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Research Article

Developing a Transfer Point Location Problem Considering Normal Demands Distribution

Ammar Mollaie [a], Soroush Avakh Darestani [b,*,a], Deneise Dadd [c]

[a] *Department of Industrial Engineering, Faculty of Industrial and Mechanical Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran*
[b] *Guidhall School of Business and Law, London Metropolitan University, London, United Kingdom .*
[c] *School of Strategy and Leadership, Faculty of Business and Law, Coventry University, Coventry, United Kingdom*
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Abstract

As related to the center location problem, transfer point location problems (TPLP) are gaining attention as a reliable prediction model. For this, one normally assumes no transformation directly from a demand point to the service facility location, which leads to the transfer point being constantly engaged. We propose a method for finding the best location for the transfer point so that the maximum expected weighted distance to all demand points through the transfer point is minimized. A mathematical solution is employed for demand points follow normal distribution, with some points of demands being in regions. Then, this model was validated using real data under real conditions. We used the Maple software to solve the optimization problem, as well as MATLAB software to solve this model numerically.

Keywords: Transfer point location problem; Distribution ; Normal distribution.

1. Introduction

Locations are highlighted in order to determine the location of a device for achieving the desired objectives. It is done to identify important criteria, such as proximity to main roads, consumption market, supplies of raw materials, availability of essential human requirements, conditions environment, developmental possibilities, regulations and state laws, and so on. The problem of locating and deploying facilities is one of the research issues that should be considered in the early stages of designing industrial systems.

Facility design defines how the constants of activity would provide the best support for its purposes. Issues around facility planning are divided into four major categories: location, routing, allocation, and design. By combining these components, location-routing, and location-allocation issues are achieved. Also, there are two attitudes to classifying location problems, traditional attitude and a new attitude.

In the realm of the transfer point location problem (TPLP), the main objective is to establish a new facility which serves n demand points. This allows the applicants to be conveyed to the transfer point at regular speed and then from there they can be taken to the facility at a greater speed. Transfer incur a cost that nevertheless may be compensated by savings both in costs of the installation and distribution (Corberána et al., 2020). Its purpose is to find the best location for the transfer point in order to minimize the sum of distance to all the demand points through the transfer point. TPLP is typically used in hospitals, which involve the transference of patients through a helicopter pad. Patients are transferred by ambulance to the transfer point (the 'helicopter pad') at regular speed. Then they are flown to the hospital at a greater speed from there. These problems include some variants: in the multiple locations of transfer points (MLTP), the location of the facility is known, and we need to cluster the points into subsets, each served via a single transfer point.

In this work, the location of the facility is assumed to be known and the objective is to make it accessible with a minimum time and cost. Demand points are locations which are taken for granted as some nodes. These points are also assumed to be known here. Hubs or transfer points are the nodes which combine the service with the demand points. Campbell et al (2002) defined them as "the facilities that are servicing many origin-destination pairs as transformation and trade off nodes, and are used in traffic systems and telecommunications". Unit traveling cost from the transfer point to the facility is reduced by a factor of 0<?<1 (Cambell et al., 2002).

In this paper, we consider a case study, where in demand points are weighted and their coordinates assumed to have a normal distribution. This scheme is employed to make the model more applicable in real world situations. The problem is to find the best location for the transfer points so that the maximum expected weighted distance to all demand points through the transfer point is minimized.

The main contribution is the consideration made on the location of the facility and the set of demand points, namely demand points being weighted based on a normal distribution.

As probabilistic parameters of model may have different type of probability distribution functions, but for simplicity (Yousefi et al., 2018), normal distribution is employed for all demand points in this work. Here, two

*Corresponding author Email address: s.avakhdarestani@londonmet.ac.uk

models are discussed; the one in which all the points are considered in one area and the second one which includes several areas. The research is to find the best location for the transfer point so that the maximum expected weighted distance to all demand points through the transfer point is minimized.

2. Literature Review

The hub location problem is employed for many real applications, including delivery, airline and telecommunication systems and so on (Avakh and Rajabi, 2018). Different kinds of location problems related to centre location problem. They can be categorized as follows: Hub and Spoke Location Model (HSLM), Location-Routing Problem (LRP) and TPLP.

2.1. The hub and spoke model

The hub and spoke model, first suggested for airline travel, seeks to select a set of hubs so that travellers will use one or more hubs as stops on the way to their destination (Toh & Higgins, 1985). The hub-and-spoke design problem is customarily called the hub location problem (HLP) (Campbell and O'Kelly 2012; Roni et al., 2017). O'Kelly (1986) presented the first recognized mathematical formulation for a HLP by studying airline passenger networks. HLP is a device that assists with settling on vital choices. An HLP includes a set of nodes: origins, destinations, and hub candidates. The HLP is concerned with locating hub facilities and allocating demand nodes to hubs in order to route the traffic between origin–destination pairs (O'Kelly, 1986; Osorio-Mora et al., 2020). Hub location modelling has many applications in airline travel and the transportation industry. Barid (2005) worked on the application of hub location modelling in transportation industry. Contreras et al. (2011) studied on stochastic uncapacitated hub location problems on the uncertainty in relationship between demand and expenses. The stochastic problem with unknown or dependent demands on transportation expenses is equal to indefinite expected value problem (Contreras et al., 2011). Remote facilities and fixed ones are the parking places, which are used to reach public transportations in order to get the final destination. Later on, Aros-Vera et al. (2013) developed a mixed linear formulation to determine the location of a number of fixed and remote facilities so that the best efficiency is achieved.

2.2. Location routing problem

Travelling to the demand point is not a traffic travelling but a rotator. These models are used in distribution systems. As an example, if in a LRP a multilayer diagram encountered in the distribution system, its analysis allocates the first layer to the factory, the second layer to stores, distribution centers, as well as demand points, and the third one to the final customers.

In most cases, the location of primary facilities and demand points are known. The numbers and location of secondary facilities should be determined as hubs in distribution systems. Albareda et al. (2007) introduced stochastic location-routing problems in two parts. In the first part, locating factory machinery is discussed and in the second, they used active sources to regulate routes in real locating.

One of the most important strategic decisions in supply is the dependence of supply systems on facility locations. Structure and management organization are other important decision points. Yong (2008) studied the composition of these two problems. He found a solution to minimize expenses through genetic algorithm (Yong, 2008). Li et al. (2009) studied LRP in three level distribution network. They located two level's facilities simultaneously and proposed related mathematical formulation. Yang and Zi-Xia (2009) proposed a two-step method based on point's optimization for LRP problems. Mingang et al. (2009) split the LRP problem into two sub-problems: locating critical facilities problem and routing critical source problem. They used a two-step heuristic algorithm to solve the problem in order to minimize total expenses. In 2010, Jafari and Golzari (2010) proposed a solution to locate store in distribution network for allocating customers and routing decision. Wang et al. (2018) provided a low-carbon and environmental protection point of view, based on the characteristics of perishable products, and combines with the overall optimization idea of cold chain logistics distribution network. Zhang et al (2018) presented an exploration of the sustainable multi-depot emergency facilities location-routing problem with uncertain information. Zhang et al (2019) studied a novel location-routing problem in electric vehicle transportation with stochastic demands.

2-3 - Transfer Point Location Problem

Berman et al. (2004b) introduced the hub locating problems. Also they proposed a heuristic algorithm to optimize the hub locating problem in the network. In 2008, a heuristic method was employed for solving MLTP but it was not an accurate technique given the problems with different sizes (Berman et al., 2008). Sasaki et al. (2008) showed that locating MLTPs problems can be formulated as p-median. In addition, they presented a new formulation of facility hub locating in the network problems.

In addition, Sylvester proposed single centre problems for the first time in 1975 (Sylvester, 1957). Wesolowsky (1977) presented probable weights in single dimension locating problems. Berman et al. (2003) considered weighted minimax single center locating problems in which weights are unknown but independent from uniform distributions. Averbakh and Bereg (2005) considered weighted center locating problems with indefinite weights and customers coordinate. Foul (2006) considered a center locating problem with demand points in a rectangular distribution.

Berman et al. (2004c) studied hub locating problem to solve a minimax optimization in situations, where no weights were devoted to demand points, and with known location. Shiode and Derezner (2003) considered a tree network with n demand points. They assumed that demand points are weighted and these weights are stochastic in arrival. Furthermore, they assumed that each customer utilizes the nearest facility to him/her; and finally introduced competitive facility location problem with stochastic weights (Shiode & Derezner, 2003).

Masson et al. (2014) investigated the dial-a-ride problem with transfers (DARPT). They extended the large neighbourhood search technique to solve the problem. They also studied the quality of servicing the applicants through constrains on the maximum ride time. Hoseseinijou and Bashiri (2012) extended a model in which demand points are weighted and the coordinates have rectangular bivarient distribution. They developed a conceptual view and some stochastic models. Their models found the best location for transfer point by minimizing the maximum expected weighted distance to all demand points through the transfer points. Two models were considered based on uniform distribution assumptions (Hosseinijou & Bashiri, 2012). Berman et al. (2007) introduced point location problem. A set of demand points providing services from one facility (such as hospital) generates demand for emergency service. Their model includes the location p helicopter pad and one facility. The location of facility (hospital) is known and the location of one transfer point that serves a set of demand point (Yusefli et al., 2018).

Yousefli et al (2018) studied stochastic transfer point location problem using a probabilistic rule-based approach. The procedure was employed to infer the optimum or near optimum values of all decision variables without solving nonlinear programming model directly. Finally, to approve the yield of the extended algorithm, a numerical example was dedicated and the results are compared with the optimum solution. Based on the studies conducted to date, it can be claimed that distribution function of demand points has not been covered thoroughly. Table 1 shows a brief review of this matter and about pertinent exploration on TPLP to date, utilizing normal characterizations.

Table 1
Some studies related to the transfer point location

Problem	**Solution**	**Objective**	**Topology**	**Modelling**	**Authors**
The transfer point location problem	Exact	Minisum/minimax	Plan/network	Determinism	Berman et al, 2007
Heuristic and lower bound for a Stochastic location –routing problem	Heuristic	Minisum	Network	Stochastic	Albareda et al., 2007
Integrated location-routing problem modelling and GA Algorithm Solving	Exact	Minisum	Network	Determinism	Yong, 2008; Yong & Zi-Xia, 2009
An improved branch and bound algorithm for location-routing problems	Heuristic	Minisum	Network	Determinism	Li et al., 2009
Two-phase particle swarm optimization for multi-depot location- routing problem	Heuristic	Minisum	Network	Determinism	Yong & Zi-Xia, 2009
Research on location-routing problem of relief system based on emergency logistic	Heuristic	Minisum	Network	Determinism	Mingang et al., 2009
Application of ranking function to solve Fuzzy Location-Routing Problem with L-R Fuzzy Numbers	Exact	Minisum	Network	Determinism	Jafari & Golzari, 2010
A competitive facility location problem on a tree network with stochastic weights	Exact	Minimax	Tree	Stochastic / Determinism	Shiode & Derezner, 2010
Stochastic uncapacitated hub location	Approximation	Minisum	Network	Stochastic / Determinism	Contreras et al., 2011
Stochastic models for transfer point location problem	Exact	Minimax	Plan	Stochastic	Hosseinijou & Bashiri, 2012
p-Hub approach for the optimal park-and-ride facility location problem	Exact	Maximize	Network	Determinism	Aros-Vera et al., 2014
Stochastic transfer point location problem: A probabilistic rule-based approach	Probabilistic Role Base	Minisum	Network	stochastic	Yousefli et al., 2018
Optimal Transfer Point Locations in Two-Stage Distribution Systems	Exact	Minimum	Plan	Determinism	Mcdougall & Otero., 2018
Developing a model transfer point location problem considering normal demands distribution	Exact	Minimax	Plan	Stochastic	This paper

3. Problem Definition and Gap of Research

The issue of location is one of the oldest research issues in operations. The study in this field began in the early twentieth century by Alfred Weber. The work is considered to be the basis of modern location principles (Rodrigue, 2020). Demand for crisis service is produced at a set of demand points who need the services of a central facility. Demand for emergency service is generated at a set of demand points who need the services of a central facility (Berman et al., 2007). Classical TPLP problems include some variants: in MLTP, the location of the facility is known. It needs to cluster the points into subsets, with each of them being served via a single transfer point. The general model is to make a facility and a set of transfer points that is called the Facility and Transfer Points Location Problem (FTPLP).

In hub center problems we deal with the following three items:

1- Facility center
2- Demand points
3- Hubs

Service facility is the point where applicants demand to be there. Based on the studies reviewed above, there is no work demand points in TPLP models by considering normal distribution. In this paper, the location of the facility is assumed to be known and the objective is to make it accessible with a minimum time and cost. Demand points are locations that are taken for granted as some nodes. These points are also assumed to be known here.

3.1. Research methodology

First of all, the mathematical model is developed due to the problem statement. Demand points follow a normal distribution with some points of demands being in regions. After that, this model is solved by replacing real number with a real condition. Maple software is used to solve this objective function as well as MATLAB software to solve this model numerically.

3.1.1. Transfer point location problem with deterministic demand points

In center location problem we have:

n: Number of demand points, α: the factor by which travel to the transfer point is multiplied by (x_0,y_0): the location of facility, (x,y): the location of transfer point, d(x,y): the distance between the transfer point and the facility, D_i: the distance between demand point i and facility, $d_i(x,y)$: the distance between demand point i and the transfer point.

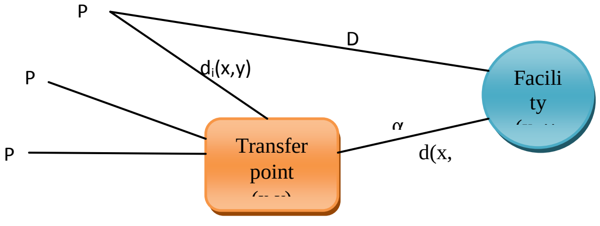

Fig. 1. Geometric display of the transfer point location (Yousefli et al., 2018)

Figure 1 is a geometric display of the transfer point location and where demands point is in one city.

This study considers the following situation. First, the location of the facility is known and each demand point has coordinates of P_i. Second, weights demand points are equal. Therefore, no weights are included in the model. Third, the demand points use the transfer point and distance is Euclidean. It is assumed that transfusion should effect the transfer point with no rectilinear motion between demand points to facility. The problem is to find the optimal location for the transfer point leading to a maximum distance between the facility so that demand points via transfer point is minimum. The problem formulization is as follows:

$$\min_{x,y} F(x,y) = \max_{1\le i\le n}\{d_i(x,y)+\alpha d(x,y)\} \qquad (1)$$

The simple model is:

$$\min_{x,y} F(x,y) = \max_{1\le i\le n}\{d_i(x,y)\}+\alpha d(x,y) \qquad (2)$$

Since the distance between d and d_i is convex function and maximum set convex function is convex, therefore a local optimization is the global one.

3.1.2. Transfer point location problem with stochastic weights at demand points

The following relations are considered for problem formulation:

w_i: Weight associated with the demand point I, $P_i(U_i,V_i)$: Coordinates of demand point i

One can assume that the location of the facility, (x,y),is known and all demand point i has coordinates of P=(U,V) so that U_i,V_i are in reliant accidental variables. Thus, we can find the optimal transfer point location. The problem formulation is as follows:

$$\mathrm{Min}_{x,y} EF(x,y) = \mathrm{Max}_{1\le i\le n}\{w_i E[d_i(x,y)]+\alpha d(x,y)\} \qquad (3)$$

Where [d(x,y)] hinge the probability distribution of demand points.

We survey the problem under situation that coordinates of demand points have a normal distribution. In the real world, it is safe to assume that occurrence demand points have a normal distribution. Two models are noted in this study. First, distribution demand points in a zone. Second, distribution demand points in several zones. In each case, distance between demand points and transfer point is orthogonal linear and distance between transfer point and facility is Euclidean.

3.1.3. Transfer point location problem when there are demand points sit in quadrangular region and repartition of demand points is normal distribution

Initially we consider the simple model in which there is one demand location with coordinates U, V that are located in dependent random variables with normal distribution in [a,b].

$$U, V \sim N(x;\mu,\sigma)$$

$f_V(v) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(v-\mu)^2}{2\sigma^2}}$, $f_V(v) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(v-\mu)^2}{2\sigma^2}}$

P = (U, V), P ϵ [a,b]×[a,b]

The target is to find optimum location for the transfer point.

$d_i(x,y)=|U-x|+|V-y|$, $d(x,y)=[(x-x_0)^2+(y-y_0)^2]^{1/2}$

Now the weight for value of traveled distance from demand i to transfer point can be frame follows:

$$E[d_i(x,y)]=E[|U-x|+|V-y|]=E[|U-x|]+E[|V-y|]$$

We have:

$$f(x) = E(|U-x|) = \int_{-\infty}^{+\infty} |u-x| \times f_U(u)du = \int_a^b |u-x| \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(u-\mu)^2}{2\sigma^2}} du \quad (4)$$

We have similarly:

$$f(y) = E(|V-y|) = \int_{-\infty}^{+\infty} |v-y| \times f_V(v)dv = \int_a^b |v-y| \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(v-\mu)^2}{2\sigma^2}} dv \quad (5)$$

If distributed demand points be normal, we have:

$$f(x) = \int_a^b |u-x| \frac{1}{\sqrt{2\pi}} e^{\frac{-u^2}{2}} du \quad (6)$$

$$f(y) = \int_a^b |v-y| \frac{1}{\sqrt{2\pi}} e^{\frac{-v^2}{2}} dv \quad (7)$$

Objective function is:

$$\min_{x,y} EF(x,y) = w[f(x) + f(y)] + \alpha[(x - x_0)^2 + (y - y_0)^2]^{1/2} \quad (8)$$

Since the Euclidean distance is convex function and maximum of bundle functions are convex. Thus, objective function model is convex and each optimal resolvent of problem is a global one.

The optimal resolvent is a function of α/w and if this factor tends to zero, the second objective function will have senseless.

$$\text{if } \frac{\alpha}{w} \to 0 \Rightarrow \min_{x,y} EF(x,y) = w[f(x) + f(y)]$$

$$\Rightarrow \begin{cases} \frac{\partial EF(x,y)}{\partial x} = wf'(x) = 0 \\ \to x^* = w \frac{\int_a^b |u-x| \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(u-\mu)^2}{2\sigma^2}} du}{\partial x} \\ \frac{\partial EF(x,y)}{\partial y} = wf'(y) = 0 \\ \to y^* = w \frac{\int_a^b |v-y| \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(v-\mu)^2}{2\sigma^2}} dv}{\partial y} \end{cases}$$

$$\text{if } \frac{\alpha}{w} \to \infty \Rightarrow \begin{cases} x^* = x_0 \\ y^* = y_0 \end{cases}$$

Condition of optimality is:

$$\begin{cases} \frac{\partial EF(x,y)}{\partial x} = wf'(x) + \alpha(x - x_0) = 0 \\ \frac{\partial EF(x,y)}{\partial y} = wf'(y) + \alpha(y - y_0) = 0 \end{cases}$$

By differentiation of function, we have:

$$f'(x) = \begin{cases} \left(2\sqrt{\pi} \times erf\left(\frac{\sqrt{2}(a-\mu)}{2\sigma}\right) - 2\sqrt{\pi} \times erf\left(\frac{\sqrt{2}(b-\mu)}{2\sigma}\right) \right) / 4\sqrt{\pi} & , x < a \\ \left(2\sqrt{\pi} \times erf\left(\frac{\sqrt{2}(a-\mu)}{2\sigma}\right) + 2\sqrt{\pi} \times erf\left(\frac{\sqrt{2}(b-\mu)}{2\sigma}\right) \right) / 4\sqrt{\pi} & , x > b \end{cases}$$

Now it allocates the optimal location transfer point as for location of facility. While region disport to four zones, we have:

$$w\left(\partial\left(-\sqrt{2}\begin{pmatrix}2\sigma\times e^{\left(\frac{-(a-\mu)^2}{2\sigma^2}\right)}\\ -\mu\sqrt{2\pi}\times erf\left(\frac{\sqrt{2}(a-\mu)}{2\sigma}\right)\\ +x\sqrt{2\pi}\times erf\left(\frac{\sqrt{2}(a-\mu)}{2\sigma}\right)\end{pmatrix}/4\sqrt{\pi}\right)/\partial x\right.$$

$$+\partial\left(\sqrt{2}\begin{pmatrix}-2\sigma\times e^{\left(\frac{-(b-\mu)^2}{2\sigma^2}\right)}\\ +\mu\sqrt{2\pi}\times erf\left(\frac{\sqrt{2}(b-\mu)}{2\sigma}\right)\\ -x\sqrt{2\pi}\times erf\left(\frac{\sqrt{2}(b-\mu)}{2\sigma}\right)\end{pmatrix}/4\sqrt{\pi}\right)/\partial x$$

$$\left.+\partial\left(\sqrt{2}\begin{pmatrix}4\sigma\times e^{\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)}\\ -2\mu\sqrt{2\pi}\times erf\left(\frac{\sqrt{2}(x-\mu)}{2\sigma}\right)\\ +2x\sqrt{2\pi}\times erf\left(\frac{\sqrt{2}(x-\mu)}{2\sigma}\right)\end{pmatrix}/4\sqrt{\pi}\right)/\partial x\right)$$

$$+\alpha(x-x_0)=0\rightarrow x^*$$

$$(\mu\le x\le b)\rightarrow$$

$$(\mu\le y\le b)\rightarrow$$

$$w\left(\partial\left(-\sqrt{2}\begin{pmatrix}2\sigma\times e^{\left(\frac{-(a-\mu)^2}{2\sigma^2}\right)}\\ -\mu\sqrt{2\pi}\times erf\left(\frac{\sqrt{2}(a-\mu)}{2\sigma}\right)\\ +y\sqrt{2\pi}\times erf\left(\frac{\sqrt{2}(a-\mu)}{2\sigma}\right)\end{pmatrix}/4\sqrt{\pi}\right)/\partial x\right.$$

$$+\partial\left(\sqrt{2}\begin{pmatrix}-2\sigma\times e^{\left(\frac{-(b-\mu)^2}{2\sigma^2}\right)}\\ +\mu\sqrt{2\pi}\times erf\left(\frac{\sqrt{2}(b-\mu)}{2\sigma}\right)\\ -y\sqrt{2\pi}\times erf\left(\frac{\sqrt{2}(b-\mu)}{2\sigma}\right)\end{pmatrix}/4\sqrt{\pi}\right)/\partial x$$

$$\left.+\partial\left(\sqrt{2}\begin{pmatrix}4\sigma\times e^{\left(\frac{-(y-\mu)^2}{2\sigma^2}\right)}\\ -2\mu\sqrt{2\pi}\times erf\left(\frac{\sqrt{2}(y-\mu)}{2\sigma}\right)\\ +2y\sqrt{2\pi}\times erf\left(\frac{\sqrt{2}(y-\mu)}{2\sigma}\right)\end{pmatrix}/4\sqrt{\pi}\right)/\partial x\right)$$

$$+\alpha(y-y_0)=0\rightarrow y^*$$

When the facility is located in A1 and demand point is located in S3 then:

$$(x>b, y>b)\rightarrow x^*=x_0, y^*=y_0$$

Proof: if demand points are located in S3 region with regards to ratio α/w tend to intolerable then the transfer point set position of the facility location. ($x^*=x_0, y^*=y_0$) It means that do not required to transfer point.

$$\begin{cases}\dfrac{\partial EF(x,y)}{\partial x}=wf'(x)+\alpha(x-x_0)=0\\ \dfrac{\partial EF(x,y)}{\partial y}=wf'(y)+\alpha(y-y_0)=0\end{cases}$$

$$\rightarrow\begin{cases}\alpha(x-x_0)=0\\ \alpha(y-y_0)=0\end{cases}\rightarrow\begin{cases}x^*=x_0\\ y^*=y_0\end{cases}$$

Similarly, we could account optimal transfer point in condition the facility located in other area (A2, A3, A4) and demand points located in S regions. Break down region solution problem is shown in Figure 2.

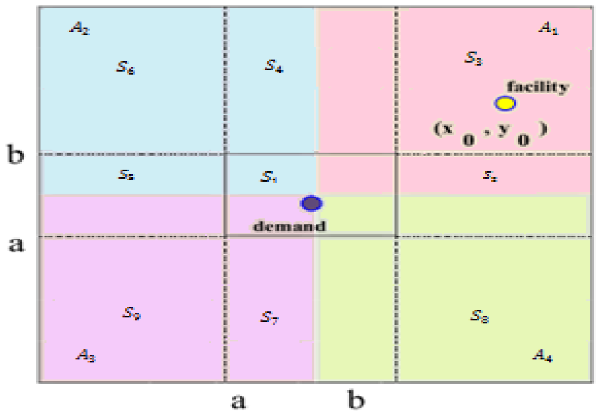

Fig. 2. Break down region solution problem

3.1.4. The transfer point location problem when the demand points in several rectangles and demand distributed is normal distribution

The demand points' coordinates are assumed to be between $[a_i,b_i]$ and $[c_i,d_i]$ by normal distribution.

All cities have demand area. Demands of each city have normal distribution. The objective is to find transfer point when distance between demand points and transfer point is Euclidean and distance between transfer point and facility is Euclidean. Figure 3 represents this case.

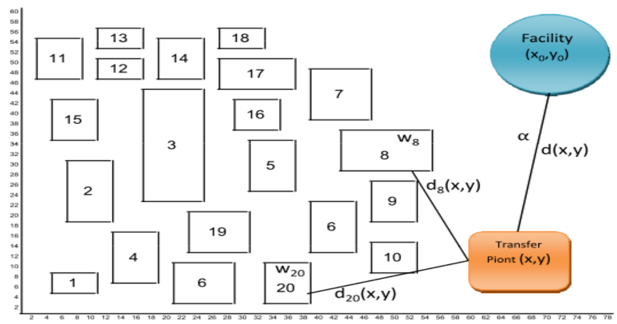

Fig. 3. The transfer point location in multiple cities

$f_i(x)$, $f_i(y)$ can be formulated as follows:

$$\text{if } (x < a_i): \tag{9}$$

$$f_i(x) = -\sqrt{2}\left(\begin{array}{l} -2\sigma \times e^{\left(\frac{-(a_i-\mu)^2}{2\sigma^2}\right)} \\ +\mu\sqrt{2\pi} \times erf\left(\dfrac{\sqrt{2}(a_i-\mu)}{2\sigma}\right) \\ -x\sqrt{2\pi} \times erf\left(\dfrac{\sqrt{2}(a_i-\mu)}{2\sigma}\right) \end{array}\right)/4\sqrt{\pi}$$

$$+\sqrt{2}\left(\begin{array}{l} -2\sigma \times e^{\left(\frac{-(b_i-\mu)^2}{2\sigma^2}\right)} \\ +\mu\sqrt{2\pi} \times erf\left(\dfrac{\sqrt{2}(b_i-\mu)}{2\sigma}\right) \\ -x\sqrt{2\pi} \times erf\left(\dfrac{\sqrt{2}(b_i-\mu)}{2\sigma}\right) \end{array}\right)/4\sqrt{\pi}$$

$$\text{if } (y < c_i): \tag{10}$$

$$f(x) = -\sqrt{2}\left(\begin{array}{l} -2\sigma \times e^{\left(\frac{-(c_i-\mu)^2}{2\sigma^2}\right)} \\ +\mu\sqrt{2\pi} \times erf\left(\dfrac{\sqrt{2}(c_i-\mu)}{2\sigma}\right) \\ -y\sqrt{2\pi} \times erf\left(\dfrac{\sqrt{2}(c_i-\mu)}{2\sigma}\right) \end{array}\right)/4\sqrt{\pi}$$

$$+\sqrt{2}\left(\begin{array}{l} -2\sigma \times e^{\left(\frac{-(d_i-\mu)^2}{2\sigma^2}\right)} \\ +\mu\sqrt{2\pi} \times erf\left(\dfrac{\sqrt{2}(d_i-\mu)}{2\sigma}\right) \\ -y\sqrt{2\pi} \times erf\left(\dfrac{\sqrt{2}(d_i-\mu)}{2\sigma}\right) \end{array}\right)/4\sqrt{\pi}$$

Objective function is formulated as follows:

$$\min_{x,y} EF(x,y) = \max_{1 \le i \le n} \{w_i[f_i(x) + f_i(y)] + \alpha[(x-x_0)^2 + (y-y_0)^2]^{1/2}\} \tag{11}$$

Function $w_i[f_i(x)+f_i(y)]+\alpha[(x-x_0)^2+(y-y_0)^2]^{1/2}$ is convex function and there for maximum of a convex function is convex function. Thus, a local optimum is the global one.

4. Computational Experiences

In this work, a numerical example is used to demonstrate the credibility and efficiency of the proposed algorithm, and the results are compared with the optimal solution.

4.1. Transfer point location determine in terms of demand points occur in the city.

Parameters for emplacement transfer point consist of the following: facility location (x_0, y_0), weights transfer demand points to transfer point (w), weight relocation from transfer point to facility (α), demand points mean (μ), demand points standard deviation (σ). Three states are considered for demand points in the city.

In first state, mean of demand points weights, demand points standard deviation and facility coordinate are specified (Table 2). In second case, demand points mean and demand points standard deviation are changed and other parameter are fixed (Table 3). In the third case, facility location is changed and other parameters are the as in the second case (Table 4).

Table 2
Initial value in the city (first case)

x_0	y_0	a	b	σ	μ
65	70	10	40	5	15

In Table 3 we assume mean and standard deviation equal 10, 20.

Table 3
Initial value in the city with variation mean and variation standard deviation (second case)

x_0	y_0	a	b	σ	μ
65	70	10	40	10	20

In third case coordinate facility is equal (50, 45).

Table 4
Initial value in the city with variation coordinate facility (third case)

x_0	y_0	a	b	σ	μ
50	45	10	40	10	20

4.2. Transfer point location determine in terms of demand points occur in multiple cities.

In this case value parameters consider according to Table 5.

Table 5
Value of parameters in multiple cities

a	b	c	d	(x_0,y_0)	σ	μ	w	Row
4	10	4	8	(70,65)	.66	6	0.3	1
6	12	18	30	(70,65)	2	24	0.2	2
16	26	22	44	(70,65)	3.7	33	0.5	3
12	18	6	16	(70,65)	1.66	11	0.6	6
30	36	24	34	(70,65)	1.66	29	0.2	5
20	28	2	10	(70,65)	1.33	6	0.3	6

4.3. Result of solution

Transfer point location problem when demands are in the city

Result of solution transfer point location problem when demands are in the city with value parameters are given in Table 6.

Table 6
Result of solution in the city

Row	w	α	x*	y*	EF*(x,y)
1	0.9	0.1	10	8.36.80	2.2406
2	0.8	0.2	9.6397	10	15.7068

3	0.7	0.3	17.9644	18.1857	25.2274
4	0.6	0.4	19.2716	19.6921	31.4832
5	0.5	0.1	16.8798	16.9723	9.9272
6	0.5	0.2	17.8220	18.0250	17.0183
7	0.5	0.3	18.8789	19.2327	23.9596
8	0.5	0.4	20.1533	20.7535	30.7239
9	0.5	0.5	21.9426	23.1105	37.2508
10	0.5	0.6	65	70	42.9607
11	0.5	0.7	65	70	42.9607
12	0.5	0.8	65	70	42.9607
13	0.5	0.9	65	70	42.9607
14	0.1	0.5	65	70	8.5921
15	0.2	0.5	65	70	17.1843
16	0.3	0.5	65	70	25.7764
17	0.4	0.5	65	70	34.3686
18	0.6	0.5	20.4013	21.0613	38.1969
19	0.7	0.5	19.5697	20.0455	38.9889
20	0.8	0.5	19.0232	19.4007	39.7050
21	0.9	0.5	18.6301	18.9452	40.3761
22	0.4	0.6	65	70	34.3686
23	0.3	0.7	65	70	125.7764
24	0.2	0.8	65	70	17.1843
25	0.1	0.9	65	70	8.5921

Figure 4 represents outcome diagram of objective function EF^* for row 4 of Table 6 when α=0.4 and w=0.6. Coordinates of facility is equal (65, 70).

Under conditions where the displacement of a point from the point of demand to a transfer point (w) and the displacement weight from the transfer point to a facilitator (α) is 0.6 and 0.4. The optimal target function is 31.4832 and the point x = 19.2716 and y = 19.6921. It would be the best point for considering the location of the transfer point. Also, the surface under the curve of its target function is as shown in Figure. 4.

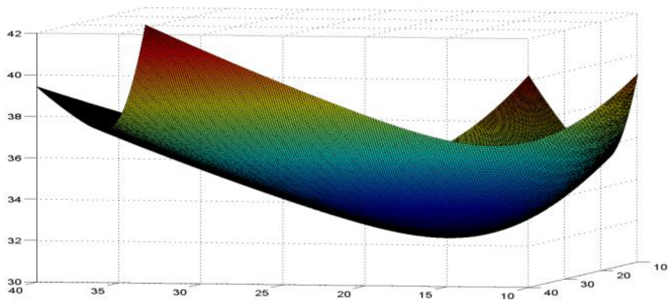

Fig. 4. Surface of objective function when α=0.4 and w=0.6

Figure 5 presented as for contour lines for regions had equal objective function. Also, it represents the defensive line of the target function in areas where the objective function has similar values. The best place can be chosen to move the point in that range.

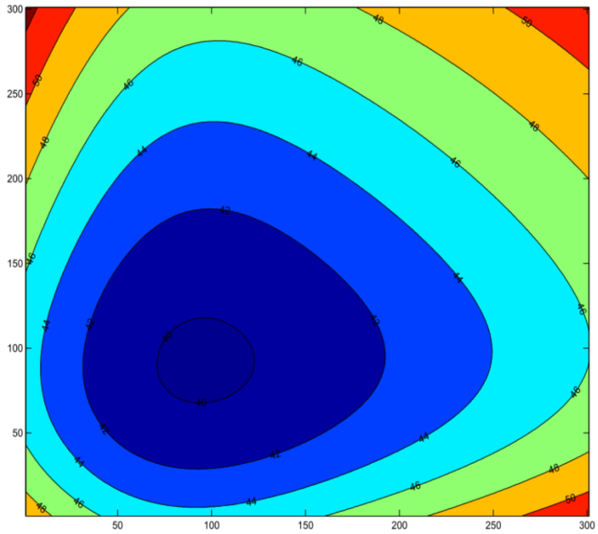

Fig. 5. Contour lines for regions when α=0.5 and w=0.8

Result of solution transfer point location problem when demands are in the city with variation mean and standard deviation of Table 3 (Table 7).

Table 7
Solution result of the city with variation mean and standard deviation.

Row	w	α	x*	y*	EF*(x,y)
1	0.9	0.1	9.7312	10	5.3095
2	0.8	0.2	9.5891	9.9998	13.8183
3	0.7	0.3	25.5649	26.0510	25.4718
4	0.6	0.4	28.0965	28.9920	30.0004
5	0.5	0.1	9.6254	9.9959	6.5902
6	0.5	0.2	10	8.8827	14.9196
7	0.5	0.3	27.3394	28.1012	23.1417
8	0.5	0.4	29.7854	31.0199	28.5930
9	0.5	0.5	33.1531	35.2590	33.5972
10	0.5	0.6	64.9999	69.9999	37.0035
11	0.5	0.7	65	70	37.0034
12	0.5	0.8	65	70	37.0034
13	0.5	0.9	65	70	37.0031
14	0.1	0.5	65	70	7.4007
15	0.2	0.5	65	70	14.8014
16	0.3	0.5	65	70	22.2021
17	0.4	0.5	65	70	29.6028
18	0.6	0.5	30.2575	31.5979	35.3548
19	0.7	0.5	28.6692	29.6726	36.8248
20	0.8	0.5	27.6178	28.4278	38.1492
21	0.9	0.5	26.8581	27.5407	39.3868
22	0.4	0.6	65	70	29.6028
23	0.3	0.7	65	70	22.2021
24	0.2	0.8	65	70	14.8014
25	0.1	0.9	65	70	7.4007

Also, outcome diagram based on the objective function EF^* for row 7 in Table 7 when α=0.3 and w=0.5 are presented in Figure 6. Figure 7 depicts contour lines for regions had equal objective function.

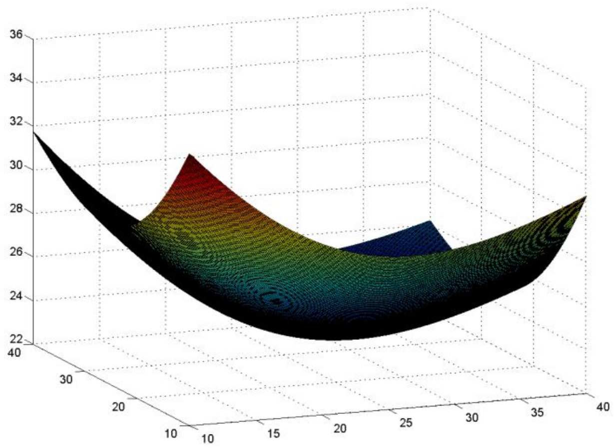

Fig. 6. Surface of objective function when α=0.3 and w=0.5

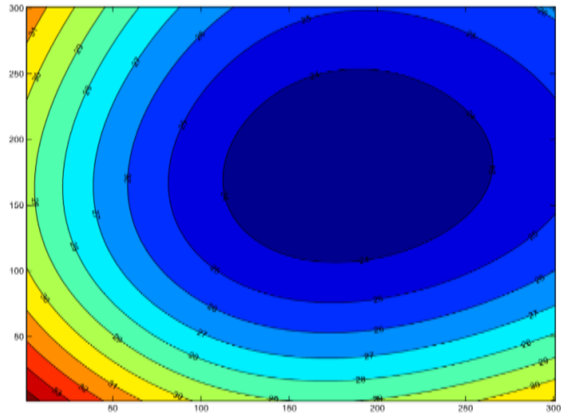

Fig. 7. Contour lines for regions when α=0.3 and w=0.5

Result on the solution transfer point location problem, when demands are in the city with variation coordinate facility of Table 4, in Table 8.

Table 8
Solution in the city with variation coordinate facility

Row	w	α	x*	y*	EF*(x,y)
1	0.9	0.1	9.9998	9.6201	2.5057
2	0.8	0.2	9.9987	9.4193	8.2096
3	0.7	0.3	26.2250	25.3706	17.0083
4	0.6	0.4	29.3099	27.7657	18.7138
5	0.5	0.1	10	9.4765	3.7847
6	0.5	0.2	25.8942	25.1079	11.7053
7	0.5	0.3	28.3731	27.0489	14.6773
8	0.5	0.4	31.4487	29.3717	17.3037
9	0.5	0.5	35.9074	32.6627	19.4754
10	0.5	0.6	50	45	20.6316
11	0.5	0.7	50	45	20.6316
12	0.5	0.8	50	45	20.6316
13	0.5	0.9	50	45	20.6316
14	0.1	0.5	50	45	4.1263
15	0.2	0.5	50	45	8.2526
16	0.3	0.5	50	45	12.3789
17	0.4	0.5	50	45	16.5052
18	0.6	0.5	32.0588	29.8241	21.2421
19	0.7	0.5	30.0270	28.3086	22.7155
20	0.8	0.5	28.7162	27.3124	24.0416
21	0.9	0.5	27.7846	26.5938	25.2800
22	0.4	0.6	50	45	16.5052
23	0.3	0.7	50	45	12.3789
24	0.2	0.8	50	45	8.2526
25	0.1	0.9	50	45	4.1263

4.4. Result of solution when demands be in multiple cities

Result of solution transfer point location problem when demands are in multiple cities are given in Table 5 and Table 9.

Figure 8 shows equal objective function for row 8 of Table 9 when α=0.5. Also, it illustrates the line between the target function and several areas where the objective function has the same values and the best location for the transmission point in that range may select.

Table 9
Result of solution in multiple cities

Row	α	x*	y*	EF(x*,y*)
1	0.001	13.2575	0.0260	2.2604
2	0.01	13.2412	3.9993	3.0098
3	0.03	13.2666	4	4.6760
4	0.1	13.1964	6	10.4967
5	0.2	13.2334	6	18.6926
6	0.3	12.3310	7.3679	26.7678
7	0.4	12.3349	7.3637	34.9209
8	0.5	58.6644	43.7747	39.1337
9	0.55	62.1714	45.3670	40.2556
10	0.6	64.3473	46.3549	41.2687
11	0.62	65.1921	46.7386	41.6550
12	0.64	70	65	41.6643
13	0.65	70	65	41.6643
14	0.7	70	65	41.6643
15	0.8	70	65	41.6643
16	0.9	70	65	41.6643

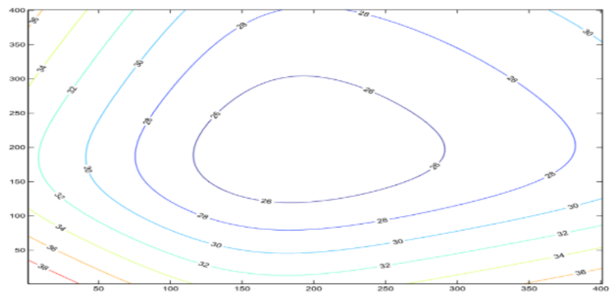

Fig. 8. Contour lies for regions with equal objective function when α=0.5

4.5. checking time solution with grow-up condition

For checking time solution with grow up conditions, we test the model when number of cities are 10 and over. Then exploit results with 1.61 GHz processor and one GB of RAM. The results are presented in Table10.

Table 10
time solution of model

Duplication Number	CPU time (s)	Number of Cities
10	0.8125	10
14	1.4219	20
18	3.2656	40
24	6.0781	80
26	7.6250	100
13	15.6406	200
30	29.5469	400
29	39.1563	800

5. Conclusion

Network transmission and distribution systems are able to deliver high quality and reliable services in such a way that under normal conditions they would meet various operating restrictions in the network. In development planning, one should consider new lines to meet the needs of consumers at the lowest cost. Applicants are transferred to the transfer point at the usual speed and then transferred to the service center at dual speeds. The goal is to find the best point for the transfer locations. Using a transfer point can increase the transfer speed from the transfer centers to the service centers in proportion to the transfer rates. One of the application domains is to determine the transfer points in cities to reach service centers. Considering the large population in the city center, the accumulation of individuals can reduce, by taking into account the normal distribution of popular locations. Accordingly, in the event of an increase in the number of people in the city center, the probability of occurrence of accidents also increases and this is very important for determining the centers for the transfer to service centers. Moreover, in the real world, this hypothesis leads to a more appropriate model than the theory

In the real world, in some applications, the demand distribution is in a normal condition. Solving TPLP can be used as a decision support system for decision makers. Tools assisting to establish transfer points ensure the best exploitation of resources and decrease costs incurred to the company as they set up distribution networks.

One limitation or challenge for future research is the lack of modeling the disaster distribution, which can be probably dealt with by investigating the distribution of demand and then solve the problem by distributing demand points.

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