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A Two-dimensional Warranty Model with Consideration of Customer and Manufacturer Objectives Solved with Non-Dominated Sorting Genetic Algorithm

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Abstract

Warranty is a powerful implement for marketing strategy that is used by manufacturers and creates satisfaction for consumers by ensuring to compensate for incorrect operation of the product. Warranty service results in a cost named warranty cost for a manufacturer. This cost is a function of warranty policy, regions, and product failures pattern. Since this service covers the cost of uncertain failure of the product, it makes some utility for customers. In this paper, we developed a novel customer utility function that is used as a customer objective to be maximized. In addition to the manufacturer objective, minimizing the warranty cost is considered simultaneously. There are four restrictions on warranty parameters such as time, usage, unit product price and the R&D expenditure to be considered. Finally, we will propose a novel bi-objective model that maximizes the utility function for customers and minimizes the warranty cost for the manufacturer. This model will be solved with an evolutionary algorithm called Non-Dominated Sorting Genetic Algorithm (NSGA-II) and non-dominated Pareto solutions will be gained from this method. To give a numerical instance, for a certain usage rate's range of custumers, different warranties are provided and compared. It is believed that the computational results can help manufacturers to determine optimal solutions for the objective functions and consequently warranty parameters.

Keywords: Two dimensional warranty; Warranty cost; Utility function; Bi-objective model; NSGA-II.

1. Introduction

In order to indicate the high level of quality and reliability of the sold products, manufacturers use warranty services at the time of sale. The warranty statement ensures costumers that the seller undertakes all or part of cost of failures for a specified region. For 1-Dimensional warranty, this region only involves the age of sold product and for 2-Dimensional one it includes usage of product in addition to age. Larger warranty region as an attractive compensation helps the manufacturer to increase the volume of sale, but without having sufficient quality and reliability, the margin profit for manufacturers will hit severely and obviously increasing warranty cost is the reason. Therefore, it is necessary to determine warranty parameters in a way that optimum values for both manufacturers and costumers' perspectives are met.

Warranty policy is a statement that is offered by the manufacturer for a product and determines how the warranty will be presented. A taxonomy for different types of warranty is available in the work of Blischke and Murthy (1994). Considering no agreement for product development after sale, policies are divided into single item and group of items. Focusing on single-item policies, they are classified into two categories of renewing and non-renewing. In renewing polices, whenever a product in the warranty region fails, it is replaced by a new item with a new warranty while in non-renewing policies the basic warranty for replaced products will be continued. Then, renewing and non-renewing warranties are subdivided into 1-Dimensional and 2-Dimensional. The one-dimensional (1D) policy is indicated by an interval called warranty period, while the two-dimensional (2D) policy is characterized by a region in 2D plane. One-dimension policy usually represents age and the other one indicates usage.

A comprehensive review of warranty policies for new products can be found in Murthy and Djamaludin (2002) who list over 186 references, but a minority of these researches considered 2-D policy.

Three major approaches are available to find the number of failures in order to calculate 2-D warranty costs. The first approach is reducing the dimension, assuming that usage and time are related to each other through a usage rate variable, so it can be said that the average usage equals to average rate multiplied by average warranty period. It is the most applicable method and will be illustrated later. The second involves reducing the dimension, relating usage and time through a formula and trying to estimate the formula parameters and then optimizing 1-D model. Kordonsky and Gertsbakh (1998) used this method to reduce the dimension of problem. The third is using a two-variable function to indicate usage and time variables. Chun and Tang (1999) suggest several regions for 2D warranty. For these regions, failure

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instance is indexed by the two variables—age and usage. Murthy et al. (1995) adopted bivariate Pareto and Beta Stacy distributions for failure models. Kim & Rao (2000) used bivariate exponential and Weibull distribution for failure modeling. In this article, like many researches, we assume that usage and age of product are related through a random variable, the usage rate, which may have a determinate probabilistic distribution that is influenced by usage behavior pattern of the society. For instance, Mitra and Patankar (2010) use this method for 2-D warranty policies, then after successful implication, they used it in the method for a multi-objective modeling. In this paper, we will reduce 2-D problem to the 1-D one.

So far, the majority of warranty problems attempt to minimize cost or maximize profit without attending widely to the important role of customer satisfaction. One of the recent researches supporting this prospective can be found in the work of Tong and et.al (2014) where extended warranty contracts are modeled to exact minimum cost to producer. On the other hand, considering consumer view, Manna and et.al (2006) struggled to insert consumer importance to their model by providing two different policies in their model at a certain cost for manufacturer; the policy will be selected that provides more extended time for consumer. Zhou et.al (2009) used an exponential function for utility function for a 1-D warranty model. Since this function can model the behavior of consumers toward probabilistic failures and incurred warranty cost very well, a major contribution of this article is extending the application of exponential function to our 2-D model. Rahman et.al (2009) also used exponential behavior for their 1-D model in which risk estimation is performed from consumer's and manufacturer's perspectives.

Mitra and Patankar (2011) developed a multi-objective model for 2-D warranty from manufacturer point of view in which warranty parameters and the amount of expenditure on product R&D are determined and the model is solved by goal programming. In this article, a multi-objective model with both manufacturer and consumer objectives is presented that considers minimizing the expected warranty cost and maximizing the costumer utility simultaneously. It is believed that considering consumer's point of view will have a positive effect on future market share of companies and consumers' dissatisfaction with product.

The model can be solved through searching within the boundary of all parameters and converting the 2-D model to a 1-D model. However, we found that in this way, it takes about one hour on Intel (R) Corei-7 CPU to obtain parameter values. Therefore, in this article the model will be solved with an evolutionary algorithm called NSGA-II. Other advantage of the algorithm will be explained later.

The rest of this article is organized as follows: Section 2 describes the model assumptions and formulation of the model in detail. In Section 3, a concise procedure of algorithm is presented and the model will be solved with NSGA-II. Section 4 includes the computational results and, finally, section 5 presents conclusions and suggestions for future researches.

2. Model Formulation

In this section, we intend to build a model and find the values of warranty parameters (U, W). A two-dimensional warranty policy is considered where the warranty parameters are time and usage. Warranty region is rectangular and the failed product is repaired or replaced free of charge, up to a time (W) or up to a usage (U), whichever occurs first from the time of the initial purchase. Warranty is not renewed on product failure. This policy is known as non- renewing FRW[1]. Fig 1 shows a two-dimensional warranty region.

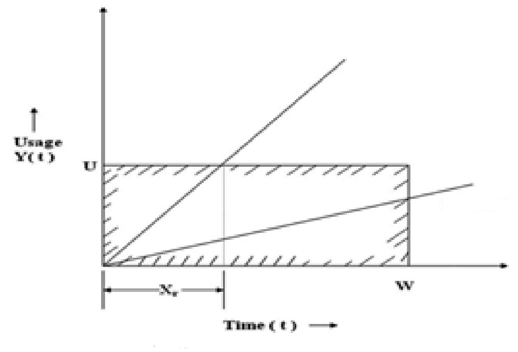

Fig.1. Two dimensional warranty rectangular region

Let indicate U/W with R_1. We assume that time and usage are related linearly through a random variable that is called usage rate. Suppose *Y(t)* to be the usage at time t and *X(t)* the corresponding age, we have

$$Y(t) = RX(t) \tag{1}$$

All failures are minimally repaired; so repair time is tiny and we assume this is equal to zero and therefore $X(t)=t$.

R is the usage rate and to model a variety of costumer is assumed to be a random variable with a determinate probability density function denoted by g(r). For modeling a variety of usage rates among the population of costumers we apply gamma distribution.

$$g(r) = \frac{e^{-r} r^{p-1}}{\Gamma(p)}, 0 \le r < \infty \tag{2}$$

Failures are assumed to occur according to a Poisson process. Since the failure rate is unaffected by minimal repair (time of repair is approximated zero),

failures over time happen according to a non-stationary Poisson process with intensity function λ (t) equal to the failure rate. It is also assumed that with expenditures on R&D we will create an improved product with a reduction in the failure rate. Conditional on the usage rate R= r, we can put failure intensity function at time t, $\lambda(t \mid r)$, as follows:

$$\lambda(t|r) = \theta_0 + \theta_1 r + (\theta_2 + \theta_3 r)t - \theta_5 RD \tag{3}$$

[1] Free repair/replace warranty

In order to obtain the expected warranty cost, first we need to find the expected number of failures. Two status could happen; the first one is finishing the time of warranty because of low usage rate and the second is expiring warranty usage before finishing time because of high usage rate. In this situation, we indicate the time that warranty finishes with, X_r, and we will have $X_r = U / r$ (see fig 1). Therefore, the number of failures are given by:

$$N(W,U|r) = \begin{cases} \int_{t=0}^{W} \lambda(t|r)dt\,, & if \quad r < R_1 \\ \int_{t=0}^{X_r} \lambda(t|r)dt\,, & if \quad r \geq R_1 \end{cases} \tag{4}$$

To obtain the expected values of this variable, we need to release the condition, so we will have

$$E[N(W,U)] = \int_{r=0}^{R_1} [\int_{t=0}^{W} \lambda(t|r)dt]g(r)dr + \int_{r=R_1}^{\infty} [\int_{t=0}^{X_r} \lambda(t|r)dt]g(r)dr \tag{5}$$

If C_s indicates the state cost of repair an item failure, we present the expected warranty cost as follows:

$$E[N(W,U)] = \int_{r=0}^{R_1} [\int_{t=0}^{W} \lambda(t|r)dt]g(r)dr + \int_{r=R_1}^{\infty} [\int_{t=0}^{X_r} \lambda(t|r)dt]g(r)dr \tag{6}$$

C shows the product unit price so the expected warranty costs per unit sales (ECU) are obtained from:

$$EWC = C_s E[N(W,U)] \tag{7}$$

By developing an expression for the average failure rate (λ_{ave}) that is influenced by R&D expenditures, we will have:

$$E[N(W,U)] = \int_{r=0}^{R_1} [\int_{t=0}^{W} \lambda(t|r)dt]g(r)dr + \int_{r=R_1}^{\infty} [\int_{t=0}^{X_r} \lambda(t|r)dt]g(r)dr \tag{8}$$

The unit product price is impacted by the average failure rate, a_4 and b_4 are constant

$$C = a_4 + \frac{b_4}{\lambda_{ave}} \tag{9}$$

Accordingly, the unit cost of repair is obtained from the following equation while a_3 and b_3 are constant.

$$C_s = a_3 + b_3 C \tag{10}$$

$$ECU = EWC/C \tag{11}$$

We assumed that low limit and high limit for parameters C,W,U,RD are determinate and respectively indicated by C_1 , C_2 , W_1 , W_2 , U_1 , U_2 , d_1 , d_2. Decision variables in this term of objective function are unit price, warranty time, usage, and unit repair cost and the objective function that we want to minimize are as follows:

$$\begin{aligned} &Minimize \quad ECU + RD \\ &a^2 + b^2 = c^2 \\ &U_1 \le U \le U_2 \\ &W_1 \le W \le W_2 \\ &d_1 \le RD \le d_2 \end{aligned} \tag{12}$$

Until now the majority of researches have tried to minimize manufacturer costs and a few researchers have worked on utility of costumer. One of the numerable works on warranty and utility can be seen in the paper of Zhou et al. (2008). They hold that the utility of the presented warranty is an exponential function of the repair cost. Here, we also define the consumer utility in relation to the recovery cost from manufacturer for failure in warranty region. The purpose is to maximize costumer utility function $ut = exp(ECU)$

In a similar research, Mitra and Patankar (2011) showed that an increase in *R&D* expenditure can decrease *ECU*, so if we want to maximize *ut*, *ECU* must be maximized then *R&D* must be minimized, while we do not want to minimize *R&D* expenditure. In order to solve this problem in terms of objective function, *R&D* is considered constant and for pessimistic view to the problem *R&D* must be equal to minimum possible value. So we will have:

$$MCU = ECU_{\min RD} \tag{13}$$

$$utility_1 = \exp(MCU) \tag{14}$$

In this model, maximum and minimum values of utility happened in (U_2,W_2) and (U_1,W_1) , respectively. Since utility has a subjective concept, to place it within common value of (0,1) standardization is required. According to one of the most prevalent scientific normalizing, we must have:

$$\begin{aligned} Utility &= \frac{utility_1 - min(utility_1)}{max(utility_1) - min(utility_1)} = \frac{exp\left(MCU_{(U,W)}\right) - exp(MCU_{(U_1,W_1)})}{exp(MCU_{(U_2,W_2)}) - exp(MCU_{(U_1,W_1)})} \\ &= \left\{\frac{exp(\left(\frac{C_s}{C}\right)E[N(W,U]) - exp(\left(\frac{C_s}{C}\right)E[N(W_1,U_1])}{exp(\left(\frac{C_s}{C}\right)E[N(W_2,U_2]) - exp(\left(\frac{C_s}{C}\right)E[N(W_1,U_1])}\right\}_{min\,RD} \end{aligned} \tag{15}$$

Considering both goals, minimizing cost for manufacturers and maximizing utility for costumers, we have:

$$\begin{aligned} &Max\ Utility \\ &Min\ ECU + RD \\ &S.t \\ &C_1 \le C \le C_2 \\ &U_1 \le U \le U_2 \\ &W_1 \le W \le W_2 \\ &d_1 \le RD \le d_2 \\ &0 \le a \le 1 \end{aligned} \tag{16}$$

3. Description of the NSGA-II Algorithm

Although the model could be solved with traditional non-linear methods or searching comprehensively feasible regions of parameter values by making problem boundaries discrete, it can decrease the quality of solutions by losing some feasible values or increase the time of obtaining the final solutions if we include more possible values for parameters. For instance, if each restriction is divided into one thousand equal parts, it will take about one hour to produce the final results. If any parameter feasible values change, this time will be significant and the efficacy of model will decline sharply. Therefore, we present a meta-heuristic approach to solve this model which is called non-dominated sorting genetic algorithm (NSGA-II). The implementation of the NSGA-II algorithm can be found in the work of Deb et.al (2002). It can work with sets of solutions simultaneously and produce accurate solutions in less than one minute in the case. In addition, we can adjust the problem with new imposing conditions easily without the need to solve the problem again. This algorithm has been demonstrated as one of the most efficient algorithms for multi-objective optimization on many problems. A brief description of NSGA-II follows.

Generally, multi-objective evolutionary algorithms utilize single-objective strategies in a similar way. However, some differences are noticed between them in terms of selection and diversity strategies. In the case of selection, NSGA-II reproduces an N-member population of offspring by an N-member parents' population. After mixing two populations by using a Fast non-dominated sorting algorithm, population categorizes into different fronts. New population formation is begun by the best frontier and continued by other frontiers, respectively, until reaching the size number of N. NSGA-II uses non-dominated sorting for fitness assignments. All individuals who are not dominated by any other individuals are assigned front number 1. All individuals only dominated by individuals in front number 1 are assigned front number 2, and so on. Selection is made, using tournament between two individuals. The individual with the lowest front number is selected between the two individuals from different fronts. The individual with the highest crowding distance is selected if they are from the same front. i.e., a higher fitness is assigned to individuals located on a sparsely populated part of the front.

One of the major aims that all the multi-objective algorithms are seeking for is an appropriate diversity between solutions. In order for NSGA-II to preserve, it tries to maintain a reasonable space between solutions, so estimation of diversity is performed for each solution to ensure that final solutions are sufficiently diverse. Hence, after ascendant sorting population based on function values, it assigns an infinite amount for the highest and lowest value and for other inbound solutions a normalized amount equal to absolute distance between its place and its adjacent solution is assigned. This procedure repeats for other objective functions as well. Therefore, the total diversity distance will be the sum of these single distances. By having total distances for all solutions, they could be compared regarding this index. The algorithm priority is to select solutions in lower frontier and for solution on the same frontier those which have a higher diversity are selected earlier. This sorting algorithm code is presented in the appendix.

There are N parents and in every iteration N new individuals (offspring) are generated. Both parents and offspring compete with each other for inclusion in the next iteration. Each iteration, crossover and mutation operates and offspring will be created. Each offspring has two values from utility function and warranty cost. For this model, probability of mutation and crossover are adjusted respectively 0.3 and 0.8.

4. Computational Results

In this article, the utility function and multi-objective model solutions are provided for the first time and the model is a novel one that can illuminate a new aspect of presenting warranty. So, the results can be used by manufacturers in order to identify how they can suggest warranties that consider consumer perspective as well. To give a numerical example for our model, we used data provided by Mitra and Patankar (2010); these values are presented in Table 1.

Table 1
Values of parameters (unit of U is (10000 km), unit of W is (year) and unit of C and R&D is (dollar)

Parameters	Values	Parameters	Values	Parameters	Values	Parameters	Values
P	4	U_1	5	d_1	2	θ_0	0.002
$P_{crossover}$	0.8	U_2	12	a_3	0.25	θ_1	2
$P_{mutation}$	0.3	W_1	2	b_3	0.2	θ_2	0.05
C_1	1	W_2	10	a_4	1	θ_3	0.05
C_2	4	d_2	0.1	b_4	0.01	θ_4	1.9

In their model, Mitra and Patankar (2011), without considering utility for costumer, obtained the optimum value for R&D expenditure equal to 2. In our model, since R&D expenditure does not have any impact on utility, the best value for R&D expenditure is 2. For a better review, we solved our model for two values of *R&D* = 1, 2. Non-dominated Pareto front for both two values of the parameter is presented in Figure 2. As seen in figure 2, the maximum value for *R&D* can create better Pareto front that for a determinate cost has a higher utility and for a determinate utility has a lower cost. So, values of the parameters according to the non-dominated Pareto front can be seen in appendix.

Table 2
Optimum problem parameters for a user with usage rate between 5000-7000 km/year, R&D=2

C	Cs	U-(1000km)	W(year)	TCU	Utility	usage rate
1.004998	0.451	5	10	3.795919	0.00021	0.5
1.004998	0.451	5	10	4.606464	0.006159	0.5
1.004906	0.450981	5.086015	9.999919	6.345124	0.215159	0.5086056
1.004521	0.450904	5.481548	9.999914	6.203921	0.166513	0.5481595
1.004216	0.450843	5.840923	9.999946	5.020025	0.01599	0.5840955
1.003965	0.450793	6.17542	9.999485	6.777301	0.460118	0.6175738
1.003904	0.450781	6.261435	9.999404	5.491267	0.042661	0.62611808
1.003555	0.450711	6.812263	9.999894	6.738759	0.430595	0.6812335

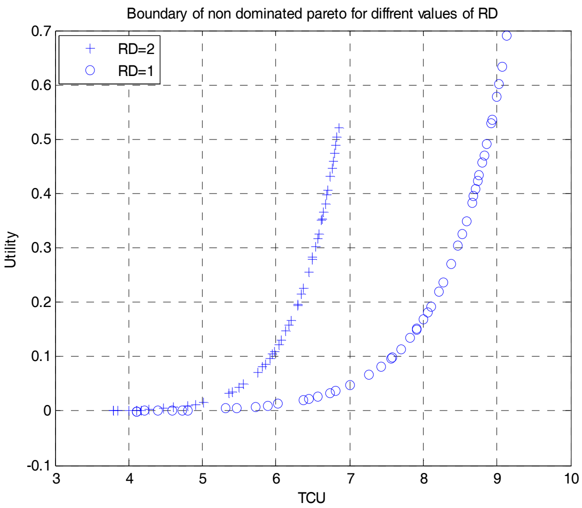

Fig. 2. Non dominated Pareto front for different values of R&D

After solving this model, there are many choices for different costumers. For instance, consider a costumer with a usage rate between 5000-7000 kilometers in a year. With respect to boundary of non-dominated Pareto we can present points that are shown in Table 2. All of these points cannot dominate each other; so selection of one point for presenting to a special costumer depends on our approach that can be accomplished by a manufacturer or a costumer.

For instance, take the lowest cost and highest utility value which are recognizable in the table. If the manager of the company decides to offer the costumer the most utility U,W are 5 and 10, respectively. However, by providing higher utility for customers, he should present higher U (6.17542-5)*10,000=1175km and it would lead to a higher TCU for manufacturer. This higher utility can show itself in lower complaint about the product and higher market share in the future. Therefore, it is up to managers to use these optimal Pareto fronts to achieve their goals.

5. Conclusion and Future Research

A two-dimensional warranty policy has been studied for FRW policy and rectangular region considering the impact of R&D expenditure on decreasing failure rate. The policy parameters are the warranty time, usage limit, unit product price and R&D expenditure per unit sale. Both approaches of costumers and manufacturers are brought in the objectives with maximization of utility for costumers and minimization of cost for manufacturers. Our Model was solved with NSGA-II and optimum non-dominated Pareto front for the problem was plotted. Finally, the best choice for a special costumer with determinate usage rate was gained. The final selection of warranty, to be accomplished by either the costumer or manufacturer, depends on the approach of the manufacturer.

Several possibilities exist for future researches on this special case of 2-D warranty. One could involve other 2-D warranty policies such as PRW [2] policy or consider other traditional 2-D warranty regions. Second, the impact of costumer utility function on market share and sale rate of products can be studied in order to extend the model. Third, costumers could be grouped in several categories because some of them are risk averse and some are risk seeking or risk natural. A new model should be created for the latest assumption. Finally, one can define other utility functions which can simulate the behavior of consumers in other societies toward product failures and compare it with the results of this study.

[2] Pro Rata Warranty

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Appendix A.

Parameters values of Non-dominated Pareto front

Row	C	Cs	U	W	TCU	Utility	row	C	Cs	U	W	TCU	Utility
1	1.004998	0.451	5	10	3.795919	0.00021	26	1.002124	0.450425	10.68379	9.999644	6.830949	0.504411
2	1.004998	0.451	5	10	4.606464	0.006159	27	1.002094	0.450419	10.81518	9.99995	6.490951	0.279187
3	1.004906	0.450981	5.086015	9.999919	6.345124	0.215159	28	1.002079	0.450416	10.88164	9.999622	6.373136	0.226279
4	1.004521	0.450904	5.481548	9.999914	6.203921	0.166513	29	1.002063	0.450413	10.95508	9.999818	6.561794	0.316383
5	1.004216	0.450843	5.840923	9.999946	5.020025	0.01599	30	1.002041	0.450408	11.05696	9.999641	6.813657	0.489747
6	1.003965	0.450793	6.17542	9.999485	6.777301	0.460118	31	1.002019	0.450404	11.15864	9.999894	6.619493	0.350046
7	1.003904	0.450781	6.261435	9.999404	5.491267	0.042661	32	1.002005	0.450401	11.22595	9.999681	5.979119	0.109775
8	1.003555	0.450711	6.812263	9.999894	6.738759	0.430595	33	1.001991	0.450398	11.29282	9.99949	5.400511	0.03553
9	1.003434	0.450687	7.024852	9.998385	6.035953	0.122096	34	1.001954	0.450391	11.47115	9.999826	4.174135	0.001805
10	1.003365	0.450673	7.154714	9.999966	6.291032	0.195132	35	1.001944	0.450389	11.52131	9.999804	4.905138	0.012408
11	1.003232	0.450646	7.415244	9.999737	6.646302	0.36682	36	1.001939	0.450388	11.54685	9.999621	5.75386	0.071483
12	1.003123	0.450625	7.642959	9.999894	4.270997	0.002469	37	1.00193	0.450386	11.59475	10	6.795534	0.474743
13	1.003022	0.450604	7.868627	9.999501	6.289799	0.194699	38	1.001921	0.450384	11.63796	9.999947	5.951219	0.104161
14	1.002868	0.450574	8.240145	9.999632	6.691694	0.396932	39	1.001919	0.450384	11.65235	9.9985	3.998029	0.0009
15	1.002664	0.450533	8.787895	9.999902	5.355567	0.032422	40	1.00191	0.450382	11.69585	9.999975	6.623131	0.352289
16	1.002632	0.450526	8.879971	9.999969	6.760678	0.447172	41	1.001903	0.450381	11.73584	9.999715	6.174326	0.157729
17	1.002588	0.450518	9.011631	9.999843	6.666885	0.380198	42	1.001895	0.450379	11.77816	9.999931	5.853298	0.086515
18	1.002545	0.450509	9.143652	9.999933	5.8201	0.081196	43	1.001888	0.450378	11.81186	9.999991	4.803577	0.009853
19	1.002515	0.450503	9.239346	10	6.576347	0.324573	44	1.001874	0.450375	11.88496	10	3.847395	0.000358
20	1.002427	0.450485	9.527706	9.99983	6.441931	0.255888	45	1.001874	0.450375	11.88496	10	6.499053	0.283223
21	1.002376	0.450475	9.703635	9.999632	6.537095	0.302916	46	1.001868	0.450374	11.91834	9.999339	4.472087	0.004368
22	1.002275	0.450455	10.07515	9.999762	6.703867	0.405397	47	1.001865	0.450373	11.93537	9.999965	6.066697	0.129288
23	1.00223	0.450446	10.24885	9.999644	4.113132	0.001452	48	1.001861	0.450372	11.95663	10	5.912878	0.096885
24	1.002227	0.450445	10.25935	9.999771	5.557459	0.048677	49	1.001854	0.450371	11.99605	10	6.849225	0.52039
25	1.002199	0.45044	10.37396	9.999581	6.136046	0.147	50	1.001854	0.450371	11.99605	10	3.795919	0.00021

Appendix B.

The Sorting MATLAB code

```
function [pop F]=NonDominatedSorting(pop)

  nPop=numel(pop);

  F{1}=[];
  for i=1:nPop
    p=pop(i);

    p.DominationSet=[];
    p.DominatedCount=0;

    for j=[1:i-1 i+1:nPop]
      q=pop(j);

      if Dominates(p,q)
        p.DominationSet=[p.DominationSet j];

      elseif Dominates(q,p)
        p.DominatedCount=p.DominatedCount+1;

      end

    end

    if p.DominatedCount==0
      p.Rank=1;
      F{1}=[F{1} i];
    end

    pop(i)=p;

  end

  f=1;
  while true

    Q=[];

    for i=F{f}
      p=pop(i);
      for j=p.DominationSet
        q=pop(j);

        q.DominatedCount=q.DominatedCount-1;

        if q.DominatedCount==0
          q.Rank=f+1;
          Q=[Q j];
        end

        pop(j)=q;
      end
    end

    if isempty(Q)
      break;
    end

    F{f+1}=Q;
    f=f+1;

  end

end
```