Journal of Optimization in Industrial Engineering
Vol.12, Issue 2, Summer & Autumn 2019, 149-154
DOI: 10.22094/JOIE.2018.350.0

Research Article

Developing a New Bi-Objective Functions Model for a Hierarchical Location-Allocation Problem Using the Queuing Theory and Mathematical Programming

Parham Azimi [a,*], Abulfazl Asadollahi [a]

[a] *Department of Industrial and Mechanical Engineering, Qazvin Branch, Islamic Azad University, Qazvin, Iran*

Received 15 June 2014; Revised 06 July 2018; Accepted 17 July 2018

Abstract

In this research, a hierarchical location-allocation problem is modeled in a queue framework. The queue model is considered as M/M/1/k, in which system capacity is finite, equals to k. This is the main contribution of the current research. Customer's enters to the system in order to find the service according to a Poisson. In this problem, the hierarchical location-allocation model is considered in two levels. Also, the model has two objective functions: maximizing the total number of demand coverage and minimizing the waiting time of customers in queues to receive services. After modeling and verifying the validity of the presented model, it is solved using NSGA II and MOPSO meta-heuristics.

Keywords: Hierarchical-allocation problem; Queue Theory; Meta-heuristic Algorithm.

1. Introduction

In nowadays competitive market, the customer desire to receive better standards and effective in all economic sectors. So, designing the service centers is a very important problem in order to meet the high level of services requested by any market. In recent years, location-allocation studies have been considered both nationally and internationally. Among them, identifying objectives and methods of solving location problems have a very high importance. Combining location-allocation problem with other approaches, such as queue structure, supply chain network, pricing strategies and etc. have been considered and absorbed the attraction of many researchers. Most attempts and researches have been carried out in real world applications to achieve proper and practical models such as health care systems, chain stores, distribution centers and so on.

Location study was formally initiated in 1909 by Webber's research (1929) about how to determine the location of one resource and then followed by Hakimi researches (1964). Since then, so many methods (precise-innovative) were presented to solve the problem. First innovative method was introduced by Cooper in 1964 known as location-allocation repeat method. Adel Ali and John White (1978) have considered the problem of a service center (police stations, firefighting stations and hospitals) in SA. They used the genetic algorithm and recooking simulation method to solve the problem.

Larson (1974-1975) has presented the first location model using queue theory. Also, Daskin (1983) has introduced probabilistic model named MCLP. In this model, it is possible to maximize population mean under coverage with a limited and specific number of providers. Maryanov & Serra (1966) have discussed about using queue theory to estimate the server occupation proportion, subsequently a QMALP model was developed. Berman et.al (1985) has determined the optimal location of servers in a queue network using an M/G/1 structure. Brandew and Chio (1990) have studied a group of location models with only one server. Siyam (2008) introduced a location-allocation model with some servers and a limitation on the reference to the nearest demand, to minimize the travel and waiting times. Melaw et.al (2009) over viewed and analyzed the facility location problem in the context of a supply chain network. Pasandideh and Akhavan Niyaki (2010) presented a two objective model for a facility location-allocation model with M/M/1 structure. Zarinpour and Seif barghi (2011) introduced a facility location model with M/M/m/k structure and solve it by a genetic algorithm and prohibitive search method.

Ethon & Chuch (1987) reviewed the hierarchical location models. Previous works were Bata (1988-1989), Bata et.al (1989), Berman & Larson (1985), Berman & Mandowski (1986), Berman et.al (1985-1987), Daskin (1993), Larson & Edeni (1981), Maryanov & Rolleh (1994-1996), Maryanov & Serra (1988-1993-1994) they also presented a hierarchical location model based on queue system for a P-Median problem which wascalled PQ-Median, in which a coherent system was considered. Maryanov & Serra (1992) also presented a hierarchical location model for competitive environment.

The main contribution of this research is to use a M/M/1/k model in the optimization process of the hierarchical location-allocation problem using a bi-objective function. According to Adel Ali and John White (1978), the problem has the complexity level of NP-Hard. Thus, two different meta-heuristics were developed to solve the problem including NSGA II and MOPSO. The section 2,

*Corresponding author Email address: p.azimi@qiau.ac.ir

the mathematical model of the problem is developed. In Section 3, the meta-heuristics have been introduced and their parameters have been tuned. In Section 4, the computational results have been reported and finally, the conclusions and suggestions for future researches have been provided in section 5.

Khodaparasti et al. (2015) a model for positioning in the healthcare sector for a long-term care network, provided they use a hierarchical planning approach were randomly assigned location. Gama et al. (2015) created a model location for the shelter during the flood proposed allocation hierarchy that the purpose of this model is to minimize the entry of people to the place of refuge in the world. Bell et al. (2015) developed a model for site selection decision-making needs of military commanders to develop their military objectives. This aims to locate the position of the arms and cover a search area to find the enemy. Başar et al. (2015) presented a model location for bank ATMs, which aims to maximize the use of ATMs is a particular bank in the city.

2. Problem Modeling

In this selection, a hierarchical location-allocation model is modeled using queue structure and mathematical programming method. The queue model for the center is considered as non-referral M/M/1/k.

2-1. Objective Functions:

- Maximizing the total number of demand coverage.
- Minimizing the waiting time of customers in all queue to receive the service.

2-2. Model Hypotheses:

System capacity is finite and equals to k. Referral rate of demand points in order to receive services for every facility, has the Poisson distribution and servicing time for each facility has the exponential distribution. Facilities or servers are supposed to be in two levels (low level and high level servers). To enter the system, customers primarily referred to low level servers and could not be referred to a high level one. When a customer refers to a service and observers that the low level server capacity is full, he overlooks to enter the system. After receiving a service in a low level server, customers may leave the system or refer a high level server.

2-3. Model Parameters:

The parameters which were used to model the problem are as follows:

I: a set of service demanding nodes indexed by i. i=1,2,…,N

J: a set of candidate nodes of low level servers introduced by index j. j=1,2,…,L

K: a set of candidate nodes of high level servers introduced by index k. k=1,2,…,H

N: maximum number of customer group nodes.

L: maximum number of nodes for low level servers.

H: maximum number of nodes for high level servers.

a_i: customer's population located in region i.

X_{ijk}: allocation variable, if equals to 1, it means that node i is related to a low level server j, and a high level server k. Otherwise, it is zero.

Y_j: location variable, it is 1, if a low level server is assigned to location node i; otherwise, it is zero.

Z_k: location variable, it is 1, if a high level server is assigned to location node k; otherwise, it is zero.

f_i: service demand rate in node i.

λ_j^L: The rate of requests received by low level server j.

μ_j^L: Service rate in low level server j.

λ_k^H: Rate of requests received by high level server k.

P_l: the number of low level centers.

P_h: the number of high level centers.

β_j: percentage of requests received by low level node j demanded high level service.

W_j: average waiting time of customers in the queue to receive service from low level server j.

W_k: average waiting time of customers in the queue to receive service from high level server k.

$\rho_j^L = \frac{\overline{\lambda_J^L}}{\mu_J^L}$, $\rho_k^H = \frac{\overline{\lambda_k^H}}{\mu_k^H}$, $\lambda_j^L = \sum_{i,k} f_i X_{ijk}$, $\lambda_k^H = \sum_{i,j} \beta_j f_i X_{ijk}$,$\overline{\lambda_J^L} = \lambda_j^L(1-\pi_k)$, $\overline{\lambda_k^H} = \lambda_k^H(1-\pi_k)$

$W_j = \frac{\frac{r_j}{1-r_j} - \frac{(k+1){r_j}^{k+1}}{1-{r_j}^{k+1}}}{\lambda_j^L} - \frac{1}{\mu_j^L}$, $r_j = \frac{\lambda_j^L}{\mu_j^L}$, $W_k = \frac{\frac{r_k}{1-r_k} - \frac{(k+1){r_k}^{k+1}}{1-{r_k}^{k+1}}}{\lambda_k^H} - \frac{1}{\mu_k^H}$, $r_k = \frac{\lambda_k^H}{\mu_k^H}$

2-4. Model description:

According to the above definitions, the mathematical problem is as follows:

Problem 1:

$$Max\ Z_1 = \sum_i \sum_j \sum_k a_i X_{ijk} \qquad (1)$$

$$Max\ Z_2 = \sum_i \sum_j f_i Y_j W_j + \sum_i \sum_k \beta_j f_i Z_k W_K \qquad (2)$$

S.T:

$$\sum_{j,k} X_{ijk} \le 1 \quad ,\forall\, i,j,k \qquad (3)$$

$$X_{ijk} \le Y_j \quad ,\forall\, i,j,k \qquad (4)$$

$$X_{ijk} \le Z_k \quad ,\forall\, i,j,k \qquad (5)$$

$$\sum_{i,k} f_i X_{ijk} W_j \le \mu_J^L \sqrt[b+2]{1-\alpha} \quad ,\forall\, j \qquad (6)$$

$$\sum_{i,j} \beta_j f_i X_{ijk} W_k \le \mu_K^H \rho_{\alpha K}^H \quad ,\forall\, k \qquad (7)$$

$$\sum_j Y_j = P_l \qquad (8)$$

$$\sum_k Z_k = P_h \qquad (9)$$

$$X_{ijk}, Y_j, Z_k \in \{0,1\} \quad ,\forall\, i,j,k \qquad (10)$$

In problem 1, objective function 1, maximizes the population coverage. Objective function 2, reduces the average waiting time of customers in queues. Constraint set 3, enforces each demand to be served at only one server. Constraint sets 4 and 5, ensure that every demand node would not be dedicated to a low or high level node. In the other words, a customer could not be simultaneously referred to a low and a high level server. Constraint set 6, is the limitation of servicing quality. A service demand is sent to a server with probability α, if the queue length is no more than b persons. Constraint set 7, is related to servicing quality, also. A service demand is sent to a server with probability α, if the customer is not

waiting more than T seconds. Constraint sets 8 and 9, show the number of centers in both levels.
The chromosome is a three-vector matrix. The first vector of length L is for top-level providers who are elected Pl The second vector is for the H to the second level providers that basic Tai Ph it will be selected. The third vector is a sequence of customers that are based on the first level and second level limits of 8 and 9 Service providers allocated.

3. Solving Methods

Problem 1 has two objective functions so it is a multi-objective decision making problem (MODM). In this paper, this model is solved using NSGA II and MOPSO algorithms and the results have been compared and analyzed.
NSGA II is one of the most useful and powerful algorithms to solve multi-objective optimization problems. This algorithm is introduced by Deb et.al in 2000. Its main features are including:

- Definition of crowding distance as an alternative feature for methods like propriety sharing.
- Using binary tournament selection operator.
- Saving and archiving non-dominant answers obtained in previous stage of algorithm (elitism).

Particle swamp optimization (PSO) algorithm is also a successful technique in artificial intelligence. PSO is a population-based algorithm that is similar to genetic algorithm in some respects. However, the searching method inside the feasible region is very different. Some similarities of these algorithms are including: both algorithms are based on population, both algorithms use propriety function to assess the obtained answers, both of them have performed specific stages with specific time before the termination of the algorithm. PSO algorithm has two operators: speed up-to-date process and situation up-to-date process to perform this operation.
3-1. Assessment Indices of the Pareto solution:
3-1-1. Distance Index:
Colet and Saiyari (2003) defined this index to compare some Pareto solutions. In this index, d_i is the Euclidean distance of two subsequent answers in the Pareto solution and $\bar{d}$, is the average distance. N is the number of Pareto solutions. The owe amount of this index is the more favorite.

$$SM = \frac{\sum_{i=1}^{N-1} \left| d_i - \bar{d} \right|}{(N-1)\bar{d}}$$

3.1.2. Distribution index

Rittzler (1999) defined this index. Assume that x,y are Pareto answers with m objective functions. The more index value is better.

$$DM = \sqrt{\sum_{i=1}^{m} \max_t (\| x_i^t - y_i^t \|)}$$

3.2. Parameters tuning

To tune the problem parameters, the L9 Taguchi's designing method was used. Where values of both objective functions are normal and unidirectional and located in a minimization form. A level is selected in which solution values are less than a predefined level for each parameter. Results obtained for different size of problem are: depicted in figures 1,2.3,4,5 and 6.

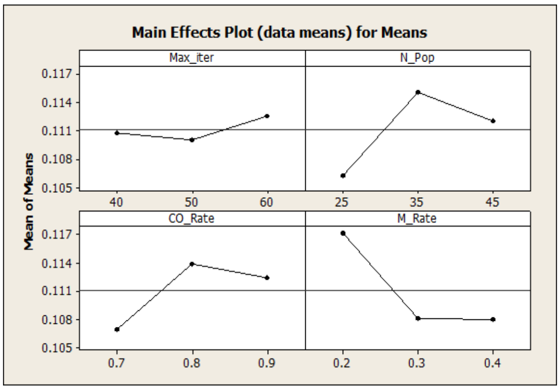

Fig. 1. Problems in small size in NSGA II algorithm

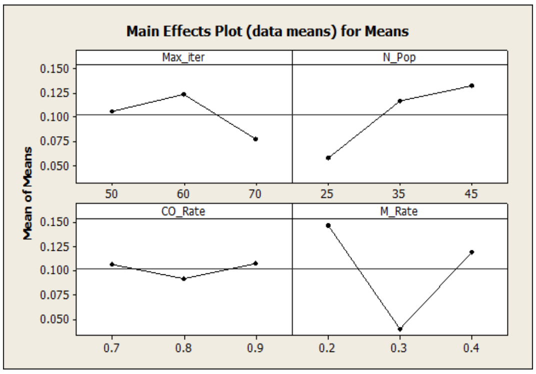

Fig. 2. Problems in medium size in NSGA II algorithm

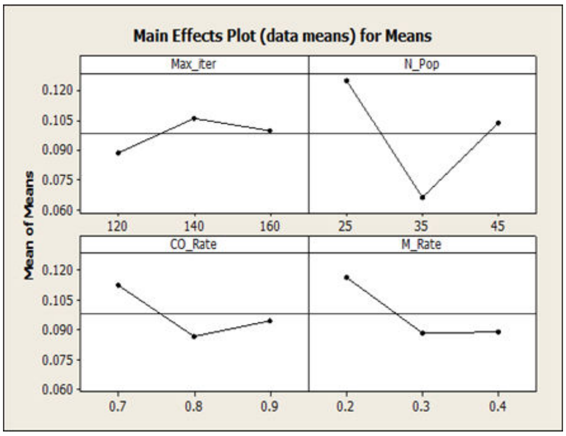

Fig. 3. Problems in large size in NSGA II algorithm

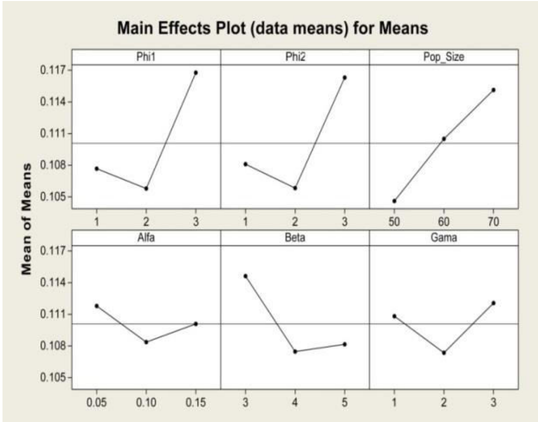

Fig. 4. Problems in small size in MOPSO algorithm

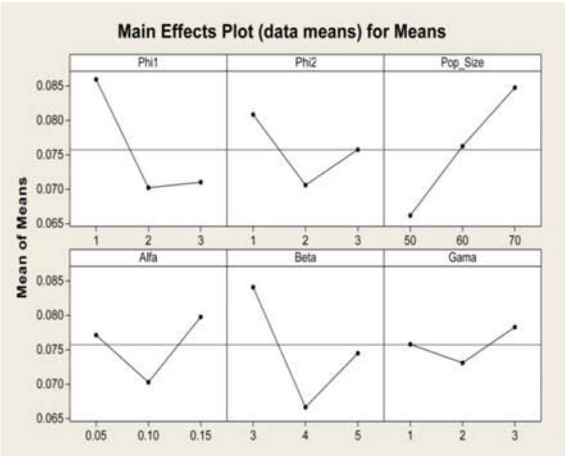

Fig 5. Problems in medium size in MOPSO algorithm

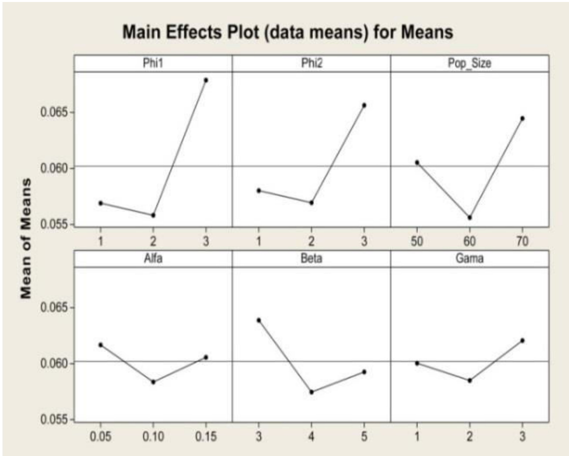

Fig 6. Problems in large size in MOPSO algorithm

Table 1
Values regulated for NSGA II algorithm

Size Matters	Max_Iter	N_Pop	CO_Rate	M_Rate
small	50	25	0.7	0.4
medium	70	25	0.7	0.3
large	120	35	0.8	0.3

Table 2
Values regulated for MOPSO algorithm

Size Matters	Phi1	Phi2	Pop Size	Alfa	Beta	Gamma
small	2	2	50	0.1	4	2
medium	2	2	50	0.1	4	2
large	2	2	60	0.1	4	2

In table 1, Max_Iter represents the number of iteration is, N_Pop is the number of population is the answer, Co_Rate is the intersection is indicated rate, M_Rate represents the mutation rate is. In table 2, Phi1 is a factor that affects the rate personal learning, Phi2: the impact factor of global learning, Pop Size: population size is, Alfa is the rating is applicable for answer, Beta is the selected parameter is the leader in algorithm, and Gamma is network swollen parameter network growth.

4. Computation Results

Problem 1 was coded in MATLAB software (version 12.6.1) and solved using NSGA II and MOPSO algorithms for randomly generated small, medium and large problems. All computations were run on a PC with Intel Core i7 using 4 GByte Ram and 3.65 GHz as SPU. The results are listed in table 3 for small scale, Table 4 for medium scale and Table 5 for large scale problems. In each table, the results were compared between NSGA II and MOPSO algorithms in terms of CPU Time in seconds and the two indices.

To produce random sample problems, the following density functions were used:

$\lambda_J^L = \text{uniform } (0.3,0.4), \mu_J^L = \text{uniform } (0.6,0.7), \lambda_K^H = \text{uniform } (0.3,0.4), \mu_K^H = \text{uniform}(0.7,0.8)$

$\beta_j = \text{uniform } (0.6,0.7), f_i = \text{uniform } (0.1,0.2), a_i = \text{uniform}(8,9)$

The numbers in column of problem name, in following tables from lift to right means that: first number is maximum number of customer group nodes. The second and three numbers is maximum number of nodes for low and high level servers. The fourth and fifth numbers is the number of low and high level servers. The sixth is number of customers in queue and last number is system capacity, that presented by N.L.H.P_l.P_h.b.k.

Table 3
Result table for small size problems

MOPSO			NSGA II			Problem name	Number
CPU Time	Distribution Index	Distance Index	CPU Time	Distribution Index	Distance Index		
1.850	4.478	1.187	5.320	0.472	1.916	10.7.5.3.2.5.5	1
1.247	4.443	0.594	5.534	2.993	1.805	10.7.5.2.2.5.5	2
2.021	5.216	1.281	5.343	NAN	0	10.7.5.3.3.5.5	3
1.715	4.463	0.740	5.274	0.757	1.916	10.7.5.3.2.10.5	4
1.281	4.594	0.623	5.342	2.890	1.916	10.7.5.3.2.5.10	5
1.384	6.638	1.026	5.524	1.529	1.916	15.10.7.5.4.15.15	6
1.258	7.538	0.981	5.532	2.890	1.916	15.10.7.4.4.15.15	7
1.279	6.514	0.620	5.515	2.980	1.916	15.10.7.5.5.15.15	8
1.368	5.791	0.346	5.437	3.008	1.916	15.10.7.5.4.20.15	9
1.280	6.656	0.502	5.485	2.868	1.916	15.10.7.5.4.15.20	10

Table 4
Result table for medium size problems

MOPSO			NSGA II			Problem number	Number
CPU Time	Distribution Index	Distance Index	CPU Time	Distribution Index	Distance Index		
3.464	6.015	0.422	8.800	3.060	1.916	40.20.15.3.2.5.5	1
3.377	6.678	0.818	8.347	3.086	1.916	40.20.15.2.2.5.5	2
3.544	7.514	0.507	8.722	3.073	1.916	40.20.15.3.3.5.5	3
3.572	6.884	0.494	8.362	1.866	1.833	40.20.15.3.2.10.5	4
3.354	6.977	0.478	8.532	3.755	1.590	40.20.15.3.2.5.10	5
8.868	12.105	0.534	12.309	3.010	1.916	60.40.20.5.4.15.15	6
8.782	10.414	0.436	12.436	4.874	1.833	60.40.20.4.4.15.15	7
9.158	11.946	0.800	12.328	3.068	1.750	60.40.20.5.5.15.15	8
9.127	11.499	0.773	12.426	2.411	1.916	60.40.20.5.4.20.15	9
9.394	11.714	0.665	12.237	3.705	1.760	60.40.20.5.4.15.20	10

Table 5
Result table for large size problems

MOPSO			NSGA II			Problem number	Number
CPU Time	Distribution Index	Distance Index	CPU Time	Distribution Index	Distance Index		
61.371	13.141	0.631	75.757	4.121	1.931	100.50.40.7.5.5.5	1
52.894	11.589	0.428	71.407	1.548	1.862	100.50.40.5.5.5.5	2
59.190	13.946	0.539	80.178	4.472	1.931	100.50.40.7.7.5.5	3
60.090	12.512	0.494	72.776	3.869	1.931	100.50.40.7.5.10.5	4
61.830	14.546	0.480	77.093	4.599	1.862	100.50.40.7.5.5.10	5
27.593	19.412	0.955	148.325	3.489	1.784	120.60.50.10.8.15.15	6
236.081	16.661	0.408	130.686	4.598	1.862	120.60.50.8.8.15.15	7
204.100	19.563	0.496	152.532	6.665	1.793	120.60.50.10.10.15.15	8
226.234	19.858	0.428	165.463	2.931	1.862	120.60.50.10.8.20.15	9
234.679	19.458	0.467	142.422	4.973	1.793	120.60.50.10.8.15.20	10

Based on previous statement, the less the Distance Index the better it is and the more the Distribution Index, the better it is. In previous table, for all problems having small, medium and large sizes, you can see, the less the Distance Index and the more the Distribution Index, the lower the CPU Time of that row.

5. Conclusion

In this research, a new model has been to address the so-called location-allocation problem with two objective functions. The main contribution of the current research is using a M/M/1/k queue model at each level of the problem, two different meta-heuristics including NSGA II and MOPSO were developed and tuned to solve several randomly generated problem in 3 scale sizes and the results were compared. The results-specially, for large scale problems show that however the NSGA II is quicker than MOPSO on average, but the solution quality of MOPSO is better than NSGA II. Also, the results show that model and the solving methods can be easily and effectively be applied in real world problems. Regarding the research constraint, it should be noted that if we could use. A more powerful PC, the results especially in terms of computational speed might be better than the current values. For future researches, studying other queue theory model such as M/M1/k, considering group arrivals or group serving are recommended.

References

Başar, A., Kabak O., İlker Topçu, Y. Bozkaya, B. Location Analysis in Banking: A New Methodology and Application for a Turkish Bank. Volume 232 of the series International Series in Operations Research & Management Science pp. 25-54

Bell, J.E., Griffis, S.E. Military Applications of Location Analysis. Volume 232 of the series International Series in Operations Research & Management Science pp. 403-433

Boffey, B., Galva˜o, R. Espejo,L. (2007). A review of congestion models in the location of facilities with immobile servers, *European Journal of Operational Research* 178, 643–662.

Chan, Y. Mahan, J. M., Chrissis, J.W., Drake, D.W.& Wang, D. (2008). Hierarchical maximal-coverage location–allocation: Case of generalized search-and-rescue, *Computers & Operations Research* 35,1886 – 1904.

Collette Y, Siarry P (2003). Multi-objective optimization: principlesand case studies. Springer, New York.

Galvao, R. D., Gonzalo Acosta Espejo, L., Boffey, B. (2003). Dual-based heuristics for a hierarchical covering location problem, *Computers & Operations Research* 30, 165–180.

Galvao, R. D., Gonzalo Acosta Espejo, L., Boffey, B. (2002). A hierarchical model for the location of perinatal facilities in the municipality of Rio de Janeiro, *European Journal of Operational Research* 138,495–517.

Galvao, R. D., Gonzalo Acosta Espejo, L., Boffey, B., Yates, D. (2006). Load balancing and capacity constraints in a hierarchical location model, *European Journal of Operational Research* 172, 631–646.

Gama, M., Santos, B.F. & Paola Scaparra, M. (2015). A multi-period shelter location-allocation model with evacuation orders for flood disasters. *EURO Journal on Computational Optimization*, pp. 1-25

Hodgson, M.J. & Kruse Jacobsen, s. (2009). A hierarchical location-allocation model with travel based on expected referral distances, Ann Oper Res 167,271–286.

Hodgson, M.J. (1986). A Hierarchical Location-Allocation Model with Allocations Based On Facility Size, *Annals of Operations Research* 6,273-289.

Jayaraman, V., Gupta, R. Pirkul, H. (2003). Selecting hierarchical facilities in a service-operations environment, *European Journal of Operational Research* 147, 613–628.

Khodaparasti, S., Jahedi, S., Bruni, M. E. & Beraldi. P. Enhancing community based health programs in Iran: a multi-objective location-allocation model. *Health Care Management Science*, pp. 1-15

Lee, J.M. & Lee, Y.H. (2012). Facility location and scale decision problem with customer preference, *Computers & Industrial Engineering* 63,184–191.

Marianov, M. & Serra, D. (1998). Probabilistic maximal covering location-allocation models for congested systems, *Journal of Regional Science,* Vol. 38, NO. 3,401-424.

Marianov, V., Serra, D. (2001). Hierarchical location allocation models for congested systems, European *Journal of Operational Research* 135,195-208.

Mohammadi, M., Jolai, F.& Rostami, H. (2011). An M/M/c queue model for hub covering location problem, *Mathematical and Computer Modelling* 54, 2623–2638.

NARUL, C. (1981). Hierarchical location-allocation problems: A classification scheme.

Rabieyan, R. & Seifbarghy, M. (2010). Maximal Benefit Location Problem for a Congested System, *Journal of Industrial Engineering* 5,73-83.

Sahin, G. & Sural, H. (2007). A review of hierarchical facility location models, *Computers & Operations Research* 34,2310 – 2331.

Teixeira, J. C. &. Antunes, A.P. (2008). A hierarchical location model for public facility planning, European *Journal of Operational Research* 185,92–104.

Vilcapoma Ignacio, A.A., Ferreira Filho, J.M. V. & Diéguez Galvao, R. (2008). Lower and upper bounds for a two-level hierarchical location.Problem in computer networks, *Computers & Operations Research* 35,1982 – 1998.

Zitzler E (1999) Evolutionary algorithm for multi-objective optimization: methods and applications. Ph.D. Dissertation, Swiss Federal Institute of Technology (ETH), Zurich, Switzerland.

This article can be cited: Azimi, P. & Asadollahi, A. (2019) Developing a New Bi-Objective Functions Model for a Hierarchical Location-Allocation Problem Using the Queuing Theory and Mathematical Programming. *Journal of Optimization in Industrial Engineering*. 12 (2), 149-154.

http://www.qjie.ir/article_543781.html
DOI: 10.22094/JOIE.2018.350.0