# Delay Model Estimation in RC-tree Circuits Based on the Power-lognormal Distribution

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**Abstract**

Computation of the second order delay in RC-tree based circuits is important during the design process of modern VLSI systems with respect to having tree structure circuits. Calculation of the second and higher order moments is possible in tree based networks. Because of the closed form solution, computation speed and the ease of using the performance optimization in VLSI design methods such as floor planning, placement and routing, the Elmore delay metric is widely implemented for past generation circuits. However, physical and logical synthesis optimizations require fast and accurate analysis techniques of the RC networks. Elmore first proposed matching circuit moments to a probability density function (PDF), which led to the widespread implementation of it in many networks. But the accuracy of Elmore metric is sometimes unacceptable for the RC interconnect problems in today's CMOS technologies. The main idea behind our approach is based on the moment matching technique with the power-lognormal distribution and proposing the closed form formula for the delay evaluation of the RC-tree networks. The primary advantages of our approach over the past proposed metrics are the ease of implementation, reduction of the complexity and proposing an efficiency formula without referring to lookup tables. Simulation results confirmed that our method illustrates a good degree of accuracy and the relative average of errors is less than 20%.

*Keywords:* Power-lognormal distribution, Elmore delay, Circuit analysis, Physical synthesis, Moment matching, Simulation.

## 1. Introduction

The wires linking transistors together are called interconnect and play a major role in the performance of the modern VLSI circuits. In the early days of the VLSI systems, wires were wide and thick and had low resistance, also transistors were slow. Under these conditions, wires could be treated as ideal nodes with lumped capacitance. However, in modern VLSI systems, transistors act as much faster switches and wires have become narrower and stay very close together; thus, the wire delay exceeds from gate delay and tends to affect its neighbour through inductance effects.

Elmore [1] proposed a method in 1948, using the means of the impulse response, to approximate the median of impulse response under the probability distribution. But, the accuracy of his metric is sometimes unacceptable for RC interconnection problems with today's CMOS technologies [2]. Elmore delay was shown to be an upper bound on the 50% step and ramp response delays for the RC trees [3]. However, it can be shown, that the first moment of the impulse response is incapable due to sensitivities with respect to changing path resistance and resistive shielding effects. For example, in the RC network shown in Figure 1, the Elmore delay to capacitor $C_1$ is independent of the resistors $R_2, R_3, R_4$ and $R_5$. The higher the values of these resistors, the more downstream capacitance are shielded, and large errors occur in the Elmore approximation.

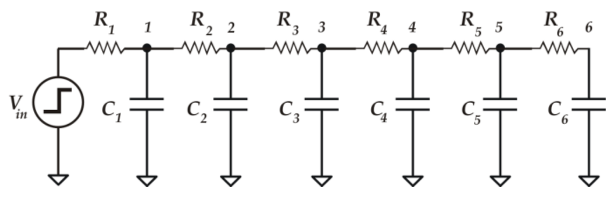


Fig. 1. An example of RC tree network.

To achieve greater accuracy than Elmore delay can provide, additional moments of the impulse response can be employed. Consequently, several delay metrics have been proposed that achieve higher degrees of accuracy than Elmore delay [4-14, 15]. The works of [10, 16] start with 2-

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pole approximation and then make simplifying assumption to derive a delay metric. In [6], Alpert et al. proposed D2M, an empirical metric that has a remarkably high accuracy at the far end nodes.

The PRIMO [13] and H-gamma [14] metrics revealed that the impulse response of an RC circuit network can be treated as the probability density function (PDF) of a statistical distribution. PRIMO and H-gamma try to match the circuit response to the Gamma statistical distribution, and then compute the median directly. Recently, [12] proposed the Weibull distribution to match the moments of impulse response of the RC circuit networks. However, to compute delay using these approaches, one needs to carefully construct a 2-dimensional lookup table in order to find the median.

In this paper, we propose a metric delay that is derived from matching the circuit moments to the Power-lognormal distribution. The primary advantage of our work is the ease of implementation and it does not require referring to a lookup table.

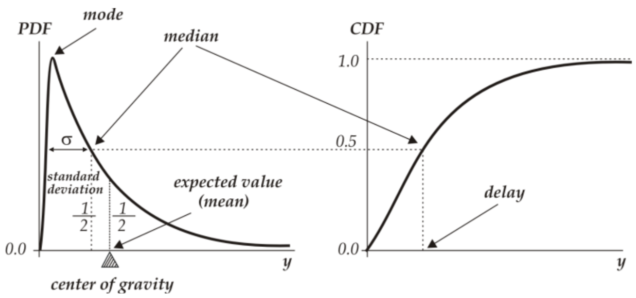



Fig. 2. Probability density function (left) and cumulative distribution function of the Power-lognormal distribution.

## 2. Extracting the Parameters of RC-tree Circuits

We assume that h(t) is the impulse response of a node in an RC circuit and H(s) is the corresponding transfer function. The moments of a circuit at s=0 are defined as [4]:

$$H(s) = \int_0^\infty h(t)e^{-st}dt = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} s^k \int_0^\infty t^k h(t)dt \quad (1)$$

Thus, the transfer function of any circuit is given by:

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1 + a_1 s + a_2 s^2 + \cdots + a_n s^n}{1 + b_1 s + b_2 s^2 + \cdots + b_n s^n}$$
$$= m_0 + m_1 s + m_2 s^2 + m_3 s^3 \quad (2)$$

where, the k-th coefficient of equation (2) is:

$$m_k = \frac{(-1)^k}{k!} \int_0^\infty t^k\, h(t)dt \qquad \forall k = 1,2,3,\ldots \quad (3)$$

In [11], it is shown that for an RC circuit without resistive path to ground, the impulse response, h(t), satisfies $h(t) \geq 0 \quad \forall t$ and :

$$\int_0^\infty h(t)dt = 1 \quad (4)$$

The condition of equation (4) is also required for a continuous function to be a PDF. Thus, we can interpret the impulse response of RC circuits as a PDF. However, there is no adequate underlying statistical distribution [2]. Now, we can describe the mean, variance and skewness from the circuit moments [17]:

$$\begin{cases} \mu_1 = m_1 \\ \mu_2 = \sigma^2 = 2m_2 - {m_1}^2 \\ \mu_3 = -6m_3 + 6m_1 m_2 - 2{m_1}^3 \end{cases} \quad (5)$$

The general formula for the calculation of the n-th central moments around the origin can be obtained from the following equation:

$$\mu_n = \int_0^\infty (t-\mu)^k h(t)dt = \sum_{k=0}^{n} \binom{n}{k} m_k (-m_1)^{n-k} \quad (6)$$

The skewness of the circuit can be expressed by:

$$\gamma = \frac{mean - median}{\mu_2^{1.5}} = \frac{\mu_3}{\mu_2^{1.5}} = \frac{-6m_3 + 6m_1 m_2 - 2{m_3}^3}{(\sqrt{2m_3 - m_1^2})^3} \quad (7)$$

The empirical relationship between mean, mode and median in unimodal curves is [17]:

$$mean - mode \approx 3(mean - median) \quad (8)$$

The main idea behind our delay metric is matching the mean, variance and skewness of the impulse response to those of the Power-lognormal distribution.

## 3. The Power-lognormal Metric

The formula for the probability density function of the standard form of the Power lognormal distribution is [18]:

$$f(y,p,B) = \left(\frac{p}{yB}\right) \emptyset \left(\frac{\ln y - A}{B}\right) \left(\varphi\left(\frac{\ln y - A}{B}\right)\right)^{p-1} \quad (9)$$

The CDF of Power-lognormal distribution is given by:

$$f(y,p,B) = \left(\varphi\left(\frac{\ln y - A}{B}\right)\right)^{p} \qquad y,p,B > 0 \quad (10)$$

where p (also referred to as the power parameter) and B are the shape parameters, Φ is the cumulative distribution function of the standard normal distribution, and φ is the probability density function of the standard normal distribution. The A is also called scaling parameter.

As with other probability distributions, the Power-lognormal distribution can be transformed with a location parameter and a scale parameter. We omit the equation for the general form of the Power-lognormal distribution. Since the general form of the probability functions can be expressed in terms of the standard distribution, all subsequent formulas in this paper are given for the standard form of the function. Figure 2 is the plot of the Power-lognormal probability density function with value of p set to 1. As we can see, the plot of the Power-lognormal's PDF is more like the shape of the impulse response of the RC circuits. In [4], it is shown that all RC interconnect networks have the following features:

- Their impulse responses, h(t), are unimodal.
- Their h(t) distributions have positive skewness.
- In their h(t) distributions always:

$$mode \leq median \leq mean.$$

The statistics for the Power-lognormal distribution are complicated and require tables. However, extracting the mean, median, mode, and standard deviation of the Power-lognormal distribution, it is feasible for p=1. Therefore, these parameters can be written as:

$$\begin{cases} Mean = \mu = E(x) = exp\left(A + \frac{B^2}{2}\right) \\ Variance = \sigma^2 = V(x) = exp(2A + B^2)(exp(B^2) - 1) \\ Skewness = \gamma = \left(exp(2A + B^2)\right)^{\frac{1}{2}}\left(2 + exp(B^2)\right) \\ Mode = exp(A - B^2) \end{cases} \tag{11}$$

As we see in equation (11), the skewness of the Power-lognormal distribution is non-negative. Therefore, this distribution is more applicable for the estimation of the RC delay metric. Hence, the next step for extracting the RC delay metric, is matching the moments of the circuit with their corresponding moments of impulse response mean and the variance of the distribution. Solving the equation (11) for A and B parameters, we will have:

$$\begin{cases} A = Ln\left(\frac{m_1^2}{\sqrt{2m_2 - m_1^2}}\right) \\ B = \sqrt{Ln\frac{2m_2 - m_1^2}{m_1^2}} \end{cases} \tag{12}$$

It is shown that, the median of the Power-lognormal distribution is exp (A). This means that half of the surface of this distribution relies in [0, exp (A)] interval. Thus, it seems to be sufficient to approximate the delay value at 50% point. For this purpose, when matching the impulse response with the expected value (median), the 50% delay metric becomes approximately:

$$exp\left(Ln\left(\frac{m_1^2}{\sqrt{2m_2 - m_1^2}}\right)\right) = \frac{m_1^2}{\sqrt{2m_2 - m_1^2}} \tag{13}$$

The D2M metric [5] was derived empirically, and is a simple function as the two first moments ration is equal to $0.8003m_1$. Thus, D2M is never as close as 2% to the Elmore delay; i.e., D2M has 80% accuracy. The D2M delay metric is given by:

$$D2M = \frac{m_1^2 Ln(2)}{\sqrt{m_2}} \tag{14}$$

*Theorem 1:* The Power-lognormal delay metric is always less than the Elmore delay.

*Proof:* We know that the second central moment (i.e. $\sigma^2$) is always non-negative. We have:

$$\sigma^2 = \mu_2 = 2m_2 - m_1^2 \geq 0 \Rightarrow 2m_2 \geq m_1^2 \Rightarrow \frac{m_1^2}{2m_2} \leq 1 \tag{15}$$

*Hence the Power-lognormal metric is given by:*

$$\begin{aligned} &\frac{m_1}{\sqrt{\frac{2m_2}{m_1^2} - 1}} \approx \frac{m_1^2}{\sqrt{2m_2}} = \frac{m_1^2}{\sqrt{m_2}} \times \frac{1}{\sqrt{2}} \\ &\cong\cong \frac{0.8003m_1}{Ln(2)} \times \frac{1}{\sqrt{2}} \approx 0.81642m_1 \end{aligned} \tag{16}$$

*Theorem 2***:** The Power-lognormal delay metric is always a non-negative value.

*Proof:* For each RC circuit network, $m_2 > 0$ and $m_1 < 0$ [20]. Since the square of $m_1$ is always in the numerator and the denominator is always positive, thus, we see that the Power-lognormal metric guarantee the return value as an estimation of delay, is non-negative.

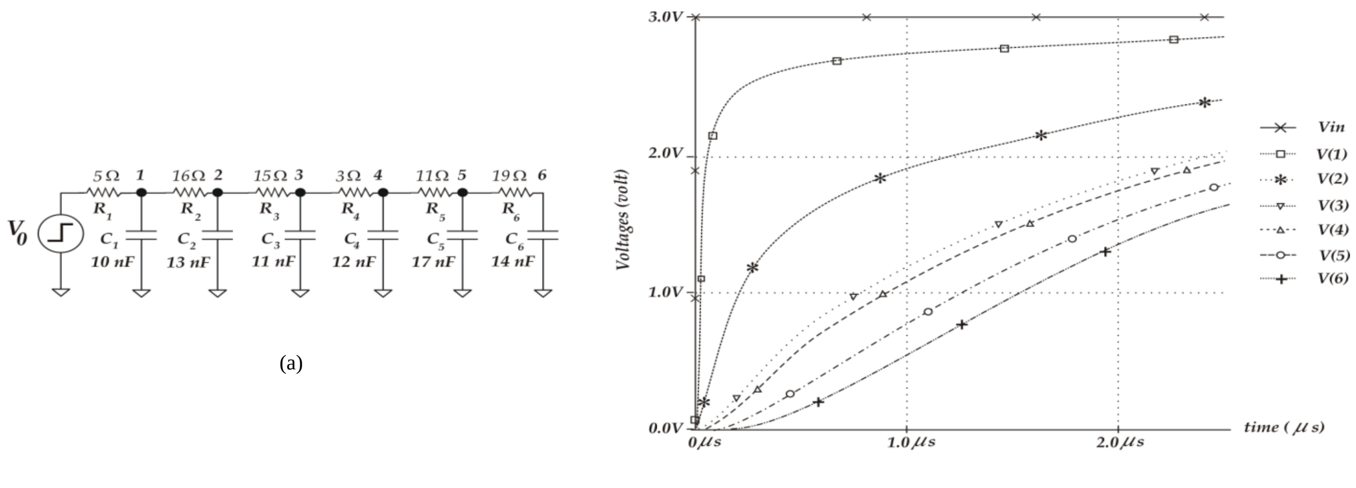
Based on the experimental results and comparison with other metrics, we can conclude that for having an accuracy of around 90% and using the optimality of the proposed metric, this value should be multiplied by an empirical parameter, α, equal to ln(2) = 0.69314. Thus, the Power-lognormal metric can be easily computed as shown in Figure 3.

1. Compute the moments $m_1$ and $m_2$.
2. Compute the parameter $r = \frac{m_1^2}{\sqrt{2m_2 - m_1^2}}$.
3. Set the empirical parameter α to ln(2) = 0.69314.
4. Return the parameter $\tau = \alpha . r$ as a 50% delay value.

Fig. 3. Algorithm for computing the Power-lognormal metric.

Table 1
Delay comparisons for the RC network shown in Figure 4(a)

| node | Elmore | D2M | PRIMO | WED | Log-normal | Power-lognormal | Spice (ns) | Relative error% |
|---|---|---|---|---|---|---|---|---|
| 1 | 385 | 113 | -39 | 45 | 115 | 83 | 40 | 107.5 |
| 2 | 1456 | 797 | 684 | 661 | 812 | 674 | 467 | 44.3 |
| 3 | 2264 | 1514 | 1486 | 1474 | 1544 | 1456 | 1437 | 1.32 |
| 4 | 2393 | 1640 | 1621 | 1620 | 1672 | 1612 | 1589 | 1.45 |
| 5 | 2733 | 1982 | 1974 | 2025 | 2021 | 2071 | 1976 | 4.8 |
| 6 | 3000 | 2258 | 2245 | 2348 | 2303 | 2479 | 2257 | 9.8 |




(b)

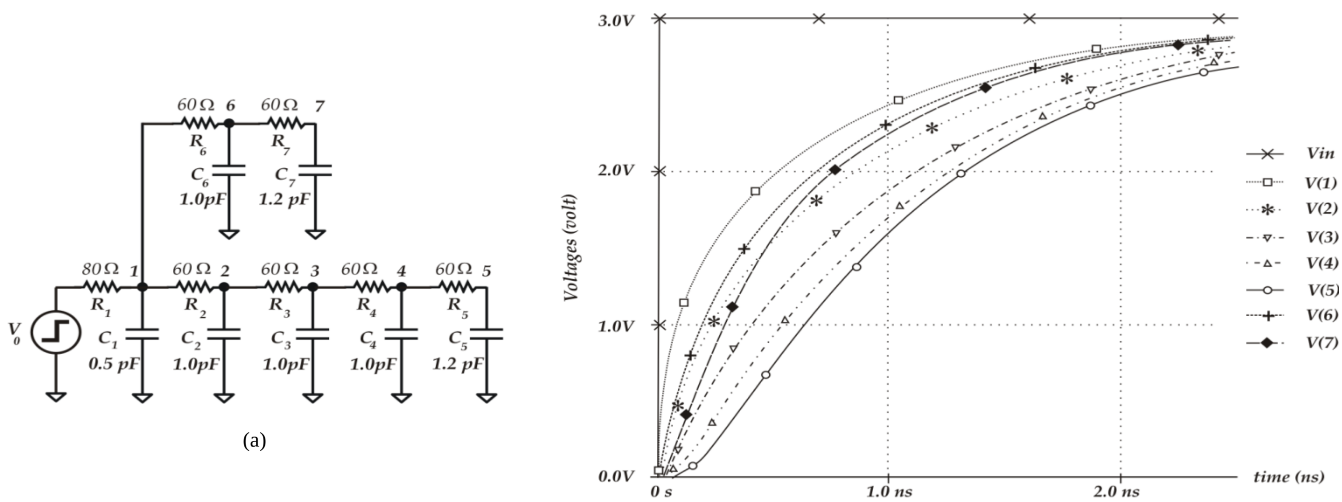
Fig. 4. (a) An RC ladder example; (b) corresponding of delay values in different nodes.

## 4. Simulation Results

We apply the Power-lognormal method in many RC-tree networks. Two examples of these types of circuits are shown in Figures 4(a) and 5(a). For exact investigation of this metric, some of the famous algorithms along with Elmore delay are calculated for these circuits. Figures 4(b) and 5(b) show the corresponding delay values in different circuit nodes at the Spice environment. We compared these approaches and listed their return values in Tables 1 and 2. We used a 3.0 volts step function with rise time equal to 0 as an input for circuit. Also, all of these simulation results are compared in Figures 6 and 7. As you can see, our proposed approach is considerably closer to Spice over all nodes and it is the most accurate metric. While the PRIMO and D2M are slightly less accurate and one can observe that for the RC shown in Figure 4, PRIMO returns a negative delay at node 1.

Table 2
Delay comparisons for the RC network shown in Figure 5(a)

| Node | Elmore | D2M | PRIMO | WED | H-gamma | Log-normal | Power-lognormal | Spice(ns) | Relative error% |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 552 | 299 | 241 | 246 | 194 | 305 | 252 | 198 | 27.3 |
| 2 | 804 | 514 | 498 | 485 | 486 | 524 | 476 | 480 | 0.84 |
| 3 | 996 | 696 | 699 | 698 | 701 | 710 | 698 | 705 | 1 |
| 4 | 1128 | 830 | 836 | 855 | 840 | 846 | 882 | 844 | 4.5 |
| 5 | 1200 | 905 | 909 | 943 | 912 | 923 | 996 | 922 | 8.03 |
| 6 | 684 | 420 | 376 | 386 | 355 | 428 | 378 | 375 | 0.8 |
| 7 | 556 | 492 | 450 | 470 | 431 | 501 | 461 | 453 | 1.77 |




(a) (b)

Fig. 5. (a) An RC- tree example; (b) corresponding of delay values in different nodes.

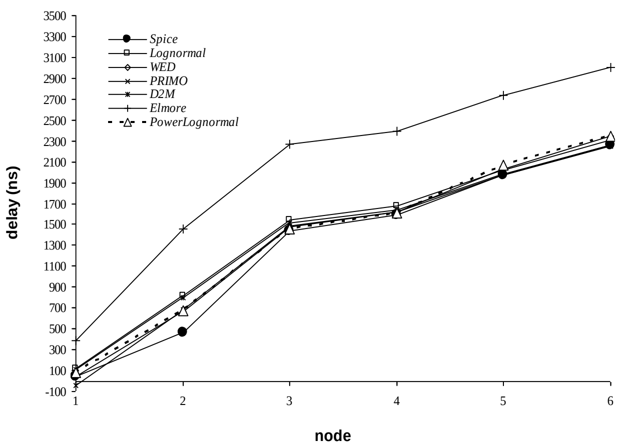



Fig. 6. Simulation results for the RC network shown in Figure 4.

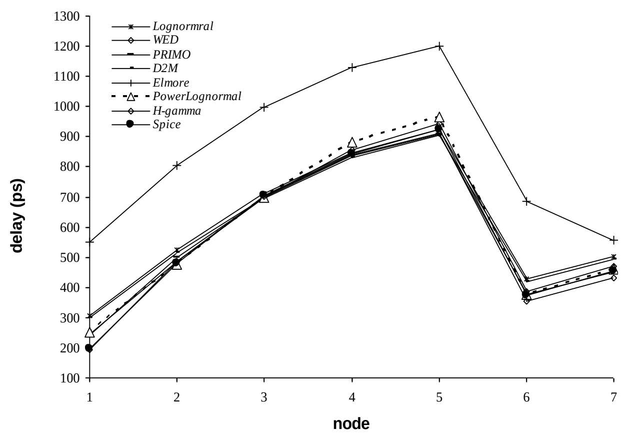



Fig. 7. Simulation results for the RC network shown in Figure 5.

## 5. Conclusion and Future Works

In this paper, we investigated some of the recent approaches that are widely accepted and implemented for a delay calculation of the RC-tree networks. Moreover, we used the Power-lognormal distribution probability function to drive a new, closed form formula delay metric for the RC circuits. This metric is straightforward and can be easily implemented via the two first moments of a circuit. Also, our approach does not require referring to lookup tables for evaluating a delay approximation in constant times. Some of the delay metrics [8, 13] may return negative values for the calculation of delays. However, we could show that our metric always returns a positive value. Future research may address other practical and feasible probability density functions and apply them into RC-tree circuit networks.

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