A Hierarchical Production Planning and Finite Scheduling Framework for Part Families in the Flexible Job-shop (with a case study)

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Abstract

Tendency towards optimization in last decades has resulted in creating multi-product manufacturing systems. Production planning in such systems is difficult, because the calculated optimal production volume must be consistent with the limitation of production system. Hence, integration has been proposed to decide about these problems concurrently. Main problem in integration is how we can relate production planning in the medium-term timeframe to scheduling in the short-term timeframe. Our contribution creates production planning and scheduling framework in the flexible job-shop environment with respect to the time-limit of each machine in order to produce different part families in the automotive industry. Production planning and scheduling have an iterative relationship. In this strategy, information flow is transformed in a reciprocative way between production planning and scheduling in order to satisfy the time-limit of each machine. The proposed production planning has a heuristic approach and renders a procedure to determine the production priority of different part families based on the safety stock. Scheduling is performed with ant colony optimization and assigns machines in order of priority to different part families on each frozen horizon. Results showed that the proposed heuristic algorithm for planning decreased parts inventory at the end of planning horizon. Moreover, the results of the proposed ant colony optimization were near the optimal solution. The framework was performed to produce sixty-four different part families in the flexible job-shop with fourteen different machines. The output of the approach determined the volume of production batches for part families on each frozen horizon and assigned different operations to machines.

Keywords: Production Planning, Finite Scheduling, Part Families, Flexible Job-shop.

1. Introduction

Current production and inventory control systems consist of make-to-order (MTO), make-to-stock (MTS) and MTS-MTO systems. The MTS-MTO system is used when there are various products. In such systems parts-making is performed by the MTS system while the assembly of products is done by the MTO system (see Figure 1). Production planning and scheduling frameworks in such systems are of two types of hierarchical and integrated. In the first approach, planning and scheduling are performed hierarchically and in the second approach, planning and scheduling are done simultaneously. In this paper, we propose a hierarchical approach for production planning and finite scheduling in MTS-MTO systems.

Our proposed production planning has a heuristic approach and is performed based on required lots of part families on the planning horizon. Parameters such as lot size, safety stock and safety lead-time are considered. Lot size is calculated based on the periodic order quantity (POQ)

The scheduling of different production batches in the Flexible Job-shop (FJS) environment is studied with regard to the Independent setup time.

Scheduling is performed with the Ant Colony Optimization (ACO) based on required lots of part families on the frozen horizon.

This paper studies hierarchical production planning and scheduling, which have an iterative relationship, for the components of different products in the automotive industry.

Our contribution creates production planning and scheduling framework in the FJS environment with the consideration of time-limit in order to produce different parts families. This framework is evaluated in *Safe Sanat*, a supplier of *Iran Khodro* and *SAIPA* Company.

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The paper is organized as follows. In the following section we review previous related work on production planning and scheduling. Section (3) contains an extended integer programming formulation of the FJS problem with considering the time-limit of each machine. Then Section (4) proposes a hierarchical production planning and finite scheduling in the FJS environment.

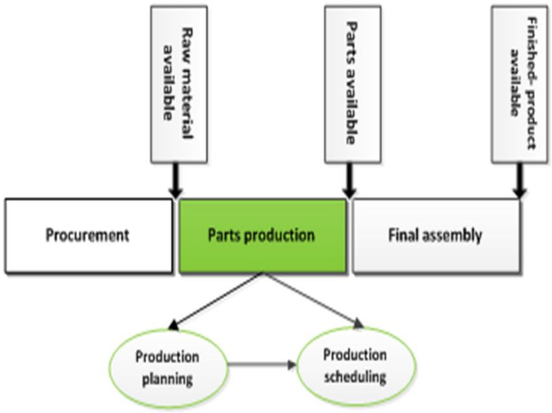

Fig. 1. Production planning and scheduling for parts

A case study in *Safe Sanat* Company is explained in Section (5), and in Section (6), we report the results of the proposed hierarchical production planning and scheduling. Finally, we conclude the paper with a summary and some directions for future research in Section (7).

2. Literature Review

2.1. Uncertainty in Production and Inventory Control Systems

Production in MTS systems without considering issues such as demand uncertainty for finished product, material procurement, fluctuation in production time, machine breakdown, poor quality of products and failure to provide material in a timely manner is not possible. Controllable parameters in uncertainty conditions are safety stock, safety lead-time, production batch, frozen horizon and planning horizon.

2.2. Important Parameters in Production Planning

2.2.1. Safety stock, Safety Lead-time and Setup time

Safety stock parameter decreases the probability of shortage, but increases holding costs. It is calculated by minimizing the sum of holding and shortage costs based on the significance level. Melnyk and Piper (1981) state that safety lead-time is equivalent to safety stock in which time is used instead of quantity. It is usually considered as a standard variance of *k* lead-times. According to Why bark and Williams (1976), safety stock is used when the uncertainty is based on quantity and safety lead-time is used when the problem is faced with the estimation of lead-time.

In this study we consider production planning and scheduling in MTS systems. For such systems, Molinder (1997) considers setup time, significance level, inventory and shortage cost. He proposes safety stock for the high variation of demand and the low variation of production lead-time. He also proposes safety lead-time for the high variation of demand and lead-time.

Internal safety lead-time is considered for machine breakdown, poor quality of products and variation in production time while external safety lead-time is considered for the activity done outside the organization. In this article we use external safety lead-time to calculate the safety stock. They consist of galvanized plating, smanth, electroplates dakrvmat and heat treatment. These operations start when internal activities end.

2.2.2. Lot-size

First, the main method of determining lot size is economic order quantity (EOQ). This method calculates the fixed order size through Wilsons' formula. But the interval between two consecutive orders may be variable. The second method is POQ in which an optimized fixed interval between two orders is calculated and using the interval, the order size of each period is calculated. The third method is Wagner Whithin algorithm. It is based on minimizing order costs for dynamic demands, without considering capacity constraints. Due to the long duration of solution, Silver Meal, Least Unit Cost (LUC) and Part Period balancing (PPB) techniques are used instead of Wagner Whithin.

In this regard, we propose a heuristic procedure to calculate production batches based on the POQ method.

2.2.3. Planning Horizon and Frozen Horizon

To maintain a balance between a suitable significance level for customer satisfaction and decreasing inventory, lots of techniques use frozen horizon for production planning. Using frozen horizon (*FH*) decreases the significance level and inventory. In this article, we divide the planning horizon (*PH*) into several frozen horizons.

2.3. Production Scheduling Systems in Job-shop and Flexible Job-shop

A classical job-shop (JS) system consists of a set of different machines that is used for operation on all the jobs. Each job specifies the processing order through the machines, i.e., a job is composed of an ordered list of operations. Each operation is determined by the machine required and the processing time of it. There is no job-preemption and each machine can handle only one operation at a time. Each operation can also be performed only on one machine. Moreover, in this JS, the sequence of operations for each job is fixed and the problem is to find the job sequences on the machines which minimize the Make-span, i.e. minimizing the maximum of the completion time for all operations. It should be pointed out that Garey et al. (1976) proved that job-shop problem is a *NP-hard*.

FJS is another approach in which each operation can be performed on more than one machine. In this strategy, all parts from different families are processed in one job center (cell) and there is no more than one shop center. It is used for non-stop production processes. In this approach, the machines with a similar usage can handle a specified operation. Because of its multi-route for assigning each operation to a machine, FJS is more complex than JS and is strongly *NP-hard*. FJS' heuristics are either hierarchical or integrated. In the hierarchical approach, assigning and sequencing operations on each machine are performed separately; in other words, assignment and sequence of operations are considered independent of each other. In the integrated approach, assigning and sequencing operations are performed simultaneously and they are interdependent. There are some studies on FJS systems. The earliest finishing time (EFT) rule with respect to alternative operation to minimize mean flow time in such a system was investigated by Nasr and Elsayed (1990).

Mahmood et al. (1990) developed dynamic scheduling heuristics to stress good due date performance while reducing overall setup time in a job-shop cell. Tsai and Li (2000) presented a due date-oriented scheduling heuristic algorithm for job-shop cell manufacturing systems based on capacity constraint resource. Li and Wang (2012) suggested a pheromone-based approach using a multi-agent cell manufacturing system in which parts families can move between flexible routes in different job centers. Fattahi et al. (2007) presented three heuristic approaches for FJS scheduling problems for parts-making industries. Ponnambalam et al. (2010), the closest research to our study, developed an ant colony optimization approach in FJS with the consideration of relative pheromone trail between different operations on a specified machine. In this paper, we develop this research for different production batches.

Finite scheduling is about assigning no more operation to a machine which is expected to execute in a given time period. There is no research done about scheduling of different parts families in flexible job-shop with the consideration of capacity constraint of machines. We, therefore, propose a heuristic for the finite scheduling of different part families in the FJS systems.

2-3-1. Setup Time in Job-shop Scheduling Systems

To use setup times, there are two approaches. In the first one, the setup time for each operation is independent of the previous operation on an identified machine. In the second approach, the setup time for each operation is dependent on the previous operation on an identified machine.

2.4. A Mixed integer linear formulation for Flexible Job-shop Scheduling

In this study, we develop an MILP model with consideration of time-limit for production batches on each frozen horizon based on Mehrabad and Fattahi (2007).

2.5. Production Planning and Scheduling Framework

The most important research about production planning and scheduling framework was done by Meybodi (1994), in which the integration of production activity control with consideration of final customer demand was studied. A heuristic algorithm with respect to family's production cycle time was proposed. Furthermore, production orders for families were presented, and the lot size was calculated using the *EOQ* model.

2.6. The Study

This paper studies production planning and scheduling for the components of different products in the automotive industry in MTS-MTO systems. There is no identified production planning and scheduling framework for part families considering time-limit. Hence, to produce different parts families, we look for creating a production planning and scheduling framework with respect to the time-limit of each machine. To do so, we go through three stages. First, we propose a heuristic to calculate the production batches of parts families in each frozen horizon. Second, we develop the ACO method in the FJS system and then compare it with the developed mixed integer linear programming. Third, we propose a hierarchical production planning and finite scheduling framework for parts families in the FJS system. This strategy is used in *Safe Sanat* to calculate its effectiveness.

3. The Mixed Integer Linear Formulation for Flexible Job-shop Scheduling

The production batch consists of *K* lots related to a certain family. Each lot pertains to an identified part. All lots are stacked to reach an equal size. Each time all parts are processed by a certain machine, and then they are added to their lots in order to reach the specified quantity. On the other hand, we have the batch availability with K lot sizes. Finally, after the production batch is prepared for next operation, it is transferred to a new machine. The assumptions of this model are as follows:

1- The production batch and transferring batch for each family are equal in a certain frozen horizon.

2- Transportation times between different machines are not considered.
3- The setup time for a new production batch on a certain machine is independent of the previous production batch on the same machine.
4- There is no job-preemption.
5- The number of different machines that can be used for each operation for an identified production batch is between 1 to 3.
6- The setup for all lots of a production batch is performed concurrently.
7- The preliminary setup for production batches is not considered.
8- The possibility of batch splitting for simultaneous production on different machines is not considered.

Here are the notations for scheduling of the production batches:

n *the number of machines*
m *the number of production batches*
$h_{j'}$ *operation number*
i' *index of machines,* $i' = 1, \dots, n$
j', k *index of production batch,* $j' = 1, \dots, m$
h, l *index of operation,* $h = 1, \dots, h_{j'}$
$p_{i',j',h}$ *process time for operation h of production batch j' on machine* i'
$s_{i',k,l}$ *setup time for operation l of production batch k* on machine i'
$RT_{i'}$ *regular time for machine* i'
$ET_{i'}$ *extra time for machine* i'
M *a large number*

And the integer programming uses the following variables:

c_{max} *maximum completion time for production batch*
$t_{j',h}$ *start time of operation h of production batch j'*
$f_{j',h}$ *finish time of operation h of j*
$y_{i',j',h}$ $equal\ to\ 1\ \ if\ operation\ h\ of\ production\ batch\ j' is\ assigned\ to\ machine\ \ i', 0\ otherwise;$
$x_{i',j',h,k,l}$ $equal\ to\ 1\ if\ operation\ l\ of\ production\ batch\ k\ after\ operation\ h\ of\ production\ batch\ j'\ is\ assigned\ to\ machine\ \ i', 0\ otherwise;$
$a_{i',j',h}$ $equal\ to\ 1\ if\ operation\ h\ of\ production\ batch\ j' can\ be\ perrformed\ on\ machine\ \ i', 0 otherwise;$

Using the above parameters and variables, we can represent our problem of minimizing Make-span to a MILP as follows:

$$min\ C_{max} \tag{1}$$

$$t_{j',h} + y_{i',j',h}.p_{i',j',h} \le f_{j',h} \quad ; \forall j = 1, \dots, m; i = 1, \dots, n; h = 1, \dots, h_{j'} \tag{2}$$

$$f_{j',h} \le t_{j',h+1} \quad ; \ \forall j' = 1, \dots, m; h = 1, \dots, h_{j'-1} \tag{3}$$

$$f_{j',h_{j'}} \le \ C_{max} \quad ; \forall j' = 1, \dots, m \tag{4}$$

$$y_{i',j',h} \le a_{i',j',h} \quad ; \ \forall j' = 0, \dots, m; \ i' = 1, \dots, n; h = 1, \dots, h_j \tag{5}$$

$$t_{j',h} + \left(p_{i',j',h} + s_{i',k,l}\right).y_{i,j',h} \le t_{k,l} + \left(1 - x_{i',j',h,k,l}\right)M \quad ; \forall j' = 0, \dots, m; \ k = 1, \dots, m; \ \forall i' = 1, \dots, n \ ; \ \forall h = 1, \dots, h_j; \forall l = 1, \dots, h_k \tag{6}$$

$$f_{j',h} + \left(s_{i',k,l}\right).x_{i',j',h,k,l} \le t_{j',h+1} + \left(1 - x_{i',k,l,j',h+1}\right)M \quad ; \forall j' = 1, \dots, m; \ k = 0, \dots, m; \forall i' = 1, \dots, n \ ; \ \forall h = 1, \dots, h_{j'} - 1; \forall l = 1, \dots, h_k \tag{7}$$

$$x_{i'j',h,k,l} \le y_{i',j',h} \quad ; \forall j' = 0, \dots, m; \ k = 1, \dots, m \ ; \forall h = 1, \dots, h_{j'}; \forall l = 1, \dots, h_k \ ; \forall i' = 1, \dots, n \tag{8}$$

$$x_{i',k,l,j',h} \le y_{i',k,l} \quad ; \forall j' = 1, \dots, m; \ k = 0, \dots, m; \forall i' = 1, \dots, n \ ; \forall h = 1, \dots, h_{j'}; \forall l = 1, \dots, h_k \tag{9}$$

$$\sum_{i'=1}^{n} y_{i',j',h} = 1 \quad ; \ \forall j' = 0, \dots, m; \ h = 1, \dots, h_{j'} \tag{10}$$

$$\sum_{h=1}^{h_j} \sum_{j=0}^{m} x_{i',j',h,k,l} = 1 \quad ; \ \forall i' = 1, \dots, n; \ \forall \ k = 1, \dots, m; \forall l = 1, \dots, h_k \tag{11}$$

$$\sum_{l=1}^{h_k} \sum_{k=0}^{m} x_{i',k,l,j',h} = 1 \quad ; \ \forall i' = 1, \dots, n; \ \forall \ j = 1, \dots, m; \ \forall h \ =, \dots, h_{j'} \tag{12}$$

$$\sum_{j'=0}^{m} (p_{i',j',h} + s_{i',j',h}) y_{i',j',h} \le RT_{i'} + ET_{i'} \quad ; \forall i' = 1, \dots, n; \ \forall h = 1, \dots, h_{j'} \ ; \forall l = 1, \dots, h_k \tag{13}$$

$$x_{i',j',h,j',h} = 0 \quad ; \ \forall j' = 0, \dots, m; \ i' = 1, \dots, n; \ h = 1, \dots, h_{j'} \tag{14}$$

$$t_{j',h} \ge 0 \ \forall \ j' = 0, \dots, m \quad ; \ \forall h = 1, \dots, h_{j'} \tag{15}$$

$$f_{j',h} \ge 0 \ \forall \ j' = 0, \dots, m \quad ; \ \ \forall h = 1, \dots, h_{j'} \tag{16}$$

$$y_{i',j',h} \in \{0, 1\} \quad ; \forall j' = 0, \dots, m; \ i' = 1, \dots, n; \ h = 1, \dots, h_{j'} \tag{17}$$

$$x_{i',j',h,k,l} \in \{0, 1\} \quad ; \forall j' = 0, \dots, m; \ k = 1, \dots, m; \ \ \forall h = 1, \dots, h_{j'} \tag{18}$$

$; \forall l = 1, \dots, h_k; \ \forall i' = 1, \dots, n$

Eq. (1) means that this problem is to minimize the Make-span. Equations (2), (3) guarantee that each production batch has an identified sequence of operations. Eq. (4) defines the Make-span, and Eq. (5) assures that the operation h of production batch j' can process on alternative machines. Equations (6), (7) assure that in any time one operation can process on each machine. Equations (8) and (9) show the possibility of sequencing operations for different families on each machine. Eq. (10) says that that the operation h of production batch j' is performed on only one machine. Equations (11) and (12) guarantee that only one operation is performed on a certain machine when the operation which is processing on it is fully performed. Finally, Eq. (13) shows the time-limit for each machine on a frozen horizon.

4. The Proposed Approach for Hierarchical Production Planning and Finite Scheduling

4.1. The Production Planning Approach for Different parts families

First, parts were divided into different families with respect to the bill of material. All parts in any family had similar processes. Production planning for families were performed based on product demand forecast and usage rate of its parts. Then the actual start of production for each family was calculated based on the run-out date of parts inventory. Finally, the production batch for each family in any frozen horizon was obtained. Internal and external safety lead-time was also considered to calculate the safety stock for each part. The assumptions of the production planning system were as follows:

1- Production planning was performed based on the MTS system.

2- Production planning was performed on the planning horizon.

3- The lot size for families was calculated based on the POQ approach.

4- There was uncertainty for finished-products demand and external lead-time.

5- Shortage was not considered on the planning horizon.

4.1.1. Steps of the Heuristic Production Planning Approach

Step (1) – Forecasting the parts demand of different products is calculated by(19).

$$_k = \sum_{i=1}^{n} U_{ijk} . D_i . \delta_{ijk} \qquad (19)$$

$; \forall j = 1, \dots, m; \ \forall k = 1,2, \dots, l_j$

where

U_{ijk} *usage rate for the part k of family j in product i*

D_i *forecasted demand of product at the beginning of planning horizon*

δ_{ijk} *equal to 1 if the part k of family j belongs to product i ,0 otherwise*

Step (2) – Calculating the safety stock for the part *k* of family *j* based on the fixed order interval system by (20), (21) and (22).

$$SS_{jk} = ROUNDUP\left[Z_\alpha \times (\sigma_{D\,\bar{L}+T})_{jk}\right] \qquad (20)$$

(20)

$; \ \forall j = 1, \dots, m; \ \forall k = 1, \dots, l_j$

$$(\sigma_{D\,L+T})_{jk} = \sqrt{\left((\bar{L}'' + T + \bar{L}') \times \sigma^2_{\overline{FD}} + \overline{FD}^2 \times \sigma^2_{(\bar{L}''+T+\bar{L}')}\right)_{jk}} \qquad (21)$$

$; \forall j = 1, \dots, m; \ \forall k = 1, \dots, l_j$

$$T = \frac{PHFD - PHSD}{Day\ Per\ Month} \qquad (22)$$

where

SS_{jk} *safety stock for the part k of family j during lead-time plus planning period*

$(\sigma_{D\,\bar{L}+T})_{jk}$ *standard deviation for the part k of family j during lead-time plus planning period*

$\bar{L}'_{jk}$ *average production lead-time for the part k of family j*

$\bar{L}''_{jk}$ *average external safety lead-time for the part k of family j*

T *planning horizon period(month)*

$PHSD$ *planning horizon start date*

$PHFD$ *planning horizon finished date*

$\sigma^2_{\overline{FD}}$ *forecasted demand variance for the part k of family j*

$\sigma^2_{(\bar{L}''+T+\bar{L}')_{jk}}$ *lead-time variance for the part k of family j during planning horizon*

$\overline{FD}_{jk}$ *forecasted demand for the part k of family j during planning horizon*

Z_α *confidence level* α

Step (3) – Calculating the run-out date of inventory for each part at the beginning of planning horizonby (23):

$$F_{jk} = \frac{FD_{jk}}{W_D} \quad (23)$$

$; \forall j = 1, \dots, m ; \forall k = 1, \dots, l_j$

where

I_{jk} *on-hand inventory for the part k of family j at the beginning of planning horizon*

F_{jk} *daily usage rate for the part k of family j at the beginning of planning horizon*

$PHSD$ *planning horizon start date*

W_D *The number of days on the planning horizon*

Step (4) – Calculating the actual lot size of each family by (24) and (25):

$$ALS_j = max_{k\epsilon 1,2,\dots,l}(TLS_{jk}) \quad ; \forall j = 1, \dots, m \quad (24)$$

$$TLS_{jk} = FD_{jk} - (I_{jk} - SS_{jk}) \quad (25)$$

$; \forall j = 1, \dots, m ; \forall k = 1, \dots, l_j$

where

TLS_{jk} *temporary lot size for the part k of family j*

Step (5) –Calculating the actual start of production for family *j* by (26):

$$ASOP_j = min_{k\epsilon 1,2,\dots,l}(FROD_{jk}) + 1 \quad (26)$$

where

$ASOP_j$ *actual start of production date*

Step (6) –Calculating the production batch per different frozen horizon by (27):

$$SD_{FH_Q} \le ASOP_j \le FD_{FH_Q} \quad ; if \ \exists Q\epsilon 1, \dots, n \quad (27)$$

$$PB_{FH_P,j} = ROUNDUP\left[\left(\frac{ALS_j}{W_D}\right)\right]$$

$; if \ FD_{FH_Q} = ASOP_j \ ; \forall j = 1, \dots, m; \ P = Q$

$$PB_{FH_P,j} = ROUNDUP\left[\left(\frac{ALS_j}{W_D}\right) * (FD_{FH_i} - ASOP_j + 1)\right]$$

$; if \ SD_{FH_Q} < ASOP_j < FD_{FH_Q} \ ; \forall j = 1, \dots, m; \ P = Q$

$$PB_{FH_P,j} = ROUNDUP\left[\left(\frac{ALS_j}{W_D}\right) * (FD_{FH_P} - SD_{FH_P} + 1)\right]$$

$; \forall j = 1, \dots, m; \forall P = Q + 1, \dots, n$

If the production batches for each family on different frozen horizons are considered equal, then we have (28):

$$SD_{FH_i} \le ASOP_j \le FD_{FH_i} \ ; \exists Q\epsilon 1, \dots, n$$

$$PB_{FH_p,j} = ROUNDUP\left[\left(\frac{ALS_j}{n - Q + 1}\right)\right]$$

$; \ \forall j = 1, \dots, m; \ \forall P = Q, Q + 1, \dots, n \quad (28)$

where

$PB_{FH_{p,j}}$ *the production batch of family j on frozen horizon p*

$PHFD$ *planning horizon finished date*

SD_{FH_Q} *start date for frozen horizon Q*

FD_{FH_Q} *finished date for frozen horizon Q*

4.2. The scheduling approach for different production batches with the developed Max- Min Ant system

Scheduling of different production batches was performed with a *Max-Min* ant system by considering the priority of assigning machines to each operation. The scheduling also had a hierarchical approach.

Notations of the ant colony optimization are as follows:

n *the number of production batch per frozen horizon*

i', i'' *index of production batch* i', $i' = 1, \dots, m$

j', j'' *index of operation*j', *j'=1,…,*j_i'

$O_{i'j'}$ *operation j of production batch' i'*

z *index of ant,* $z = 1, \dots, u$

4.2.1. Steps of the scheduling algorithm

For each ant in hierarchical approach, the steps are described as follows:

Step (1) – Obtaining the initial solution for assigning $O_{i'j'}$ to machine ***r*** by (29) an (30).

$$p_{i'j'r}^{z}(tn) = \frac{[\alpha_{i'j'r}.\tau_{i'j'r}(tn)]^{a}.[\eta_{i'j'r}]^{\beta}}{\sum_{i'=1}^{R_{i'j'}}[\alpha_{i'j'r}.\tau_{i'j'r}(tn)]^{a}.[\eta_{i'j'r}]^{\beta}} \quad (29)$$

where

$p'^{z}_{ij'r}(tn)$ *probability of allocation* $O_{i'j'}$ *to route r*

r *index of route r,* $r = 1, \dots, r_{i'j'}$

$K_{i'j'r}$ *machine number of* $O_{i'j'}$ *for route r*

$\alpha_{i'j'r}$ *priority ratio of assigning machine K to* $O_{i'j'}$ *in route r*

β *heuristic the parameters for controlling the relative importance of the pheromone trial and the heuristic information*

$\tau_{i'j'r}(tn)$ *pheromone trail of* $O_{i'j'}$ *on route r in iteration tn*

$R_{i'j'}$ *the number of alternative machine for* $O_{i'j'}$

$$\eta_{i'j'r} = \frac{1}{T_{i'j'r} + S_{i'j'r}} \quad (30)$$

where

$\eta_{i'j'r}$ *heuristic information from* $T_{i'j'r}$ *and* $S_{i'j'r}$

$T_{i'j'r}$ *process time of route r for* $O_{i'j'}$

$S_{i'j'r}$ *setup time of route r for* $O_{i'j'}$

Step (2) - Allocating $O_{i'j'}$ to the machine

In the current iteration, if the selected $O_{i'j'}$ based on the earliest finishing time is unique on a machine, the ant will assign $O_{i'j'}$ to the machine. Then the assigned operation to a special machine will save in $Q_k^{(z)}(s)$ at each step. If there is conflict between at least two operations in one step, so that one of the operations has the earliest finishing time on the same machine, a operation will be assigned to the machine based on the following problem rule by (31).

$$p''^{(z)}_{ki'j'i''j''}(tn) = \frac{[\xi_{ki'j'i''j''}(tn)]^{\gamma}.[\psi_{i'}(s)]^{\omega}}{\sum_{i'=1}^{R_{i'j'}}[\xi_{ki'j'i''j''}(tn)]^{\gamma}.[\psi_{i'}(s)]^{\omega}}$$

$$\forall\,(i',j') :\ O_{i'j'} \in G_k(s) \quad (31)$$

where

$p''^{(z)}_{ki'j'i''j''}(tn)$ *probability of allocation* $O_{i''j''}$ *on machine k after* $O_{i'j'}$

$\xi_{ki'j'i''j''}(tn)$ *pheromone trail of* $O_{i'j'}$ *relative to* $O_{i''j''}$ *on machine* k *in iteration* tn

$\psi_i(s)$ *sum of the processing time of all unassigned operation of production batch j' at step s*

ω, γ *heuristic parameters for controlling the pheromone trial*

$G_k(s)$ *collection of operations with conflict on machine i'*

If there is conflict between some operations on a machine that has an operation with the earliest finishing time, firstly operations will be sorted randomly, and then the pheromone trail for each operation will be considered relative to the previous operation. Finally, after calculating the probability of allocation through Eq. (31), the operation will be selected based on generating a random number between 0 to 1 (see Figure 2). For each ant with these steps, a feasible scheduling is obtained that is considered as a Make-span.

Step (3) – The datum time for each operation to perform on machine *k* is obtained in each step as follows by (32):

$$Datum\,Time(O_{i'j'k}) = (T_{i'j'k} + S_{i'j'k}) + Max(DO_{i',j'-1}, DM_{i',j',k}) \quad (32)$$

where

$DO_{i',j'-1}$ *datum time for* $O_{i',j'-1}$

$DM_{i',j',k}$ *datum time for the latest operation from a production batch that is assigned to machine k, before assigning* $O_{i',j'}$ *to this machine*

Step (4) - Sorting the answers:

The best solution in the current iteration (*ibest*) for all ants and the best solution from the beginning of the iteration (*gbest*) are sorted separately.

Step (5) – Terminating the check module:

A specified number of iterations with respect to the problem size is considered. When the number of iterations is reached to the specified number, the scheduling is terminated. Otherwise, the pheromone trials are updated and the scheduling procedure for production batches is repeated.

Step (6) – Updating the pheromone trials:

Step (6) /stage (1) – Updating the pheromone trail for operations:

For ibest at the end of each iteration, if $O_{i'j'}$ is assigned to route *r,* pheromone updating is performed by Eq. (33) and (35). Otherwise, it is performed by Eq. (34) and (35).

$$\tau_{i'j'r}(tn+1) = \tau_{i'j'r}(tn+1) + \Delta\tau_{i'j'r}(best)$$

$; \forall i' = 1, \ldots, m; \forall j' = 1, \ldots, j'_i; \forall r = 1, \ldots, r_{i'j'},$ (33)

$$\tau_{i'j'r}(tn+1) = \rho.\ \tau_{i'j'r}(tn) \quad ; \forall i' = 1, \ldots, m; \forall j' = 1, \ldots, j_{i'}; \forall r = 1, \ldots, r_{i'j'} \quad (34)$$

$$\Delta\tau_{i'j'r}(best) = \frac{1}{f(sbest)} \quad (35)$$

where

ρ	*evaporation factor between 0,1*
$f(sbest)$	*ibest* or *gbest*

The pheromone trial range for each operation is obtained through Equations (36), (37), (38), (39), (40), (41) and (42) as follows:

$$\tau_{i'j'r}(tn) = \tau_{max}(tn) \quad if \quad \tau_{i'j'r}(tn) \geq \tau_{max}(tn) \quad (36)$$

$$\tau_{i'j'r}(tn) = \tau_{min}(tn) \quad if \quad \tau_{i'j'r}(tn) < \tau_{min}(tn) \quad (37)$$

$$\tau_{i'j'r}(tn) = \tau_{i'j'r}(tn) \quad (38)$$

$$if\ \tau_{min}(tn) \leq \tau_{i'j'r}(tn) \leq \tau_{max}(tn) \quad (39)$$

$$\tau_{ij'r}(1) = \tau_{max}(1) \quad (40)$$

$$\tau_{max}(tn+1) = \frac{1}{(1-\rho).f(gbest)} \quad (41)$$

$$\tau_{min}(tn+1) = \frac{\tau_{max}(tn+1)}{y} \quad (42)$$

where

$\tau_{max}(tn)$	*maximum pheromone trial for each route in iteration* tn
$\tau_{min}(tn)$	*minimum pheromone trial for each Route in iteration* tn

Step (6) /stage (2) – Updating the relative pheromone trail between operations

In the *ibest* at the end of iterations, if $O_{i'j'}$, $O_{i''j''}$ are processed sequentially on machine *k,* the relative pheromone trail between them is updated by Eq. (43) and (45). Otherwise, the relative pheromone trail is obtained by Eq. (44).

$$\xi_{ki'j'i''j''}(tn+1) = \xi_{ki'j'i''j''}(tn+1) + \Delta\xi_{ki'j'i''j''}(best)$$

$$; \forall (i',j'),(i'',j''),k,s{:}\ O_{i'j'} \in Q_k^{(z)}(s),\ O_{i''j''} \in Q_k^{(z)}(s+1) \quad (43)$$

$$\xi_{ki'j'i''j''}(tn+1) = \rho.\ \xi_{kij'i'j''}(tn) \quad (44)$$

$; \forall i, i' = 1, \ldots, m; \forall j', j'' = 1, \ldots, j'_{i}; \forall r = 1, \ldots, r_{ij'}$

$$\Delta\xi_{ki'j'i''j''}(best) = \frac{1}{f(sbest)} \quad (45)$$

where

ρ	*evaporation factor between 0,1*

The relative pheromone trials are reviewed after being updated by Eq. (46), (47) and (48). The pheromone trial range for each operation is obtained through Equations (49), (50), and (51):

$$\xi_{ki'j'i''j''}(tn) = \xi_{min}(tn) \quad if\ \xi_{ki'j'i''j''}(tn) < \xi_{min}(tn) \quad (46)$$

$$\xi_{ki'j'i''j''}(tn) = \xi_{ki'j'i''j''}(tn)\ if\ \xi_{min}(tn) \leq \xi_{ki'j'i''j''}(tn) \leq \xi_{max}(tn) \quad (47)$$

$$\xi_{ki'j'i''j''}(tn) = \xi_{max}(tn) \quad if\ \xi_{ki'j'i''j''}(tn) \geq \xi_{max}(tn) \quad (48)$$

$$\xi_{ki'j'i''j''}(1) = \xi_{max}(1) \quad (49)$$

$$\xi_{max}(tn+1) = \frac{1}{(1-\rho).f(gbest)} \quad (50)$$

$$\xi_{min}(tn+1) = \frac{\xi_{max}(tn+1)}{y} \quad (51)$$

where

$\xi_{max}(tn)$	*maximum pheromone trial between two operations in iteration* tn
$\xi_{min}(tn)$	*minimum pheromone trial between two operations in iteration* tn
ρ	*evaporation factor between 0,1*

Step (7) – output module

This answer is the *gbest* for all iterations and represents the best Make-span for the problem.

4.3. Integrated production planning and scheduling framework

The planning horizon was divided into equal frozen horizons. The scheduling assigned machines in order of priority to each operation of parts families on each frozen horizon. If all of production batches were processed in a normal period of time, the problem would be solved; otherwise, changing the shift in order to assign the production batches on each frozen horizon would be done. After this step, if there was no enough time to assign the operation to the machines in this step, the problem would be continued with enlarging the frozen horizon and determining the size of production batches on each new frozen horizon again (see Figure 3).

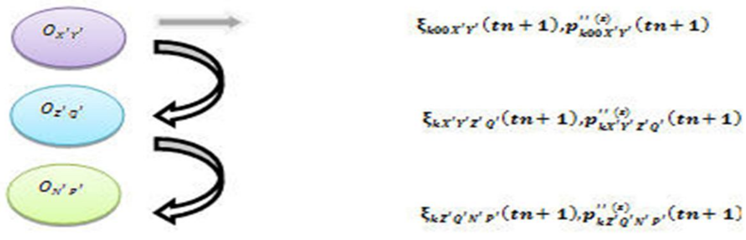

Fig. 2. The usage of relative pheromone trials on a machine

5. Case Study

To examine the proposed approach, a job center in *Safe Sanat* Company was investigated. The company, founded in 1992, is a supplier of the automotive industry in Iran and is expert in producing side door locks.

To produce thirteen different products, eighty-six parts in the form of sixty-four families were processed in the job center. Parts in each family had a similar process. Also, each family consisted of 1 to 5 different parts.

The production batch consisted of K lots related to a certain parts family. Each lot pertained to an identified part. All lots were stacked to reach an equal size. Each time all parts were processed by a certain machine, and then they were added to their lots in order to reach the specified quantity

The job center had fourteen machines. These machines consisted of hydraulic and kick press. The normal time for each machine in shifts (1) and (2) was seven hours and a quarter. Shift (3) was also considered as extra time for each machine with the same normal time.

The current condition for production planning and scheduling in this job center involved determining the lot size for each part on the planning horizons. Then parts families were produced only based on experiences and without respect to the optimal production batch, Make-span and time-limit for each kind of machine. In this situation, there was fluctuation in the production and the end inventory of the parts. Therefore, there were a lot of parts more or less than they were needed at the end of planning horizon.

In order to improve the planning at this work center, alternative routes for each operation, if it was possible, were considered. Corresponding to each family, there was a production batch ($PB_{FH_p,j}$) in every frozen horizon, in case it was produced. The production batch for each family was considered equal on any frozen horizon. Also, the number of operations for each family was between 1 to 5.

Safety stocks were calculated for each part based on the planning horizon, lead-time, and 85% significance level in a normal distribution.

6. Results of the Proposed Production Planning and Scheduling

6.1. The production lead-time and external safety lead-time

In order to calculate the safety stock, production lead-time and external safety lead-time of each part are calculated as follows by (52):

$$\bar{L}'_{ik} \cong \sum_{j=1}^{n} \frac{\overline{FD}_{ik} * \overline{UPT_{ijr}} + \overline{S_{ijr}}(n')}{(\sum_{s=1}^{2} NWTD_S) * (Day\ Per\ Month)} + \frac{n_i}{Day\ Per\ Month} \quad (52)$$

$$; \forall i = 1,2,\dots,n ; \forall k = 1,2,\dots,l_i ; \forall r = 1,2\dots,r_j ; s = 1,2$$

where

$\bar{L}'_{ik}$ *mean production lead-time for the part k of family i*

$\overline{UPT}_{ijr}$ *unit production time of route r for operation j of family i*

$\overline{S_{ijr}}$ *mean setup time for operation j of family i for a different route*

$NWTD$ *normal production time for shift s in a day (in seconds)*

$\overline{FD}_{ik}$ *average forecasted demand for the part k of family j during planning horizon*

n' *the number of initial frozen horizon*

n_i *the number of internal operation for family i*

The external safety lead-time for each part is also calculated by Eq. (53):

$$\bar{L}''_{ik} = \frac{n'_i}{\text{Day Per Month}} \quad (53)$$

Where

$\overline{L}''_{ik}$ mean external safety lead-time for the park k of family i(in month)

n'_i the number of external operation for family i

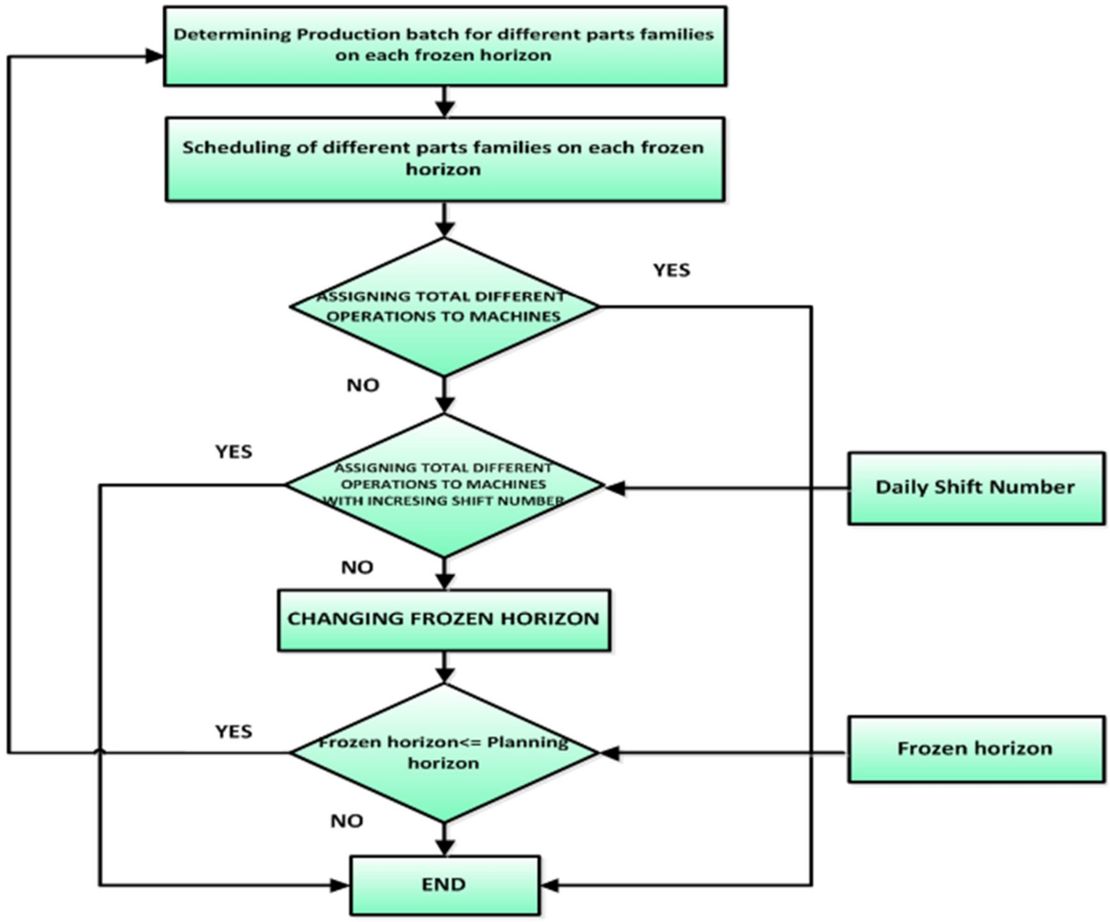

Fig. 3. The integrated approach for production planning and scheduling

6.2. *Results of the proposed framework for part families*

Table A-1 in the appendix presents the forecasted demand (*FDjk*), safety stock (*SSjk*) and Temporary lot size (TLS_{jk}) for each part. Table A-2 in the appendix shows the actual lot size (ALS_j), Actual start of Production ($ASOP_j$), and Production Batch ($PB_{FH_p,j}$) for each family.

6.3. *Results of the proposed approach for scheduling on optimal frozen horizons*

PHSD was January 1st 2012 and *PHFD* was January 30th 2012. Our *FH* consisted of six, ten, fifteen, and thirteen days. The non-working days were January 6th 2012, January 13th 2012, January 20th 2012, January 21st 2012, and January 27th 2012. Our case study had sixty seven families with the maximum of five operations at each family. The number of machines was fourteen.

This case study was a large size problem. Therefore, the time to reach the final answer was directly related to increasing the number of ants and their iterations. The number of ants was 50 (z=50) and the number of iterations was 60 (n=60). Parameters for scheduling were $\rho = 0.9, \omega = 2, \gamma = 1, \beta = 2, a = 1$. Moreover, $\tau_{max}(1) = \xi_{max}(1) = 0.1$. The time for solving was between two and a half hours and three hours. The $f(sbest)$ is also calculated as follows.

For each tn between the interval [1,100], the $f(sbest)$ is equivalent to the $f(ibest)$. For each tn between the interval [100, 200], after every four iterations, we used the $f(gbest)$ instead of the $f(sbest)$. For each tn between the interval [200, 300], after every three iterations, we replaced the $f(gbest)$ instead of the $f(sbest)$. For each tn between the interval [300,400], after every two iterations, we used the $f(gbest)$ instead of the $f(sbest)$. For each tn between the interval [400,500], after every iteration, the $f(sbest)$ is equivalent to the $f(gbest)$. Finally, for $(500 < tn)$, after every iteration, we replaced only the $f(ibest)$ instead of the $f(sbest)$.

6.4. Comparing the results of ant colony optimization with the mixed integer programming

Table 1 shows that results of the proposed ACO were near the optimal solution. Indexes used for this comparing are D_1 and D_2. The D_1 was equivalent to the deviation between the best result of the proposed meta-heuristic and the lower bound of the Lindo Software for the same problem. The D_2 was equivalent to the deviation between the results of the proposed meta-heuristics that is used by (54). The sample size of each problem was also (13).

Figure 4 depicts the D_2 for different problems. Comparing the CPU time for different methods was also done in the figure 5. Therefore, we conclude that the proposed algorithm has efficiency to minimize the Make-span in FJS.

$$D_2 = \frac{\sum_{i=1}^{n}(f_i - f^*)}{n.f^*} \tag{54}$$

where

f^* *the best results from the proposed algorithm in the specified sample*

Table 1

Comparing the results of ant colony optimization with the mixed integer linear programming

Row	J	M	Opr	Result of Lindo Software LB	UB	CPU time(s)	Result of Proposed Algorithm Best of Cmax CPU time $\overline{Cmax}$			D1	D2
1	2	3	7	55	55	10	55.31	55	40	0	0.0005
2	2	3	6	62	62	8	62.46	62	50	0	0.0007
3	2	2	5	88	88	20	88.77	88	35	0	0.0008
4	3	3	5	54	54	15	54.62	54	40	0	0.011
5	3	3	10	69	69	280	69.77	69	75	0	0.012
6	3	3	8	140	140	700	143.15	140	90	0	0.023
7	3	4	10	80	95	3600	90.85	85	2000	5	0.069
8	4	5	7	79	140	4000	123.85	117	2220	38	0.059
9	3	4	10	78	200	4500	141.46	138	1800	60	0.055
10	5	5	12	100	150	5010	136.23	131	2300	31	0.05
11	5	5	14	N/A	N/A	-	129	118	2350	-	0.093
12	6	5	17	N/A	N/A	-	195	185	2500	-	0.088
13	8	7	25	N/A	N/A	-	287	260	2987	-	0. 103
14	7	6	26	N/A	N/A	-	320	292	3432	-	0.095
15	10	7	30	N/A	N/A	-	660	630	4700	-	0.215

J: number of family- M: number of machines- Opr: number of operation

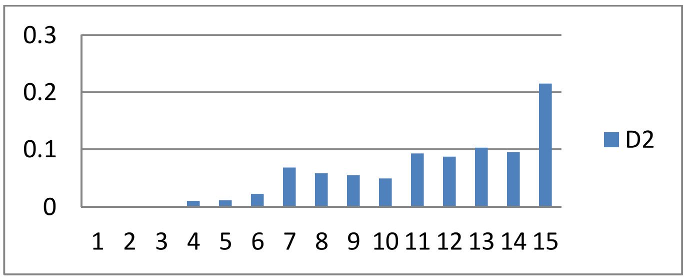

Fig. 4. Index D2 for different problems

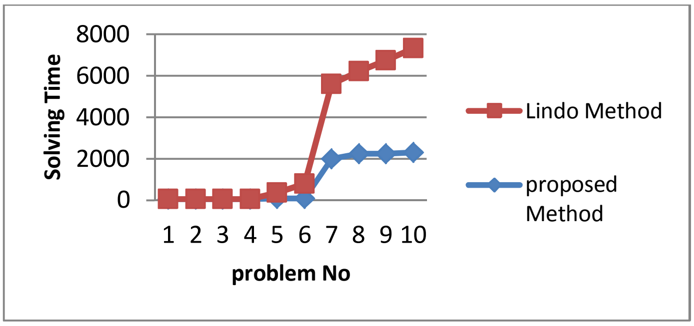

Fig. 5. Comparison of the CPU time of the proposed algorithm and the LINDO software

Besides, in order to survey the proposed algorithm, we used t statistic with the infinity upper bound. The mean for this hypothesis test is obtained by the Lindo Software. Table2 indicates the results of the hypothesis test for different problems. The sample size is 13 (N=13) and the confidence level is 95%.

6.5. Final results of production scheduling on the frozen horizons

The results of scheduling on the frozen horizons are presented in TableA-3 in the appendix. The make-span is also presented in the table 3. In addition, the optimal shifts are presented in Tables 4, 5, and 6.

6.6. The parts inventory of the proposed framework at the end of planning horizon and evaluating its effectiveness

The number of different parts in each family at the end of planning horizon by the proposed framework was calculated by Eq. (55). Also, Eq. (56) calculated the efficiency of the proposed framework at the end of planning horizon:

Table 2
A statistical comparison between the results of the proposed algorithm and LINDO results

Row	J	M	Opr	N	μ_0	p-value	Selected Hypothesis $H_0: \mu = \mu_0$ OR $H_1: \mu > \mu_0$
1	2	3	7	13	55	0.0519	$H_0: \mu = \mu_0$
2	2	3	6	13	62	0.055	$H_0: \mu = \mu_0$
3	2	2	5	13	88	0.0532	$H_0: \mu = \mu_0$
4	3	3	5	13	54	0.0519	$H_0: \mu = \mu_0$
5	3	3	10	13	69	0.0634	$H_0: \mu = \mu_0$
6	3	3	8	13	140	0.0016	$H_1: \mu > \mu_0$

Table 3
Make-span for the optimal frozen horizon

Optimized Frozen Horizon	10	10	10
Frozen Horizon Number	1	2	3
Make Span(Sec.)	515205.6	514466.9	572348.1

Table 4
Shifts for each machine in the frozen horizon (10-1)

Machine number	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12	M12	M14
Machine shifts	3	2	2	2	2	3	2	-	3	3	3	2	1	1

Table 5
Shifts for each machine in the frozen horizon (10-2)

Machine number	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12	M12	M14
Machine shifts	3	3	3	3	2	2	3	3	3	3	1	1	1	1

Table 6
Shifts for each machine in the frozen horizon (10-3)

Machine number	M1	M2	M3	M4	M5	M6	M7	M8	M9	M10	M11	M12	M12	M14
Machine shifts	3	3	3	3	2	2	3	3	3	3	1	1	1	1

$$I''_{jk} = ALS_j - TLS_{jk} \quad (55)$$

$$\Delta I = I''_{jk} - I'_{jk} \quad (56)$$

where

I'_{jk} *inventory for the part k of family j at the end of planning horizon that is calculated by the operator*

I''_{jk} *inventory for the part k of family j at the end of planning horizon that is calculated by the proposed framework*

Table A-4 in the appendix shows the ΔI for different parts families. This indicator indicates a more decrease in the parts inventory at the end of planning horizon than the traditional method.

We coded this problem in *Visual C#*. We also designed a database in SQL Server (2000). A PC with Core 2 Duo CPU, a 2.53GHZ processor and 4 *GIG Ram* was used for running the problem.

7. Future Research

A hierarchical framework for production planning and scheduling in FJS with respect to priority for production families was presented in this paper. This approach resulted in more decrease in the parts inventory at the end of planning horizon than the traditional method. This decrease led to the falling cost of material and human resources. Moroever, the utilization of the machines with this framework was increased. Finally, the results of the proposed scheduling algorithm were near the optimal solution.

With regard to future research, we recommend that researchers may investigate scheduling algorithms where production batches can be processed concurrently on identical machines. Also, investigations on presenting an MILP model, in a way that machines can be assigned to operations in order of priority, are more desirable.

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Appendix

Table A-1
The calculated parameters for each part

K	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
J	1	1	2	2	3	3	4	4	5	6	7	8	9	9	10	10	11	11	12	12	13	13
FDjk	20000	20000	20000	20000	20000	20000	20000	20000	40000	40000	40000	80000	40000	40000	40000	40000	40000	40000	40000	40000	20000	20000
SSjk	4210	5351	3698	3698	3794	4807	3239	3239	8537	6687	8242	14565	7328	8564	6776	7895	6773	7890	8471	6666	3930	4981
TLSjk	15325	6377	15928	17168	19652	20792	10619	18360	48537	40687	42503	82565	41328	41044	45091	16624	42695	29323	48471	24700	21750	23510

K	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45
J	15	15	16	16	17	17	18	19	19	20	20	21	21	22	22	22	22	22	23	24	24	25
FDjk	20000	20000	20000	20000	20000	20000	40000	20000	20000	20000	20000	20000	20000	20000	20000	40000	40000	40000	11640	11640	11640	3000
SSjk	4798	3788	3811	4813	3594	4539	8410	3809	4826	3195	3195	3175	3175	3212	3212	8176	6450	6450	5543	5240	527	5240
TLSjk	6727	10315	-46996	9646	8636	147	5281	23809	24826	3999	13612	6508	-8498	9087	10727	40176	46450	20407	17183	5449	1658	2870

K	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68
J	27	28	29	30	31	32	33	34	34	35	36	37	38	38	39	40	41	42	43	44	45	46
FDjk	8640	8640	8640	8640	8640	8640	12000	12000	12000	12000	12000	5000	5000	5000	5000	5000	5000	5000	5000	5000	40000	10000
SSjk	1543	2108	1991	2004	2057	2135	2770	2598	2598	2329	3727	3931	1985	1985	1936	2004	1970	1906	1932	1934	3351	1967
TLSjk	7894	9598	-1369	10644	-1303	-1225	9479	13239	13988	13029	12287	4480	2520	1411	6936	1916	1823	-7753	5937	-5066	43351	11967

K	70	71	72	73	74	75	78	79	80	81	82	83	84	85	86	88	89
J	48	49	50	51	52	53	56	57	58	59	60	61	62	63	64	66	67
FDjk	5000	5000	5000	5000	5000	30000	15000	1200	1200	1200	1200	1200	2400	1200	1200	1200	1200
SSjk	1749	1929	1880	1975	2100	2692	1852	305	291	295	311	303	329	286	296	285	295
TLSjk	-7117	6929	-248	6975	7100	17882	4855	-1315	1491	1000	-679	-1597	-7117	1138	-3287	-845	-308

Table A-2
Calculated parameters for each family

Row	J	ALSj	ASOPj	Optimized Frozen Horizon(Day)	Production Batch Per Frozen Horizon ($PB_{FH_p,j}$)		
1	1	15325	01/08/2012	10	5109	5109	5109
2	2	17168	01/05/2012	10	5723	5723	5723
3	3	20792	12/31/2011	10	6931	6931	6931
4	4	18360	01/03/2012	10	6120	6120	6120
5	5	48537	12/26/2011	10	16179	16179	16179
6	6	40687	01/01/2012	10	13563	13563	13563
7	7	42503	12/31/2011	10	14168	14168	14168
8	8	82565	01/01/2012	10	27522	27522	27522
9	9	41328	01/01/2012	10	13776	13776	13776
10	10	45091	12/29/2011	10	15031	15031	15031
11	11	42695	12/30/2011	10	14232	14232	14232
12	12	48471	12/26/2011	10	16157	16157	16157
13	13	23510	12/27/2011	10	7837	7837	7837
14	14	35094	01/04/2012	10	11698	11698	11698
15	15	10315	01/15/2012	10	-	5158	5158
16	16	9646	01/16/2012	10	-	4823	4823
17	17	8636	01/18/2012	10	-	4318	4318
18	18	5281	01/27/2012	10	-		5281
19	19	24826	12/25/2011	10	8276	8276	8276
20	20	13612	01/01/2012	10	4538	4538	4538
21	21	6508	01/21/2012	10			6508
22	22	46450	12/28/2011	10	15484	15484	15484
23	23	17183	12/18/2011	10	5728	5728	5728
24	24	5449	01/16/2012	10	-	2725	2725
25	25	1658	01/14/2012	10	-	829	829
26	26	1260	01/18/2012	10	-	630	630
27	27	7894	01/03/2012	10	2632	2632	2632
28	28	9598	12/29/2011	-	3200	3200	3200
29	29	-	-	-	-	-	-
30	30	10644	12/26/2011	10	3548	3548	3548
31	31	-	-	-	-	-	-
32	32	-	-	-	-	-	-

Table A-2 (CONTINUED)
Calculated parameters for each family

Row	J	ALSj	ASOPj	Optimized Frozen Horizon(Day)	Production Batch Per Frozen Horizon ($PB_{FH_{p},j}$)		
33	33	9479	01/07/2012	10	3160	3160	3160
34	34	13988	12/28/2011	10	4663	4663	4663
35	35	13029	12/30/2011	10	4343	4343	4343
36	36	12287	01/01/2012	10	4096	4096	4096
37	37	4480	01/04/2012	10	1494	1494	1494
38	38	2520	01/15/2012	10	-	1260	1260
39	39	6936	12/21/2011	10	2312	2312	2312
40	40	1916	01/19/2012	10	-	958	958
41	41	1823	01/20/2012	10	-	912	912
42	42	-	-	10	-	-	-
43	43	5937	12/27/2011	10	1979	1979	1979
44	44	-	-	10	-	-	-
45	45	43351	12/30/2011	10	14451	14451	14451
46	46	11967	12/27/2011	10	3989	3989	3989
47	47	574	01/27/2012	10	-	-	574
48	48	-	-	-	-	-	-
49	49	6929	12/21/2011	10	2310	2310	2310
50	50	-	-	10	-	-	-
51	51	6975	12/21/2011	10	2325	2325	2325
52	52	7100	12/20/2011	10	2367	2367	2367
53	53	17882	01/13/2012	10	-	8941	8941
54	56	4855	01/21/2012	10	-	-	4855
55	57	-	-	-	-	-	-
56	58	1491	12/25/2011	10	497	497	497
57	59	1000	01/06/2012	10	334	334	334
58	60	-	-	-	-	-	-
59	61	-	-	-	-	-	-
60	62	-	-	-	-	-	-
61	63	1186	01/01/2012	10	396	396	396
62	64	-	-	-	-	-	-
63	66	-	-	-	-	-	-
64	67	-	-	-	-	-	-

Table A-3(contain table 1-33)
The result of production scheduling for each machine on each frozen horizon

Frozen Horizon=10

N=1

Table 1

Steps	11	15	24	29	35	48	53	63
Datum time(Sec.)	33649.4	81330.6	111498.6	158921.8	266861.8	350117.8	466005.6	515205.6
sequence of operations for machine (1)	o21	o511	o11	o491	o111	o91	o101	o281

Table 2

Steps	1	4	10	17	20	28	45	49	54	56
Datum time(Sec.)	11299.9	16565.9	60605.9	69308.7	130305.0	261090.0	282770.0	360271.2	405844.3	427747.3
sequence of operations for machine (2)	o631	o591	o41	o592	o131	o521	o361	o451	o522	o523

Table 3

Steps	9	22	30	36	46
Datum time(Sec.)	80272.2	166852.5	192149.3	216875.8	303995.8
sequence of operations for machine (3)	o61	o141	o201	o301	o231

Table 4

Steps	6	27	33	43	47	51	55
Datum time(Sec.)	127881.6	186498.8	249634.8	268658.8	322596.7	358729.0	425866.6
sequence of operations for machine (4)	o51	o331	o221	o271	o13	o31	o32

Table 5

Steps	25	41
Datum time(Sec.)	149538.3	264853.0
sequence of operations for machine (5)	o23	o121

Table 6

Steps	2	5	14	16	34	42	50	59
Datum time(Sec.)	9267.6	49503.1	59978.5	68812.9	207622.9	299862.9	357692.9	479315.8
sequence of operations for machine (6)	o581	o341	o582	o371	o81	o332	o24	o374

Table7

Steps	3	18	26	32	38	52	57
Datum time(Sec.)	9286.0	89190.2	161230.2	210876.6	346703.6	398393.5	433355.6
sequence of operations for machine (7)	o351	o22	o71	o12	o342	o14	o372

Table 8

Steps	7	21	37	40	61
Datum time(Sec.)	19096.0	155319.7	235779.5	255145.9	498038.0
sequence of operations for machine (9)	o391	o192	o461	o134	o375

Table 9

Steps	12	19	23	44	60
Datum time(Sec.)	60621.6	87347.2	254251.7	277971.8	495365.8
sequence of operations for machine (10)	o191	o392	o142	o393	o92

Table 10

Steps	62	**Steps**	39	58
Datum time(Sec.)	512028.7	**Datum time**(Sec.)	247492.6	453108.6
sequence of operations for machine (11)	o452	**sequence of operations for machine (12)**	o133	o373

Table 11

Steps	13	31	**Steps**	8
Datum time(Sec.)	56414.0	192033.6	**Datum time**(Sec.)	23694.9
sequence of operations for machine (13)	o432	o132	**sequence of operations for machine (14)**	o431

Frozen Horizon=10

N=2

Table 12

Steps	13	22	31	40	47	64	78
Datum time(Sec.)	47681.2	95104.3	178360.3	227560.3	257728.3	365668.3	481556.1
sequence of operations for machine (1)	o511	o491	o91	o281	o11	o111	o101

Table 13

Steps	2	15	25	42	51	60	65	67	68	69	75	79
Datum time(Sec.)	33649.4	94645.7	225430.7	271003.7	348504.9	378067.5	383333.4	405236.5	416536.4	425239.2	469279.3	490959.3
sequence of operations for machine (2)	o21	o131	o521	o522	o451	o201	o591	o523	o631	o592	o41	o361

Table 14

Steps	7	27	43	46	55	62	70	81
Datum time(Sec.)	86580.3	214461.9	230861.2	311133.5	340444.7	407582.3	427346.9	514466.9
sequence of operations for machine (3)	o141	o51	o171	o61	o151	o32	o381	o231

Table 15

Steps	5	32	34	35	39	49	56	58	66	72
Datum time(Sec.)	115314.7	127603.6	146627.6	182759.8	207486.2	267834.3	330970.3	338213.9	383688.5	435378.5
sequence of operations for machine (4)	o121	o251	o271	o31	o301	o23	o221	o262	o152	o14

Table 16

Steps	9	23	26	44	63
Datum time(Sec.)	58617.2	80826.4	102186.5	240996.6	361312.6
sequence of operations for machine (5)	o331	o241	o161	o81	o13

Table 17

Steps	4	8	12	21	29	38	45
Datum time(Sec.)	8834.3	18102.0	28577.4	74035.3	166275.3	201237.4	247197.6
sequence of operations for machine (6)	o371	o581	o582	o531	o332	o372	o374

Table 18

Steps	1	6	18	37	52	57	59	61	71	74	80
Datum time(Sec.)	10821.8	51057.3	186884.3	281582.4	328193.9	331313.9	345172.6	417212.6	432479.5	443906.3	501736.3
sequence of operations for machine (7)	o401	o341	o342	o192	o162	o261	o402	o71	o403	o252	o24

Table 19

Steps	17	28	76
Datum time(Sec.)	51422.4	106881.4	447565.1
sequence of operations for machine (8)	o412	o133	o382

Table 20

Steps	3	11	14	30	48	54	77
Datum time(Sec.)	19096.0	28382.0	36960.0	114534.7	307374.7	326096.9	477854.4
sequence of operations for machine (9)	o391	o351	o411	o134	o12	o375	o452

Table 21

Steps	16	36	50	53	73	**Steps**	24
Datum time(Sec.)	156374.3	243773.4	270499.1	415747.1	439467.1	**Datum time**(Sec.)	89190.2
sequence of operations for machine (10)	o132	o142	o392	o92	o393	**sequence of operations for machine (11)**	o22

Table 22

Steps	20	33	41	**Steps**	13	19	**Steps**	10
Datum time(Sec.)	80459.7	141081.4	220990.4	**Datum time**(Sec.)	23094.9	55214.0	**Datum time**(Sec.)	23694.9
sequence of operations for machine (12)	o461	o191	o373	**sequence of operations for machine (13)**	o431	o432	**sequence of operations(14)**	o431

Frozen Horizon=10

N=3

Table 23

Steps	9	15	26	39	47	64	67	76
Datum time(Sec.)	21680.0	51848.0	135104.0	182785.2	290725.3	324374.7	440262.5	487685.6
(sequence of operations for machine (1)	o361	o11	o91	o511	o111	o21	o101	o491

Table 24

Steps	2	25	30	41	44	50	58	61	66	68	78	82	85
Datum time(Sec.)	77501.2	107063.7	168060.1	173326.0	236462.1	285764.6	297064.6	346264.6	365384.3	496169.3	504872.1	550445.1	572348.1
sequence of operations for machine (2)	o451	o201	o131	o591	o221	o531	o631	o281	o171	o521	o592	o522	o523

Table 25

Steps	8	14	24	31	35	42	46	53	65	73	80
Datum time(Sec.)	22209.2	59823.2	95955.4	128841.4	153567.9	173332.5	227270.4	254502.5	341082.8	440263.6	527383.7
sequence of operations for machine (3)	o241	o41	o31	o181	o301	o381	o13	o211	o141	o23	o231

Table 26

Steps	6	33	37	43	62	70	75	77	83	86
Datum time(Sec.)	115314.7	144625.8	156914.7	284796.3	365068.5	423685.8	475375.8	496735.9	554565.9	575125.3
sequence of operations for machine (4)	o121	o151	o251	o51	o61	o331	o14	o161	o24	o262

Table 27

Steps	34	45	52
Datum time(Sec.)	138810.0	205947.6	251422.2
sequence of operations for machine (5)	o81	o32	o152

Table 28

Steps	4	16	21	23	27	32	38	79
Datum time(Sec.)	43796.3	54618.2	63885.8	74361.2	93385.2	118745.7	163040.7	515925.8
sequence of operations for machine (6)	o372	o401	o581	o582	o271	o561	o562	o332

Table 29

Step	3	7	18	22	2	36	57	6
Datum time(Sec.)	8834.3	17412.4	57647.9	89756.5	139402.9	153261.7	289088.7	379915.5
sequence of operations for machine (7)	o371	o411	o341	o374	o12	o402	o342	o22

Table 30

Steps	56
Datum time(Sec.)	285247.7
sequence of operations for machine (8)	o133

Table 31

Steps	1	10	13	28	40	51	54	55	59	60	63	71	72	84
Datum time(Sec.)	19096.0	33558.4	94180.0	166220.0	246679.8	258106.6	276828.8	297047.0	302239.0	309892.3	404590.4	419857.3	565105.4	568225.4
sequence of operations for machine (9)	o391	o412	o191	o71	o461	o252	o375	o382	o471	o134	o192	o403	o92	o261

Table 32

Steps	5	12	19	81	**Steps**	48
Datum time(Sec.)	9286.0	36011.6	59731.6	543347.3	**Datum time**(Sec.)	229258.7
sequence of operations for machine (10)	o351	o392	o393	o162	**sequence of operations for machine (11)**	o452

Table 33

Steps	20	49	**Steps**	11	17	74
Datum time(Sec.)	63549.4	229788.7	**Datum time**(Sec.)	23694.9	56414.0	428481.9
sequence of operations for machine (12)	o373	o132	**sequence of operations for machine (13)**	o431	o432	o142

Table 4
ΔI For different parts families

K	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
J	1	1	2	2	3	3	4	4	5	6	7	8	9	9	10	10	11	11	12	12
I"jk	0	8948	1258	0	1140	0	7741	0	0	0	0	0	0	284	0	28467	0	13372	0	23771
ΔI	0	948	-4305	0	-860	0	-259	0	0	-7715	-7465	0	0	284	-24189	-31620	-20974	-17571	0	20771
K	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
J	13	13	14	15	15	16	16	17	17	18	19	19	20	20	21	21	22	22	22	22
I"jk	1760	0	0	3588	0	56642	0	0	8489	0	1017	0	9613	0	0	15006	37363	35723	6274	0
ΔI	-2850	0	0	-20870	-18699	52188	-12064	-30211	-39825	-14606	1017	0	-17731	-19054	-8530	-8698	33172	-24582	1734	0
K	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
J	22	23	24	24	25	26	27	28	29	30	31	32	33	34	34	35	36	37	38	38
I"jk	26043	0	0	3791	0	0	0	0	1369	0	1303	1225	0	749	0	0	0	0	0	1109
ΔI	13491	-1775	-7287	3791	-763	0	-4783	-4420	-3703	0	-5697	-275	0	749	0	0	-6078	-3513	-5293	-4327
K	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	78	79	80	81	82
J	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	56	57	58	59	60
I"jk	0	0	0	7753	0	5066	0	0	0	7117	0	248	0	0	0	0	1315	0	0	679
ΔI	-2273	-6271	-7455	-6906	0	5066	-580	0	-3812	351	0	148	0	0	-6983	-6997	-1505	0	0	479
K	83	84	85	86	88	89														
J	61	62	63	64	66	67														
I"jk	1597	7117	0	3287	845	308														
ΔI	-1503	-2729	0	-1496	-34	-1495														