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Research Article

A Note On "An Interval Type-2 Fuzzy Extension Of The TOPSIS Method Using Alpha Cuts"

Amir Mohamadghasemi

Department of Management, Zabol Branch, Islamic Azad University, Zabol, Iran

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Abstract

The technique for order of preference by similarity to ideal solution (TOPSIS) is a method based on the ideal solutions in which the most desirable alternative should have the shortest distance from positive ideal solution and the longest distance from negative ideal solution. Depending on type of evaluations or method of ranking, different approaches have been proposing to calculate distances in the TOPSIS method. In a recent paper, Dymova et al. (2015) extended the TOPSIS approach using interval type 2 fuzzy sets (IT2FSs) in which distances were calculated using alpha cuts. When investigating their paper, we found out that the extended method has some drawbacks such that it leads to the incorrect calculations and results when solving an IT2FSs-based multi-criteria decision making (MCDM) problem. In this note, the corrected version of extended TOPSIS method is being presented to eliminate its limitations. In order to show effectiveness and possibility of the proposed approach, it is also implemented in two illustrative examples and one case study. The results have showed that the optimal alternative obtained by the corrected TOPSIS approach has the similar rank to the others, whereas it is different from the results of existing TOPSIS approach.

***Keywords*: TOPSIS; Interval type 2 fuzzy sets; Multi-criteria decision making; Alpha cuts**

1. Introduction

A multi-criteria decision making (MCDM) problem includes the evaluation of alternatives with respect to a set of qualitative and quantitative criteria such that the most desirable alternative should have the highest measure and the lowest measure with respect to benefit and cost criteria, respectively. Depending on type of data, situation of ideal solutions (reference points), distances calculation's method, type of ranking approach, etc. there are the different methods for solving the MCDM problems. The technique for order of preference by similarity to ideal solution (TOPSIS) is one of the most popular techniques of MCDM. It was first developed by Hwang and Yoon (1981). The main goal of method is that the best alternative should have the shortest distance from the positive ideal solution and the longest distance from the negative ideal solution. In the classical TOPSIS method, the appraisals and weights of criteria are stated as the precise values. However, in the real world, the crisp data are not suitable because the measures of membership function (MF) for the crisp data are zero and only the discrete crisp numbers with MF=1 are chosen for evaluations. To explain the ambiguity in the real world problems, the fuzzy data have been utilized instead of crisp data in many MCDM techniques including TOPSIS. In fuzzy TOPSIS (FTOPSIS), all the ratings and weights are expressed as the fuzzy numbers. However, an expert may have doubt about the constant of MF. In other words, in type-1 fuzzy sets, a decision-maker is not able to consider any flexibility regarding the size of the MF. However, there are situations where it is impossible to satisfactorily assess MF and it should be stated as an interval. Hence, the type-2 fuzzy sets were suggested by Zadeh (1975) in order to relieve MF measure's uniqueness of the type-1 fuzzy sets. The type-2 fuzzy sets are depicted in three-dimensional space including finite non-empty set *X*, secondary grade $f_x(u)$, and the domain of the secondary MF (J_x) where MF is defined by a fuzzy set at the interval [0, 1]. Interval type-2 fuzzy sets (IT2FSs) are a particular version of type-2 fuzzy sets characterized through the interval MF. IT2FSs show some interesting properties for handling uncertain information and high imprecision (Liu et al., 2018). There are the known versions for IT2FSs such as trapezoidal interval type-2 fuzzy sets (TraIT2FSs) and triangular interval type-2 fuzzy sets (TriIT2FSs).

The rest of this paper is organized as follows: In Section 2, the literature review is studied related to the MCDM techniques including TOPSIS. Section 3 presents the existing drawbacks in the TOPSIS method adopted by Dymova et al. (2015). In Section 4, these drawbacks are removed and the corrected form of the extended TOPSIS method is presented. The corrected version is implemented in two illustrative examples in Section 5. Section 6 gives an industrial case and finally, conclusions obtained by the corrected approach are summarized in Section 7.

2. Literature Review

Since the distance or preference of evaluations from the ideal solutions can be calculated using the different expressions or formulas, there are the several

*Corresponding author Email address: amir_mohamadghassemi@zaboliau.ac.ir

generalizations of the TOPSIS method based on IT2FSs in the literature. Saremi and Montazer (2008) adopted a new generalization to the TOPSIS technique based on type-2 fuzzy environment in website structures selection. Ghaemi Nasab and Rostamy-Malkhalifeh (2010) applied an extension of FTOPSIS to handle fuzzy MCDM (FMCDM) where the fuzzy positive and negative ideal solutions were obtained in the form of IT2FSs without ranking the elements of decision matrix. Yong et al. (2015) introduced a new decision making approach integrating the IT2FSs and TOPSIS method. Then, it was implemented in the metro station dynamic risk assessment. Erdoğan and Kaya (2014) created a ranking method for selection of university where the TOPSIS method with the type-2 fuzzy sets was used. Chen (2015) extended the interval type-2 fuzzy TOPSIS procedure to specify the ranked orders of the alternatives under the multiple criteria evaluation by adopting a likelihood-based comparison approach with the approximate ideals. Çebi and Otay (2015) proposed a comprehensive and systematic approach for multi-criteria and multi-stage facility location selection problem where TOPSIS was solved using IT2FSs. Temur et al. (2014) proposed the type-2 fuzzy TOPSIS methodology for the selection of the most appropriate reverse logistics facility location. Unfortunately, despite the various applications of TOPSIS with IT2FS, these have some limitations. For example, Ashtiani et al. (2009) and Mokhtarian et al. (2014) incorporated triangular interval type-2 fuzzy numbers (TriIT2FNs) into the TOPSIS method. In their approach, the positive and negative ideal solutions were selected as $A^{+}=(1,...,1)$ and $A^{-}=(0,...,0)$, respectively. These might not be accessible in the MCDM matrix. Chen and Lee (2010) applied TraIT2Ss to a MCDM matrix and then used the heuristic expressions including average and standard deviation to prioritize the assessments. Rashid et al. (2014) introduced an approach based on the generalized trapezoidal interval-valued fuzzy value in the framework of TOPSIS method. The weakness of this approach is that the reference points were taken into account for determining the positive and negative ideal solutions. Ghorabaee (2016) developed a MCDM method with TriIT2FNs for robot selection. The limitation of this methodology is that the centroid point expression was used in the decision making matrix and then these measures were applied to rank expressions. The weights of criteria in the FTOPSIS methodology were also incorporated into the distances. Abbasimehr and Tarokh (2016) introduced a hybrid MCDM technique to prioritize the reviewers in online communities in which the interval type-2 fuzzy analytic hierarchy process (AHP) and TOPSIS were used in order to obtain the weights of features and the final ranking of reviewers, respectively. They used a heuristic expression to calculate the criteria weights. In addition, the interval type-2 fuzzy numbers were first defuzzified and then, the defuzzified pairwise comparison matrices were applied to consider the consistency index. Baykasoğlu and Gölcük (2018) adopted the interval type-2 fuzzy decision making trial and evaluation laboratory (DEMATEL) method to notice interdependencies among problem attributes and then utilized the hierarchical interval type-2 fuzzy TOPSIS method to rank Strengths, Weaknesses, Opportunities, and Threats (SWOT)-based strategies. They first defuzzified TraIT2FSs and then determined ideal solutions. Deveci et al. (2018) proposed IT2FSs-based weighted aggregated sum product assessment (WASPAS)-based TOPSIS to determine the best location for a new car-sharing station among four alternatives. They used the Euclidean distance to calculate the distances of assessments from the ideal solutions such that the weights of criteria were incorporated into it. Wang et al. (2018) extended the TOPSIS method to the environment of interval normal type-2 fuzzy set (INT2FS) where the expected value of INT2FN was defined to determine the ideal solutions and the Euclidean distance of INT2FN was handled by measure the similarity among each alternative and them. The proposed approach is only usable for the normal distribution. In order to remove the above disadvantages, Mohamadghasemi et al. (2018) presented a new approach for ranking Gaussian interval type-2 fuzzy sets (GIT2FSs). It determines the maximum and minimum reference limits of GIT2FSs as positive and negative ideal solutions and then calculates the distances between assessments and them. In the next step, the TOPSIS method was extended through a new ranking approach. Recently, Gündoğdu and Kahraman (2019) applied the interval-valued spherical fuzzy sets to develop TOPSIS. They used the interval-valued spherical fuzzy TOPSIS method in order to solve the three dimensional printers' selection problem.

In a recent paper, Dymova et al. (2015) generalized the TOPSIS method with IT2FSs using the concept of alpha cuts. When investigating their extended TOPSIS method, we found that the extended TOPSIS method has some drawbacks such that application of it to MCDM problems will result in the incorrect calculations. In this note, we intend to remove the limitations of the extended TOPSIS method.

3. The Drawbacks of Extended TOPSIS Method

There is a set of limitations including indices, expressions, and calculations in the TOPSIS method extended by Dymova et al. (2015) such that a researcher may obtain the incorrect results when using this publication as a reference. In the following, we will explain these drawbacks for each step.

Dymova et al. (2015) have assumed that $A_1, A_2, ..., A_m$, $C_1, C_2, ..., C_n$ and $w_1, w_2, ..., w_n$ are alternatives, criteria, and weights of criteria, respectively, such that $\sum_{i=1}^{n} w_i = 1$. Then, they have supposed that $D = \left[\hat{X}_{ij}\right]_{n\times m}$ is a decision matrix where $\hat{X}_{ij}$ is the perfectly normal interval type-2 fuzzy value (*IT2FV*), representing the rating of alternative A_i with respect to the criterion C_j. Obviously, the first definition of indices is contradicted with the indices expressed in the definition of decision matrix. In other words, it is better to consider indices i and j for alternatives and criteria, respectively. In addition, they have taken into account $\bar{x}_{ij}^L$, $\underline{x}_{ij}^L$, x_{ij}^M, $\underline{x}_{ij}^U$, and $\bar{x}_{ij}^U$ as the reference points of perfectly normal *IT2FV* $\hat{X}_{ij}$ and then it has been showed as follows:

$$\hat{X}_{ij} = \left\{ \left[\bar{x}_{ij}^L, \underline{x}_{ij}^L \right], x_{ij}^M, \left[\underline{x}_{ij}^U, \bar{x}_{ij}^U \right] \right\}, \quad (1)$$

Next, the following two expressions have been presented for type 1 fuzzy numbers with respect to benefit and cost criteria, respectively:

$$r_{ij} = \left(\frac{x_{ij}^L}{x_j^+}, \frac{x_{ij}^M}{x_j^+}, \frac{x_{ij}^U}{x_j^+} \right), \quad i = 1,\ldots,m, \quad j \in K_b, \quad (2)$$

where x_{ij}^L, x_{ij}^M, and x_{ij}^U are the reference points of a type 1 fuzzy number, $j \in K_b$ is the set of benefit criteria, and $x_j^+ = \max_j (x_{ij}^U)$.

$$r_{ij} = \left(\frac{x_j^-}{x_{ij}^U}, \frac{x_j^-}{x_{ij}^M}, \frac{x_j^-}{x_{ij}^L} \right), \quad i = 1,\ldots,m, \quad j \in K_c, \quad (3)$$

where $x_j^- = \min_j (x_{ij}^L)$ and $j \in K_c$ is the set of cost criteria.

since index j is related to criteria, $\max_i$ and $\min_i$ should be used in $x_j^+ = \max_j (x_{ij}^U)$ and $x_j^- = \min_j (x_{ij}^L)$, respectively. In addition, $i = 1,\ldots,n$ should be used in Eqs. (2-3).

Similarly, the above normalization method can be extended for perfectly normal *IT2FV* as follows:

$$\hat{r}_{ij} = \left(\left[\frac{\bar{x}_{ij}^L}{x_j^+}, \frac{\underline{x}_{ij}^L}{x_j^+} \right], \frac{x_{ij}^M}{x_j^+}, \left[\frac{\underline{x}_{ij}^U}{x_j^+}, \frac{\bar{x}_{ij}^U}{x_j^+} \right] \right), \quad i = 1,\ldots,m, \quad j \in K_b, \quad (4)$$

where $x_j^+ = \max_j (\bar{x}_{ij}^U)$.

$$\hat{r}_{ij} = \left(\left[\frac{x_j^-}{\bar{x}_{ij}^U}, \frac{x_j^-}{\underline{x}_{ij}^U} \right], \frac{x_j^-}{x_{ij}^M}, \left[\frac{x_j^-}{\underline{x}_{ij}^L}, \frac{x_j^-}{\bar{x}_{ij}^L} \right] \right), \quad i = 1,\ldots,m, \quad j \in K_c, \quad (5)$$

where $x_j^- = \min_j (\bar{x}_{ij}^L)$. again, since index j is related to criteria, $\max_i$ and $\min_i$ should be used in $x_j^+ = \max_j (\bar{x}_{ij}^U)$ and $x_j^- = \min_j (\bar{x}_{ij}^L)$, respectively. Moreover, $i = 1,\ldots,n$ should be utilized in Eqs. (4-5).

The next step is to obtain the positive (A^+) and negative (A^-) ideal solutions, respectively, as follows:

$$\begin{aligned} A^+ &= \left\{ \hat{v}_1^+, \hat{v}_2^+, \ldots, \hat{v}_n^+ \right\} = \left\{ \left(\left[\bar{v}_1^{+L}, \underline{v}_1^{+L} \right], v_1^{+M}, \left[\underline{v}_1^{+U}, \bar{v}_1^{+U} \right] \right), \ldots, \left(\left[\bar{v}_n^{+L}, \underline{v}_n^{+L} \right], v_n^{+M}, \left[\underline{v}_n^{+U}, \bar{v}_n^{+U} \right] \right) \right\} \\ &= \left\{ \left(\max_i \left\{ \hat{r}_{ij} \right\} \right) \middle| j \subset K_b, \left(\min_i \left\{ \hat{r}_{ij} \right\} \right) \middle| j \subset K_c \right\} \end{aligned} \quad (6)$$

and

$$\begin{aligned} A^- &= \left\{ \hat{v}_1^-, \hat{v}_2^-, \ldots, \hat{v}_n^- \right\} = \left\{ \left(\left[\bar{v}_1^{-L}, \underline{v}_1^{-L} \right], v_1^{-M}, \left[\underline{v}_1^{-U}, \bar{v}_1^{-U} \right] \right), \ldots, \left(\left[\bar{v}_n^{-L}, \underline{v}_n^{-L} \right], v_n^{-M}, \left[\underline{v}_n^{-U}, \bar{v}_n^{-U} \right] \right) \right\} \\ &= \left\{ \left(\min_i \left\{ \hat{r}_{ij} \right\} \right) \middle| j \subset K_b, \left(\max_i \left\{ \hat{r}_{ij} \right\} \right) \middle| j \subset K_c \right\} \end{aligned} \quad (7)$$

where $\hat{r}_{ij} = \left(\left[\bar{r}_{ij}^L, \underline{r}_{ij}^L \right], r_{ij}^M, \left[\underline{r}_{ij}^U, \bar{r}_{ij}^U \right] \right)$.

the number of expressions in A^+ and A^- should be equal to number of criteria, i.e. m. In addition, $\max_i \left\{ \hat{r}_{ij} \right\} = \left(\left[\max_i \bar{r}_{ij}^L, \max_i \underline{r}_{ij}^L \right], \max_i r_{ij}^M, \left[\max_i \underline{r}_{ij}^U, \max_i \bar{r}_{ij}^U \right] \right)$ and $\min_i \left\{ \hat{r}_{ij} \right\} = \left(\left[\min_i \bar{r}_{ij}^L, \min_i \underline{r}_{ij}^L \right], \min_i r_{ij}^M, \left[\min_i \underline{r}_{ij}^U, \min_i \bar{r}_{ij}^U \right] \right)$ should be used in Eqs. (6-7) based on Rashid et al. (2014)

The next step is to calculate the distances between the normalized measures and the positive and negative ideal solutions. These can be calculated as follows:

$$S_i^+ = \sum_{j\in K_b} w_j \Delta(\hat{v}_j^+ - \hat{r}_{ij}) + \sum_{j\in K_c} w_j \Delta(\hat{r}_{ij} - \hat{v}_j^+), \qquad i=1,2,..,m, \tag{8}$$

$$S_i^- = \sum_{j\in K_c} w_j \Delta(\hat{r}_{ij} - \hat{v}_j^-) + \sum_{j\in K_b} w_j \Delta(\hat{v}_j^- - \hat{r}_{ij}), \qquad i=1,2,..,m, \tag{9}$$

where

$$\Delta(\hat{r}_{ij} - \hat{r}_{oj}) = \frac{\sum_\alpha \alpha \Delta(r_{ij\alpha} - r_{oj\alpha})}{\sum_\alpha \alpha}, \tag{10}$$

$$\Delta(r_{ij\alpha} - r_{oj\alpha}) = \frac{1}{8}(\bar{r}_{ij\alpha}^L + \underline{r}_{ij\alpha}^U + \underline{r}_{ij\alpha}^L + \bar{r}_{ij\alpha}^U - (\bar{r}_{oj\alpha}^U + \underline{r}_{oj\alpha}^L + \underline{r}_{oj\alpha}^U + \bar{r}_{oj\alpha}^L)), \tag{11}$$

In the above Eq. (9), $w_j\Delta(\hat{r}_{ij} - \hat{v}_j^-)$ and $w_j\Delta(\hat{v}_j^- - \hat{r}_{ij})$ are related to $j \in K_b$ and $j \in K_c$, respectively. Moreover, $i = 1,...,n$ should be used in Eqs. (8-9).

Finally, the relative closeness to the ideal alternatives is calculated as follows:

$$RC_i = \frac{S_i^-}{S_i^+ + S_i^-}, \qquad i = 1,...,m, \tag{12}$$

the bigger RC_i, the better alternative A_i.

4. The Corrected TOPSIS Method

In order to remove the drawbacks discussed above, this section presents the improved version of the TOPSIS method as follows:

Let $A_1, A_2,..., A_n$, $C_1, C_2,..., C_m$, and $w_1, w_2,..., w_m$ be alternatives, criteria, and weights of criteria, respectively, such that $\sum_{j=1}^m w_j = 1$. In addition, let $D = \left[\hat{X}_{ij}\right]_{n\times m}$ be the decision matrix, where $\hat{X}_{ij}$ is the perfectly normal *IT2FV* representing the rating of the alternative A_i with respect to the criterion C_j.

In addition, let $\bar{x}_{ij}^L$, $\underline{x}_{ij}^L$, x_{ij}^M, $\underline{x}_{ij}^U$, and $\bar{x}_{ij}^U$ be the reference points of perfectly normal *IT2FV* $\hat{X}_{ij}$ as follows:

$$\hat{X}_{ij} = \left\{ \left[\bar{x}_{ij}^L, \underline{x}_{ij}^L\right], x_{ij}^M, \left[\underline{x}_{ij}^U, \bar{x}_{ij}^U\right] \right\}, \tag{13}$$

Eqs. (2-3) are transformed into the following expressions:

$$r_{ij} = \left(\frac{x_{ij}^L}{x_j^+}, \frac{x_{ij}^M}{x_j^+}, \frac{x_{ij}^U}{x_j^+}\right), \qquad i=1,...,n, \quad j\in K_b, \tag{14}$$

where $x_j^+ = \max_i(x_{ij}^U)$.

$$r_{ij} = \left(\frac{x_j^-}{x_{ij}^U}, \frac{x_j^-}{x_{ij}^M}, \frac{x_j^-}{x_{ij}^L}\right), \qquad i=1,...,n, \quad j\in K_c, \tag{15}$$

where $x_j^- = \min_i(x_{ij}^L)$.

As a result:

$$\hat{r}_{ij} = \left(\left[\frac{\bar{x}_{ij}^L}{x_j^+}, \frac{\underline{x}_{ij}^L}{x_j^+}\right], \frac{x_{ij}^M}{x_j^+}, \left[\frac{\underline{x}_{ij}^U}{x_j^+}, \frac{\bar{x}_{ij}^U}{x_j^+}\right]\right), \qquad i=1,...,n, \quad j\in K_b, \tag{16}$$

where $x_j^+ = \max_i(\bar{x}_{ij}^U)$.

$$\hat{r}_{ij} = \left(\left[\frac{x_j^-}{\bar{x}_{ij}^U}, \frac{x_j^-}{\underline{x}_{ij}^U}\right], \frac{x_j^-}{x_{ij}^M}, \left[\frac{x_j^-}{\underline{x}_{ij}^L}, \frac{x_j^-}{\bar{x}_{ij}^L}\right]\right), \qquad i = 1,\dots,n, \quad j \in K_c, \tag{17}$$

where $x_j^- = \min_i(\bar{x}_{ij}^L)$.

The next step is to obtain A^+ and A^-, respectively, as follows:

$$\begin{aligned} A^+ &= \{\hat{v}_1^+, \hat{v}_2^+, \dots, \hat{v}_m^+\} = \left\{\left(\left[\bar{v}_1^{+L}, \underline{v}_1^{+L}\right], v_1^{+M}, \left[\underline{v}_1^{+U}, \bar{v}_1^{+U}\right]\right), \dots, \left(\left[\bar{v}_m^{+L}, \underline{v}_m^{+L}\right], v_m^{+M}, \left[\underline{v}_m^{+U}, \bar{v}_m^{+U}\right]\right)\right\} \\ &= \left\{\left(\max_i \{\hat{r}_{ij}\}\right) j \subset K_b, \left(\min_i \{\hat{r}_{ij}\}\right) j \subset K_c\right\} \\ &= \left\{\begin{array}{l} \left\{\left(\left[\max_i \bar{r}_{ij}^L, \max_i \underline{r}_{ij}^L\right], \max_i r_{ij}^M, \left[\max_i \underline{r}_{ij}^U, \max_i \bar{r}_{ij}^U\right]\right)\right\} \big| j \subset K_b \\ , \left\{\left(\left[\min_i \bar{r}_{ij}^L, \min_i \underline{r}_{ij}^L\right], \min_i r_{ij}^M, \left[\min_i \underline{r}_{ij}^U, \min_i \bar{r}_{ij}^U\right]\right)\right\} \big| j \subset K_c \end{array}\right\} \end{aligned} \tag{18}$$

and

$$\begin{aligned} A^- &= \{\hat{v}_1^-, \hat{v}_2^-, \dots, \hat{v}_n^-\} = \left\{\left(\left[\bar{v}_1^{-L}, \underline{v}_1^{-L}\right], v_1^{-M}, \left[\underline{v}_1^{-U}, \bar{v}_1^{-U}\right]\right), \dots, \left(\left[\bar{v}_n^{-L}, \underline{v}_n^{-L}\right], v_n^{-M}, \left[\underline{v}_n^{-U}, \bar{v}_n^{-U}\right]\right)\right\} \\ &= \left\{\left(\min_i \{\hat{r}_{ij}\}\right) j \subset K_b, \left(\max_i \{\hat{r}_{ij}\}\right) j \subset K_c\right\} \\ &= \left\{\begin{array}{l} \left\{\left(\left[\min_i \bar{r}_{ij}^L, \min_i \underline{r}_{ij}^L\right], \min_i r_{ij}^M, \left[\min_i \underline{r}_{ij}^U, \min_i \bar{r}_{ij}^U\right]\right)\right\} \big| j \subset K_b \\ , \left\{\left(\left[\max_i \bar{r}_{ij}^L, \max_i \underline{r}_{ij}^L\right], \max_i r_{ij}^M, \left[\max_i \underline{r}_{ij}^U, \max_i \bar{r}_{ij}^U\right]\right)\right\} \big| j \subset K_c \end{array}\right\} \end{aligned} \tag{19}$$

where $\hat{r}_{ij} = \left(\left[\bar{r}_{ij}^L, \underline{r}_{ij}^L\right], r_{ij}^M, \left[\underline{r}_{ij}^U, \bar{r}_{ij}^U\right]\right)$. The next step is to calculate the distances between the normalized measures and the positive and negative ideal solutions as follows:

$$S_i^+ = \sum_{j \in K_b} w_j \Delta(\hat{v}_j^+ - \hat{r}_{ij}) + \sum_{j \in K_c} w_j \Delta(\hat{r}_{ij} - \hat{v}_j^+), \qquad i = 1,2,..,n, \tag{20}$$

$$S_i^- = \sum_{j \in K_b} w_j \Delta(\hat{r}_{ij} - \hat{v}_j^-) + \sum_{j \in K_c} w_j \Delta(\hat{v}_j^- - \hat{r}_{ij}), \qquad i = 1,2,..,n, \tag{21}$$

Finally, the relative closeness for each alternative is calculated as follows:

$$RC_i = \frac{S_i^-}{S_i^+ + S_i^-}, \qquad i = 1,\dots,n, \tag{22}$$

5. Implementation of the Corrected TOPSIS Method

In order to show the effectiveness of the corrected TOPSIS method, we apply it to the illustrative examples introduced by Dymova et al. (2015). In order to study more details, the interested reader can refer to Dymova et al. (2015). First, a MCDM problem including three alternatives and four criteria is being used in which C_1 and C_3 are the cost criteria and C_2 and C_4 are the benefit criteria with the weights vector: $w_1 = 0.35$, $w_2 = 0.05$, $w_3 = 0.05$, and $w_4 = 0.55$. Table 1 shows perfectly normal interval type-2 fuzzy numbers.

Tables 2 and 3 present the normalized decision matrix by using Eqs. (16-17).

Table 1
The decision matrix

	C_1	C_2	C_3	C_4
A_1	([1,2],4,[5,6])	([4,5],6,[7,9])	([1,2],3,[7,8])	([1,2],4,[5,6])
A_2	([2,4],7,[9,10])	([2,3],4,[5,9])	([1,2],2,[7,8])	([1,2],3,[7,8])
A_3	([2,3],4,[7,8])	([3,4],7,[8,9])	([4,5],8,[9,11])	([2,3],7,[9,10])

Table 2
The normalized decision matrix

	C_1	C_2
A_1	([0.17,0.2],0.25,[0.5,1])	([0.44,0.56],0.67,[0.78,1])
A_2	([0.1,0.11],0.15,[0.25,0.5])	([0.22,0.33],0.44,[0.56,1])
A_3	([0.13,0.15],0.25,[0.33,0.5])	([0.33,0.44],0.78,[0.89,1])

Table 3
The normalized decision matrix

	C_3	C_4
A_1	([0.13,0.14],0.33,[0.5,1])	([0.1,0.2],0.4,[0.5,0.6])
A_2	([0.13,0.14],0.5,[0.5,1])	([0.1,0.2],0.3,[0.7,0.8])
A_3	([0.09,0.11],0.13,[0.2,0.25])	([0.2,0.3],0.7,[0.9,1])

Based on Tables 2 and 3, the measures A^+ and A^- are presented in Table 4 for both Eqs. (6-7) and Eqs. (18-19):

Table 4
The measures A^+ and A^-

	By using Eqs. (6-7)		By using Eqs. (18-19)	
	A^+	A^-	A^+	A^-
$\hat{v}_1^+$	([0.1,0.11],0.15,[0.25,0.5])		([0.1,0.11],0.15,[0.25,0.5])	
$\hat{v}_2^+$	**([0.33,0.44],0.78,[0.89,1])**		**([0.44,0.56],0.78,[0.89,1])**	
$\hat{v}_3^+$	([0.09,0.11],0.13,[0.2,0.25])		([0.09,0.11],0.13,[0.2,0.25])	
$\hat{v}_4^+$	([0.2,0.3],0.7,[0.9,1])		([0.2,0.3],0.7,[0.9,1])	
$\hat{v}_1^-$		([0.17,0.2],0.25,[0. 5,1])		([0.17,0.2],0.25,[0. 5,1])
$\hat{v}_2^-$		([0.22,0.33],0.44,[0.56,1])		([0.22,0.33],0.44,[0.56,1])
$\hat{v}_3^-$		([0.13,0.14],0.5,[0. 5,1])		([0.13,0.14],0.5,[0. 5,1])
$\hat{v}_4^-$		([0.1,0.2],0.3,[0.7,0.8])		([0.1,0.2],0.3,[0.5,0.6])

Based on Table 4, one can calculate the distances of ratings from A^+ and A^- by using Eqs. (20-21). Tables 5 and 6 show the distances by using Eqs. (8-9) and Eqs. (20-21), respectively.

Table 5
The distances of the evaluations from A^+ and A^- by using Eqs. (8-9)

S_1^+	S_2^+	S_3^+	S_1^-	S_2^-	S_3^-
0.0200	0.0042	0.0072	0.0183	0.0059	0.0057

Table 6
The distances of the evaluations from A^+ and A^- by using Eqs. (20-21)

S_1^+	S_2^+	S_3^+	S_1^-	S_2^-	S_3^-
0.1181	0.0143	0.0000	0.1868	0.0249	0.8914

Table 7 demonstrates the ranked results of the existing and corrected TOPSIS methods for data in Tables 5 and 6, respectively. Moreover, the measures RC_i obtained by the Mohamadghasemi et al. (2018) and Rashid et al. (2014) approaches have been presented in this table. The similar results exist between the recent two methods. According to Table 8, there are high correlation coefficients between the corrected TOPSIS, Mohamadghasemi et al. (2018), and Rashid et al. (2014) methods whereas these have the negative or low correlation coefficients with the existing TOPSIS method. Finally, although A_3 has rank 3 based on the existing TOPSIS method, it is selected as the optimal alternative by noting Table 8 and Figure 1.

Table 7
The final ranking of the alternatives based on the existing TOPSIS, corrected TOPSIS, Mohamadghasemi et al. (2018), and Rashid et al. (2014) methods

Alternatives	RC_i (the existing TOPSIS method)	Ranking	RC_i (the corrected TOPSIS method)	Ranking	RC_i (Mohamadghasemi et al., 2018)	Ranking	RC_i (Rashid et al., 2014)	Ranking
A_1	0.4778	2	0.1868	2	0.3027	3	0.1188	3
A_2	0.5842	1	0.0249	3	0.4255	2	0.3939	2
A_3	0.4597	3	0.8914	1	0.8842	1	0.7745	1

Table 8
The correlation between the different approaches

Alternatives	RC_i (the existing TOPSIS method)	RC_i (the corrected TOPSIS method)	RC_i (Mohamadghasemi et al., 2018)	RC_i (Rashid et al., 2014)
RC_i (the existing TOPSIS method)	-	-0.7414	-0.4411	0.2256
RC_i (the corrected TOPSIS method)	-	-	0.9292	0.9738
RC_i (Mohamadghasemi et al., 2018)	-	-	-	0.9738
RC_i (Rashid et al., 2014)	-	-	-	-

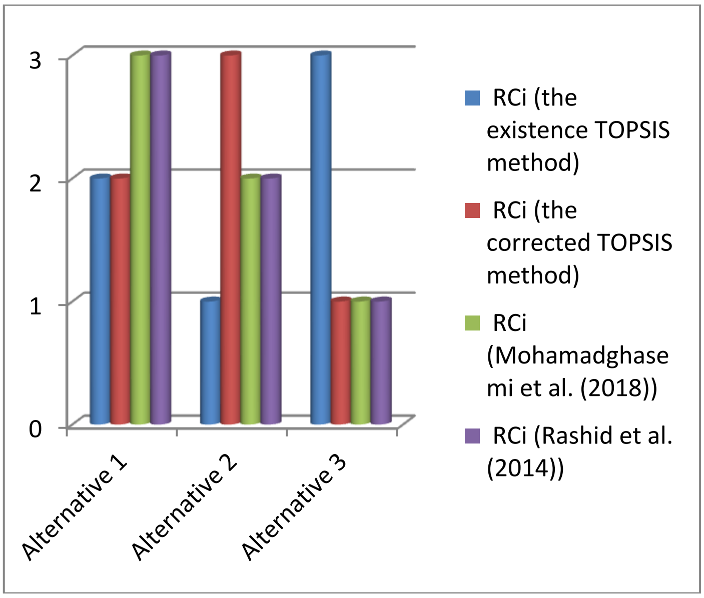

Fig. 1. Ranking the alternatives using different methods

As another example, we compare the existing TOPSIS with corrected TOPSIS method for normal IT2FVs in which the following expression should be used instead of Eq. (11) to determine distances for $\alpha \succ h$:

$$\Delta(r_{ij\alpha} - r_{oj\alpha}) = \frac{1}{2}(r^L_{ij\alpha} + r^U_{ij\alpha} - (r^L_{oj\alpha} + r^U_{oj\alpha})), \qquad (23)$$

Thus, we should determine lower heights of MF. Table 9 represents the heights of lower MF for the perfectly normal interval type-2 fuzzy numbers based on data in Table 1.

Table 9
Lower MF's heights of normal IT2FVs for evaluations of Table 1

	C_1	C_2	C_3	C_4
A_1	0.8	0.5	0.7	0.2
A_2	0.7	0.8	0.5	0.6
A_3	0.8	0.2	0.6	0.7

Tables 10 and 11 demonstrate the distances by using Eqs. (8-9) and Eqs. (20-21), respectively, for normal IT2FVs (by using Eq. (23) for $\alpha \succ h$).

Table 10
The distances of the evaluations from A^+ and A^- by using Eqs. (8-9)

S_1^+	S_2^+	S_3^+	S_1^-	S_2^-	S_3^-
0.0129	0.0029	0.0039	0.0093	0.0014	0.0020

Table 11
The distances of the evaluations from A^+ and A^- by using Eqs. (20-21)

S_1^+	S_2^+	S_3^+	S_1^-	S_2^-	S_3^-
0.2144	0.0484	0.0000	0.1901	0.0178	0.9399

According to Tables 10 and 11, Table 12 represents the ranked results of the existing TOPSIS, corrected TOPSIS, Mohamadghasemi et al. (2018), and Rashid et al. (2014) methods. As can be seen from Table 12, the corrected TOPSIS, Mohamadghasemi et al. (2018), and Rashid et al. (2014) approaches have the identical ranked results whereas these have many differences from the existing TOPSIS method. This fact is also presented in Table 13 where the high correlation coefficients exist between the above three approaches. On the other hand, alternative 3 is the most desirable alternative based on Figure 2 while this alternative has rank 2 in the existing TOPSIS method.

Table 12
The final ranking of the alternatives based on the existing TOPSIS, corrected TOPSIS, Mohamadghasemi et al. (2018), and Rashid et al. (2014) methods

Alternatives	RC_i (the existing TOPSIS method)	Ranking	RC_i (the corrected TOPSIS method)	Ranking	RC_i (Mohamadghasemi et al., 2018)	Ranking	RC_i (Rashid et al., [14])	Ranking
A_1	0.4189	1	0.1901	2	0.3131	3	0.0831	3
A_2	0.3256	3	0.0178	3	0.4328	2	0.4970	2
A_3	0.3390	2	0.9399	1	0.8213	1	0.8807	1

Table 13
The correlation between the different approaches

Alternatives	RC_i (the existing TOPSIS method)	RC_i (the corrected TOPSIS method)	RC_i (Mohamadghasemi et al., 2018)	RC_i (Rashid et al., 2014)
RC_i (the existing TOPSIS method)	-	-0.2121	-0.5790	-0.8051
RC_i (the corrected TOPSIS method)	-	-	0.9195	0.7504
RC_i (Mohamadghasemi et al., 2018)	-	-	-	0.9497
RC_i (Rashid et al., 2014)	-	-	-	-

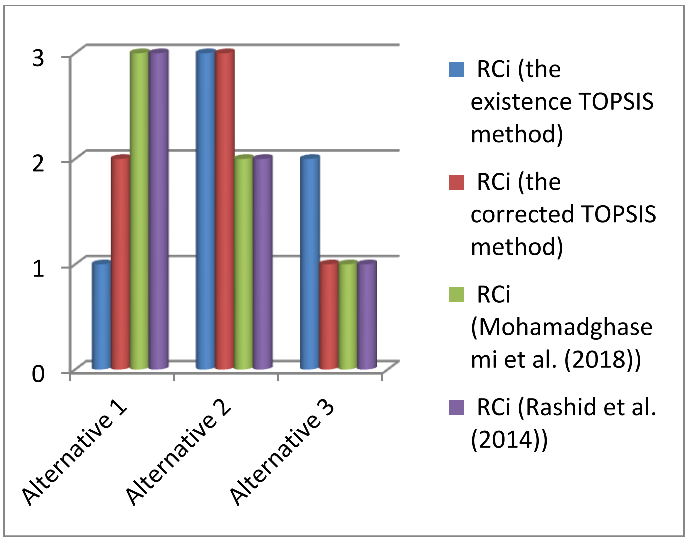

Fig. 2. Ranking the alternatives using different methods

6. Case Study

In this section, the proposed method is implemented in an industrial study. Let four machines A_i $(i = 1,2,3,4)$ need to be evaluated under four criteria C_j $(j = 1,2,3,4)$ for maintenance services where mean time between failures (MTBF), total cost, the availability of spare parts, and the repairability are the criteria. In addition, suppose that their weights are $w_1 = 0.1$, $w_2 = 0.45$, $w_3 = 0.20$, and $w_4 = 0.25$. The maintenance section's manager of a fishing net factory wants to prioritize the fishing net machines for maintenance services. On the other hand, the evaluations of machines under the criteria are stated using perfectly normal interval type-2 fuzzy numbers (as shown in Table 1). Tables 14 and 15 show the evaluations stated as perfectly normal interval type-2 fuzzy numbers and normalized measures of machines related to the criteria for one year (300 working days or 2400 hours) based on standpoints of maintenance section's manager.

Table 14
The evaluations of machines with respect to different criteria

	Criteria			
Machines	*MTBF* (hr.)	Total cost ($/per maintenance)	Availability of spare parts	Repairability
Machine 1(A_1)	([2,3],4,[5,9])	([3,4],7,[8,9])	([3,4],7,[8,9])	([4,5],8,[9,11])
Machine 2 (A_2)	([1,2],4,[5,6])	([2,3],4,[5,9])	([4,5],8,[9,11])	([1,2],3,[7,8])
Machine 3 (A_3)	([4,5],8,[9,11])	([4,5],6,[7,9])	([3,4],7,[8,9])	([2,3],4,[5,9])
Machine 4 (A_4)	([3,4],7,[8,9])	([1,2],4,[5,6])	([1,2],4,[5,6])	([3,4],7,[8,9])

Table 15
The normalized measures

	C_1	C_2
A_1	([0.11,0.20],0.25,[0.33,0.50])	([0.33,0.44],0.77,[0.88,1.00])
A_2	([0.16,0.20],0.25,[0.5,1.00])	([0.22,0.33],0.44,[0.55,1.00])
A_3	([0.09,0.11],0.13,[0.2,0.25])	([0.44,0.55],0.66,[0.77,1.00])
A_4	([0.11,0.12],0.14,[0.25,0.33])	([0.11,0.22],0.44,[0.55,0.66])

	C_3	C_4
A_1	([0.11,0.12],0.14,[0.25,0.33])	([0.09,0.11],0.13,[0.2,0.25])
A_2	([0.09,0.11],0.13,[0.2,0.25])	([0.12,0.14],0.33,[0.5,1.00])
A_3	([0.11,0.12],0.14,[0.25,0.33])	([0.11,0.20],0.25,[0.33,0.50])
A_4	([0.16,0.20],0.25,[0.5,1.00])	([0.11,0.12],0.14,[0.25,0.33])

According to Table 15, A^{+} and A^{-} are determined, as shown in Table 16.

Table 16
The measures A^{+} and A^{-}

	By using Eqs. (6-7)		By using Eqs. (18-19)	
	A^{+}	A^{-}	A^{+}	A^{-}
$\hat{v}_1^{+}$	([0.09,0.11],0.13,[0.2,0.25])		([0.09,0.11],0.13,[0.2,0.25])	
$\hat{v}_2^{+}$	([0.33,0.44],0.77,[0.88,1.00])		([0.44,0.55],0.77,[0.88,1.00])	
$\hat{v}_3^{+}$	([0.09,0.11],0.13,[0.2,0.25])		([0.09,0.11],0.13,[0.2,0.25])	
$\hat{v}_4^{+}$	([0.09,0.11],0.13,[0.2,0.25])		([0.09,0.11],0.13,[0.2,0.25])	
$\hat{v}_1^{-}$		([0.16,0.20],0.25,[0.5,1.00])		([0.16,0.20],0.25,[0.5,1.00])
$\hat{v}_2^{-}$		([0.11,0.22],0.44,[0.55,0.66])		([0.11,0.22],0.44,[0.55,0.66])
$\hat{v}_3^{-}$		([0.16,0.20],0.25,[0.5,1.00])		([0.16,0.20],0.25,[0.5,1.00])
$\hat{v}_4^{-}$		([0.12,0.14],0.33,[0.5,1.00])		([0.12,0.14],0.33,[0.5,1.00])

Now, the distances of machines from ideal solutions are computed by using Eq. (11). Tables 17 and 18 show the distances by using Eqs. (8-9) and Eqs. (20-21), respectively.

Table 17
The distances of the evaluations from A^{+} and A^{-} by using Eqs. (8-9)

S_1^{+}	S_2^{+}	S_3^{+}	S_4^{+}	S_1^{-}	S_2^{-}	S_3^{-}	S_4^{-}
0.0093	0.1001	0.0331	0.0980	-0.0651	-0.0322	-0.0992	-0.0343

Table 18
The distances of the evaluations from A^{+} and A^{-} by using Eqs. (20-21)

S_1^{+}	S_2^{+}	S_3^{+}	S_4^{+}	S_1^{-}	S_2^{-}	S_3^{-}	S_4^{-}
0.0104	0.1012	0.0342	0.0991	0.1231	0.0322	0.0992	0.0343

Table 19 presents the ranked results of different approaches. The similar results exist between the recent two methods, as presented in this table. According to Figure 3, machine 1 is the optimal selection for maintenance services based on the last three approaches, whereas it has rank 2 based on the existing TOPSIS method.

Table 19
The final ranking of the machines based on the existing TOPSIS, corrected TOPSIS, Mohamadghasemi et al. (2018), and Rashid et al. (2014) methods

Machines	RC_i (the existing TOPSIS method)	Ranking	RC_i (the corrected TOPSIS method)	Ranking	RC_i (Mohamadghasemi et al., 2018)	Ranking	RC_i (Rashid et al., 2014)	Ranking
A_1	1.1668	2	0.9216	1	0.9489	1	0.8819	1
A_2	-0.4755	3	0.2416	4	0.5606	4	0.3490	3
A_3	1.5010	1	0.7433	2	0.8896	2	0.8171	2
A_4	-0.5404	4	0.2575	3	0.6754	3	0.1774	4

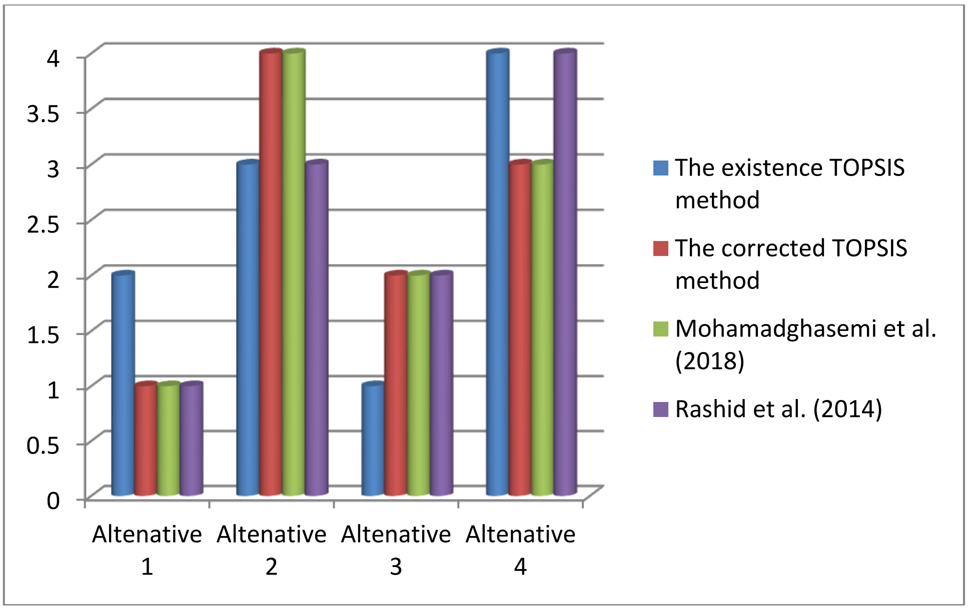

Fig. 3. Ranking the machines using different methods

7. Conclusions

In this paper, the TOPSIS method's drawbacks done by Dymova et al. (2015) have been corrected. As demonstrated in the Tables 4-8 and the Tables 10-13, there are many differences in the calculations results between the existing TOPSIS, the corrected TOPSIS, Mohamadghasemi et al. (2018), and Rashid et al. (2014) methods. Based on Table 4, the measures A^+ for $\hat{v}_2^+$ and A^- for $\hat{v}_4^-$ obtained by Eqs. (18-19) are different from those of Eqs. (6-7). There is no criterion for the maximum and minimum reference limits in Eqs. (6-7). Thus, a reference was presented for it by using Eqs. (18-19). There are the distinct differences between the distances of the assessments and the positive and negative ideal solutions in Tables 5 and 6, due to drawbacks in Eqs. (8-9). The set of these difficulties result in the different results (as shown in Table 7). In other words, the corrected TOPSIS method presents more logical results than the existing TOPSIS method. To prove this claim, consider alternative 2 (A_2). Although it has relatively lower perfectly normal interval type-2 fuzzy numbers than the others regarding the criteria, it has higher rank than the others. However, it was selected as the least important alternative based on the corrected TOPSIS method. On the other hand, the similar situation exists in the case of normal IT2FVs (as presented in Tables 10-13). A_2 has the lowest RC_i in the above methods (see Table 12). Furthermore, the measures of the positive and negative ideal solutions are not equal for all alternatives based on the existing and corrected TOPSIS method (see Tables 10 and 11). In addition, there are the negative correlation coefficients between the existing TOPSIS method and the corrected TOPSIS, Mohamadghasemi et al. (2018), and Rashid et al. (2014) methods in Tables 8 and 13 and high correlation coefficients between the last three methods. On the other hand, machine 1 was chosen as the most important machine for maintenance services based on the last three approaches, whereas this conclusion is different from orders obtained by the existing TOPSIS method. This fact implies that the TOPSIS method proposed by Dymova et al. (2015) is inefficient and cannot present the correct ranking. Thus, the corrected TOPSIS technique should be adopted instead of the existing TOPSIS method when solving MCDM problems. It is implementable to rank machines, equipment, managers, departments, etc. within factories or organizations with respect to a set of criteria in which evaluations and weights are stated as IT2FSs.

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