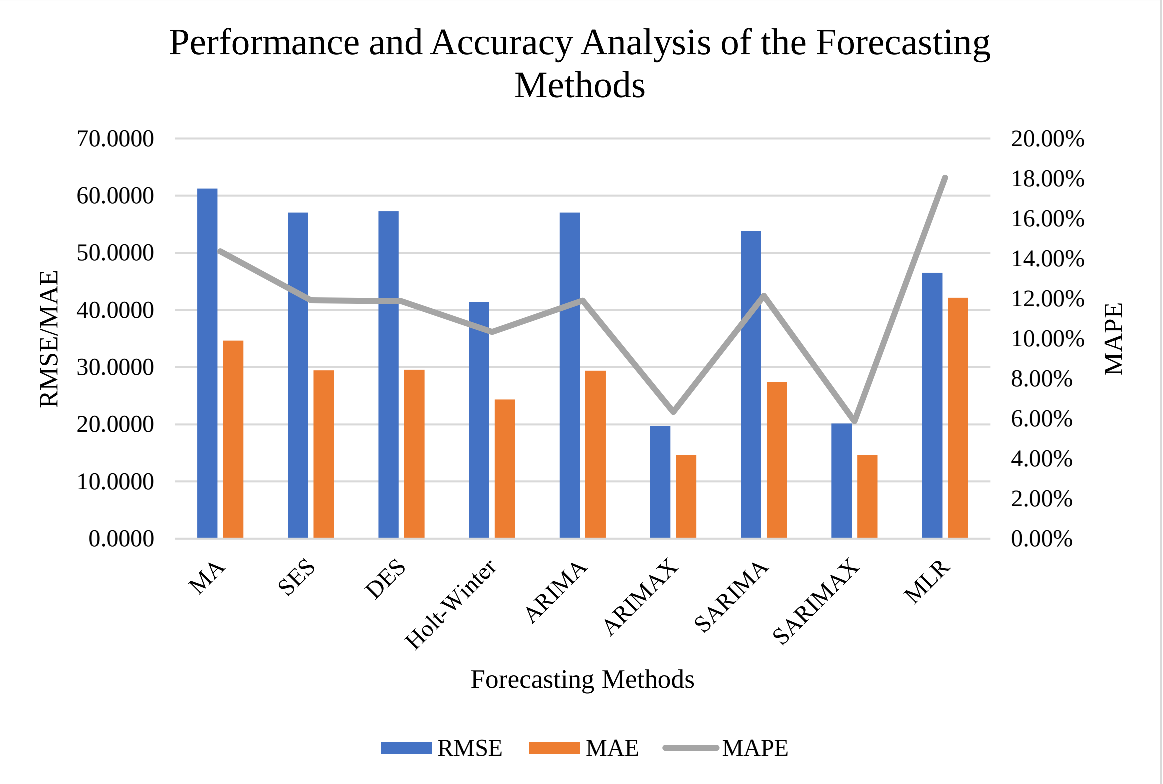

Fig. 6. Comparison Bar Graph

7. Conclusion

This study aimed to compare the performance of several classical quantitative methods, namely, Moving Average, Single Exponential Smoothing, Holt's Double Exponential Smoothing with a trend, Holt-Winter's Triple Exponential Smoothing with a trend and seasonality, ARIMA, ARIMAX, SARIMA, SARIMAX, and Multiple Linear Regression method. The methodology employed an out-of-sample validation procedure, which is a common approach in comparative analyses of forecasting techniques across various disciplines.
The dataset used in this study comprised weekly sales data for a specific product collected from February 2010 to October 2012. The data contained 143 observations describing the total units sold weekly, along with information about holidays and promotion periods.
The top-performing models were selected based on the lowest value of three accuracy metrics (MAPE, RMSE, and MAE).
The results of the analysis showed that all traditional methods performed well with MAPE values of less than 20%, classifying their forecasting ability as good to excellent in the training period. For the test period, two methods performed well, producing MAPE values of less than 10%, classifying their forecasting ability as excellent. The SARIMAX model proved to be the best fit for our dataset, registering the lowest values of RMSE, MAE, and MAPE compared to other methods. The ARIMAX model also performed well, registering the lowest values of RMSE and MAE. The Exponential Smoothing (ES) techniques came in third place with a good forecasting ability.
The results indicated that the SARIMAX models are superior in modelling multivariate datasets with trend and seasonality to some extent.
In conclusion, the study provides valuable insights into the performance of various classical forecasting methods when applied to real-world sales data. The findings can guide practitioners in selecting the most suitable forecasting method for their specific needs.

Perspectives

Forecasting is a complex task that requires many steps to build a suitable model able to predict future values for a specific dataset. This process is extremely challenging in the case of trend and seasonal driven dataset.

The findings of this comparison analysis have demonstrated that traditional forecasting techniques can produce statistically valid and trustworthy predictions.

However, these techniques as all forecasting techniques have their advantages and disadvantages that could be further investigated.

Looking ahead, there are several promising directions for future research. While this study focused on classical forecasting methods, there is potential to explore the integration of these methods with modern machine learning techniques. Hybrid models that combine the strengths of classical and modern forecasting techniques could offer improved forecasting performance.

Additionally, the use of these techniques could be broadened to include data sets other than sales, such supply chain demand, energy use, or financial market movements. The research's application and significance would increase as a result.

Finally, there is room to look further into the variables that affect how well various forecasting techniques function. Understanding these elements could result in the creation of recommendations for choosing the best method based on the particulars of the dataset at hand.

References

Anderson, E. G., Fine, C. H., & Parker, G. (2000). *Upstream Volatility in the Supply Chain: The Machine Tool Industry as a Case Study* (SSRN Scholarly Paper No. 1938442). https://doi.org/10.2139/ssrn.1938442

Arshad, M. W., & Tahir, S. F. (2022). Sales Prediction of Cardiac Products by Time Series and Deep Learning. *International Journal of Innovations in Science & Technology*, *4*(5), Article 5.

Ayad, S. (2022). Modeling and forecasting air temperature in Tetouan (Morocco) using SARIMA model. *Journal of Earth Sciences and Geotechnical Engineering*, 1–13. https://doi.org/10.47260/jesge/1221

Ayyagari Venkata, R. (2018). *Comparative Analysis of Different Forecasting Techniques for Ford Mustang Sales Data*. http://ieomsociety.org/forefront/ViewPaper.php?PaperID=3090

Box, G.E., Jenkins, G.M., Reinsel, G.C., Ljung, G.M. (n.d.). *Time Series Analysis. Forecasting and control* (Fifth Edition).

Ceylan, Z. (2020). Estimation of COVID-19 prevalence in Italy, Spain, and France. *Science of The Total Environment*, *729*, 138817. https://doi.org/10.1016/j.scitotenv.2020.138817

Dallasega, P., & Rauch, E. (2017). Sustainable Construction Supply Chains through Synchronized Production Planning and Control in Engineer-to-Order Enterprises. *Sustainability*, *9*(10), Article 10. https://doi.org/10.3390/su9101888

Dash, R., McMurtrey, M., Rebman, C., & Kar, U. K. (2019). Application of Artificial Intelligence in Automation of Supply Chain Management. *Journal of Strategic Innovation and Sustainability*, *14*(3), Article 3. https://doi.org/10.33423/jsis.v14i3.2105

Dinesh Kumar, U. (2017). *Business analytics: The science of data-driven decision making*. Wiley India.

Frank, C., Garg, A., Sztandera, L., & Raheja, A. (2003). Forecasting women's apparel sales using mathematical modeling. *International Journal of Clothing Science and Technology*, *15*, 107–125. https://doi.org/10.1108/09556220310470097

Hyndman, R. J., & Athanasopoulos, G. (2013). *Forecasting: Principles and practice*. OTexts.

Jafari-Samimi, A., Shirazi, B., & Fazlollahtabar, H. (2007). A Comparison Between Time Series, Exponential Smoothing, and Neural Network Methods To Forecast GDP of Iran. *Iranian Economic Review*, *12*(19), 19–35. https://doi.org/10.22059/ier.2007.32639

Jaramillo, M., & Carrión, D. (2022). An Adaptive Strategy for Medium-Term Electricity Consumption Forecasting for Highly Unpredictable Scenarios: Case Study Quito, Ecuador during the Two First Years of COVID-19. *Energies*, *15*(22), Article 22. https://doi.org/10.3390/en15228380

Jebb, A. T., Tay, L., Wang, W., & Huang, Q. (2015). Time series analysis for psychological research: Examining and forecasting change. *Frontiers in Psychology*, *6*. https://www.frontiersin.org/article/10.3389/fpsyg.2015.00727

Kochkina, E., Radkovskaya, E., & Denezhkina, K. (2021). Analysis and forecasting of performance indicators of air transport facilities. *E3S Web of Conferences*, *291*, 08009. https://doi.org/10.1051/e3sconf/202129108009

Makridakis, S. G., Wheelwright, S. C., & Hyndman, R. J. (1998). *Forecasting: Methods and applications* (3rd ed). John Wiley & Sons.

McCleary, R., & Hay, R. (1980). *Applied time series analysis for the social sciences*. Sage Publications.

Mohamad, M., Ali, M., Abdullah, A. S., & Elfiky, A. S. (2018). The Usage of Social Media and E-Reputation System in Global Supply Chain: Comparative Cases from Diamond & Automotive Industries. *International Journal of Communications, Network and System Sciences*, *11*(05), 69–103. https://doi.org/10.4236/ijcns.2018.115006

Poole, M. S., McPhee, Robert. D., & Canary, Daniel. J. (2002). Hypothesis testing and modeling perspectives on inquiry. In *Handbook of interpersonal communication* (3rd ed, pp. 23–72). SAGE Publications.

Rahbari, M., Khamseh, A. A., & Mohammadi, M. (2023). A novel multi-objective robust fuzzy stochastic programming model for sustainable agri-food supply chain: Case study from an emerging economy. *Environmental Science and Pollution Research*, *30*(25), 67398–67442. https://doi.org/10.1007/s11356-023-26305-w

RAWAL, P. (2021). Suzuki Swift Marketing Data Comparative Study of Different Forecasting Methods. *Journal of Contemporary Issues in Business and Government*, *27*(3). https://doi.org/10.47750/cibg.2021.27.03.144

Rublíková, E. (1998). Development of employment and unemployment in hotels and restaurants of Slovakia. *Tourism and Hospitality Management*, *4*(1), 107–112.

Saxena, M. A. (2020). An innovative time series based method of forecasting monthly sales of champagne. In *Samvakti Journal of Research in Information Technology* (202007.50.281; Vol. 1, Issue 1, pp. 77–88). Samvakti Journals. https://www.samvaktijournals.com/journal/202007.50.281

Sharma, V. K., & Nigam, U. (2020). Modeling and Forecasting of COVID-19 Growth Curve in India. *Transactions of the Indian National Academy of Engineering*, *5*(4), 697–710. https://doi.org/10.1007/s41403-020-00165-z

Syaharuddin, S., Ahmad, A., Mandailina, V., Pramita, D., Mahsup, M., & Abdillah, A. (2020). Amount of Poverty as Policy Basis: A Forecasting Using The Holt Method. *Jurnal Varian*, *4*(1), Article 1. https://doi.org/10.30812/varian.v4i1.849

Tang, Y. M., Chau, K. Y., Lau, Y., & Zheng, Z. (2023). Data-Intensive Inventory Forecasting with Artificial Intelligence Models for Cross-Border E-Commerce Service Automation. *Applied Sciences*, *13*(5), Article 5. https://doi.org/10.3390/app13053051

Ullah, M., Su, K., & Jan, B. (2016). *Forecasting, Cointegration and Causality Analysis of Unemployment Using Time Series Models* (SSRN Scholarly Paper No. 2863673). Social Science Research Network. https://doi.org/10.2139/ssrn.2863673

Wang, S., & Chaovalitwongse, W. (2011). Evaluating and Comparing Forecasting Models. In *Wiley Encyclopedia of Operations Research and Management Science*. https://doi.org/10.1002/9780470400531.eorms0307

Widowati, -, Sutrisno, -, & Utomo, R. H. S. (2023). The Joint Decision-Making Support through Piecewise Objective Optimization Model for Integrated Supplier Selection, Inventory Management, and Production Planning Involving Discounts. *JOIV : International Journal on Informatics Visualization*, *7*(1), Article 1. https://doi.org/10.30630/joiv.7.1.1328

Xu, S., & Tan, A. (2023). The Impact of Diversity, Inclusion and Equity Product Offering and Forecast Bias on Supply Chain Decisions. *International Journal of Economics, Business and Management Studies*, *10*(1), 1–9. https://doi.org/10.55284/ijebms.v10i1.821

You, F., Wassick, J. M., & Grossmann, I. E. (2009). Risk management for a global supply chain planning under uncertainty: Models and algorithms. *AIChE Journal*, *55*(4), 931–946. https://doi.org/10.1002/aic.11721

Zhang, X., Wu, X., Zhu, G., Lu, X., & Wang, K. (2022). A seasonal ARIMA model based on the gravitational search algorithm (GSA) for runoff prediction. *Water Supply*, *22*(8), 6959–6977. https://doi.org/10.2166/ws.2022.263