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Design and Simulation of Adaptive Neuro Fuzzy Inference Based Controller for Chaotic Lorenz System

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Abstract

Chaos is a nonlinear behavior that shows chaotic and irregular responses to internal and external stimuli in dynamic systems. This behavior usually appears in systems that are highly sensitive to initial condition. In these systems, stabilization is a highly considerable tool for eliminating aberrant behaviors. In this paper, the problem of stabilization and tracking the chaos are investigated. In fact, this problem can be divided into two categories, regulation and tracking. These kinds of stabilization have been studied, first regardless of the chaos and then considering the chaos. For this purpose, smart and powerful adaptive neuro fuzzy inference system (ANFIS) technique is used because intelligent approaches unlike the classical methods do not require complex mathematical equations and do not need to acquire the dynamics. Moreover, ANFIS is a complete and optimized fuzzy approach that has both advantages of neural network and fuzzy network. Furthermore, it can acquire fuzzy membership functions automatically. The proposed technique is examined by a famous example of a chaos system called Lorenz system. The simulation results show the ability of the proposed technique and its effectiveness in comparison with PID controller in the system.

1. Introduction

There are a large number of systems which have chaotic behavior and are very sensitive to initial condition. Study of these chaos dynamic systems has quickly spread in the last three decades, and it has become a very attractive area of research to remove dynamic chaotic behaviors and make the nonlinear systems stable. Chaos Control in a broader meaning can be divided into two categories, first, removing chaos dynamic behaviour and the other, producing or improving chaos behaviour in nonlinear systems (known as destabilizing or anti-chaos control) [1]. In this paper, stabilization has been considereddishab as one of the most significant methods to eliminate aberrant behaviors of chaotic system. Stabilization is divided into two categories: regulation and tracking. As regards, regulation, system is stabled by designing proper control signals to one of the available balance point or one of the alternate unstable routes on strange attractor tracking system. This type of stabilization is used in control of reactors, chemical, oscillators and electric circuits, laser, secure communications, noise elimination and so on [2]. Other category of stabilization in chaotic systems is, tracking [3]. In this type of stabilization, a reference signal is considered which is changeable with time and consider a designing control signals that follows responses of this signal [3].

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Chaotic systems have high sensitivity to any system parameters, existence of non-linear factors in the routes of control inputs decreases the efficiency of designed controllers even in some cases these controllers completely lose their functionality, and in some cases cause the instability of closed loop chaotic system with controller [4]. So, designing the controller which can challenge these uncertainties seems crucial. In addition, neuro -phase system played a key role as powerful tool for dealing with uncertainty in solving control issues in recent years [5]. In this paper we use the popular intelligent and useful algorithm of comparative neuro-phase to stabilize the chaotic system. And we analyse, compare and assess the ability of it in the Stabilization of Lorentz chaos system. The general block diagram for the suggested system has been shown below:
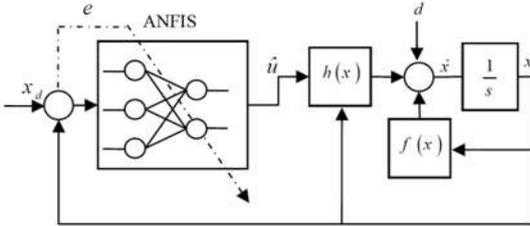

Fig.1. the general block diagram for the suggested system

2. Proposed Method

These method is on basis of data, and neuro networks which are named black box, means that they just depend on data and do not give any information about the internal system, they identify the system with input and output data [6], [7]. Now the phase algorithm illustrates the system on the basis of date functions. These functions are Qualitative and derived from the knowledge of the experts. In addition, with combination of these two methods we can use both advantages, which can be flexible and do not need the knowledge of an expert, due to this, using this method in the controllers can increase the function and resistance of them to unwanted changes and uncertainties in system [8].

2.1. Hybrid Method

Hybrid method is used for ANFIS training, in this method, a combination of descending gradient method and the least square error (LSE) is used. To describe the method, we first study the LSE.

2.1.1. Least Square Error

In general, the output of a linear model is as follows: Which "u" is the input vector and "f"s are known functions and "θ"s are unknown parameters that should be estimated. For parameter identification we require training data which are expressed as follows. Replacing the input and output pairs in original equation we reach to "m" linear equation as follows:

$$\begin{cases} \{f_1(u_1)\theta_1 + f_2(u_1)\theta_2 + \cdots + f_n(u_1)\theta_n = y_1 \\ \{f_1(u_2)\theta_1 + f_2(u_2)\theta_2 + \cdots + f_n(u_2)\theta_n = y_2 \\ \vdots \\ \{f_1(u_m)\theta_1 + f_2(u_m)\theta_2 + \cdots + f_n(u_m)\theta_n = y_m \end{cases} \quad (1)$$

Which are as follows in matrix form:

$$A\theta = Y \quad (2)$$

Matrix "A" is in the following form:

$$A = \begin{bmatrix} f_1(u_1) & \cdots & f_n(u_1) \\ \vdots & \vdots & \vdots \\ f_1(u_m) & \cdots & f_n(u_m) \end{bmatrix} \quad (3)$$

"θ" is a "$n \times 1$" vector.

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix} \quad (4)$$

"Y" is the output vector of "$m \times 1$".

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} \quad (5)$$

In this case, the "θ" vector can be calculated as follows:

$$\theta = A^{-1}Y \tag{6}$$

Of course in practical matters, the number of input/output data is usually higher than the number of equations. In general, the data may be accompanied by noise or model may not be able to accurately determine the output. In this case, the equation is as follows:

$$A\theta + e = Y \tag{7}$$

In this case we are looking for "$\theta = \hat{\theta}$" to minimize the sum of squared errors.

$$E(\theta) = \sum_{i=1}^{m}(y_i - a_i^T\theta)^2 = e^T e = (y - A\theta)^T(y - A\theta) \tag{8}$$

If AA^T is reversible then:

$$\hat{\theta} = (A^T A)^{-1} A^T y \tag{9}$$

There is another way to calculate the parameters which is a recursive method (RLSE); this method has been created for two reasons:

1. Sometimes AA^T matrix may not be invertible.

2. Suppose that we have estimated "θ", if a new pair of input/output is added to the system, in the previous method the whole parameters should be calculated again; but in this method, only the new parameter value is obtained. Returning formulas are observed below.

$$\begin{aligned} S_{k+1} &= S_k - \frac{S_k a_{k+1} a_{k+1}^T S_k}{1 + a_{k+1}^T S_k a_{k+1}} \\ \theta_{k+1} &= \theta_k + S_{k+1} a_{k+1}(y_{k+1} - a_{k+1}^T \theta_k) \\ st \quad & k = 0,1,\cdots m-1 \end{aligned} \tag{10}$$

For the initial conditions "$\theta = 0$" and "$S_0 = \gamma I$" are considered where "I" is the unit matrix with "$m \times m$" dimensions and "γ" is a large integer. Having expressed these two algorithms, we can get to ANFIS training by combined method. In each repeat, we move forward until the matrix A (which was explained in LSE method) is obtained. We also have the outputs. Then parameters are obtained by the combined method. It should be noted that all educational data should be applied and the Premise parameters should be kept constant. Then the Conclusion parameters are kept constant, and the Premise parameters are set by descending gradient [7], [9].

3. Simulation Results

In this section simulation results in regulation and tracking in the presence of turbulence have been assessed for the Lorenz system without control. Parameters are considered r=28, b=8/3 and δ=10 which cause the system behaves chaotic and makes three unstable balance point.

3.1. Output Regulation (Without Disturbances)

With choosing the Gaussian membership function and using the error input and error derivative and using the error input and error integral, controller behaviour is shown in Fig.2 to 4. It can be seen that in this state regulating error in nominal and have high convergence speed.
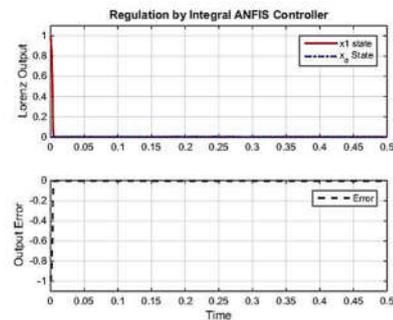

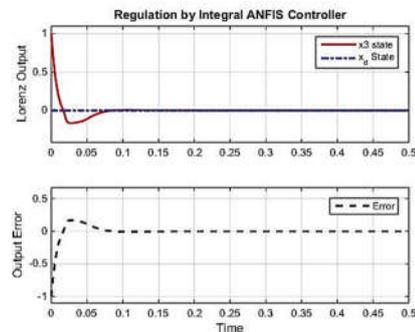Fig. 2. ANFIS system output(error and derivative error)

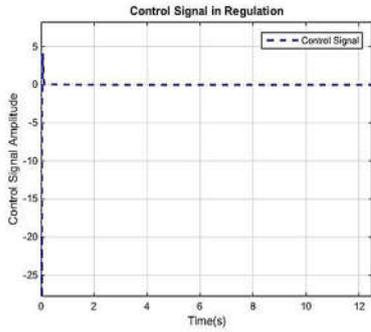Fig. 3. ANFIS system output(error and integral error)

Fig. 4. signal control for x regulation

3.2. Output Tracking (Without Disturbances)

With using the error input and error integral, closed loop output in this state is showed is Fig. 5. It can be seen that this state controller causes the output track source step function with maximum overshoot.
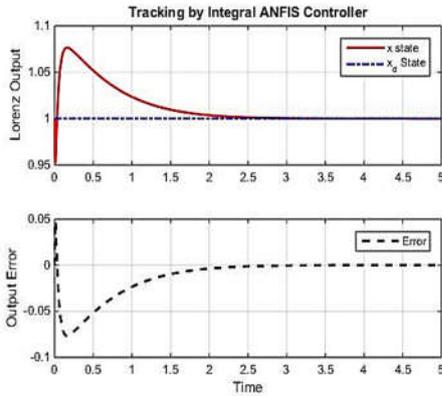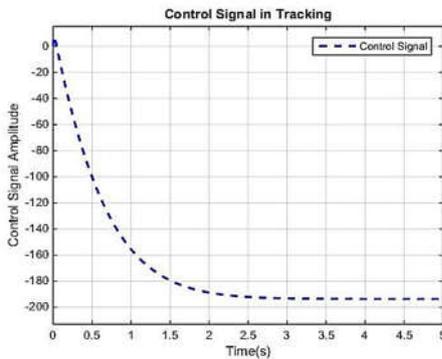

Fig. 5. ANFIS system output

Fig. 6. control signal for x tracking

3.3. Selecting Error Output and Error Integral for Regulation and Tracking

Dynamic Lorenz equations in general with disturbances are shown below:

$$\begin{cases} \dot{x} = \delta(y-x) + d_1 \\ \dot{y} = rx - y - xz + d_2 \\ \dot{z} = xy - bz + d_3 \end{cases} \quad (11)$$

It is supposed that, in these equations the amplitude of uncertainties is limited to upper bound Bi (Bi>0):

$$|d_i| < B_i, d_i = 1, 2, 3 \quad (12)$$

In this paper both of Lorentz dynamic equations with and without disturbance di=0 is used. For instance, disturbance with limited range can be used in simulation:

$$\begin{cases} d_1 = \cos 3t \\ d_2 = 0.5 \sin t \\ d_3 = 0.5 \cos 8t \end{cases} \quad (13)$$

3.3.1. Output Regulation with Disturbance

Closed loop results are showed in Fig.7. It can be seen that in this state regulating error is nominal and convergence speed to zero output is higher in comparison with the previous one.
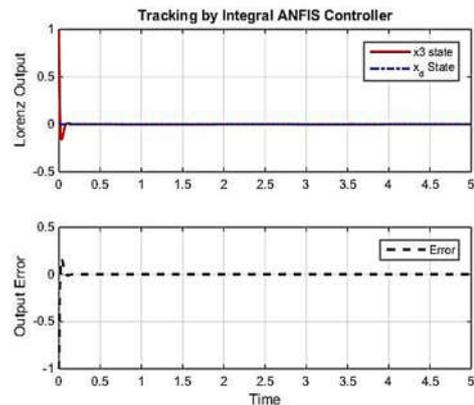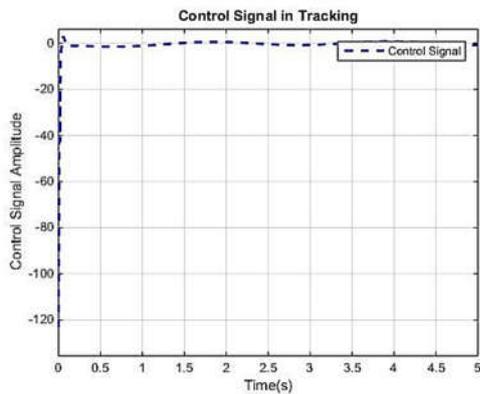

Fig. 7. ANFIS system output

Fig.8.control signal for x regulation

3.3.2. Output Tracking with Disturbance

After assessing the controller behavior in regulation and keeping them near zero we came to analysing controller behavior in source input tracking. In this basis we use step function as a source signal.
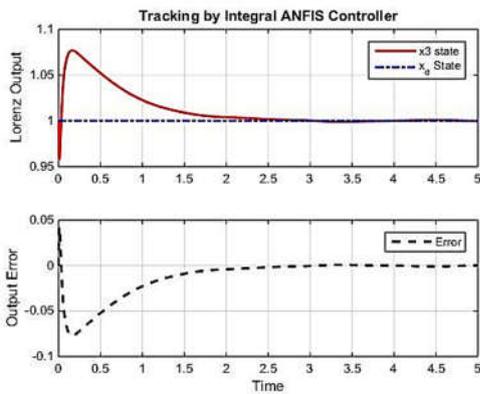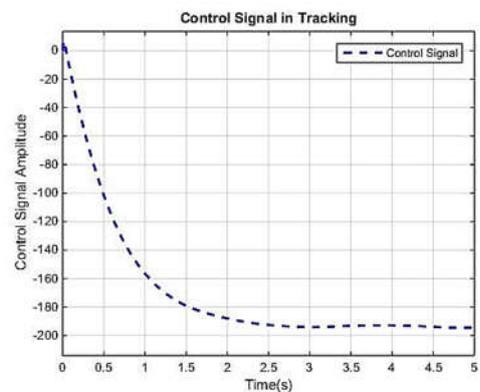

Fig. 9. ANFIS system output

Fig. 10. control signal for x tracking

4. Comparison with PID Optimized Controller

In this part from the perspectives of regulation and tracking we compare the suggested optimized controller with PID optimized controller in which in both states suggested controller functioned better. Optimization is based on mean square error in gradient descent algorithm which have been used to acquire the PID coefficients.
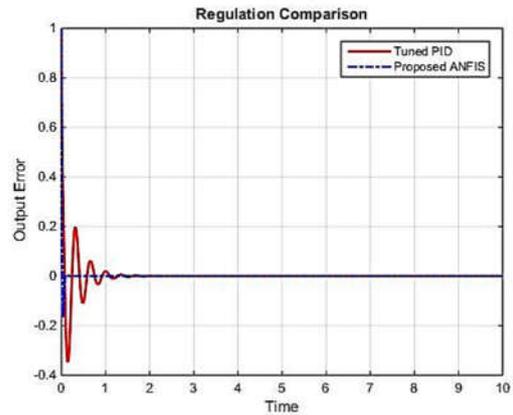

Fig. 11. comparison between PID optimized controller and suggested optimized controller in regulation state
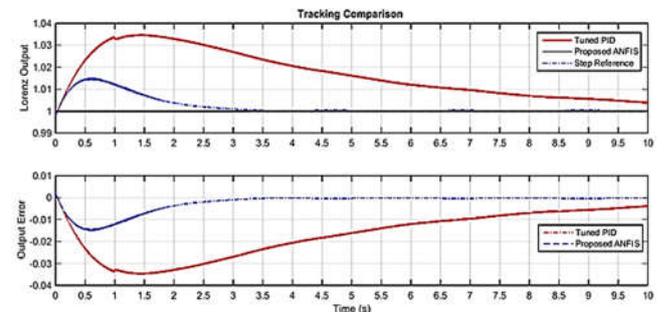

Fig. 12. comparison between PID optimized controller and suggested optimized controller in tracking state

Table. 1. Comparison between PID controller and ANFIS in tracking

maximum	Settling time by criteria 2%	The maximum overshoot	The Type of controller
1.031	0 s	3.1%	Tuned PID
1.015	0.5 s	1.5%	ANFIS

5. Conclusion

In the past, the artificial neural network was used to control the lorenz chaotic system. This approach was unsuccessful in dealing with uncertainty and external

disturbances and has less accuracy rather than our proposed technique. In this paper, the ANFIS control method was proposed to regulate the response. It, also, showed an efficient response in tracking mission. This method was compared with optimized PID controller to control the lorenz system. The effectiveness of our approach, its better accuracy and less fluctuations were shown by simulations.

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