A Continuous Review Inventory Control Model within the Batch Arrival Queuing Framework: A Parameter-Tuned Imperialist Competitive Algorithm

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Received 12 September, 2011; Revised 3 December, 2011; Accepted 22 March, 2012

Abstract

In this paper, a multi-product continuous review inventory control problem within the batch arrival queuing approach ($M^{Qr}/M/1$) is modeled to find the optimal quantities of the maximum inventory. The objective function is to minimize the total costs of ordering, holding and shortage under warehouse space and service level, and expected lost-sales shortage cost constraints from retailer and warehouse viewpoints. Since the proposed model is NP-hard, an efficient imperialist competitive algorithm (ICA) is developed to solve the model. Moreover, to justify the proposed ICA, a simulated annealing algorithm is utilized, and to determine the best values of algorithm parameters that may result in a better solution, a fine-tuning procedure is followed. Finally, the performance of the proposed ICA is assessed through some numerical examples.

Keywords: Continues review Inventory control; Queuing theory; Imperialist Competitive Algorithm; Simulated Annealing.

1. Introduction

In inventory control problems, determining the ordering times and the order quantities of products are the two strategic decisions to either minimize total costs or maximize total profits. The main policy of the inventory control is that when supply and demand are not in the same size and non-uniform,an inventory is established(Breuerb and Baum, 2005). In this regard, a number of studies were performed in the past decade (Chuang et al., 2004; Vijayan and Kumaran, 2008; Chang, 2009).

The objective of inventory management is to balance conflicting goals like keeping stock levels down in order to have cash available for other purposes and having high stock levels for the continuity of the production and for providing a high service level to customers (Arda and Hennet, 2006).A relevant concept is stochastic modelingwhich is the application of probability theory to the description and analysis of real world phenomena. One of the most important domains in stochastic modeling is the field of queuing theory. Many real systems can be reduced to components which can be formulated by the concept called queue. A queue in a more exact scientific sense consists of a system into which there comes a stream of users who demand some capacity of the system over a certain time interval before they leave the system again. Thus, a queuing system can be described by a stochasticspecification of the arrival stream and of the system demand for every user as well as a definition of the service mechanism (Arda and Hennet, 2006). In this paper, the inventory control problem is considered within the queuing frameworkin order to make the mathematical model more realistic. The connection between the queuing theory and inventory control systems and the use of them incombination areinvestigated by several researchers in recent years(Bylka, 2005; Kim, 2005; Hill, 2007).

Many researchers expanded the inventory models to make them more reliable and closer to reality.In this respect, ElHafsi (2009)investigated a pure assemble-to-order system subject to multiple demand classes where customer orders would arrive according to a compound Poisson process. He showed that the optimal production policy of each component is a state-dependent base-stock policy and the optimal inventory allocation policy is a multi-level state-dependent rationing policy. Xiaoming and Lian(2008) considered the cost-effective inventory control of work-in-process (WIP) and finished products in a two-stage distributed manufacturing system. They first used a network

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of inventory-queue model to evaluate the inventory cost and service level achievable for the given inventory control policy, and then found a very simple algorithm to find an optimal inventory control policy that minimizes the overall inventory holding cost and satisfies the given service level requirements.

Azad et al. (2008) presented a complex distribution network design problem in the supply chain system which included location and inventory decisions. Customers' demandswere generated randomly and each distribution center maintained a certain amount of safety stock to achieve a certain service level for the customers.Since the modelwasin a non-linear integer programming mode,the researchers proposed a hybrid heuristic tabu search with simulated annealing (SA) sharing the same tabu list developed for solving the problem.In another study, Taleizadeh et al. (2008) investigated a stochastic replenishment multi-product inventory model and proposed two models for two cases of uniform and exponential time distribution between two replenishments. They showed that the models were integer-nonlinear programming problems and developed a SA algorithm to solve them.

Alfa et al. (2008)presenteda discrete time GI[x]/G[y]/1 queuing system. To do so, some general results were obtained about the stability condition, stationary distributions of the queue lengths and waiting times. In addition, a GI/M/1 type Markov chain associated with the age process of the customers in service was also developed. Hill (2007) also investigated continuous-review lost-sales inventory models with no fixed order cost and a Poisson demand process. The objective of the study, which included a holding cost per unit per time unit and a lost sales cost per unit, was to minimize the long-run total cost and explore alternative approaches which might offer better solutions. Kiesmuller et al. (2006)studies a single node in a supply chain that faced stochastic demand. They investigated the waiting time in an (R,s,Q) inventory system under compound renewal demand. At the end, they provided an approximation for the distribution function of the customer waiting time and determined the minimal reorder level subject to the maximum average waiting time. Dong et al. (2005) developed a network of inventory-queue models for the performance modeling and analysis of an integrated logistic network.Thestudy extended the previous work done on the supply network model with base-stock control and service requirements. Instead of one-for-one base stock policy, batch-ordering policy and lot-sizing problems were considered in the study. Moreover, as in practice the assumption of incapacitated production often does notholdtrue, $GI^X/G/1$ queuing analysis was used to replace the $M^X/G/\infty$ queue based method. In addition, to include the lot-sizing issue in the analysis of stores, a fixed-batch target-level production authorization mechanism was employed to explicitly obtain performance measures of the logistic chain queuing model.

Maiti et al. (2005)proposed a deterministic inventory model of a damageable item with variable replenishment rate and unit production cost. In the study,the replenishment rate and unit production cost were dependent on demand while demand and damageability were stock- dependent; the dependency could be linear or non-linear. The optimum inventory level was evaluated by the profit maximization principle through an SA algorithm. Arslan et al. (2001) proved the optimal inventory policy structure for both continuous and discrete-time M/G/1 and G/M/1 models with an alternate source of goods and make-to-order productions. They also provided an expression from which inventory costs could be calculated for an M/M/1 model although no closed-form expression for the optimal policies was possible. Gallien et al. (2001) examined the component procurement problem in a single-item, make-to-stock assembly system. The suppliers were incapacitated and had independent but non-identically distributed stochastic delivery lead times. The assembly was instantaneous, the product demand followed a Poisson process, and the unsatisfied demand was backordered. The aim of the studywas to minimize the sum of steady state holding and backorder costs over a pre-specified class of replenishment policies. Combining the existing results ofthe queuing theory with the original results concerning distributions that are closed under maximization and translation,the researchersoffered a simple approximate solution for the problem when lead time variances were identical.

Since the proposed model is a non-linear integer mathematical programming and then is overly Np-hard, utilizing meta-heuristic algorithms to solve itis one of the best ways. In this respect, many meta-heuristic algorithms such as genetic algorithm, simulated annealing (Pasandideh et al., 2011), particle swarm optimization (Poli, 2007; Hajipour and Pasandideh, 2012), Tabu search (Zarrinpoor and Seifbarghy, 2011)are proposed. Nowadays, it is quite common to develop new meta-heuristic algorithms and apply them to various optimization problems. As an example, Taleizadeh et al. (2011) proposed a multiproduct inventory control problem in which the periods between the two replenishments of the products wereconsidered independent random variables, and increasing and decreasing functions were assumed to model the dynamic demands of each product. Furthermore, the quantities of the orders wereregardedas integer-type, space and budget were constraints, the service-level was a chance-constraint, and the partial back-ordering policy was taken into account for the shortages. Besides, the costs of the problem were holding, purchasing, and shortage. Having considered all these conditions, the researchers presented a harmony search algorithm (introduced by Geem, 2001) to solve the model.

Recently, a new meta-heuristic algorithm named imperialist competitive algorithm (ICA) was developed by Atashpaz-Gargari and Lucas (2007). The proposers of the algorithm drew inspiration from the socio-political evolution of human. The suitability of this algorithm is demonstrated in some problems such as flow shop scheduling (Behnamian and Zandieh 2011), Game theory (Rajabioun et al., 2008), integrated product mix-outsourcing problem (Nazari-Shirkouhi et al. 2010), K-means data clustering (Niknam et al., 2011), hub covering location problem (Mohammadi et al., 2011), and so on.

Using the ICA, the main contributions of this study are (1) presenting a new mathematical model in the area of continues review inventory control within the queuing framework under limited warehouse spaces, number of shortage, service level, and cost of expected shortageand (2) proposing a parameter-tuned ICA algorithm to solve the model.In this paper, we presentthe ICA to solve the proposed multi-product continues review inventory control model within the batch arrival queuing approach. Moreover, the validity of the proposed ICA is demonstrated via one of the common algorithms,namely the SA.

The rest of this paper is organized as follows: In section 2, the problem is defined and then the parameters, indices, and decision variables are introduced to formulate the corresponding mathematical model. Section 3 presents both proposed meta-heuristic algorithms including ICA and SA to solve the model. In section 4, the process of calibrating the algorithms by the Taguchi method is illustrated. Section 5 presents the analysis of the outputs of the algorithms by some numerical examples statistically and graphically. Finally, the conclusion and some suggestions for further research are provided in section 6.

2. The Proposed Mathematical Model

2.1. Problem definition

In this section, first,the continuous review inventory control problem is defined and then our proposed mathematical model is illustrated in details. The goal is to determine the optimal quantities of the maximum inventory with minimizing the total cost of the inventory system. In order to clarify the problem, we schematically showthe elements of the system including warehouse, retailer, external supplier, and customer with batch arrivals in Figure 1.

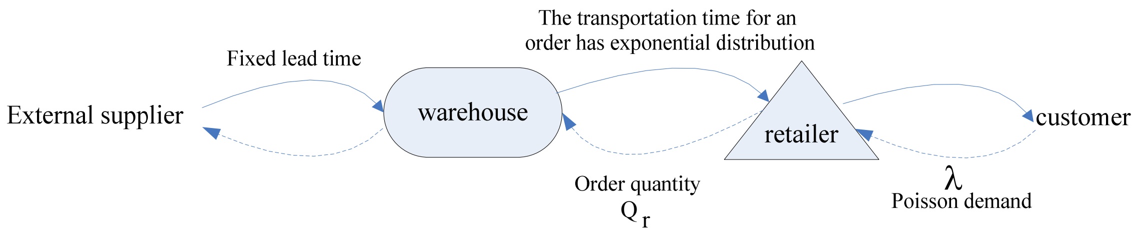

Fig. 1. A system in $M^{Qr}/M/1$

In our system, customers give their orders to the retailer and then the corresponding retailer orders the customers' demands to the warehouse in a stochastic time interval. Thus, we consider the retailers' orders as stochastic variables. Since the customers' demands are stochastic, there is a harden inventory control and accordingly the safety stock are in the warehouse. It should be pointed out that the warehouse demand is considered within T intervals and Qr quantity.

To make the model more realistic, in this research we discussthe continuous review inventory control problem within the queuing framework. As products arrive to the retailer as batch arrivals, $M^{Qr}/M/1$ queuing system is used in the study. In this queuing, the time between products arrivals and service time have exponential distributions, and one retailer is a server. When the service time ofa previous customer is not finished, the service time is longer than the arrival time of the next demand.Therefore, we encounter the formation of a queue. In order to formulate the proposed mathematical model, we make the following assumptions:

- The retailer faces Poisson demand.
- The warehouse faces a stochastic demand.
- Unsatisfied demands by the retailer are as lost sales.
- Shortage is not allowed in the warehouse.
- Shortage is allowed at the retailer.
- There is no lot-splitting in the warehouse.
- The transportation time for an order to arrive at the retailer from the warehouse is an exponential distribution.
- The warehouse orders to an external supplier with infinite capacity.
- The retailer's service times for customer *j* are independent and exponential random variables.
- The lead time for an order to arrive at the warehouse is constant.

In the following subsection, the mathematical model is illustrated in details.

2.2. The mathematical model formulation

To formulate the proposed model, firstlyits notations, parameters, and decision variables are defined, and then the non-linear mixed integer programming model is presented.

j	The index of products; j=1,...,n
n	The number of products
h_{wj}	The holding cost rate in the warehouse for product j
A_{wj}	The fixed cost of ordering related to the warehouse for product j
T_{wj}	The time interval between two consecutive orders of the warehouse for product j

Q_{wj} The order quantity of the warehouse for product *j*
h_{rj} The holding cost rate at the retailer for product j
A_{rj} The fixed cost of ordering related to the retailer for product *j*
T_{rj} The time interval between two consecutive orders of the retailer for product *j*
φ_j The arrival rate of the customer for product *j*
μ_j The service rate of the server for product *j*
ρ_j The productivity coefficient of product *j*
$\bar{I}_j$ The average inventory level at the retailer between (0,*T*) during the lost sales period for product *j* which is equivalent to the queue length for product *j*
π_j The fixed shortage cost of product *j*
L_j The length of lead time of product *j* is assumed to be constant
F The available warehouse space for the retailer in all products
f_j The space occupied by each unit of product *j*
G The number of allowed shortage
P_j The service level for product *j*
S The expected allowable shortage cost in lost sales state
Γ_j The maximum inventory in the warehouse for product *j*
SS_j The safety stock for product *j*
Q_{rj} The stockpile amount random variable in the batch arrival queuing system for product *j* which is equivalent to the order quantity of the retailer for product *j*
m_j The coefficient of the retailer's order quantity into the warehouse
$E[Q_{rj}]$ The average stockpile amount of product *j*
y_{1j} The random demand in period *T* for product *j* which acts as Poisson distribution $y_{1j} \sim pp(\lambda_{1j})$

y_{2j} The random demand in period *L* for product *j* which acts as Poisson distribution $y_{2j} \sim pp(\lambda_{2j})$

$y_j=(y_1+y_2)_j$ The random demand in period *L+T* for product *j* which acts as Poisson distribution with parameter $\lambda_j=\lambda_{1j}+\lambda_{2j}$

R_j The maximum inventory position after order for product *j*
$P(y_j)$ The demand probability density function

$\bar{b}(R_j)$ The average shortage for product *j*
ECH_r The expected holding cost per time unit at the retailer in the steady state
ECL_r The expected shortage cost per time unit at the retailer in the steady state in lost sales state
ETC_r The expected total cost per time unit at the retailer in the steady state
ETC_w The expected total cost per time unit at the warehouse in the steady state
ETC_B The expected total system (retailer and warehouse) cost per time unit in the steady state
$k_1(R,T)$ The expected total cost per time unit at the retailer in the steady state in the (*R,T*) system

It should be mentioned that the demand in the (*R,T*) system in the period *L+T* is as

$$D_{rL+T} \sim pp(\lambda_1+\lambda_2) \quad (1)$$

$$L_r+T_r \sim \text{Erlang}(2,\lambda) \quad (2)$$

In order to formulate the mathematical model, firstthe main parameters of the proposed model should be determined. The expected total cost per time unit at the retailer which includes the ordering, holding and shortage costs in the (R,T) system is as follows(Vijayan and Kumaran, 2008):

$$k_1(R,T)=\frac{1}{Tr}A + h\bar{I} + \frac{\pi}{Tr}\bar{b}(R) \quad (3)$$

Since *T* has an exponential distribution under the assessment system, we use $\frac{1}{E[Tr]}$ instead of $\frac{1}{Tr}$ to determine $\bar{b}(R_j)$ in Eq. (4).

$$\bar{b}(R_j) = \sum_{yj=Rj}^{\infty} P(yj).(yj - Rj) \quad (4)$$

Where $P(y_j)$ are as

$$P(y_j)=P(N(L+T)=n)=\lambda e^{-\lambda n}(n+\frac{1}{\lambda}) \quad (5)$$

Now, we can calculate $\bar{b}(R_j)$ as follows:

$$\bar{b}(R_j) = \sum_{yj=Rj}^{\infty} \lambda j e^{-\lambda j yj}(yj + \frac{1}{\lambda j}).(yj - Rj) \quad (6)$$

With regard to this subject and the periodic order system as the inventory order policy, the average number of customers in the system for a long time can be considered to calculate the average inventory in the periodic order inventory system. Therefore, the average number of customers in the system for a long time in the$M^{Qr}/M/1$system is (Pirayesh and Haji, 2007):

$$\bar{I}_j=\frac{\rho j}{1-\rho j}+\frac{\rho j(\frac{E[Q_{rj}^2]-1}{E[Qrj]})}{2(1-\rho j)}+\bar{b}(R_j) \quad (7)$$

where$\rho_j = \frac{\varphi j E[Qrj]}{\mu j}$.It should be pointed out that in steady state,$\varphi j E[Qrj] < \mu j$should be observed. Since the batch size is a random variable withthe Poisson distribution per *T* time,$E[Q_{rj}]$ and $E[Q_{rj}^2]$are respectively obtained from Eq. (8) and Eq. (9) as follows:

$$E[Q_{rj}]= \sum_{i=0}^{Rj} \sum_{Qrj=Rj-1}^{\infty} (Rj-1) \times \frac{e^{-\lambda 1 j} \lambda 1 j^{Qrj}}{Qrj!} \quad (8)$$

$$E[Q_{rj}^2]= \sum_{i=0}^{Rj} \sum_{Qrj=Rj-1}^{\infty} (Rj-1)^2 \times \frac{e^{-\lambda 1 j} \lambda 1 j^{Qrj}}{Qrj!} \quad (9)$$

In this paper, we proposethe continues review inventory control with considering the queue framework at the retailer. To do so, the $M^{Qr}/M/1$queuing systemis considered to determine the optimum R_j. The objective function is minimizing the expected total cost of the inventory system including holding, ordering,and shortage costs at the retailer as well as the holding and ordering costs in the warehouse.

In order to solve the model, the order arrival at the warehouse and the order delivery into the retailer should be same.Thus,Q_{wj} and T_{wj} arecalculated as $(m_j\text{-}1)Q_{rj}$ and $m_jE[T_{rj}]$, respectively (Pirayesh and Haji, 2007). Since no shortage is allowed in the warehouse, the safety stock is determined as follows (Sherbrooke, 2004):

$$SS_j=\Gamma_j+\mu_j L+T \quad (10)$$

Thenaccording toEq. (5) andEq. (10), the corresponding safety stock is determined by Eq. (11). Following that, the expected total cost per time unit in the warehouse in the steady state is the sum of ordering and holding costs which is formulated as Eq. (12).

$$SS_j=\frac{-ln\frac{\lambda j}{2}}{\lambda j}-\lambda j \quad (11)$$

$$TC_{wj}=\frac{A_{wj}}{mjTrj}+h_{wj}\left(\frac{-ln\frac{\lambda j}{2}}{\lambda j}-\lambda j+\frac{(m_j-1)Q_{rj}}{2}\right) \quad (12)$$

Finally, the proposed non-linear mixed integer programming model is formulated to determine the inventory position up to *R*. Then the concepts of objective function and the constraints are explained.

$$TC_B=\sum_{j=1}^{n}\left(\frac{1}{E[Tj]}A_j+h_j\bar{I}_j+\frac{\pi_j}{E[Tj]}\bar{b}(R_j)\right)$$

$$+\sum_{j=1}^{n}\frac{A_{wj}}{mjE[Tj]}+h_{wj}\left(\frac{-ln\frac{\lambda j}{2}}{\lambda j}-\lambda j+\frac{(m_j-1)Q_{rj}}{2}\right) \quad (13)$$

Subject to:

$$\sum_{j=1}^{n}\bar{b}(R_j)\leq G \quad (14)$$

$$\sum_{y_j=0}^{R_j}\lambda_j e^{-\lambda_j y_j}\left(y_j+\frac{1}{\lambda j}\right)\geq P_j \quad (15)$$

$$\sum_{j=1}^{n}\pi_j\times\sum_{y_j=R_j}^{\infty}\lambda_j e^{-\lambda_j y_j}\left(y_j+\frac{1}{\lambda j}\right)(y_j-R_j))\leq S \quad (16)$$

$$\lambda_j Q_{jr}<T_{jr} \quad (17)$$

$$R_j\geq 0$$

Qrj and mj are as positive integer (18)

The objective (13) above minimizes the inventory system cost which is divided into two parts: retailers and warehouse. From the retailer viewpoint, the total cost of ordering, holding and shortage is minimized while the ordering and holding costs are considered to be minimized in the warehouse. The constraint (14) ensures the maximum number for the shortage. Moreover, the constraint (15) is a kind of service level which indicates the ability to meet the customers' demand by the available inventory. In fact, it shows a key factor in computing reliability in the supply chain. The constraint (16) ensures the maximum cost for encountering with the shortage, and the constraint (17) shows the stability of the considered queuing system. Finally, the constraint (18) considers the range of decision variables.

3. Proposed Meta-Heuristic Algorithms

Nowadays, the use of meta-heuristic algorithms as a common and efficient way to solve mathematical programming models is justified. In this regard, a parameter-tuned imperialist competitive algorithm (ICA) is proposed in the present paper. Following this, to demonstrate the performance of the proposed algorithm, an efficient random search algorithm called simulated annealing (SA) is employed which is described in details in the following subsections.

3.1. The imperialist competitive algorithm

As a strong optimization strategy, the ICA is inspired by the socio-political evolution of human being. Like other population-based meta-heuristic algorithms, the initialization phase is the first step to generate population as countries in the world. Among these, the best countries in the population are selected to be imperialists and the rest form the colonies of these imperialist. Then all colonies should be evaluated based on their power (the fitness value) and then divided among the imperialists. After dividing all colonies, the colonies start moving towards their relevant imperialist country.It should be noted that the total power of an empire depends on both the power of the imperialist country and the power of its colonies. Then the imperialistic competition begins among all the empires. The empires which do not win the competition will be out of the competition. Gradually, the imperialistic competition results in an increase in the power of powerful empires and a decrease in the power of weaker ones. Finally, all the countries will be converted to a state including just one empire in the world and all the other countries are the colonies of that empire (Atashpaz-Gargari and Lucas, 2007). Having described the idea behind the ICA, now in order to clarify the trend of our proposed ICA, we schematically plot the flowchart of the algorithms in Figure2.

In the following subsections, the steps of the proposed ICA are described in details.

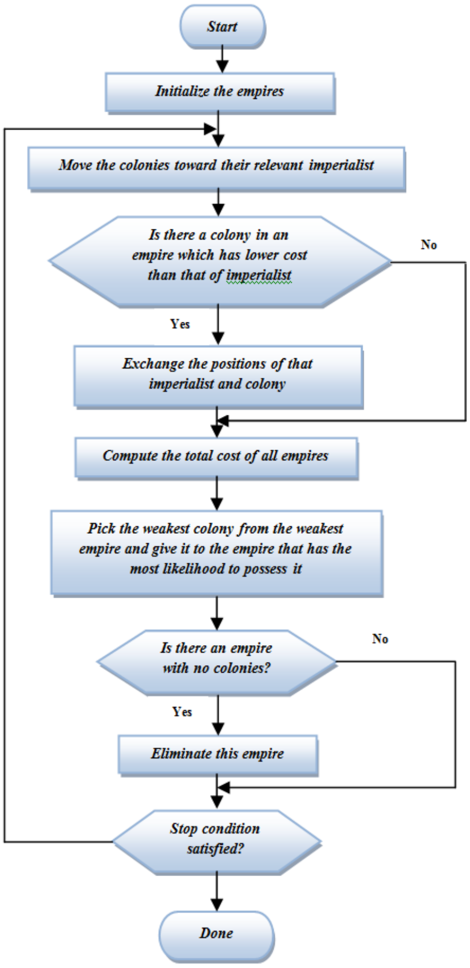

Fig. 2. Flow chart of the proposed ICA

3.1.1. Generating initial countries

In this subsection, an array of decision variable values is formed to determine the optimal values in the search area. In the ICA, the number of countries ($N_{country}$), imperialists (N_{imp}), and colonies (N_{col}) should be determined. The relationship between these algorithm parameters is $N_{country} = N_{imp} + N_{col}$.

In the ICA optimization, the aforementioned array is called '*country*'. A country is an $1 \times N$ array which is defined as:

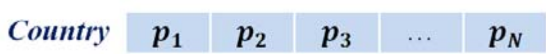

Fig. 3. A country structure

Where p_N is the normalized power of imperialist N which is defined as Eq. (19). In fact, the normalized power of an imperialist is determined by the proportion of colonies it possesses. Besides, to form initial empires, all colonies should be assigned to the imperialists based on their power. The normalized cost of an imperialist is, therefore, computed as Eq. (20).

$$p_t = \left|\frac{C_t}{\sum_{t=1}^{N_{imp}} C_t}\right| ; t = 1,2, \dots , N_{imp} \quad (19)$$

$$C_t = a_t - \max_i\{a_i\} \quad (20)$$

where a_t is the cost of the t^{th} imperialist and C_t is its normalized cost. Obviously, the colonies will be randomly chosen the size of $N_{col} \times p_t$, and then will be assigned to the imperialists. To clarify the process of initialization phase, we show it schematically in Figure4.

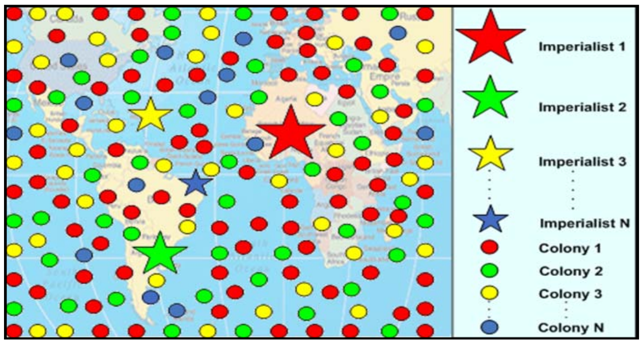

Fig. 4. A scheme of initialized empires and colonies (Atashpaz-Gargariand Lucas, 2007)

3.1.2. Movement of the colonies

After initializing the countries and selecting the empires, the imperialist countries attempt to enhance the number of their colonies by moving all the colonies toward the imperialists. This process is carried out by generating a random variable (x) which acts as the uniform distribution $X \sim Uniform(0, \beta \times d); \beta > 1$ (Atashpaz-Gargari and Lucas, 2007) where the parameter d indicates the distance between a colony and an imperialist. To explore variations around an imperialist, the concept of the deviation of a path (θ)which acts as the uniform distribution $\theta \sim U(-\gamma, \gamma)$ (Atashpaz-Gargariand Lucas, 2007) is necessary. In this concept, the parameter γ is defined as the deviation from the original direction.

It is worthy to mention that while moving toward an imperialist, a colony may reach a position with lower costs in comparison with the imperialists. In such cases, the imperialist moves to the position of that colony and vice versa. In the ICA, this process is called 'exchanging positions of the imperialist and a colony'.

3.1.3. Empires power evaluation

In this step, the total power of each empire is calculated as the sum of the imperialist cost and the average of colonies cost as follows:

$$TP_t = Cost(\text{imperialist}_t) + [rand() \times mean\{Cost(colonies\ of\ \text{empire}_t)\}] \quad (21)$$

Where TP_t is the total cost of the t^{th} empire.

3.1.4. Imperialists competition

In this phase, all empires attempt to take possession of more colonies and control them. This imperialistic competition gradually brings about a reduction in the power of weaker empires and an increase in the power of more powerful ones by choosing a number (usually one) of the weakest colonies of the weakest empires and allowing the empires to compete for having the chosen colonies (Atashpaz-Gargari and Lucas, 2007). In order to start the competition, we first calculate the probability of each empire's success to determine colonies instead of its total power as follows:

$$NTP_t = TP_t - \max_i\{TP_i\} \quad (22)$$

where TP_t and NTP_t are the total cost and the normalized total cost of the t^{th} empire, respectively. Then the possession probability of each empire is computed as Eq. (23). According to this equation, we form the vector P ($P=[P_{p1}, P_{p2}, \dots, P_{pN_{imp}}]$) to divide the colonies among the empires.

$$Pp_t = \left| \frac{NTP_t}{\sum_{i=1}^{N_{imp}} NTP_i} \right| \quad (23)$$

Following this, the vector (R) is generated the size of P based on the following policy.

$$R=[r_1, r_2, \dots, r_{N_{imp}}]; r_1, r_2, \dots, r_{N_{imp}} \sim Uniform(0,1) \quad (24)$$

Next, the vector D will be calculated simply by subtracting R from P in Eq. (25).

$$D=P-R$$
$$=[D_1, D_2, \dots, D_{N_{imp}}]$$
$$=[P_{p1}-r_1, P_{p2}-r_2, \dots, P_{pN_{imp}}-r_{N_{imp}}] \quad (25)$$

Then the colonies are assigned to the empires based on the vector D. The empires with higher Ds are more powerful. Finally, the powerless empires will be out of the imperialistic competition and their colonies will be divided among the remaining empires.

At the end, when all the empires except the most powerful ones collapse and all the colonies are under the control of certain empires, the algorithm will be stopped.

3.2. The proposed parameter-tunic SA

Simulated annealing (SA) is a well-known local search meta-heuristic introduced by Kirkpatrick et al. (1983). This algorithm is based on the process of physical annealing in which a crystalline solid is heated and then allowed to cool very slowly (L) until it has its most regular crystal lattice configuration possible. The SA establishes the connection between this type of thermodynamic behavior and the search for global minima for discrete optimization problems. In the main loop of SA, a single solution (s) is generated and after evaluating it, the neighborhood structure is executed to determine a new solution. Then according to the objective function value of both obtained solutions, the better solution is selected although with regard to the SA's probability function, the worst solution may also be chosen (Glover and Kochenberger, 2003).The pseudo-code of the proposed SA is depicted in Figure 5.

```
Initializethe SA control parameters (T0, L)
Generate an initial solution, S0
Set T=T0, S=S0, and S*=S0
Evaluate f(S0)
   While the stop criterion is not reached do:
         Set n=1
         While n<L do:
                  Generate solution Sn as the
         neighborhood solution of S0
         Calculate V=f(Sn)-f(S);
                  If V≤ 0
                     S=Sn
                  Else
         Generate a random number,
                  r∈ (0,1)
                  If (r≤ p = e^(-V/T))
                        S=Sn
                     End
                  End
                  If f(S)<f(S*)
                     S*=Sn
                  End
         End
Reduce the temperature T
   End
```

Fig. 5.The pseudo-code of the proposed SA

It is worth mentioning that the solution representation and evaluation in the SA are similar to those of the proposed ICA. Moreover, the neighborhood structure is carried out by the swap strategy (Haupt and Haupt, 2004).

4. Meta-heuristics Calibration

In this section, we focus on tuning the input parameters of both proposed algorithms. Since all meta-heuristic algorithms heavily depend on their parameters, a Taguchi method is used to enhance the performance of both ICA and SA. The Taguchi categorizes the objective functions into three groups: (I) smaller-the-better type, (II) larger-the-better type, and (III) nominal-is-the-best type. As almost all the objective functions in the inventory control problem are classified as the smaller-the-better type, the corresponding S/N ratio (SNR) is as follows(Peace, 1993):

$$\text{SNR}= -10 \log(objective\ function)^2 \qquad (26)$$

In order to apply the Taguchi method, firstly the levels of all the parameters should be determined (Tables 1 and 2).It should be noted that according to the sensitivity of the factor to the problem size, we determine the best value of them separately.

Table 1
Parameters levels of the proposed ICA for different problems

Factor	Symbol	Problem	Level (1)	Level (2)	Level (3)
Beta	A	All Problems	1	1.25	1.5
Sigma	B	All Problems	0.05	0.3	0.6
Lambda	C	All Problems	0.2	0.3	0.6
MaxScapeAngle	D	All Problems	8	12	16
(Npop, Nimp)	E	Problem 1	(20,2)	(30,2)	(70,3)
		Problem 2	(30,3)	(40,3)	(80,4)
		Problem 3	(40,4)	(50,4)	(90,5)
		Problem 4	(50,5)	(60,5)	(100,5)
$MaxIT_{ICA}$	F	Problem 1	20	40	60
		Problem 2	30	50	70
		Problem 3	40	60	80
		Problem 4	50	70	90

Table 2
Parameters levels of the proposed SA for different problems

Factor	Symbol	Problem No.	Level (1)	Level (2)	Level (3)	Level (4)
T_0	A	All Problems	800	1200	1600	2000
$MaxIT_{SA}$	B	Problem 1	50	90	130	170
		Problem 2	60	100	140	180
		Problem 3	70	110	150	190
		Problem 4	80	120	160	200

The Taguchi designs for the SA and ICA are $L_{16}(4^2)$ and $L_{27}(3^6)$, respectively. Tables 3 and 4 report the SNRs and the mean ratiosofthe four experimental problems based on the ICA and SA.

With regard to the outputs of the MINITAB software, the optimal values of the ICA and SA parameters can be determined according to the maximum SNR and minimum MEAN rules (Figures6 and 7). To do so, we provide all the optimal values in Table 4. It should be mentioned that here this process is reported for problem number 1.

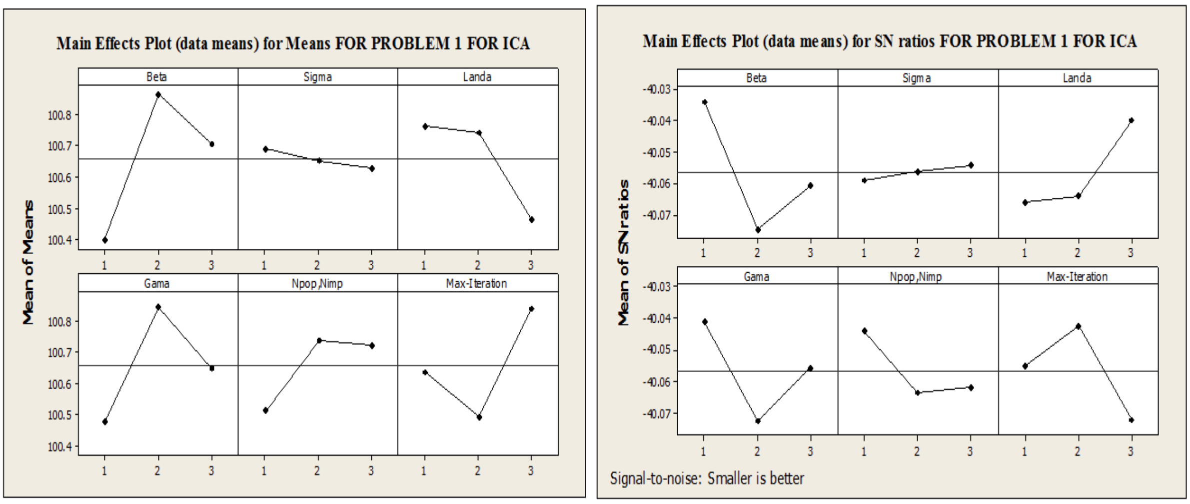

Fig. 6.The SNR and Mean ratio for the ICA in problem No. 1

Table 3
The SNR and mean ratio of all the experimental problems based on the SA (Orthogonal array)

Run No.	Factors		Problem 1		Problem 2		Problem 3		Problem 4	
	A	B	SNR	MEAN	SNR	MEAN	SNR	MEAN	SNR	MEAN
1	1	1	-40.0435	100.502	-50.3534	329.361	-53.7236	485.488	-56.1992	645.593
2	1	2	-40.0502	100.58	-50.3824	330.462	-53.6442	481.07	-56.0835	637.052
3	1	3	-40.045	100.52	-50.3824	330.462	-54.559	534.501	-56.272	651.029
4	1	4	-40.0499	100.576	-50.5155	335.562	-53.6167	479.55	-57.2445	728.16
5	2	1	-40.0443	100.512	-50.52	335.738	-54.1503	509.934	-56.741	687.144
6	2	2	-40.0502	100.579	-50.4274	332.178	-53.6216	479.82	-54.2179	513.921
7	2	3	-40.0435	100.502	-50.5726	337.778	-54.1578	510.377	-55.3658	586.53
8	2	4	-40.051	100.589	-50.3462	329.087	-53.7434	486.597	-57.2372	727.548
9	3	1	-40.0501	100.578	-50.3824	330.462	-55.2822	580.912	-56.245	649.01
10	3	2	-40.0499	100.576	-50.5155	335.562	-55.0859	567.929	-56.1407	641.261
11	3	3	-40.0582	100.672	-50.5695	337.657	-53.8134	490.536	-57.1154	717.413
12	3	4	-40.05	100.577	-50.4795	334.174	-55.3952	588.521	-57.1293	718.561
13	4	1	-40.0453	100.523	-50.5733	337.803	-53.8429	492.202	-55.2164	576.53
14	4	2	-40.0519	100.599	-50.5478	336.815	-55.0859	567.929	-56.9951	707.548
15	4	3	-40.0458	100.529	-50.52	335.738	-53.8234	491.098	-56.8897	699.01
16	4	4	-40.0481	100.556	-50.5729	337.788	-55.3197	583.427	-56.4075	661.261

Table 4
The SNR and mean ratio of all the experimental problems based on the ICA (Orthogonal array)

Run No.	Factors						Problem1		Problem2		Problem3		Problem4	
	A	B	C	D	E	F	SNR	MEAN	SNR	MEAN	SNR	MEAN	SNR	MEAN
1	1	1	1	1	1	1	-40.0087	100.101	-50.4499	333.04	-53.6188	479.669	-54.2838	517.833
2	1	1	1	1	2	2	-40.0686	100.793	-50.4553	333.248	-53.6238	479.944	-54.2768	517.417
3	1	1	1	1	3	3	-40.0152	100.175	-50.464	333.58	-53.6142	479.414	-54.2103	513.472
4	1	2	2	2	1	1	-40.0647	100.747	-50.4628	333.534	-53.662	482.06	-54.2719	517.123
5	1	2	2	2	2	2	-40.0319	100.368	-50.4868	334.457	-53.6246	479.986	-54.2215	514.133
6	1	2	2	2	3	3	-40.0765	100.885	-50.4882	334.511	-53.4091	468.224	-54.2305	514.663
7	1	3	3	3	1	1	-39.9963	99.957	-50.4888	334.536	-53.2164	457.951	-54.2391	515.173
8	1	3	3	3	2	2	-40.0188	100.216	-50.4525	333.137	-53.4907	472.645	-54.2143	513.708
9	1	3	3	3	3	3	-40.0288	100.332	-50.4595	333.407	-53.2283	458.579	-54.2777	517.472
10	2	1	2	3	1	2	-40.0116	100.134	-50.475	334.003	-53.2541	459.942	-54.2808	517.656
11	2	1	2	3	2	3	-40.1159	101.344	-50.4685	333.755	-53.2839	461.527	-54.256	516.176
12	2	1	2	3	3	1	-40.1243	101.441	-50.502	335.041	-53.2214	458.214	-54.3276	520.451
13	2	2	3	1	1	2	-40.0026	100.03	-50.5044	335.134	-53.3616	465.669	-54.3061	519.167
14	2	2	3	1	2	3	-40.1032	101.196	-50.4966	334.835	-53.4696	471.497	-54.5159	531.857
15	2	2	3	1	3	1	-40.0214	100.247	-50.5419	336.587	-53.2389	459.141	-54.572	535.306
16	2	3	1	2	1	2	-40.0834	100.964	-50.5026	335.067	-53.308	462.806	-54.5435	533.548
17	2	3	1	2	2	3	-40.082	100.949	-50.5312	336.17	-53.2705	460.816	-54.2533	516.017
18	2	3	1	2	3	1	-40.128	101.484	-50.5224	335.831	-53.374	466.34	-54.7691	547.591
19	3	1	3	2	1	3	-40.0242	100.279	-50.4679	333.73	-53.4505	470.46	-54.4	524.81
20	3	1	3	2	2	1	-40.1193	101.382	-50.5343	336.291	-53.4854	472.358	-54.305	519.097
21	3	1	3	2	3	2	-40.0456	100.526	-50.4795	334.177	-53.3351	464.254	-54.2028	513.026
22	3	2	1	3	1	3	-40.1011	101.171	-50.4768	334.073	-53.3622	465.705	-54.3366	520.994
23	3	2	1	3	2	1	-40.0136	100.156	-50.4595	333.406	-53.4433	470.072	-54.3331	520.782
24	3	2	1	3	3	2	-40.093	101.076	-50.4796	334.182	-53.415	468.542	-54.2076	513.312
25	3	3	2	1	1	3	-40.1049	101.215	-50.4645	333.598	-53.3269	463.814	-54.2433	515.422
26	3	3	2	1	2	1	-40.0188	100.216	-50.4517	333.107	-53.2982	462.286	-54.1879	512.145
27	3	3	2	1	3	2	-40.0266	100.307	-50.4661	333.661	-53.3221	463.558	-54.254	516.057

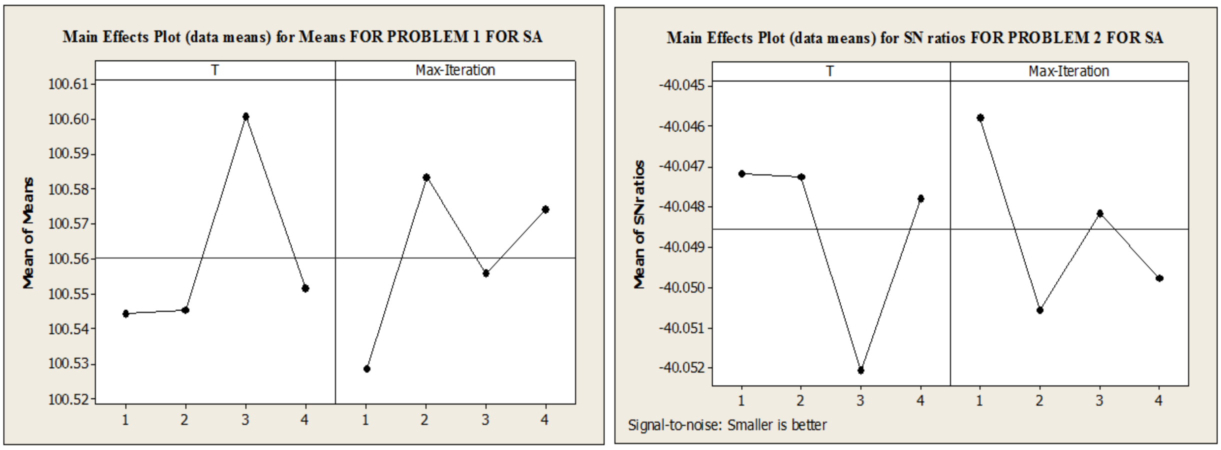

Fig. 7.The SNR and Mean ratio for the SA in problem No. 1

Table 5
Optimal values of the meta-heuristic parameters

Solving Methodologies	Factor	Optimal Value Problem 1	Problem 2	Problem 3	Problem 4
ICA	Beta	1	1	1.25	1
	Sigma	0.6	0.05	0.6	0.05
	Lambda	0.6	0.3	0.6	0.3
	MaxScapeAngle	8	16	16	16
	(Npop, Nimp)	(20,2)	(30,3)	(90,5)	(60, 5)
	$MaxIT_{ICA}$	40	50	80	70
SA	T_0	800	800	800	1200
	$MaxIT_{ICA}$	50	60	150	120

5. Results and Comparisons

In this section, we first provide our four numerical examples in Table 6. Then to demonstrate the performance of the proposed algorithms, we analyze the results statistically and graphically. For each numerical example, 10 independent runs are performed by the proposed ICA and SA to decrease the uncertainty of generated runs. The reported value is based on the algorithms outputs in these 10 runs provided in Table 7. The first column of Table 7 indicates the problem number (according to the number of products) and the 2th-7th columns show the number of runs for the RPD, MIN, and TIME criteria. The experimental tests of this study were carried out on a personal computer with a Pentium processor (1.86 GHz) and one GB RAM, and the algorithms were coded by MATLAB (Version 7.10.0.499, R2010a).

The algorithms outputs are compared with each other in the following terms:

(I) Relative percentage deviation (RPD): This criterion is well-developed for measuring the efficiency of mathematical programming models. The RPD is obtained as Eq. (27):

$$RPD=[(MIN_{stage} - MIN_{total})/ MIN_{total}]\times 100\% \quad (27)$$

where MIN_{stage} and MIN_{total} are the best cost of the algorithm in each stage and the best cost that it has had up to now, respectively. Obviously, the algorithms with the lowest RPD are the best.

(II) Best cost (MIN): The algorithms with better objective functions are the best ones.

(III) Computational time (TIME): The computational time of running the algorithm to reach the best solutions.

The outputs of all the criteria for each problem are reported in Table 8.In order to compare the algorithms, we run a T-paired statistical analysis at the 95% confidence level. Finally, to determine the best solving methodologies, based on the role of accepting H_0 hypotheses, the value of test statistic must be in the acceptance region $[-t_{\alpha,n-1},+\infty]$ or $P-value>\alpha$. The statistical analysis and comparisons are done by MINITAB and summarized in Table 9. According to the Table 19, the proposed ICA significantly works better than the SA in terms of the RPD and MIN criteria. Yet, proposed SA shows a better performance based on the Time criterion. To clarify these results, graphical comparisons are illustrated in Figs.8 and 9.

6. Conclusion and Suggestions for Future Research

In this study, a multi-product continues review inventory control problem within the batch arrival queuing approach ($M^{Qr}/M/1$) was formulated to determine the optimal quantities of maximum inventory. The objective function was to minimize the summation of ordering, holding and shortage costs under the warehouse space, service level, and the expected lost-sales shortage cost constraints from the retailer and warehouse viewpoints. Since the proposed model is Np-Hard, an efficient imperialist competitive algorithm (ICA) was proposed to solve it. To justify the proposed ICA, a simulated annealing algorithm (SA) was used to demonstrate the applicability of the proposed ICA. Moreover, a parameter tuning procedure was followed to find the best outputs of the algorithm. The results showed that the proposed ICA significantly works better than the SA in terms of the RPD and MIN criteria while the proposed SA shows a better performance based on the Time criterion. In further studies, the multi-objective version of the model may be developed. Moreover, Pareto-based meta-heuristic algorithms such as NSGA-II or MOPSO can be used to solve multi-objective mathematical models.

Table 6
General Data of the numerical examples

Problem No.	Number of products	λ_{1j}	λ_{2j}	h_j	A_j	$E[T_j]$	f_j	π_j	F	G	P_j	S	φ_j	μ_j
1	1	[1]	[4]	[3]	[20]	[0.2]	[2]	[0.2]	10000	10	[0.95]	1000	[5]	[70]
2	3	[1 2 3]	[4 5 6]	[3 4 7]	[20 19 10]	[0.2 0.1 0.3]	[2 3 4]	[0.2 0.3 0.1]	10000	10	[0.95 0.92 0.9]	1000	[5 10 3]	[60 110 70]
3	5	[1 2 3 2 5]	[4 5 6 4 7]	[3 4 7 4 9]	[20 19 10 15 22]	[0.2 0.1 0.3 0.2 0.5]	[2 3 4 5 1]	[0.2 0.3 0.1 0.01 0.9]	10000	10	[0.95 0.92 0.9 0.9 0.98]	1000	[5 10 3 9 5]	[60 110 70 50 100]
4	10	[1 4 7 2 4 8 9 5 3 6]	[4 2 4 1 7 5 8 4 6 5]	[3 5 7 2 9 5 1 8 5 10]	[20 10 32 12 30 49 19 30 46 33]	[0.2 0.1 0.3 0.8 0.4 0.6 0.2 0.55 0.9 0.34]	[2 5 3 8 4 3 9 5 3 5]	[0.2 0.4 0.2 0.7 0.4 0.6 0.7 0.9 0.3 0.5]	10000	10	[0.95 0.9 0.97 0.98 0.93 0.92 0.98 0.95 0.96 0.94]	1000	[5 3 5 4 8 6 10 3 9 21]	[70 40 59 82 78 29 48 77 81 94]

Table 7
The runs of the algorithms for the experimental problems

ICA			SA			RUN	PROBLEM
RPD	TIME	MIN	RPD	TIME	MIN		
0.001439	3.7639	100.1007	0.005451	3.88538	100.5018	1	PROBLEM 1
0.00836	3.8686	100.7925	0.006234	2.70084	100.58	2	
0.002181	3.8799	100.1749	0.005633	4.100994	100.52	3	
0.007907	4.0251	100.7473	0.00619	3.843987	100.5756	4	
0.004111	3.864	100.3678	0.007158	3.764688	100.6724	5	
0.009284	3.9842	100.8849	0.006226	3.868943	100.5792	6	
0	3.5898	99.9569	0.005661	3.392587	100.5228	7	
0.002594	3.9873	100.2162	0.006426	3.941883	100.5992	8	
0.003755	4.1085	100.3322	0.006226	3.234731	100.5792	9	
0.001773	3.8119	100.1341	0.00619	4.093915	100.5756	10	
0.01201	12.4694	333.0395	0.000831	9.448316	329.3608	1	PROBLEM 2
0.012642	12.0967	333.2477	0.004178	10.079401	330.4621	2	
0.010615	11.7437	332.5804	0.004178	11.586842	330.4621	3	
0.013513	13.4689	333.5344	0.019676	10.748621	335.5623	4	
0.016317	11.7792	334.4569	0.020211	9.704158	335.7384	5	
0.016481	11.5188	334.5109	0.009392	12.96601	332.178	6	
0.013517	14.6966	333.5355	0.026409	9.277427	337.7783	7	
0	11.4481	329.0873	0.009268	8.785978	332.1372	8	
0.013125	12.5192	333.4066	0.004178	10.361679	330.4621	9	
0.014939	11.281	334.0034	0.019676	8.947472	335.5623	10	
0.047423	59.0444	479.6686	0.060131	15.388689	485.4884	1	PROBLEM 3
0.048024	59.8008	479.944	0.050483	14.43013	481.0699	2	
0.046867	58.7419	479.4143	0.047163	13.184825	534.5014	3	
0.052645	59.2687	482.0603	0.047163	17.251074	479.5495	4	
0.048116	58.7312	479.986	0.113511	14.250209	509.9339	5	
0.022433	58.2641	468.2243	0.047752	16.065968	479.8196	6	
0	58.6309	457.9513	0.114478	16.857634	510.3765	7	
0.032086	60.1365	472.6452	0.062552	15.529153	486.5971	8	
0.00137	59.4095	458.5785	0.508701	14.302164	690.9118	9	
0.004346	58.9115	459.9417	0.240152	14.717272	567.929	10	
0.007561	132.2879	516.0169	0.260569	32.80812	645.5933	1	PROBLEM 4
0.069213	133.394	547.5914	0.243891	32.404455	637.052	2	
0.024731	133.861	524.8103	0.271182	28.685864	651.0287	3	
0.013575	132.1272	519.0967	0.421786	29.415257	728.1598	4	
0.001722	132.2369	513.0264	0.341699	32.942456	687.144	5	
0.017278	132.5622	520.9935	0.003469	25.881626	513.9213	6	
0.016864	131.098	520.7815	0.145244	30.183525	586.5304	7	
0.00228	133.8993	513.3121	0.420591	33.225511	727.548	8	
0.0064	132.1485	515.422	0.267239	33.528925	649.0097	9	
0	131.165	512.1445	0.25211	31.371941	641.2611	10	

Table 8
Computational results of the SA and ICA for all the problems

Algorithms	Criterion	Problem1	Problem2	Problem3	Problem4
SA	RPD	0.00614	0.01179	0.12920	0.262778
ICA		0.00414	0.01231	0.03033	0.015962
SA	MIN	100.571	332.97	522.618	646.72
ICA		100.371	333.14	471.841	520.32
SA	TIME	3.682	10.190	15.197	31.044
ICA		3.888	12.302	59.094	132.478

Table 9
Statistical analyses of all the criteria

Criterion	***Test Statistic***	***P-value***	***Result***
RPD	$T - \text{value} = 1.49 < t_{0.05,9} = 1.833$	$0.884 > 0.05$	$D \geq 0; RPD_{SA} > RPD_{ICA}$
Min	$T - \text{value} = 1.48 < t_{0.05,9} = 1.833$	$0.883 > 0.05$	$D \geq 0; MIN_{SA} > MIN_{ICA}$
TIME	$T - \text{value} = -1.55 < t_{0.05,9} = 1.833$	$0.109 > 0.05$	$D < 0; TIME_{SA} < TIME_{ICA}$

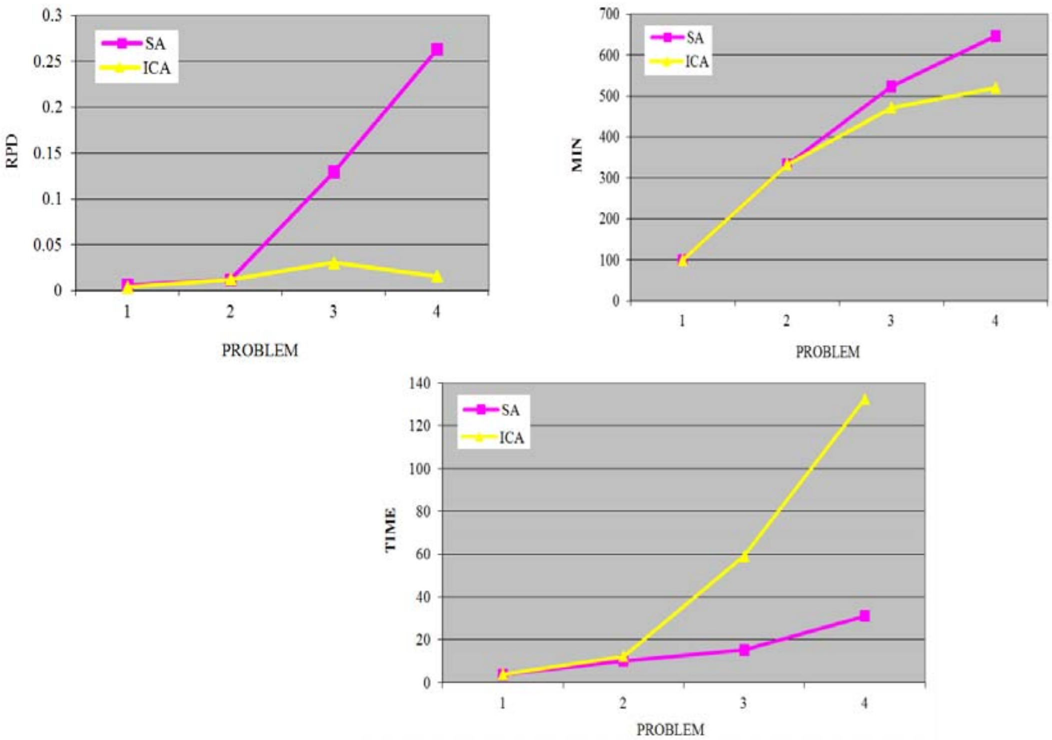

Fig. 8. Graphical comparisons of the SA and ICA based on all the criteria for all the problems

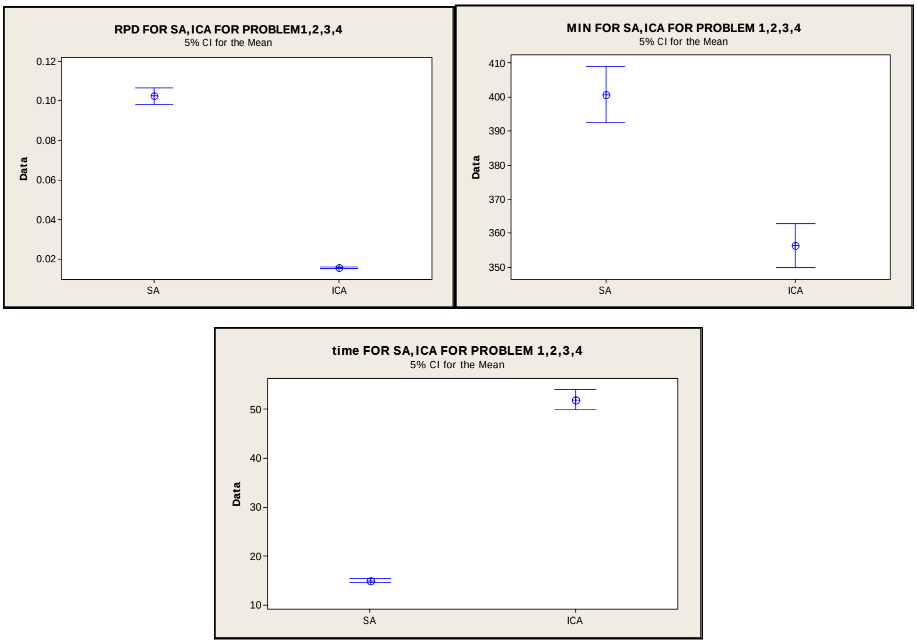

Fig. 9. The box plot of all the criteria with significant differences

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