

Monitoring Process Variability: A Hybrid Taguchi Loss and Multi-objective Genetic Algorithm Approach

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Abstract

The common consideration on economic model is that there is knowledge about the risk of occurrence of an assignable cause and the various cost parameters that does not always adequately describe what happens in practice. Hence, there is a need for more realistic assumptions to be incorporated. In order to reduce cost penalties for not knowing the true values of some parameters, this paper aims to develop a bi-objective model of the economic-statistical design of the S control chart to minimize the mean hourly loss cost while minimizing out-of-control average run length and maintaining reasonable in-control average run length considering Taguchi loss function. The purpose of Taguchi loss function is to reflect the economic loss associated with variation in, and deviations from, the process target or the target value of a product characteristic. In contrast to the existing modeling approaches, the proposed model and given Pareto-optimal solution sets enables the chart designer to obtain solutions that is effective even for control chart design problems in uncertain environments. A comparison study with a traditional economic design model reveals that the proposed chart presents a better approach for quality system costs and the power of control chart in detecting the assignable cause.

Keywords: Economic-Statistical design, Taguchi loss function, NSGA-II Algorithm, Process variability, Immeasurable costs.

1. Introduction

When a production process faces with an assignable cause, it may shift the process variance to out-of-control states. Factors such as faculty (variable quality), raw material, unskilled/careless operators, and loosening of machine settings may lead to an increase in process variability without necessarily influencing the level of the process mean (Collani and Sheil, 1989). The S control chart is useful for monitoring a change in the process variance.

The usual approach in economic design of control charts is to develop a cost model for a particular type of industrial process and finding the optimal parameters that minimize the long-run expected cost per hour using the optimization methodologies. Several researchers studied the economic design of control charts (Collani and Sheil, 1989, Yeong et al., 2013, Faraz and Saniga, 2013, Asadzadeh and Khoshalhan, 2009, Amiri et al., 2014, Safaei et al., 2015). However, the supposition that everything is all right inside the specification and all wrong outside does not correspond to this world. Deming (1982) believes that the Taguchi loss function in which there is a minimum loss at the nominal value and an ever-increasing loss with departure either way from the nominal value is a better description of the world.

Economic design model involving Taguchi's loss function takes the advantages of both Taguchi concepts and SPC in which any deviation from the target is penalized whether the system is in-control or out-of-control (Safaei et al., 2015). Taguchi *et al.* (1989) provided an economic design to determine the diagnosis interval and control limits. The loss function as a rational approach for the minimization of the process variation has been widely studied. Yang (1998) developed the first economic statistical design of S chart which embellished with Taguchi loss function. Several researchers have applied the loss function approach in the economic design of control charts (see, e.g., ((Ben-Daya and Duffuaa, 2003); (Jiao and Helo, 2008)). Using the multivariate Taguchi loss function approach, Niaki *et al.* (2010) recently extended the economic and economic-statistical models of the MEWMA control chart in monitoring the mean vector of a process.

In reality, a certain function exists for each quality characteristic that uniquely defines the relationship between the economic loss and the deviation of the quality characteristic from its target value. The concept involved in Taguchi methods is that useful results must be obtained quickly and at low cost. Use of a quadratic,

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parabolic approximation for the quality loss function is consistent with this philosophy. Kim and Liao (1994) suggested liquid products in containers such as juice, soda, and medicine as potential applications of symmetric quadratic loss function.

In this paper, an S control chart considering a quadratic loss function, single-type and symmetric around the target value, is proposed. The optimized chart scheme is obtained by a multi objective economic-statistical design such that the mean hourly loss-cost is minimized while maintaining reasonable in-control average run length (ARL_0) and minimizing out-of-control average run length (ARL_1).

The remainder of this paper is organized as follows. In section 2, economical-statistical design and quality loss function applications in design of control charts are introduced. Motivations to multi objective economic-statistical design of control charts are then explained in section 3. Next the model of the economic-statistical design of S chart is developed. Section 5 describes the proposed solution algorithm to find the optimal solution of the model. In section 6, an illustrative example is given to demonstrate the applicability of the proposed methodology. Furthermore, some sensitivity analysis and a comparative study are performed in this section to evaluate the performance of the procedure and to determine the effects of model parameters on the performance of the proposed model. Finally, the paper is concluded in section 7.

2. The Economic Cost Function

The cost model used for determining the optimal values of chart parameters is built upon the general cost function of Lorenzen and Vance (1986). The cycle consists of an in-control phase followed by the out-of-control phase. The total cost in a cycle includes sampling inspection, search, and repair costs in addition to the cost due to nonconformities produced. It is assumed the in-control time for the process is exponentially distributed with mean $1/\lambda$. When the process goes to an out-of-control situation, the process variance (the variance of variable X), becomes $\sigma_1^2 = \rho\sigma_0^2$ ($\rho \geq 1$), where σ_0 is in-control standard deviation. Then, the false alarm rate (α) and the detection power of the control chart (P_S) are (McWilliams et al., 2001):

$$\alpha_S = 1 - G((n-1)L_S^2) \tag{1}$$

$$P_S = 1 - G((n-1)L_S^2 / (\sigma_1^2 / \sigma_0^2)) \tag{2}$$

Where $G(0)$ is the cdf of the chi-squared probability distribution with $n-1$ degrees of freedom and L_S is the chart control limit. Equation (3) gives the expected cost per unit time (hour), $E(A)$, associated with a control chart [3]:

$$E(A) = \frac{E(C)}{E(T)} = \frac{\frac{C_0}{\lambda} + C_1[h(ARL_1) - \tau + nE + \gamma_1 T_1 + \gamma_2 T_2] + (a_1 + a_2 n) \frac{E[T]}{h} + \frac{sa'_3}{ARL_0} + a_3}{\frac{1}{\lambda} + (1 - \gamma_1) \frac{sT_0}{ARL_0} - \tau + nE + h(ARL_1) + T_1 + T_2} \tag{3}$$

Where,

- C_0 is the cost per hour due to nonconformities produced while the process is in-control,
- C_1 is the cost per hour due to nonconformities produced while the process is out-of-control,
- τ is the expected time between the occurrence of the assignable cause and the time of the last sample taken before the assignable cause given in (4)

$$\tau = \frac{\int_{jh}^{(j+1)h} \lambda(t - jh)e^{-\lambda t} dt}{\int_{jh}^{(j+1)h} \lambda e^{-\lambda t} dt} = \frac{1 - (1 + \lambda h)e^{-\lambda h}}{\lambda(1 - e^{-\lambda h})} \tag{4}$$

- E is the time to sample and chart one item,
- ARL_0 is the average run length while in-control ($ARL_0 = 1/\alpha$, where α is the type-I error probability)
- ARL_1 is the average run length while out-of-control ($ARL_1 = 1/(1-\beta)$ where β is the type-II error probability)
- T_0 is the expected search time when the signal is a false alarm,
- T_1 is the expected time to discover the assignable cause,
- T_2 is the expected time to repair the process,
- γ_1 is 1 if production continues during searches and 0 if production ceases during searches,
- γ_2 is 1 if production continues during repair and 0 if production ceases during repair,
- s is the expected number of samples taken while in control ($s = e^{-\lambda h} / (1 - e^{-\lambda h})$)
- a'_3 is the cost per false alarm,
- a_3 is the cost to locate and repair the assignable cause,
- a_1 is the fixed cost per sample, and
- a_2 is the cost per unit sampled.

The average run length (ARL), a function of the process characteristic, is a measure of the expected number of consecutive samples taken until the sample statistic falls outside the control limits. To reduce the total cost, ARL should be large when the process is in-control, and should be small when the process is out-of-control. The in-control ARL for the S chart is (McWilliams et al., 2001):

$$ARL_0 = 1 / \left[1 - G((n-1)L_S^2) \right] \quad (5)$$

and the out-of-control ARL is

$$ARL_1 = 1 / \left[1 - G((n-1)L_S^2 / (\sigma_1^2 / \sigma_0^2)) \right] \quad (6)$$

3. Multi-objective S Chart

The primary assumption in economic models of control charts is that various cost parameters and the risk of occurrence of assignable causes are known (Pignatiello and Tsai, 1988, Reynolds and Cho, 2006, Chen and Liao, 2004). However, some cost parameters are difficult to accurately estimate in practice. For instance, hourly cost due to nonconformities produced while the process is out-of-control (C_1) is difficult to estimate because it involves an immeasurable diminishment in customer goodwill, besides measurable freight and indemnity. Similarly, the cost of investigating a false alarm a_3 also involves immeasurable portion (Chen and Liao, 2004).

Moreover, Multicriteria optimization algorithms are studied in economic design of control chart. Celano and Fichera (1999) developed an \bar{X} chart considering the optimization of the costs and at the same time the statistical proprieties, whereas in the multiobjective configuration the fitness is the expected loss per hour multiplied by a coefficient function of the weighed sum of α and $(1-\beta)$. Bakir and Altunkaynak (2004) applied such an algorithm to develop $\bar{X}-R$ chart. Asadzadeh and Khoshalhan (2009) developed a procedure to derive a multiobjective decision-making model with multiple assignable causes.

Recently, Safaei *et al.* (2012) formulated an optimal design of \bar{X} control chart as a multiple objective decision-making problem. Faraz *et al.* (2013) studied a multiobjective Genetic Algorithm Approach to the Economic Statistical Design of Control Charts with an Application to \bar{X} and S^2 Charts. Morabi *et al.* (2015) considered a multiobjective design of \bar{X} control charts with fuzzy process parameters.

The optimal design of an S control chart involves determining the optimal values of the chart parameters, i.e., the sample size, the interval between successive samples, and the control limit. In this study, a model that considers both measurable and immeasurable costs in a multiobjective economic-statistical design of control chart is considered. Correspondingly, Minimization of ARL_1 is cooperated as quality performance indices in the model to give another consideration to the aspect of immeasurable costs.

Let T be the target value for the quality characteristic monitored the quality loss is zero only when the quality characteristic X equals the target T . The loss increases as the deviation from the target increases. Suppose that μ_0 and σ_0 are in-control mean and standard deviation, respectively. When the process goes to an out-of-control situation, the process variance, becomes $\sigma_1^2 = \rho^2 \sigma_0^2$ ($\rho \geq 1$).

Further, Assume also the probability density function (pdf) of the quality characteristic X to be $f(x)$. If the loss function, $L(X)$, is symmetric around the target, the loss coefficient K should be estimated such that the loss can be obtained as

$$L(X) = K(x - T)^2 \quad (7)$$

To design control charts based on a symmetric quadratic loss function, J_0 is calculated as

$$\begin{aligned} J_0 &= \int_{-\infty}^{+\infty} K(x - T)^2 f(x) dx = \\ &= \int_{-\infty}^{+\infty} K(x - \mu_0 + \mu_0 - T)^2 f(x) dx \\ &= K \left[\sigma_0^2 + (\mu_0 - T)^2 \right] \end{aligned} \quad (8)$$

Where $f(x)$ is probability density function (pdf) of a normal variable $N(\mu_0, \sigma_0^2)$.

The expected cost per unit under quadratic loss function when the process is out-of-control is

$$J_1 = K \left[\sigma_1^2 + (\mu_0 - T)^2 \right] \quad (9)$$

The expected external costs of each product are shown by J_0 and J_1 . If p units are produced per hour, C_0 and C_1 in cost function can be computed as, $C_0 = J_0 p$ and $C_1 = J_1 p$.

The optimal values of the control parameters are determined such that the ATL of the company is minimized and the detection power of the chart is maximized. In other words, the multiobjective economic-statistical model for S chart becomes:

$$\begin{aligned} &Min \ ATL(L_S, n, h) \\ &Min \ ARL_1 \\ &Subject \ to \\ &ARL_0 > ARL_L \\ &n \ is \ a \ positive \ integer \\ &h \ and \ L_S > 0 \end{aligned} \quad (10)$$

In the next section, a solution algorithm is proposed to solve the multiobjective optimization model given in (10).

4. The Solution Algorithm

The economic models are of nonlinear programming (NLP) type with linear constraints and there are different methods available in literature to solve such models. Molnau *et al.* (2001) used the algorithm of Hooke and Jeeves (1961) to solve the nonlinear programming model of economic design of control charts. However, since one of the model parameters, L_S , is indirectly used in the model and is solely used for calculation of $ARLs$, a search-based algorithm (for example genetic algorithm) can be applied to solve the model. These algorithms by searching different quantities of decision variables and using appropriate penalty or barrier functions can converge to the best solutions.

The solution vector of the design parameters of this research is (L_S, n, h) based on which the economic objective function with both internal and external costs along with statistical properties is used to evaluate and compare different solutions. The Pareto optimization is utilized and an elitist non-dominated sorting genetic algorithm (NSGA-II) is developed to optimally determine the solution vector.

4.1. Pareto Optimization

There are different solution algorithms to optimally solve the multi-objective optimization model at hand. However, the proposed Pareto optimization method searches for non-dominated solutions, rather than attempting to combine all objectives into a single objective. Optimization through Pareto dominance compares each objective only with itself, removing the need for standardization of objectives, as well as the arbitrary nature that it adds to the optimization process.

4.2. Non-Dominated Sorting Genetic Algorithm (NSGA-II)

One of the multi-objective evolutionary algorithms (MOEA) that has been effective in finding the Pareto optimal solutions is the elitist non-dominated sorting genetic algorithm (NSGA-II) developed by Deb (2001). The adapted steps involved in this algorithm are as follows:

1. Randomly initialize population (designs in the variable space) of size $npop$.
2. Compute ATL , ARL_1 and constraints for each design.
3. Rank the population using non-domination criteria (many individuals can have same rank and rank-1 is the best).
4. Compute crowding distance (this distance finds the relative closeness of a solution to other solutions in the function space and is used to differentiate between the solutions on same rank).
5. Employ genetic operators – selection, crossover, and mutation – to create intermediate population of size $npop$.
6. Evaluate objectives and constraints for this intermediate population.
7. Combine the two (parent and intermediate) populations, rank them and compute the crowding distance.
8. Select new population of $npop$ best individuals based on the rank and crowding distance.
9. Go to step 3 and repeat till termination criterion is reached. In this research this criterion is chosen to be the number of generations.

In this research, a chromosome is composed of three genes and each gene represents a decision variable. The decision variables of the model are n, h, L_S . Imported chromosomes from the previous steps will be inputs of the crossover operation with the probability of 0.2 (This

value was reached by trial-and-error using numerical examples). The mutation step in each loop creates the mutated children using adaptive mutation in which genes satisfy linear constraints. It is very important to maintain the diversity of population for convergence to an optimal Pareto front. This is performed by controlling the elite members of the population as the algorithm progresses. For controlling the elitism Pareto fraction, 0.2 Pareto fraction is applied. The Pareto fraction limits the number of individuals on the Pareto front (elite members). Crowding distance measure function that finds the relative closeness of a solution to other solutions in the function space employed to differentiate between the solutions on same rank. The crowding distance measure function has argument to calculate distance either in function space (phenotype) or in design space (genotype).

Here we take advantage of “genotype” for distance function. Population will be evaluated based on ATL , ARL_1 and constraints. After ranking and computing the crowding distance, new population of best individuals will be selected. Finally, steps repeat until the termination criterion is reached. At the end of these steps, chromosomes with Pareto optimality are reported as the solution for economic-statistical design of S control chart.

5. Illustrative Example

In order to illustrate the application of the developed multiobjective economic-statistical S control chart, the data which is borrowed from Serel (2009), are used in this section.

5.1. An Example

The cost and process parameters, with some modifications, are: the fixed cost per sample $a_1=\$5$, the cost per unit sampled $a_2=\$1$, the cost to locate and repair the assignable cause $a_3=\$900$, the cost per false alarm $a'_3= \$300$, the time to sample and chart one item $E=0.05$, the expected search time when the signal is a false alarm $T_0=2$, the expected time to discover the assignable cause $T_1=2$, the expected time to repair the process $T_2=0$, and when the process goes out-of-control, the variance of the process increases by $\rho=1.5$.

Assuming a quadratic loss function with $K=1$, $\sigma_0^2=1$, $\mu_0= T$, the expected cost per product is \$1 and \$3.25 while producing in-control and out-of-control items, respectively. The average production rate per hour is planned 300, and on the average, the process stays in-control for 100 hours. The production continues during the search for an assignable cause, but it ceases during repair, i.e., $\gamma_1=1$ and $\gamma_2=0$.

Furthermore, the following limits are imposed on the decision-variables: $2 \leq n \leq 30$, $h \leq 40$ and $L_S \leq 4$. The minimum allowable in-control average run length (ARL_0) is 105.

The optimal S chart parameters minimizing both the ATL and ARL_1 are given in Table (1). The Pareto front for ATL and ARL_1 of the multi objective economic-statistical design is shown in Figure (1).

From the economic viewpoint, the solution vector at minimum ATL is (1.60,9,1.54) with \$344.68 with an ARL_1 of 1.34. However, a slight movement from the

minimum ATL (\$0.3), as revealed in Table (1) and Figure (1), solution (1.55,11,1.74) results improvement in the both statistical properties with ARL_1 of 1.22 and ARL_0 142.4 where it receives higher product quality. Accordingly, this multiple objective design of S control chart present a better approach for quality engineers to improve the process.

Table 1
Optimal design of S chart

L_S	n	h	ATL	ARL_1	ARL_0
1.60	9	1.54	344.68	1.34	130.6
1.60	9	1.54	344.68	1.34	130.6
1.59	9	1.55	344.68	1.33	123.5
1.59	9	1.55	344.68	1.33	123.5
1.57	10	1.64	344.77	1.27	125.7
1.57	10	1.64	344.77	1.27	125.7
1.56	10	1.76	344.81	1.26	105.7
1.55	11	1.74	344.98	1.22	142.4
1.53	12	1.86	345.29	1.18	127.0
1.53	12	1.86	345.29	1.18	127.0
1.53	12	1.86	345.29	1.18	126.5
1.50	13	2.00	345.68	1.14	127.1
1.50	14	2.00	346.11	1.13	146.7
1.49	15	2.08	346.60	1.11	167.0
1.45	15	2.15	346.69	1.09	121.1
1.46	18	2.32	348.29	1.07	265.3
1.45	19	2.53	348.97	1.05	237.4
1.38	21	2.49	350.51	1.02	105.0
1.37	26	3.31	354.10	1.01	200.6
1.37	27	4.36	356.67	1.01	237.0
1.32	28	5.68	360.99	1.00	102.8
1.32	28	5.68	360.99	1.00	102.3

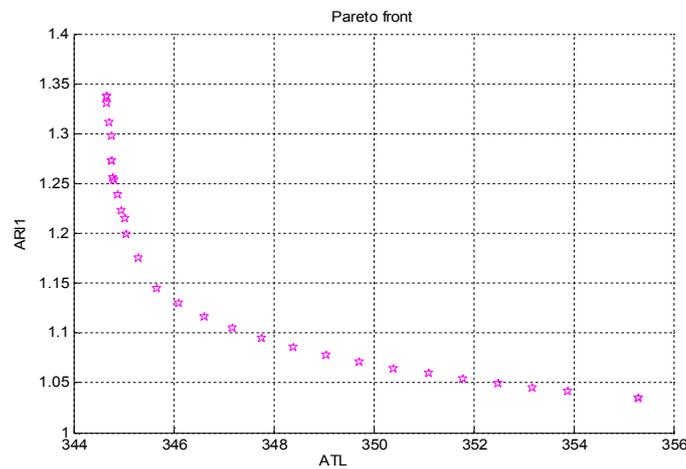


Fig. 1. Pareto front for ATL and ARL_1

5.2. Sensitivity Analysis

In this section, the sensitivity of the chart parameters to variations in process parameters is explored. The parameters are set as follow. Two values for cost per false alarm, $a_3 \in \{300,900\}$, and two values for cost to locate and repair the assignable cause, $a_3 \in \{150,900\}$, are

considered. Moreover, the process variance shifts are of the sizes of $\rho=1.5, 2, 2.5$. Figure (2) summarize the Pareto front for 12 runs of the parameter values. In Figure (2), $a_3= \$150$ is shown in purple, $a_3= \$900$ is exposed by blue color, and three iso-chromatic curve in the chart are based upon the sizes of process shift (small, moderate and large).

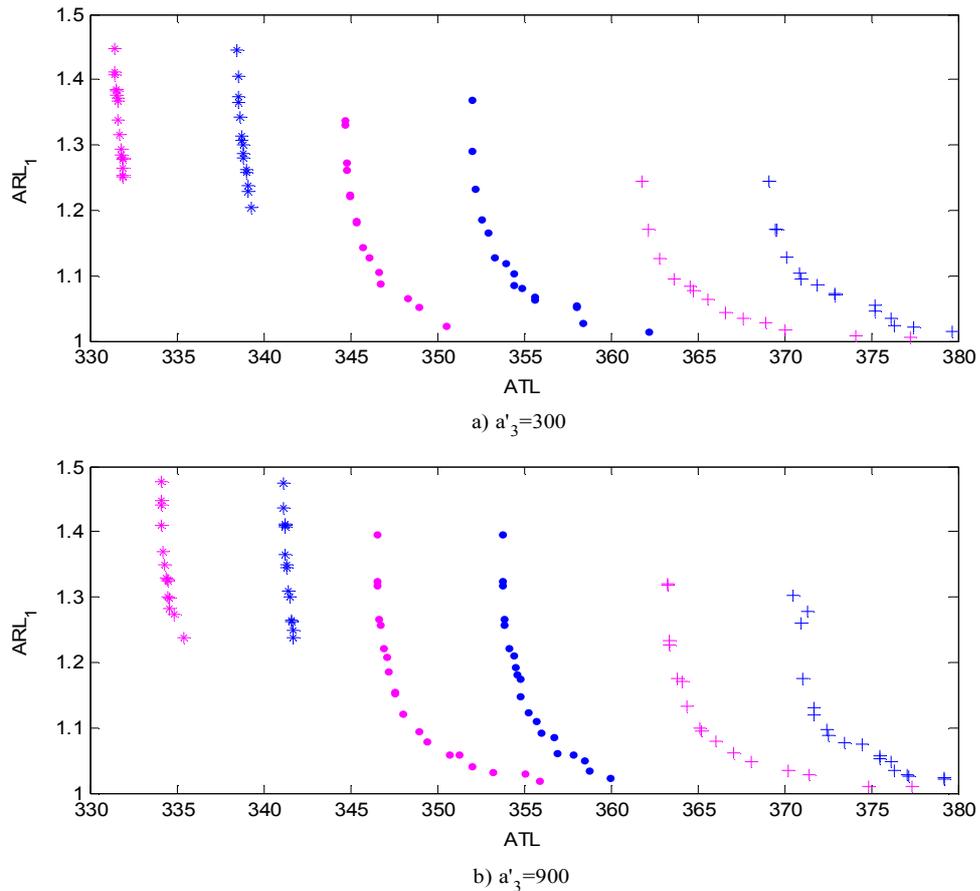


Fig. 2. Sensitivity Analysis of multi objective economic-statistical design of S chart

As it is visible from Figure (2), in sections (a) and (b), the cost per false alarm (a'_3) and ARL_1 are positively related, i.e., a large a'_3 leads to a larger ARL_1 . For all shift sizes, ATL increases slightly with increasing a'_3 . At the point, cost to locate and repair the assignable cause (a_3) has no significant effect on ARL_1 , where ATL slightly increases at steady state with increasing a_3 .

Generally, when the process incurred a small shift in the variance, the quality engineer can improve the chart performance with a slightly increase in its ATL . Moreover, in moderate shift in the variance, one can see good improvement with a small increase in ATL , which is harder to reach for the large shift size.

5.3. Comparisons

In this section, the Pareto optimal solutions of the proposed multi objective economic-statistical design of S chart parameters that minimizes both the ATL and ARL_1 are compared with the traditional approach of control chart design. According to the proposed model, each setting is subjected to a lower bound for reported ARL_0 .

Table (2) summarizes the comparison performed between the minimum ATL within the Pareto solutions of the proposed S chart (two solutions in each setting sorted

by ATL and ARL_1) and economic design of S chart reported in Serel (2009). In Table (2), the asterisk denotes solutions with minimum ATL and ARL_1 and plus sign denotes solutions which are alike reported in Serel (2009).

The results in Table (2) show that the proposed economic-statistical design has better performance than the traditional approach in all settings. It shows that a slight increase in the hourly expected cost can lead to the faster detection of assignable causes. Therefore, the better quality may be achieved and the probability of the production in the out-of-control state will be decreased. Consequently, the results reveal that the proposed methodology has been profitable and valuable.

6. Conclusion

A multi objective economic-statistical design of the S chart in which quadratic loss function that uniquely defines the relationship between the economic loss and the deviation of the quality characteristic from its target value was taken into account. The Pareto-optimal solution sets enabled the designer to obtain solutions that were effective even for control chart design problems.

Table 2
The results of comparison study with Serel (2009) S chart

Multiobjective economic-statistical design of S chart s.t. $ARL_0 \geq ARL_L$								Traditional S chart		
a_3	a_3	ρ	L_S	n	h	ATL	ARL_1	ATL	ARL_1	ARL_0
300	150	1.5	1.35	16	2.92	331.40 ⁺	1.5	331.40	1.5	38.3
			1.34	17	3.04	331.41	1.4			
		2	1.6	9	1.54	344.68*	1.3			
			1.55	11	1.74	344.98	1.2			
		2.5	1.78	7	1.09	361.84*	1.2			
			1.69	9	1.27	362.84	1.1			
	900	1.5	1.34	17	3.09	338.50	1.4*	338.50	1.5	38.3
			1.32	20	3.39	338.65	1.3			
		2	1.62	9	1.49	351.95*	1.4			
			1.58	10	1.61	352.01	1.3			
		2.5	1.78	7	1.1	369.11*	1.2			
			1.69	9	1.27	370.10	1.1			
900	150	1.5	1.3	22	3.26	334.08*	1.5	334.10	1.5	115.8
			1.37	23	3.31	334.11	1.4			
		2	1.66	10	1.54	346.51*	1.4			
			1.63	11	1.64	346.52	1.3			
		2.5	1.88	7	1.05	363.27*	1.3			
			1.82	8	1.15	363.40	1.2			
	900	1.5	1.37	22	3.3	341.14*	1.5	341.15	1.5	115.8
			1.36	23	3.32	341.16	1.4			
		2	1.66	10	1.54	353.73	1.4			
			1.63	11	1.65	353.74*	1.3			
		2.5	1.88	7	1.06	370.54*	1.3			
			1.83	8	1.15	370.66	1.2			

Comparisons with traditional economic design model reveal that the proposed multi objective model is superior to the traditional models.

The model allows the easy and fast optimization of quality costs and statistical performance measures. The resulting Pareto-optimal solution set can be used to extract knowledge that could not be determined using single objective approaches, including the trade-off relationships between ATL and ARL_1 . This provides a variety of choices to arrive at the requirement of long run quality of product or minimal cost concurrently for quality engineers. It is interesting to extend the current work for other control charts or to consider other loss function policies for the construction of the total cost model.

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