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Research Article

Stochastic Analysis of k-out-of-n: G Type of Repairable System in Combination of Subsystems with Controllers and Multi Repair Approach

Vijay Vir Singh [a,*], Praveen Kumar Poonia[b]

[a] *Department of Mathematics, Yusuf Maitama Sule University, Kofar Nassarawa, Kano State, Nigeria*
[b] *Department of General Requirements, Ibri College of Applied Sciences, Ibri-516, Oman*

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Abstract

This paper discusses the study of different reliability measures of a complex system composed of two subsystems with controllers connected in a series arrangement, which is an interesting possibility for specific design problems. Subsystem-1 is made up of n units that operate under the policy k-out-of-n: G; policy, whereas subsystem-2 is made up of m units that operate under the r-out-of-m: G; policy. Both subsystems' failure rates are constant and expected to obey an exponential distribution; two types of distributions are permitted to repair: general and Gumbel-Hougaard family copula distributions. During the process the partially failed states are mended via general repair, while the completely failed states are fixed using copula repair. After repair, both subsystems are "as good as new." Both subsystems are controlled by a controller, and if the controller fails, the whole system fails. If the operator is dissatisfied with the organization, he/she may purposefully fail the system. The problem is modelled using the supplementary variable technique, Laplace transforms and copula repair. Traditional system reliability measures, such as availability, reliability, and expected profit, are calculated for various arbitrary values of failure and repair parameters.

Keywords: k-out-of-n: G; Controller; Reliability physiognomies; Expected profit; Gumbel-Hougaard family copula distribution.

1. Introduction

Everyone is reliant on machinery, automated equipment, robots, appliances, and other items in contemporary diverse society. To overcome this challenge, we produce realistic, high-quality products and develop incredibly dependable systems. The most effective method of increasing system performance is to incorporate redundant components into the design that sustain the system and mitigate the effects of failure. A popular type of redundancy is k-out-of-n, which is commonly recognized by a wide variety of sectors and organizations. The word k-out-of-n is frequently used to refer to a successful (G) or failure (F) system, or to both. The k-out-of-n: G system is reasonable only if at least k of its n components functions correctly. It fails if less than k of its n components are operational, that is, if at least (n-k+1) of its n components fail. Numerous examples of systems can be presented in realistic situations to illustrate the application and necessity of such setups in repairable systems. The k-out-of-n: G mode of operation of the system can be compared to a huge truck with 18 tires coupled at each wheel. If less than eighteen tires are functioning, the system's performance will suffer. As long as ten tires are operational, rearranging of the tire configuration will result in satisfactory performance. Thus, the system can exist as an 8-out-of-18: G with a major degraded mode or a 10-out-of-18: G with a minor degraded mode. Complex devices such as a multi-display system in the cockpit, a multi-engine system in space shuttles, or a multi-pump system in hydraulic control systems are all examples of systems. The systems are constructed using a variety of components to do special task. Many downright deadly accidents have happened recently in the world where systems have been large-scale and complex. They not only caused massive damage but also brought an unrecoverable lousy effect on the living environment. The accident in a Self-driving car, the crash of a Boeing 737 max eight aircraft, the explosion at an oil refinery, and the disappearance of an Indian air force AN-32 plane, etc., are due to failure of more units/over the required components. These all are due to occurred deterioration of efficiency and lack of maintenance. A pipeline system with n pump stations is an example of a consecutive k-out-of-n: F system. Each pump is powerful enough to transport oil to the following k stations. If fewer than k successive stations fail, the oil flow will not be disrupted, and the pipeline system will continue to operate normally. Another important application of k-out-of-n: G/F systems in like transmission relay systems, electricity distribution systems, power plant systems, project management systems, autonomous car parking systems, quality control systems for accepting or rejecting

*Corresponding author Email address: singhvijayvir68@gmail.com

the samples and many similar systems can be present for the utility of such configuration.

Many systems are big and complex in their design, and it is almost impossible to create universal theories for general systems of this size and complexity. Researchers have investigated statistically and stochastically complex phenomena of k-out-of-n: G/F type of systems to improve their performances through reliability theories like evaluating availability together with other performance-based indices and developing formulas. To cite the contributions of some scholars, including Fawzi (1991) Jieyu (1992), Yam (2003), Liang (2010), and Sharma (2017) have studied reliability physiognomies under (k-out-of-n); G/F, with a constant failure rate and general repair. In addition to the above, researchers Kullstam (1981), Bai (1991), Moustafa (1996), and El-Damcese (2014) studied the repairable systems under k-out-of-n: G/F types of configuration and general repair facilities. A lot of scholars have been studying the consistency of the series system, looking at things like the controllers, in order to boost the reliability of the system's operation. The controller is a device that monitors the working conditions of a given dynamical system. It can be used not only in engineering systems (in electronics as a microprocessor, in computers as a peripheral device, in software architecture to form an interface, game controller, etc.). The controller device is also used in non-engineering systems like linguistics (control the verb) and aviation (control the air traffic). The key components of complex engineering systems are digital computers because they enable control of the variables. Ogata (2009) has elucidated the idea of controllers in current engineering systems. k-out-of-n: G/F system configurations are critical in the functioning of industrial systems because they provide a high degree of fault tolerance. With the study of k-out-of-n configuration, researchers have analyzed a system with one or more than one active redundant unit. Further, the study was extended to the k-out-of-n: G/F system with constant failure and general repair. Later on, the researchers study the system performances with a combination of more than one subsystem using general repair and also with copula repair. To investigate the contributions of a few of them, Singh et al. (2013) conducted a cost analysis of two subsystems in a series configuration. The first subsystem used the k-out-of-n: G scheme, the second subsystem used two identical units in a parallel configuration, and the units of both subsystems were controlled via the controller. With the help of the k-out-of-n: G scheme with Gumbel-Hougaard family copula distribution for fixing fully failed units, Singh et al. (2013a) investigated the availability, mean time to failure, and cost of a complex system that consisted of two units in a series configuration with controllers and human failure under various conditions. They came to the conclusion that all of these requirements had been met. Lado et al. (2018) has evaluated the repairable complex system's reliability and sensitivity with two subsystems coupled in series mode.

Regular maintenance is essential to guarantee that the repairable system continues to operate at peak functionality. Maintenance may be classified in a variety of ways, including priority maintenance and preventative maintenance, among others. The preventative maintenance of the system has been completed prior to the system failing in order to minimize the consequences of the system failing. Authors Singh and Ram (2014) probed the reliability of a system with two subsystems running in a k-out-of-n: G configuration, which is handled by a human operator. The authors used two different kinds of repairs using the copula technique. The case study was performed for subsystem-1, which consists of three parallel units and operates under the 2-out-of-3: F policy. The authors, Kumar and Gupta (2007), Kumar et al. (2017), Singh et al. (2018, 2020) and Raghav et al. (2020) studied various reliability characteristics of complex systems in series with controllers and catastrophic failure. Lin et al. (2019) have evaluated the system reliability for a freeway system. The speed on each division is stochastic residual to user behavior, accidents, tunnels, road gradient, and road repair. Author Davoudpour (2019) utilized a hierarchical Bayesian network to juxtapose maintenance techniques based on cost and reliability using onshore wind turbines as a case study.

Some new equipment groups need to add some natural systems such as satellite transmission systems or computer systems because of the system's requirement for better output. One real example that has led to this study is improving a system that initially consisted of one equipment group. Other equipment groups are added to it so that the new system is composed of two equipment groups because of more requirements. In addition to these, the authors Goel (1984), Galikowsky (1996), Levitin (2001), and Ram (2008) studied the k-out-of-n: G/F types of systems. The authors that were identified focused on the functioning of units in a parallel configuration or a circular design with catastrophic failure and preventative maintenance. They did not consider the series arrangement of two or more subsystems operating in accordance with the k-out-of-n policy. As a result of recognizing the importance of this arrangement in the current article, we examined a complex system comprised of subsystems 1 and 2 linked in a series configuration. Subsystem-1 consists of n units that operate under the policy k-out-of-n: G, whereas subsystem-2 consists of m units that operate under the policy r-out-of-m: G. In the paradigm, each component has three situations: perfect functioning, partial failure, and total failure. Both subsystems are linked together by means of a controller, which may or may not be handy depending on the circumstances. The controller's purpose is to ensure that "as long as the controller fails, the whole system instantly fails". Failure rates of units in both subsystems are constant and follow an exponential distribution, while the repair follows two types of distribution. In order to repair them, they must go through two kinds of distributions namely general and copula distributions. Partially failed units are repaired via general repair distribution and fully failed units are repaired via copula distribution. The supplementary variable technique (Cox, 1955 and Oliveira, 2005) was used in this research to analyse the various reliability characteristics like system availability, system reliability, and profit analysis. Additionally, some specific cases have been addressed in detail for a range of various failure rates. The findings are shown graphically, and conclusions have been made.

The following is the structure of the paper: Section-1 examined the relevant work provided by many scholars and dubbed it the model's introduction. The system description that includes transition diagrams is explained in section-2. In sections-3 &4, assumptions and notations have been elaborated. Section-5 & 6 presents the model's formulation and solution with an analytical study that includes availability, reliability, and profit analysis. Conclusions of the proposed analysis are given in Section-7.

2. System Organization, State Transition Diagram, and System Description

From the system architecture in figure 1 (a) and the state transition diagram in figure 1(b), we can confer that the entire system has three types of states, i.e., perfect state, partial failed, and fully failed state represented by the circle, diamond, and squares respectively. In the partial failure situation, the state is repaired using general repair. The complete failed state is repaired employing copula distribution due to copula repair's implication as beneficial over the general repair. As a result, the state description emphasizes that S_0 is a state in which both subsystems are operating normally; S_1, S_3, and S_5 are degraded states and the general repair is being used. S_2, S_4, S_6, S_7, S_8, S_9, and S_{10} are fully failed states and copula repair used to restore system. The Gumbel-Hougaard family copula distribution is used to restore complete failed states, despite the fact that the literature specifies the many types of copula functions, and therefore owing to its simplicity and appropriateness for computational tasks, we chose this distribution for our purposes.

Table 1

State Description

S_0	In this state, all units of subsystems-1 and subsystem-2 are in excellent functioning condition, and the system is in its ideal state.
S_1	The indicated state epitomizes that the system has deteriorated but still operational, since at least k units in subsystem-1 and all units in subsystem-2 are in excellent operational condition. Regular maintenance is currently being performed on the system.
S_2,S_6	The states show that the system is in completely failed mode since more than (n-k) units in subsystem-1 have failed. Copula distribution is being utilized for the restoration of the system while it is being repaired.
S_3	The indicated state epitomizes that the system has deteriorated but still operational, since at least r units in subsystem-2 and all units in subsystem-1 are in excellent operational condition. Regular maintenance is currently being performed on the system.
S_4,S_7	The states show that the system is in completely failed mode since more than (m-r) units in subsystem-2 have failed. Copula distribution is being utilized for the restoration of the system while it is being repaired.
S_5	The indicated state signifies that the system is degraded but is nonetheless operational since at least k units of subsystem-1 and r units of subsystem-2 are functioning. The system is presently undergoing routine maintenance.
S_8,S_9,S_{10}	The states epitomize that the system is in totally failed mode because of controller's failure in subsystem-1/controller in subsystem-2/deliberate failure by the operator. Copula distribution is being utilized for the restoration of the system while it is being repaired.

3. Assumptions

Each of the two subsystems satisfies the following assumptions:

1. Initially in state S_0, Subsystem-1 and subsystem-2 are both in excellent functioning order, and all of their components are in fantastic functional order as well.
2. The subsystem-1 operates till k units are functioning well and fails if more than (n-k) units fail.
3. The subsystem-2 operates till r units are functioning well and fails if more than (m-r) units fail.
4. The units in the subsystems are in parallel mode and on hot standby, ready to start within a short period of time if any unit in the subsystems fails.
5. The repairman is on call around the clock with the system and may be called as soon as the system reaches a partially or fully failing state.
6. Regardless of whether it's manufacturing or service-related, it always follows an exponential distribution. A system that's meant to fail or has completely failed has to be repaired quickly. Copula distribution is being utilized for the restoration of the system while it is being repaired
7. There have been no reports of harm due to the repair of the system.
8. Once fixed, the failed device is ready to function as well as new.

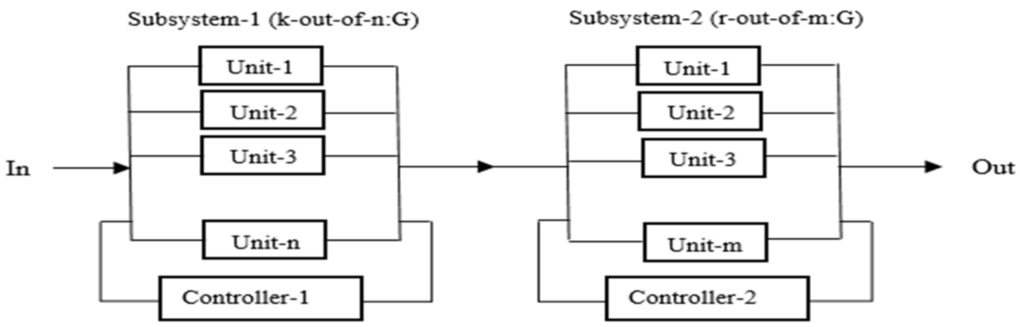

Fig. 1. (a) System Configuration

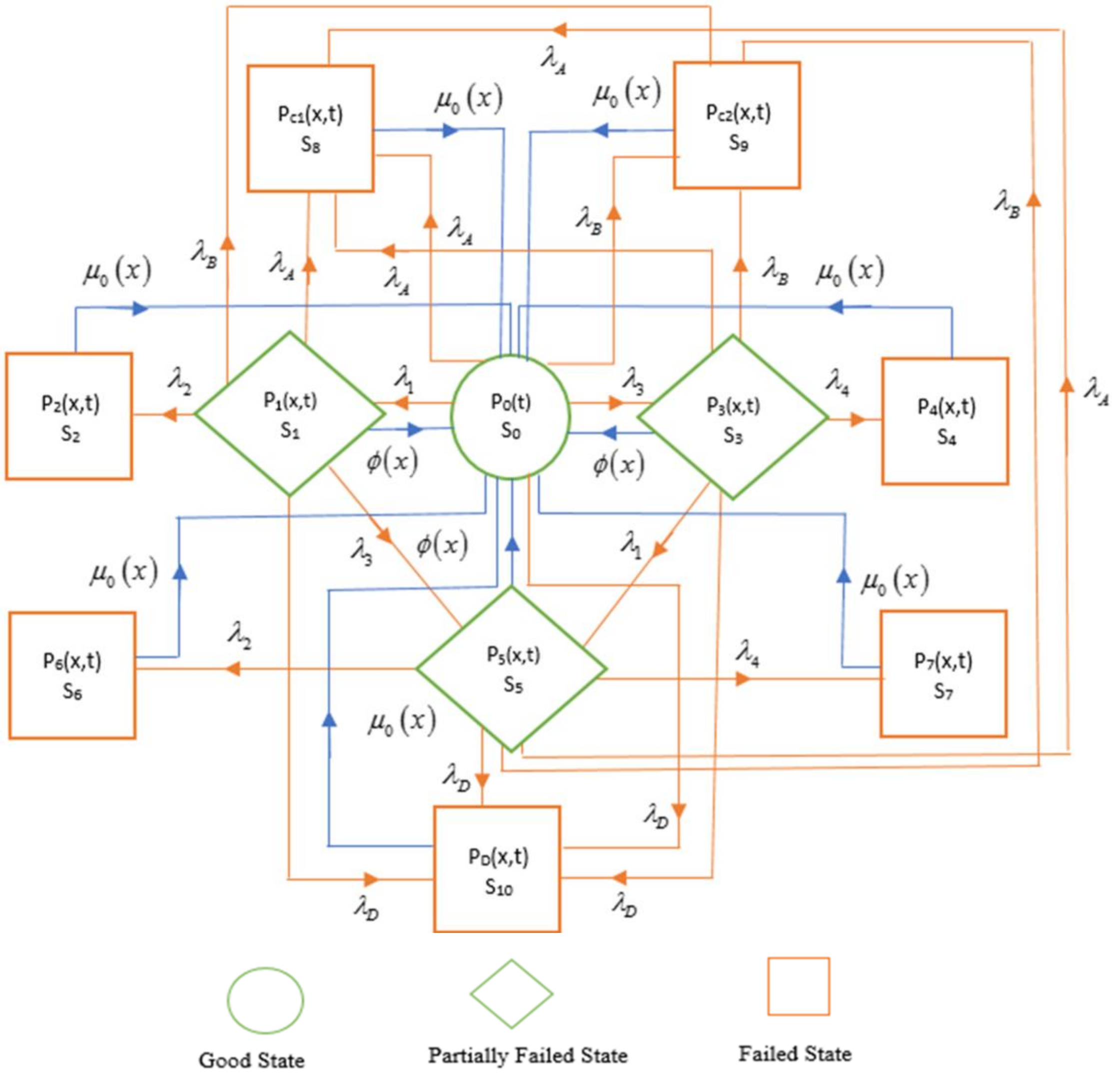

Fig. 1. (b) State transition diagram of the model

4. Notations

s, t Laplace transform and Time scale variable

λ_1, λ_2 Subsystem-1 failure rates if at least k units/more than (n-k) units fail during the operation.

λ_3, λ_4 Subsystem-1 failure rates if at least k units/more than (n-k) units fail during the operation.

$\lambda_A, \lambda_B, \lambda_D$ The failure rate of the controller of subsystem-1/controller of subsystem-2/ due to deliberate failure.

$\varphi(x)$ Repair rate of the units in subsystem-1 / subsystem-2.

$P_0(t)$ The state transition probability that the system is in S_i stated at an instant $i = 0$.

$\bar{P}(s)$ Laplace transformation of the state transition probability $P(t)$.

$P_i(x,t)$ The probability that the system is in the state S_i for $i = 1 \ to \ 10$.

$E_p(t)$ Profitability in the interval $[0, t)$.

$K_1 \ K_2$ Revenue made and service cost per unit time, respectively.

$\mu_0(x)$ Repair rate for completely failed states for supplementary variable x. It is a joint probability function derived from the copula family that ranges from the completely failed state Si to the perfect state S_0.

5. Formulation of Mathematical Model

Using probability theory, considerations, and continuity arguments, we may derive the following set of difference-differential equations that are connected with the current mathematical model. If the system is currently in state S0, it will stay in state S0 for the period (t, t+Δt), and it will not progress to any other state and if the system is currently in failed state, it will be on its way back to state S0 once it has been repaired. For an instant, the system is in state S_0 and will remain in state S_0 is that it must not go to state $S_1, S_3, S_8, S_9, S_{10}$, and it must come to state S_0 from the states $S_2, S_4, S_5, S_6, S_7, S_8, S_9, S_{10}$.

$$P_0(t+\Delta t) = (1-\lambda_1\Delta t)(1-\lambda_3\Delta t)(1-\lambda_A\Delta t)(1-\lambda_B\Delta t)(1-\lambda_D\Delta t)P_0(t) + \sum_i \int_0^\infty \varphi(x)P_i(x,t)dx\Delta t \ \{i = 1,3\} + \sum_i \int_0^\infty \exp\left[x^\theta + \{log\varphi(x)\}^\theta\right]^{1/\theta} P_j(x,t)dx\Delta t \ \{j = 2,4,6,7,8,9,10\}$$

$$\lim_{\Delta t\to 0}\frac{P_0(t+\Delta t)-P_0(t)}{\Delta t} + (\lambda_1+\lambda_3+\lambda_A+\lambda_B+\lambda_D)P_0(t) = \sum_i \int_0^\infty \varphi(x)P_i(x,t)dx \ \{i = 1,3\} + \sum_i \int_0^\infty \exp\left[x^\theta + \{log\varphi(x)\}^\theta\right]^{1/\theta} P_j(x,t)dx \ \{j = 2,4,6,7,8,9,10\}$$

$$\left(\frac{\partial}{\partial t}+\lambda_1+\lambda_3+\lambda_A+\lambda_B+\lambda_D\right)P_0(t) = \sum_i \int_0^\infty \varphi(x)P_i(x,t)dx \ \{i = 1,3\} + \sum_i \int_0^\infty \exp\left[x^\theta + \{log\varphi(x)\}^\theta\right]^{1/\theta} P_j(x,t)dx \ \{j = 2,4,6,7,8,9,10\} \quad (1)$$

The above equation is associated with the state S_0 in which $\lambda_1, \lambda_3, \lambda_A, \lambda_B, \lambda_D$ represents the failure rates, while $\varphi(x)$ and $\exp\left[x^\theta + \{log\varphi(x)\}^\theta\right]^{1/\theta}$ represents repair rates for general and copula distribution respectively. Similarly, the differential equations for the other states represented from (2 to 7) can be obtained.

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_2+\lambda_3+\lambda_A+\lambda_B+\lambda_D+\varphi(x)\right)P_1(x,t) = 0 \quad (2)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\exp\left[x^\theta + \{log\varphi(x)\}^\theta\right]^{1/\theta}\right)P_2(x,t) = 0 \quad (3)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_1+\lambda_4+\lambda_A+\lambda_B+\lambda_D+\varphi(x)\right)P_3(x,t) = 0 \quad (4)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\exp\left[x^\theta + \{log\varphi(x)\}^\theta\right]^{1/\theta}\right)P_4(x,t) = 0 \quad (5)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_2+\lambda_4+\lambda_A+\lambda_B+\lambda_D+\varphi(x)\right)P_5(x,t) = 0 \quad (6)$$

$$\left(\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\exp\left[x^\theta + \{log\varphi(x)\}^\theta\right]^{1/\theta}\right)P_i(x,t) = 0 \quad \{i = 6,7,8,9,10\} \quad (7)$$

Boundary Conditions

$$P_1(0,t) = \lambda_1 P_0(t) \quad (8)$$

$$P_2(0,t) = \lambda_2 P_1(0,t) = \lambda_1\lambda_2 P_0(t) \quad (9)$$

$$P_3(0,t) = \lambda_3 P_0(t) \quad (10)$$

$$P_4(0,t) = \lambda_4 P_3(0,t) = \lambda_3\lambda_4 P_0(t) \quad (11)$$

$$P_5(0,t) = \lambda_3 P_1(0,t) + \lambda_1 P_3(0,t) = 2\lambda_1\lambda_3 P_0(t) \quad (12)$$

$$P_6(0,t) = \lambda_2 P_5(0,t) = 2\lambda_1\lambda_2\lambda_3 P_0(t) \quad (13)$$

$$P_7(0,t) = \lambda_4 P_5(0,t) = 2\lambda_1\lambda_3\lambda_4 P_0(t) \quad (14)$$

$$P_8(0,t) = \lambda_A[P_0(t)+P_1(0,t)+P_3(0,t)+P_5(0,t)] \quad (15)$$

$$P_9(0,t) = \lambda_B[P_0(t)+P_1(0,t)+P_3(0,t)+P_5(0,t)] \quad (16)$$

$$P_{10}(0,t) = \lambda_D[P_0(t)+P_1(0,t)+P_3(0,t)+P_5(0,t)] \quad (17)$$

Initial conditions

$P_0(t) = 1$ and $P_i(t) = 0 \ \forall i$ at $t = 0$. (18)

The equations (1) to (17) may be transformed as follows after taking Laplace transformation of equations and using equation (18):

$(s+\lambda_1+\lambda_3+\lambda_A+\lambda_B+\lambda_D)\bar{P}_0(s)=$
$\sum_i \int_0^\infty \varphi(x)\bar{P}_i(x,s)dx\ \{i=1,3\}+\sum_i \int_0^\infty \exp\left[x^\theta+\{log\varphi(x)\}^\theta\right]^{1/\theta}\bar{P}_j(x,s)dx\ \{j=2,4,6,7,8,9,10\}$ (19)

$$\left(s+\frac{\partial}{\partial x}+\lambda_2+\lambda_3+\lambda_A+\lambda_B+\lambda_D+\varphi(x)\right)\bar{P}_1(x,s)=0 \quad (20)$$

$$\left(s+\frac{\partial}{\partial x}+\exp\left[x^\theta+\{log\varphi(x)\}^\theta\right]^{1/\theta}\right)\bar{P}_2(x,s)=0 \quad (21)$$

$$\left(s+\frac{\partial}{\partial x}+\lambda_1+\lambda_4+\lambda_A+\lambda_B+\lambda_D+\varphi(x)\right)\bar{P}_3(x,s)=0 \quad (22)$$

$$\left(s+\frac{\partial}{\partial x}+\exp\left[x^\theta+\{log\varphi(x)\}^\theta\right]^{1/\theta}\right)P\bar{P}_4(x,s)=0 \quad (23)$$

$$\left(s+\frac{\partial}{\partial x}+\lambda_2+\lambda_4+\lambda_A+\lambda_B+\lambda_D+\varphi(x)\right)\bar{P}_5(x,s)=0 \quad (24)$$

$$\left(s+\frac{\partial}{\partial x}+\exp\left[x^\theta+\{log\varphi(x)\}^\theta\right]^{1/\theta}\right)\bar{P}_i(x,s)=0\ \{i=6,7,8,9,10\} \quad (25)$$

Boundary Conditions

$$\bar{P}_1(0,s)=\lambda_1\bar{P}_0(s) \quad (26)$$

$$\bar{P}_2(0,s)=\lambda_2\bar{P}_1(0,s)=\lambda_1\lambda_2\bar{P}_0(s) \quad (27)$$

$$\bar{P}_3(0,s)=\lambda_3\bar{P}_0(s) \quad (28)$$

$$\bar{P}_4(0,s)=\lambda_4\bar{P}_3(0,s)=\lambda_3\lambda_4\bar{P}_0(s) \quad (29)$$

$$\bar{P}_5(0,s)=\lambda_3\bar{P}_1(0,s)+\lambda_1\bar{P}_3(0,s)=2\lambda_1\lambda_3\bar{P}_0(s) \quad (30)$$

$$\bar{P}_6(0,s)=\lambda_2\bar{P}_5(0,s)=2\lambda_1\lambda_2\lambda_3\bar{P}_0(s) \quad (31)$$

$$\bar{P}_7(0,s)=\lambda_4\bar{P}_5(0,s)=2\lambda_1\lambda_3\lambda_4\bar{P}_0(s) \quad (32)$$

$$\bar{P}_8(0,s)=\lambda_A[\bar{P}_0(s)+\bar{P}_1(0,s)+\bar{P}_3(0,s)+\bar{P}_5(0,s)] =\lambda_A[1+\lambda_1+\lambda_3+2\lambda_1\lambda_3] \quad (33)$$

$$\bar{P}_9(0,s)=\lambda_B[\bar{P}_0(s)+\bar{P}_1(0,s)+\bar{P}_3(0,s)+\bar{P}_5(0,s)] =\lambda_B[1+\lambda_1+\lambda_3+2\lambda_1\lambda_3] \quad (34)$$

$$\bar{P}_{10}(0,s)=\lambda_D[\bar{P}_0(s)+\bar{P}_1(0,s)+\bar{P}_3(0,s)+\bar{P}_5(0,s)] =\lambda_D[1+\lambda_1+\lambda_3+2\lambda_1\lambda_3] \quad (35)$$

Now solving equations (20)- (25) with help of boundary conditions (26)- (35), one may get (See appendix 1 for solution)

$$\bar{P}_0(s)=\frac{1}{D(s)} \quad (36)$$

$$\bar{P}_1(s)=\frac{\lambda_1}{D(s)}\frac{(1-P)}{(s+\lambda_2+\lambda_3+\lambda_A+\lambda_B+\lambda_D)} \quad (37)$$

$$\bar{P}_2(s)=\frac{\lambda_1\lambda_2}{D(s)}\frac{(1-S)}{s} \quad (38)$$

$$\bar{P}_3(s)=\frac{\lambda_3}{D(s)}\frac{(1-Q)}{(s+\lambda_1+\lambda_4+\lambda_A+\lambda_B+\lambda_D)} \quad (39)$$

$$\bar{P}_4(s)=\frac{\lambda_3\lambda_4}{D(s)}\frac{(1-S)}{s} \quad (40)$$

$$\bar{P}_5(s)=\frac{1}{D(s)}\frac{2\lambda_1\lambda_3(1-Q)}{(s+\lambda_2+\lambda_4+\lambda_A+\lambda_B+\lambda_D)} \quad (41)$$

$$\bar{P}_6(s)=\frac{2\lambda_1\lambda_2\lambda_3}{D(s)}\frac{(1-S)}{s} \quad (42)$$

$$\bar{P}_7(s)=\frac{2\lambda_1\lambda_3\lambda_4}{D(s)}\frac{(1-S)}{s} \quad (43)$$

$$\bar{P}_8(s)=\frac{\lambda_A[1+\lambda_1+\lambda_3+2\lambda_1\lambda_3]}{D(s)}\frac{(1-S)}{s} \quad (44)$$

$$\bar{P}_9(s)=\frac{\lambda_B[1+\lambda_1+\lambda_3+2\lambda_1\lambda_3]}{D(s)}\frac{(1-S)}{s} \quad (45)$$

$$\bar{P}_{10}(s)=\frac{\lambda_D[1+\lambda_1+\lambda_3+2\lambda_1\lambda_3]}{D(s)}\frac{(1-S)}{s} \quad (46)$$

where,

$$D(s)=s+\lambda_1+\lambda_3+\lambda_A+\lambda_B+\lambda_D-\lambda_1P-\lambda_3Q-2\lambda_1\lambda_3R-(\lambda_1\lambda_2+\lambda_3\lambda_4+2\lambda_1\lambda_2\lambda_3+2\lambda_1\lambda_3\lambda_4)S-(\lambda_A+\lambda_B+\lambda_D)([1+\lambda_1+\lambda_3+2\lambda_1\lambda_3)$$

$$P=\bar{S}_\varphi(s+\lambda_2+\lambda_3+\lambda_A+\lambda_B+\lambda_D)=\frac{\varphi}{s+\lambda_2+\lambda_3+\lambda_A+\lambda_B+\lambda_D+\varphi}$$

$$Q=\bar{S}_\varphi(s+\lambda_1+\lambda_4+\lambda_A+\lambda_B+\lambda_D)=\frac{\varphi}{s+\lambda_1+\lambda_4+\lambda_A+\lambda_B+\lambda_D+\varphi}$$

$$R=\bar{S}_\varphi(s+\lambda_2+\lambda_4+\lambda_A+\lambda_B+\lambda_D)=\frac{\varphi}{s+\lambda_2+\lambda_4+\lambda_A+\lambda_B+\lambda_D+\varphi}$$

$$S=\bar{S}_{\mu_0}(s)=\frac{\mu_0}{s+\mu_0}$$

In the case when the system is in operational mode at some moment and in failed state at another, the total of Laplace transformations of the state transitions is as follows.

$$\bar{P}_{up}(s)=\bar{P}_0(s)+\bar{P}_1(s)+\bar{P}_3(s)+\bar{P}_5(s) \quad (47)$$

$$\bar{P}_{down}(s) = 1 - \bar{P}_{up}(s) \quad (48)$$

6. Numerical Analysis

6.1 Availability Analysis

When repair follows both general and copula distributions, then we have

$$\bar{S}_{\mu_0}(s) = \bar{S}_{\exp[x^\theta + \{log\varphi(x)\}^\theta]^{1/\theta}}(s) = \frac{\exp[x^\theta + \{log\varphi(x)\}^\theta]^{1/\theta}}{s + \exp[x^\theta + \{log\varphi(x)\}^\theta]^{1/\theta}}$$

Setting

$$\bar{S}_{\mu_0}(s) = \frac{\varphi_i}{s + \varphi_i}; i = 1, 2, 3, and\ 4$$

Since the subsystem-1 is working on k-out-of-n: G policy having n identical units with failure rate α_1, then $\lambda_1 = k_1\alpha_1$and $\lambda_2 = (k - k_1)\alpha_1$. Similarly, the subsystem-2 is working on r-out-of-m: G policy having m identical units with failure rate α_2, then $\lambda_3 = r_1\alpha_2$and $\lambda_4 = (r - r_1)\alpha_2$. Let us fix the values of parameter arbitrarily as $\alpha_1 = 0.01$, $\alpha_2 = 0.01$, $\lambda_A = 0.04, \lambda_B = 0.05, \lambda_D = 0.03, \varphi_i = 1, \theta = 1, x = 1$ $\{i = 1, 2, 3, and\ 4\}$ in (47) and taking inverse Laplace transformation for availability. Here we consider three cases for various values of k, k_1, r, k_1.

Case-I: $k = 10, k_1 = 5, r = 10, k_1 = 5$ (5-out-of-10 : G configuration)

(a) $P_{up}(t) = -0.002004e^{-51.2200t} - 0.000223e^{-91.0800t} + 0.064442e^{-2.8881t} + 0.075129e^{-1.3736t} + 0.000236e^{-1.2627t} + 0.000509e^{-1.2121t} + 0.864416e^{-0.0417t} - 0.002507e^{-41.2200t}$ (49)

Case-II: $k = 15, k_1 = 10, r = 15, k_1 = 10$ (10-out-of-15 : G configuration)

(b) $P_{up}(t) = 0.078416e^{-2.9195t} + 0.095048e^{-1.4664t} + 0.001342e^{-1.2247t} + 0.829504e^{-0.0677t} - 0.004008e^{-76.2700t} - 0.000301e^{-151.0800t}$ (50)

Case-III: $k = 20, k_1 = 15, r = 20, k_1 = 15$ (15-out-of-20 : G configuration)

(c) $P_{up}(t) = 0.093339e^{-2.9519t} - 0.107279e^{-1.5465t} + 0.002383e^{-1.2271t} + 0.801407e^{-0.1028t} - 0.004008e^{-101.3200t} - 0.000401e^{-201.0800t}$ (51)

Fixing $t = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and\ 10$ units of time in the above three cases (a), (b) and (c), One may get the values of system availability $P_{up}(t)$. The corresponding figure-2 shows the variation of availability as a function of time t for different system configuration types. The predicted graph shows that the system's availability is best for case-I and low for case-III, informing the operators for adopting the policy for future perspectives and performance purposes.

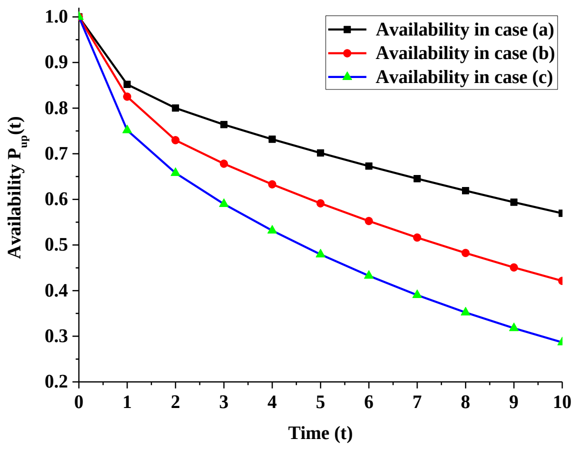

Fig. 2. Availability of the system

6.2 Reliability of the system

Reliability of the system is defined as a performative measure of a non-repairable system. Therefore, putting all repairs to zero and obtaining the inverse Laplace transform of $\bar{P}_{up}(s)$ presented in equation (47) in section-5, one can acquire expression for the system's reliability. Taking the parametric values of failure rates as $\alpha_1 = 0.01$, $\alpha_2 = 0.01$, $\lambda_A = 0.04, \lambda_B = 0.05, \lambda_D = 0.03, \varphi_i = 1, \theta = 1, x = 1$ $\{i = 1, 2, 3, and\ 4\}$ in (47) and get reliability expressions for system reliability same like availability for different configurations as-

Case-I: $k = 10, k_1 = 5, r = 10, k_1 = 5$ (5-out-of-10 : G configuration)

(a) $R(t) = 1.004733e^{-0.3200t} - 0.000223e^{-90.0800t} - 0.002506e^{-40.22000t} - 0.002004e^{-50.2200t}$ (52)

Case-II: $k = 15, k_1 = 10, r = 15, k_1 = 10$ (10-out-of-15 : G configuration)

(b) $R(t) = -0.000301e^{-150.0800t} - 0.004008e^{-75.2700t} + 1.004309e^{-0.4200t}$ (53)

Case-III: $k = 20, k_1 = 15, r = 20, k_1 = 15$ (15-out-of-20 : G configuration)

(c) $R(t) = -0.000401e^{-200.0800t} + 1.004409e^{-0.5200t} - 0.004008e^{-100.3200t}$ (54)

Fixing $t = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and\ 10$ units of time in the above three cases (a), (b) and (c), One may get the values of reliability of the system $R(t)$. The variation of reliability has graphically shown in figure -3. It has been demonstrated via the graph in figure-3 that reliability variations for the configuration (5-out- of- 10: G) are best and worst for (15-out- of- 20: G), which attract reader attention for knowing the factors corresponding to failure effects which need to be controlled.

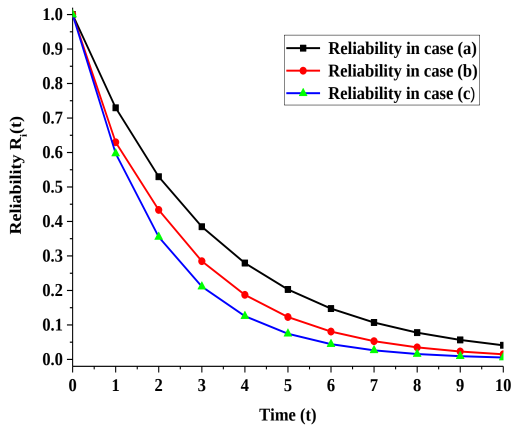

Fig. 3. Reliability of the system

6.3 Expected profit analysis

Let us allow access to the service facility at all times, with K_1 and, K_2 as revenue generation from unit production and service cost per unit time in the interval [0, t) than net profit from the system operations can be computed as follows:

$$E_p(t) = K_1 \int_0^t P_{up}(t)dt - K_2 t \quad (55)$$

For the same set of parameters defined in (49), for a particular case, i.e., (5-out-of- 10: G configuration) can obtain the expression for expected profit function presented in equation (56). Therefore,

$$E_p(t) = K_1\{20.7917 + 0.000061e^{-41.2200t} - 0.022313e^{-2.8881t} - 0.054696e^{-1.3736t} - 0.000187e^{-1.2627t} - 0.000420e^{-1.2121t} - 20.714227e^{-0.0417t} + 0.000039e^{-51.2200t} + 0.000002e^{-91.0800t}\} - K_2 t \quad (56)$$

Fixing the value for revenue generation at $K_1 = 1$ and varying service costs $K_2 = 0.6, 0.5, 0.4, 0.3, 0.2, and\ 0.01$ and taking $t = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, and\ 10$ units of time and putting in (56). The expected profit analysis is shown graphically in figure -4. It goes without saying that when service costs drop, the expected profit increases with time t. The figure-4 shows that the expected profit for the configuration (5-out-of-10: G) is maximum when K_1 =1 and K_2 =0.1and minimum for K_1 =1 and K_2 =0.6.

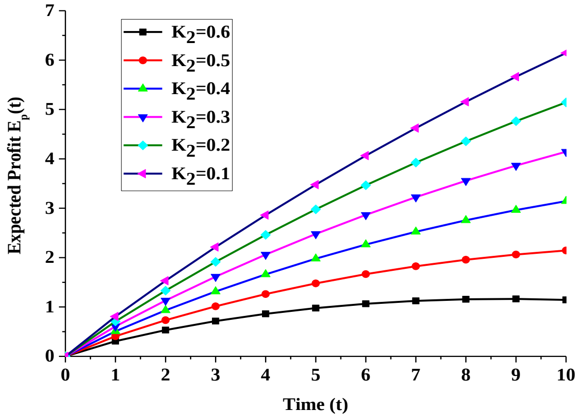

Fig. 4. Expected profit for various t

7. Result Discussion and Conclusion

The reliability, availability, and expected profit of a complex system comprised of two subsystems connected in series through a switching device and monitored by a human operator were investigated. The supplementary variable technique was used to generate explicit expressions. Several special cases are proposed for various values of k and r in order to evaluate the impact on availability and reliability. On the basis of the research performed in this article, the following conclusions may be drawn:

1. Table-2 and figure-2 detail the availability of both subsystems for various values of k and r. As can be seen, availability is higher for lower values of k and r, and unavailable for bigger values. Additionally, in all three instances, the system's availability diminishes as the value of time t rises.
2. Table-3 and figure-3 illustrate the reliability of the system at various time points by changing the values of k and r. As a result, for any given set of input parameters, one may confidently forecast the future behaviour patterns of a complex system at any point in time, as shown by the model's graphical representation.
3. Tables 2 and 3 demonstrate that availability values are more significant than reliability values, emphasizing the need of routine maintenance for high-performance of repairable systems.
4. A close study of table-4 and figure-4 shows that expected profit rises when service cost K_2 lowers, while revenue cost per unit time remains constant at K_1=1. The computed predicted profit is greatest when K_2=0.1 and smallest for K_2=0.6. With the passage of time, we notice that as service costs drop, profit increases. When service costs are low, the projected profit margin is large in comparison to the high service costs.
5. The authors have analysed this model using constant failures and two types of repairs employing copula distribution. Further the model can also extend to a comparative study of performance evaluation of system for copula repair and general repair.

In this paper, the models developed were highly advantageous to engineers, maintenance managers, system designers, and plant management in terms of performing appropriate maintenance analysis, decision-making, and performance assessment.

Conflicts of interest

The authors declare that they have no conflict of interest.

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Appendix-1

Solution of equation set (26) to (48); The equations (26) to (35) are the Laplace transform of equations (8) to (17). From the equation (20), we have

$$\left(s+\frac{\partial}{\partial x}+\lambda_2+\lambda_3+\lambda_A+\lambda_B+\lambda_D+\emptyset(x)\right)\bar{P}_1(x,s)=0$$

$$\left(s+\lambda_2+\lambda_3+\lambda_A+\lambda_B+\lambda_D+\emptyset(x)\right)\bar{P}_1(x,s)+\frac{\partial}{\partial x}\bar{P}_1(x,s)=0$$

$$\frac{\partial}{\partial x}\bar{P}_1(x,s)=-\left(s+\lambda_2+\lambda_3+\lambda_A+\lambda_B+\lambda_D+\emptyset(x)\right)\bar{P}_1(x,s)$$

$$\therefore \frac{\frac{\partial}{\partial x}\bar{P}_1(x,s)}{\bar{P}_1(x,s)}=-\left(s+\lambda_2+\lambda_3+\lambda_A+\lambda_B+\lambda_D+\emptyset(x)\right)$$

Integrating from 0 to x,

$$\int_0^x \frac{\frac{\partial}{\partial x}\bar{P}_1(x,s)}{\bar{P}_1(x,s)}dx=-\int_0^x ((s+\lambda_2+\lambda_3+\lambda_A+\lambda_B+\lambda_D+\emptyset(x))dx$$

$$\log[\bar{P}_1(x,s)]_0^x=-\int_0^x(s+\lambda_2+\lambda_3+\lambda_A+\lambda_B+\lambda_D)dx-\int_0^x\emptyset(x)dx$$

$$\log\bar{P}_1(x,s)-\log\bar{P}_1(o,s)=-(s+\lambda_2+\lambda_3+\lambda_A+\lambda_B+\lambda_D)x-\int_0^x\emptyset(x)dx$$

$$\log\left[\frac{\bar{P}_1(x,s)}{\bar{P}_1(o,s)}\right]=-(s+\lambda_2+\lambda_3+\lambda_A+\lambda_B+\lambda_D)x-\int_0^x\emptyset(x)\,dx$$

$$\therefore \frac{\bar{P}_1(x,s)}{\bar{P}_1(o,s)}=e^{-(s+\lambda_2+\lambda_3+\lambda_A+\lambda_B+\lambda_D)x+\int_0^x\emptyset(x)dx)}$$

$$\bar{P}_1(x,s)=\bar{P}_1(o,s)\{\exp(-(s+\lambda_2+\lambda_3+\lambda_A+\lambda_B+\lambda_D)x).e^{-\int_0^x\emptyset(x)dx}\}$$

Where $\bar{P}_1(o,s)=\lambda_1\bar{P}_0(s)$ from equation (26). Similarly, we can solve other equation from (21) to (28).

The solutions of the above equations can be modified by the use of the following slandered notations:

$S_\emptyset(x)=\emptyset(x).e^{-\int_0^x\emptyset_1(x)dx}$ and

$$S_{\mu_0}(x)=\mu_0(x).e^{-\int_0^x\mu_0(x)dx}$$

$$L[S_\emptyset(x)]=\int_0^\infty e^{-sx}\left\{\emptyset(x).e^{-\int_0^x\emptyset_1(x)dx}\right\}dx$$

$$=\int_0^\infty e^{-sx}.S_\emptyset(x)dx=\bar{S}_\emptyset(s)=\frac{\emptyset}{s+\emptyset}$$

$$L[S_{\mu_0}(x)]=\int_0^\infty e^{-sx}\left\{\mu_0(x).e^{-\int_0^x\mu_0(x)dx}\right\}dx$$

$$=\int_0^\infty e^{-sx}.S_{\mu_0}(x)dx=\bar{S}_{\mu_0}(s)=\frac{\mu_0}{s+\mu_0}$$

Also, $\int_0^\infty e^{-sx}\,.e^{-\int_0^x\emptyset(x)dx}dx=\frac{1-S_\emptyset(s)}{s}=\frac{1}{s+\emptyset}$

$$\int_0^\infty e^{-sx}\,.e^{-\int_0^x\mu_0(x)dx}dx=\frac{1-S_{\mu_0}(s)}{s}=\frac{1}{s+\mu_0}$$

Using the solutions in equation (19) one can get the solution of as; $D(s)\bar{P}_0(s)=1$, D(s) is the coefficient of $\bar{P}_0(s)$ in $D(s)\bar{P}_0(s)=1$ conclusively solution of (19) is given as;

$$\bar{P}_0(s)=\frac{1}{D(s)} \quad (36)$$

$$\bar{P}_1(s)=\int_0^\infty\bar{P}_1(x,s)\,dx=\int_0^\infty\bar{P}_1(0,s)\{\exp(-(s+\lambda_2+\lambda_3+\lambda_A+\lambda_B+\lambda_D)\,x)\,.e^{-\int_0^x\emptyset(x)dx}\}\,dx$$

$$=\lambda_1\bar{P}_0(s)\left[\frac{1-\bar{S}_\emptyset(s+\lambda_2+\lambda_3+\lambda_A+\lambda_B+\lambda_D)}{s+\lambda_2+\lambda_3+\lambda_A+\lambda_B+\lambda_D+\emptyset(x)}\right]$$

(using shifting property of Laplace Transforms)

$$\bar{P}_1(s)=\lambda_1\frac{\lambda_1}{D(s)}\frac{(1-P)}{s+\lambda_2+\lambda_3+\lambda_A+\lambda_B+\lambda_D} \quad (37)$$

where, $P=\frac{\emptyset(x)}{s+\lambda_2+\lambda_3+\lambda_A+\lambda_B+\lambda_D+\emptyset(x)}$

Similarly, $\bar{P}_2(s)$, $\bar{P}_3(s)$, $\bar{P}_4(s)\ldots\ldots\ldots.\bar{P}_{10}(s)$ presented in equations (38), (39), …… (46) can be obtained. Putting values in (47) the expression (48) can be obtained.

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