Switching H2/H∞ Controller Design for Linear Singular Perturbation Systems

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Abstract

This paper undertakes the synthesis of a logic-based switching H2/H∞ state-feedback controller for continuous-time LTI singular perturbation systems. Our solution achieves a minimum bound on the H2 performance level, while also satisfying the H∞ performance requirements. The proposed hybrid control scheme is based on a fuzzy supervisor managing the combination of two controllers. A convex LMI-Based formulation of two fast and slow subsystem controllers leads to a structure which ensures a good performance in both transient and steady-state phases. The stability analysis leverages on the Lyapunov technique, inspired from the switching system theory, to prove that a system with the proposed controller remains globally stable in the face of changes in configuration (controller).

Keywords: Continuous-time LTI singular perturbation system, Fuzzy supervisor, Switching H_2/H_∞ state-feedback control, Linear Matrix Inequality (LMI).

1. Introduction

The systems with both slow and fast dynamics, described mathematically by singular perturbations, have been studied extensively in numerous papers and books; the works reported in [3,4,8,12,13] are a few representatives of the relevant studies existing in the literature. For the robust control of singular perturbation systems, the controller is usually derived from indirect mathematical programming approaches (e.g. solving Riccati equations), which encounter serious numerical problems linked with the stiffness of the equations involved in the design. To avoid this difficulty, several approaches [6,9] have been developed to transform the original problem into ϵ-independent sub-problems. For instance, the time-scale decomposition [6] is a commonly adopted technique in this context. As an alternative to the solution of the Riccati equation, LMI formulation has been attracting more and more attention within the robust control research community. However, solving mixed H2/H∞ control problems for singular perturbation systems through the LMI approach has, up to the present, remained an open research area. Garcia et al. [4] have extended the results reported in [14] and have proposed a solution to the infinite-time near-optimal regulator problem (H_2 control) for singular perturbation systems through an LMI formulation. Time-scale decomposition has been employed in the overall system as well. A different approach to this problem has been discussed in [15]; in particular, by proposing a new lemma, the problem is formulated into a set of inequalities independent of ϵ. An algorithm is then given to solve this set of inequalities through LMI formulation. However, the extension of this method to a mixed H2/H∞ control is very difficult. In [16], a similar approach has been adopted for solving the problem with a static output rather than a state feedback. Today, it is a common practice to adopt a combination of different techniques to obtain different performance levels ([7,10,11]). Within this perspective, hybrid dynamical systems have emerged featuring continuous and discrete dynamics together with a mechanism (supervisor) for managing the interaction among these dynamics [2]. Compared to the sole H∞ control, the mixed H2/H∞ control is more attractive in engineering practice, since the former is essentially a pessimistic design which tends to be overly conservative, whereas the latter optimizes the average performance with a guaranteed worse-case performance. In this paper, we deal with the switching mixed H2/H∞ state feedback control problems for continuous-time linear singular perturbation systems. The

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simple design methods discussed in [5] have also been adopted here to derive the state-feedback gains, separately for the fast and slow sub-systems. A fuzzy supervisor is proposed for the hybrid combination of these controllers to harvest their advantages and to ensure the required performance and the stability of the closed-loop system. The major contribution of the presented work is coming up with a combination of the fast and slow sub-system controllers using a supervisor managing the gradual transition from one controller to another. This method is primarily intended to reap the benefits associated with each controller. The control signal is obtained via a weighted sum of the two signals produced by the slow and fast sub-system controllers. This weighted sum is managed thanks to a fuzzy supervisor which is adapted to obtain the desired performance of the closed-loop system. More specifically, the fast sub-system controller acts mainly in the transient phase, providing a fast dynamic response and enlarging the stability limits of the system. The slow sub-system controller, on the other hand, is pivotal to the steady-state operation, reducing chattering and maintaining the tracking performance. Furthermore, the global stability of the system is guaranteed even if the system switches from one configuration to another (i.e. transient to steady-state and vice versa).

The organization of the remainder of the paper is as follows: Section 2 presents the system definition and elaborates on the specifics of the controllers used. In Section 3, the fuzzy supervisor and the proposed control law are described. Stability analysis is conducted in Section 4. The design procedure is explained in Section 5 along with an example illustrating the efficiency of the proposed method. The paper concludes in Section 6.

2. Problem Statement

Consider the following linear singularly perturbed system Σ with slow and fast dynamics described in the "singularly perturbed" form:

$$\Sigma:\begin{cases}\dot{x}_1 = A_1x_1 + A_2x_2 + B_{w_1}w + B_1u\\ \epsilon\dot{x}_2 = A_3x_1 + A_4x_2 + B_{w_2}w + B_2u\\ z = C_{z_1}x_1 + C_{z_2}x_2 + D_zu\\ y = C_1x_1 + C_2x_2\end{cases} \tag{1}$$

where $x_i \epsilon R^{n_i}, i = 1, 2$, are the states; $u\epsilon R^{m_1}$ is the control input; $w\epsilon R^{m_2}$ is the disturbance input; $y\epsilon R^{l_1}$ is the measured output; $z\epsilon R^{l_2}$ is the output to be regulated; and ϵ is a small positive parameter. By introducing the following notation:

$$x = \begin{bmatrix}x_1\\x_2\end{bmatrix}, A_\epsilon = \begin{bmatrix}A_1 & A_2\\ \frac{1}{\epsilon}A_3 & \frac{1}{\epsilon}A_4\end{bmatrix},\quad B_\epsilon = \begin{bmatrix}B_1\\ \frac{1}{\epsilon}B_2\end{bmatrix}, B_{w_\epsilon} = \begin{bmatrix}B_{w_1}\\ \frac{1}{\epsilon}B_{w_2}\end{bmatrix},\quad C_z = [C_{z_1} \quad C_{z_2}],\ C = [C_1 \quad C_2] \tag{2}$$

The system Σ can be rewritten as the following compact form:

$$\Sigma:\begin{cases}\dot{x} = A_\varepsilon x + B_{w_\varepsilon}w + B_\varepsilon u\\ z = C_zx + D_zu\\ y = Cx\end{cases} \tag{3}$$

Applying a static state feedback control:

$$u = Kx \tag{4}$$

leads to the following closed-loop system:

$$\Sigma_{cl}:\begin{cases}\dot{x} = A_{cl}x + B_{cl}w\\ z = C_{cl}x\end{cases} \tag{5}$$

where:

$$\begin{aligned}A_{cl} &= A_\varepsilon + B_\varepsilon K\\ B_{cl} &= B_{w_\varepsilon}\\ C_{cl} &= C_z + D_zK\end{aligned} \tag{6}$$

We denote the transfer function of the closed-loop system Σ_{cl} from w to z as:

$T(s,K) = C_{cl}(sI - A_{cl})^{-1}B_{cl}$.

The generalized H_2 norm of $T(s,K)$ is defined by:

$$\begin{aligned}&\min_K \|T(s,K)\|_2 =\\ &\min_K Sup\left\{\|z(T)\|: x_{cl} = 0, T \geq 0, \int_0^T \|w(t)\|^2 dt \leq 1\right\}\\ &= \min_K \frac{\|z\|_\infty}{\|w\|_2}\end{aligned} \tag{7}$$

with the H_∞ norm of $T(s,K)$ defined as:

$$\|T(s,K)\|_\infty = \frac{\|z\|_2}{\|w\|_2} \tag{8}$$

the norm $\|E\|$ of a complex matrix E is defined as the largest singular value of E.

2.1. Slow and Fast Sub-systems

Supposing that A_4 is a nonsingular matrix, we can decompose the original singularly perturbed system (1) into two slow and fast subsystems. The slow subsystem is defined by letting $\varepsilon = 0$ in the second equation of set (1), computing x_2 in terms of x_1, w and u, to be ultimately substituted in the first equation. Therefore, the slow subsystem can be obtained as follows:

$$\begin{cases}\dot{x}_s = A_sx_s + B_{w_s}w + B_{1_s}u_s\\ z_s = C_{z_s}x_s + D_{w_s}w + D_su_s\\ y_s = C_sx_s + D_{w_{s1}}w + D_{s1}u_s\end{cases} \tag{9}$$

where: $A_s = A_1 - A_2A_4^{-1}A_3$

$$\begin{aligned}&C_{z_s} = C_{z_1} - C_{z_2}A_4^{-1}A_3 \quad D_{w_s} = -C_{z_2}A_4^{-1}B_{w_2}\\ &B_{w_s} = B_{w_1} - A_2A_4^{-1}B_{w_2} \quad B_{1_s} = B_1 - A_2A_4^{-1}B_2\\ &D_s = D_z - C_{z_2}A_4^{-1}B_2 \quad C_s = C_1 - C_2A_4^{-1}A_3\\ &D_{w_{s1}} = -C_2A_4^{-1}B_{w_2} \quad D_{s1} = -C_2A_4^{-1}B_2\end{aligned} \tag{10}$$

On the other hand, the fast subsystem associated with (1) is defined by:

$$\begin{cases} \varepsilon \dot{x}_f = A_{22}x_f + B_{w_2}w + B_2u_f \\ z_f = C_{z_2}x_f + D_zu_f \\ y_f = C_2x_f \end{cases} \tag{11}$$

Therefore, according to (9) and (11) we can decompose the overall full order system (1) into two slow and fast subsystems. Later, we use these two subsystems in the context of slow and fast controller design. Also, their mixture, generated with the help of a fuzzy supervisor, is leveraged to produce the proposed controller for the overall system. In this paper, we focus on the suboptimal mixed H_2/H_∞ static state feedback control problem. Later, we express the suboptimal *H2, H∞* and mixed H_2/H_∞ problems in terms of linear matrix inequalities (LMI).

● *Lemma 2.1.* [1] (The suboptimal overall H2 static state feedback control problem):

Consider the overall system described by (1). The static state feedback control law (4) stabilizes the closed-loop system (5) and achieves a prescribed H2-norm bound $\upsilon > 0$ for the system (5) if and only if there exists $Q = Q^T > 0, T$ and Z with appropriate dimensions such that:

$$\begin{pmatrix} AQ + QA^T + B_\epsilon T + T^T B_\epsilon{}^T & QC_z^T + T^T D_z{}^T \\ C_zQ + D_zT & -I \end{pmatrix} < 0$$

$$\begin{pmatrix} Q & B_{w_\varepsilon} \\ B_{w_\varepsilon}{}^T & Z \end{pmatrix} > 0 \tag{12}$$

$$\text{Trace}(Z) < \nu$$

By solving the LMIs, Q, T and Z will be found, and control law (4) is calculated as:

$$K = TQ^{-1} \tag{13}$$

The application of (13) to the system characterized by (1) guarantees that the closed-loop system (5) is asymptotically stable and that the H2-norm (7) is less than $\upsilon > 0$.

● *Lemma 2.2.* [1] (The suboptimal overall H∞ static state feedback control problem):

Consider the overall system specified by (1). The static state feedback control law (4) stabilizes the closed-loop system (5), yielding a prescribed H∞-norm bound $\gamma > 0$ for the closed-loop system (5) if and only if there exists $Q = Q^T > 0$ and T with appropriate dimension such that:

$$\begin{pmatrix} AQ + QA^T + B_\epsilon T + T^T B_\epsilon{}^T & B_{w_\varepsilon} & QC_z^T + T^T D_z{}^T \\ B_{w_\varepsilon}{}^T & -I & D_{cl}^T \\ C_zQ + D_zT & D_{cl} & -\gamma^2 I \end{pmatrix} < 0 \tag{14}$$

By solving LMI (14), Q and T will be found, and the control law (4) is calculated from (13).

The application of this controller to the system defined by (1) guarantees that the closed-loop system (5) is asymptotically stable and that the H∞-norm (8) is less than $\gamma > 0$.

● *Lemma 2.3.* [1] (The suboptimal overall Mixed H_2/H_∞ static state feedback control problem):

Consider the overall system described by (1). The static state feedback control law (4) satisfies the mixed H_2/H_∞ control problem if and only if the following LMIs for $Q = Q^T, T, Z$ and a given positive scalar $\gamma > 0$ are satisfied:

$$\begin{aligned} &Min\ \nu \\ &\text{Subject to: (12) and (14)} \end{aligned} \tag{15}$$

By solving (15), we can find Q, T, Z and ν, with the control law (4) computed from (13).

3. Fuzzy Supervisor

In this paper, we take on a different approach to solving the mixed H2/H∞ control problem for the linear singular perturbation system. We start with an overall linear singular perturbation system and decompose it into slow and fast subsystems. Next, we solve the mixed H2/H∞ control problem for both the slow and fast subsystems, and find K_{fast} and K_{slow} by solving out the corresponding LMIs. It is well known that the fast subsystem can be a good approximation for the transient time of the overall system response and that the slow subsystem can be a good model for the steady-state time of the overall system response. Therefore, the fast subsystem controller, K_{fast}, can accordingly be used during the transient time and the slow subsystem controller, K_{slow}, can be used during the steady-state. Their control actions are combined by means of a weighting factor, $\propto \in [0\ 1]$, representing the output of a fuzzy logic supervisor that takes the tracking error e and its time derivatives $\dot{e}, \ddot{e}, \dots, e^{n-1}$ as inputs.

The fuzzy system is constructed from a collection of fuzzy rules whose jth component can be given in the form:

$$\text{If } e \text{ is } H_1^j \text{ And} \ldots \text{And } e^{n-1} \text{ is } H_n^j \text{ Then } \alpha = \alpha_j \tag{16}$$

where H_j^i is a fuzzy set and $\propto_j$ is a singleton.

It is easy to note that the above rule can be considered as a fuzzy rule of a Takagi–Sugeno fuzzy system. The fuzzy implication uses the product operation rule. The connective AND is implemented by means of the minimum operation, whereas fuzzy rules are combined by algebraic addition. Defuzzification, on the other hand, is performed using the centroid method which generates the gravity center of the membership function of the output set. Since the membership functions that define the linguistic terms of the output variable are singletons, the output of the fuzzy system is given by:

$$\alpha = \frac{\sum_{i=1}^{m} \alpha_i \prod_{j=1}^{n} \mu_i^j}{\sum_{i=1}^{m} \prod_{j=1}^{n} \mu_i^j} \tag{17}$$

where μ_j^i is the degree of membership for H_j^i and m is the number of used fuzzy rules. The objective of this fuzzy supervisor is to determine the weighting factor, $\propto$, which specifies the participation rate of each control signal. Indeed, when the norm of the tracking error e and its time derivatives $\dot{e}, \ddot{e}, \dots, e^{n-1}$ are small, the plant is governed by the slow subsystem controller K_{slow} ($\propto = 1$). Conversely, if the error and its derivatives are large, the plant is governed by the fast subsystem controller K_{fast} ($\propto = 0$). The control action, u, is determined by:

$$u = (1 - \alpha)u_{fast} + \alpha u_{slow} \tag{18}$$

where

$$u_{slow} = K_{slow}\, x_1, u_{fast} = K_{fast}\, x_2 \tag{19}$$

Figure 1 shows the structure of the proposed controller with a fuzzy supervisor.

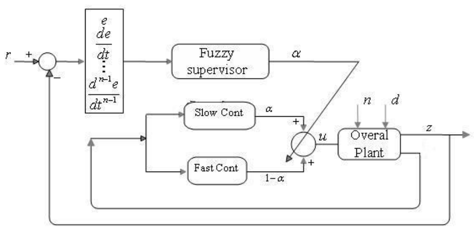

Fig. 1. Structure of the proposed controller.

4. Stability Analysis

We resort to the theorem proposed by Essounbouli et al. [7] to prove the global stability of the system governed by the control law (18). We rewrite the theorem in [7] as follows:

Theorem 4.1. Consider a combined fuzzy logic control system as described in the previous section. If:

1. There exists a positive definite, continuously differentiable and radially unbounded scalar function V for each subsystem,

2. Every fuzzy subsystem gives a negative definite $\dot{V}$ in its active region,

3. The weighted sum defuzzification method is used such that for any control input u ,

$$\min(u_{slow}, u_{fast}) \leq u \leq \max(u_{slow}, u_{fast})$$

then, the resulting control u, given by (18), guarantees the global stability of the closed-loop system.

Proof: The two first conditions guarantee the existence of a Lyapunov function in the active region, which, in turn, gives a sufficient condition for ensuring the asymptotic stability of the system during transition from the fast to the slow subsystem controller. Consider the Lyapunov function $V_{fast} = \xi^T P_{fast}\, \xi$, where P_{fast} is a positive definite matrix corresponding to the solution of (15) for fast subsystem (11) and we have $\lambda_{min}(P_{fast})\xi^T\xi \leq \xi^T P_{fast}\, \xi$, where $\lambda_{min}(P_{fast})$ is the minimal eigenvalue of P_{fast}. Also, consider the Lyapunov function $V_{slow} = \xi^T P_{slow}\, \xi$, where P_{slow} is a positive definite matrix corresponding to the solution of (15) for slow subsystem (9). We have $\xi^T P_{slow}\, \xi \leq \lambda_{max}(P_{slow})\xi^T\xi$, where $\lambda_{max}(P_{slow})$ is the maximal eigenvalue of P_{slow}. To satisfy the second condition of the theorem, it is enough to choose P_2, P_∞ such that:

$$\lambda_{max}(P_{slow}) \leq \lambda_{min}(P_{fast}) \tag{20}$$

This condition guarantees that within the neighborhood of the steady-state (slow subsystem controller), the value of the Lyapunov function V_{fast} is greater than that of V_{slow}. To guarantee the third condition, the balancing term $\propto$ takes its values from the interval [0 1]. Consequently, the three conditions of the aforementioned theorem are satisfied and the global stability of the system is guaranteed. Hence, the Problem formulation (switching H2/H∞ control) will be as follows:

$$\begin{aligned} &Minimize\ (\|T(s,K)\|_2)_{slow} \\ &subject\ to: (\|T(s,K)\|_\infty)_{slow} < \gamma_{slow} \\ &Minimize\ (\|T(s,K)\|_2)_{fast} \\ &and\ subject\ to: (\|T(s,K)\|_\infty)_{fast} < \gamma_{fast} \\ &while:\ \lambda_{max}(P_{slow}) \leq \lambda_{min}(P_{fast}) \end{aligned} \tag{21}$$

5. Design Procedure

We can summarize the design procedure as follows: Compute the slow and fast subsystems of the overall system (1) with reference to (9) and (11). Solve the control problem (21) for the slow and fast subsystems (9) and (11) with the given positive scalars γ_{slow} and γ_{fast} to find K_{slow} and K_{fast} from (13). Compute u_{slow} and u_{fast} from (19). Calculate the overall control signal u from $u = (1 - \alpha)u_{fast} + \alpha u_{slow}$ that $\alpha \in [0,1]$ is governed by the fuzzy supervisor according to error and its derivatives. Apply this control signal to (1) and construct the closed-loop system (5). To construct the fuzzy supervisor, the fuzzy sets are initially defined for each input (the error and its derivatives) and output; then, the rule base is elaborated. The error vector is computed and is then injected into the supervisor to determine the value of $\propto$ for applying to the global control signal.

Table 1

Results for example 5.1.

	H_2	H_∞
Overall	0.1734	0.0286
Switching	0.0410	0.0058

Table 2

The rules of the proposed fuzzy supervisor

	|de/dt|		
|e|	Z	M	L
Z	VL	L	M
M	S	S	Z
L	Z	Z	Z

Example. 5.1. [8] To demonstrate the solvability of various LMIs, as well as the simplicity and low conservatives of the proposed method, a fourth-order, four-output, two-input example is considered, seeking for a switching static state feedback controllers:

Consider a singularly perturbed system described by (1) with:

$$A_1 = \begin{bmatrix} -0.195378 & -0.676469 \\ 1.478265 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} -0.91716 & 0.109033 \\ 0 & 0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -0.051601 & 0 \\ 0.013579 & 0 \end{bmatrix}, A_4 = \begin{bmatrix} -0.367954 & 0.438041 \\ -2.102596 & -0.21464 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} -0.023109 & -16.945030 \end{bmatrix}', B_2 = \begin{bmatrix} -0.048184 & -3.810954 \end{bmatrix}'$$

$$B_{w_1} = \begin{bmatrix} -0.223371 \times 10^{-2} & 0 \end{bmatrix}', B_{w_2} = \begin{bmatrix} -0.279448 \times 10^{-2} & 1.596845 \times 10^{-2} \end{bmatrix}'$$

$$C_1 = \begin{bmatrix} 10 & 0 \\ 0 & 10 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, C_2 = \begin{bmatrix} 0 & 0 \\ 9.21022 & -1.61179 \\ 10 & 0 \\ 0 & 10 \end{bmatrix}$$

$$C_{z_1} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, C_{z2} = \begin{bmatrix} 0.921022 & -0.161179 \\ 0 & 1 \end{bmatrix}$$

$$D_Z = \begin{bmatrix} 1 & 1 \end{bmatrix}, \varepsilon = 0.0835$$

Tracing the proposed design method in section 5, the following results are obtained:

$$K_{overal} = \begin{bmatrix} 0.3680 & 3.0164 & 3.5976 & 3.1751 \end{bmatrix}$$
$$K_{fast} = \begin{bmatrix} 2.5759 & 9.0772 \end{bmatrix}$$
$$K_{slow} = \begin{bmatrix} -0.1668 & 2.8659 \end{bmatrix}$$
$$\boldsymbol{K_{switching}} = [(\mathbf{1}-\boldsymbol{\alpha})\boldsymbol{K_{fast}} \quad \boldsymbol{\alpha K_{slow}}]$$

The fuzzy supervisor is constructed by using the three fuzzy sets zero, medium and large for the tracking error and its time derivative. The corresponding membership functions are triangular. As for the output, five singletons are selected; very large (VL), large (L), medium (M), small (S) and zero (Z), corresponding to 1, 0.75, 0.5, 0.25 and 0, respectively.

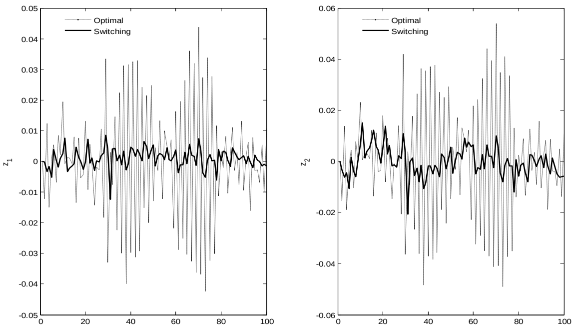

Fig. 2. State response for Example 5.1.

Rules are defined within a table; for example, a given rule in the table can be stated as follows: "IF the norm of the error is medium AND the norm of the error's derivative is large, then α is zero". From the obtained simulation results in Table 1, it is clear that the proposed method yields a better response compared to its conventional overall design counterparts. In our proposed switching method, with a smaller γ for the H_∞ constraint, we have a smaller H_2 norm; whereas, both H_2 and H_∞ norms are increased in the conventional overall method. As can be easily inferred from Figure 2, the state regulation associated with our proposed controller is superior compared to the conventional overall controller.

6. Conclusion

In this paper, we have taken on a convex optimization approach to designing a logic-based switching H_2/H_∞ controller for a linear singular perturbation system. The proposed controller guarantees stability for a closed-loop system, and satisfies the prescribed level of performance indexes for both H_2 and H_∞ norms. Our reliance on two reduced-order fast and slow mode controllers in lieu of one full-order overall controller is the main contribution of this paper. A fuzzy supervisor efficiently manages the performance of both fast and slow controllers. In reality, the fast mode controller exhibit a satisfactory performance (i.e. fast dynamic response and low energy impulse response) in transient interval, while the slow mode controller plays a key role in the steady-state operation via attenuating the interaction of low frequency disturbances. Simulation results show that the proposed controller achieves a considerable improvement over the performance of a closed-loop system.

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