International Journal of Decision Inelligence
Vol 1, Issue 3, Summer 2024 , 35-47

Ranking the Generalized Fuzzy Numbers Based on the Center of the Area

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Received 28 October 2023; Accepted 28 April 2024

Abstract

In decision-making contexts marked by uncertainty, the application of fuzzy numbers has emerged as a crucial tool. These numbers offer a mathematical framework for representing imprecise information, enabling a more nuanced approach to decision-making. Fuzzy numbers find widespread application in quantifying the inherent uncertainty present in decision-making contexts. When incorporating fuzzy numbers into decision-making procedures, the necessity to compare these fuzzy numbers becomes an unavoidable occurrence. Ranking fuzzy numbers is a challenging topic. In this paper, we propose a new method for ranking generalized fuzzy numbers based on the center of the area concept. First, we present the concepts of the presented method. Additionally, the proposed method can rank symmetric fuzzy numbers relative to the y-axis easily. Then the advantages of the proposed method are illustrated through several numerical examples. The results demonstrate that this approach is effective for ranking generalized fuzzy numbers and overcomes the shortcomings in recent studies. Finally, we checked the result of the presented method with other existing methods. The results show that the presented method has consistent results with less computational complexity.

Keywords: Generalized fuzzy numbers; Ranking fuzzy numbers; Center of area; Decision making; Uncertainty

1.Introduction

In decision-making contexts marked by uncertainty, the application of fuzzy numbers has emerged as a crucial tool. These numbers offer a mathematical framework for representing imprecise information, enabling a more nuanced approach to decision-making. Within the domain of fuzzy set theory, the ranking of fuzzy numbers holds paramount significance, allowing for the systematic arrangement of imprecise quantities based on their relative importance. One notable method, focused on the centroid point, seeks to refine the ranking process originally. This approach, though promising, is not without its challenges, as it tends to produce identical rankings for fuzzy numbers and their respective inverse images, potentially limiting its practical applicability.

While the centroid-based ranking approach has contributed valuable insights, it faces critiques that have spurred further exploration. Alternative ranking indices, such as those centered on the area between the centroid point and the original value, have been proposed to provide complementary perspectives on the importance of fuzzy numbers. These indices aim to capture the spatial relationship between the centroid and the original point, offering potential refinements to the ranking process. However, the implications of these methodologies in complex, real-world decision-making scenarios necessitate careful scrutiny. Additionally, considerations of the relative weights of horizontal and vertical components in the ranking process underscore the dynamic nature of these methodologies, driving ongoing efforts to enhance the effectiveness of generalized fuzzy number ranking. The process of ranking generalized fuzzy numbers based on the center of the area, while a valuable approach in decision-making under uncertainty, is not without its limitations. One notable concern arises from the tendency of this method to yield identical rankings for fuzzy numbers and their corresponding inverse images. This inherent limitation potentially hinders its practical applicability in complex decision-making scenarios. Furthermore, there are debates regarding the appropriate weighting of horizontal and vertical components in the ranking process, which can significantly impact the outcomes. Recognizing these drawbacks, there is a pressing need to address and eliminate these limitations to refine the methodology and enhance its effectiveness in real-world applications. This pursuit of refinement is crucial for advancing the field of generalized fuzzy number ranking and ensuring its practical utility in a wide range of decision-making contexts.The subsequent sections of this paper are structured as follows: Section 2 includes a literature review and the suggested drawbacks. Section 3 provides a concise overview of fundamental concepts and crucial definitions pertinent to our discourse. Section 4 outlines the centroid point method and expounds on its limitations. In Section 5, a novel approach for ranking generalized fuzzy numbers, grounded in the center of area principle, is introduced.

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Additionally, we offer numerical examples to demonstrate the merits of this proposed methodology. Lastly, Section 6 encapsulates the conclusion drawn from our findings.

2.Literature Review

In many instances, the data used for decision-making is only known approximately [1]. In 1965, Zadeh [2] introduced the concept of fuzzy set theory to address this issue. The ranking of fuzzy numbers holds significant importance within fuzzy set theory, decision-making processes, data analysis, and practical applications [3]. Jain explored methods for comparing and ranking fuzzy numbers in 1976 [4], employing the notion of maximizing sets for ordering them. Since then, various techniques for ranking quantities have been proposed by different researchers. Yager was the pioneer in utilizing the concept of the centroid for ranking fuzzy numbers in [5], where he employed the horizontal coordinate of the centroid point, $\bar{x}$, as the ranking index. However, this method does not accurately rank fuzzy numbers when $\bar{x}$ is the same for different fuzzy numbers but their $\bar{Y}$ values differ, where $\bar{Y}$ represents the vertical coordinate of the centroid point of the fuzzy number [6]. Cheng [7] introduced a centroid index ranking method, which involves computing the distance between the centroid point of each fuzzy number and the original point, aiming to enhance Yager's [5] approach.

Cheng's method was not without its flaws. In this approach, the ranking of fuzzy numbers $\tilde{A}$ and $\tilde{B}$ corresponds to the ranking of their respective images, i.e., $-\tilde{A}$ and $-\tilde{B}$ (refer to Example 1). To address this limitation, Chu and Tsao [8] introduced the use of the area between the centroid point and the original point as a ranking index for fuzzy numbers. In this scheme, a larger area signifies a higher rank for the corresponding fuzzy number.

Wang and Lee [9] argued that in the method proposed by Chu and Tsao [8], the multiplication of values on the horizontal and vertical axes often diminishes the significance of the horizontal axis in fuzzy number ranking. They initially advocated for $\bar{x}$ as a ranking criterion. Specifically, a larger $\bar{x}$ corresponds to a higher-ranked fuzzy number, and in cases where $\bar{x}$ is equal for two fuzzy numbers, $\bar{Y}$ should be utilized.

Wang et al. [10] illustrated that Wang and Lee's [9] approach was unable to distinguish between two fuzzy numbers sharing the same centroid point. They introduced the L-R deviation degree of a fuzzy number and proposed a ranking rule: the greater the left deviation degree and the smaller the right deviation degree, the higher the rank of the fuzzy number.

Nejad and Mashinchi [6] contended that the approach presented by Wang et al. [10] was not capable of correctly ranking fuzzy numbers in instances where either the left deviation degree, right deviation degree, or transfer coefficient of the fuzzy number is zero, or the transfer coefficient is one. They opted to use the areas on the left and right sides of fuzzy numbers for ranking. However, it's worth noting that this method fails to yield accurate rankings for symmetric fuzzy numbers, resulting in identical ranking orders [11].

Abbasbandy and Hajjari [12] proposed a ranking method for fuzzy numbers based on the center of gravity. Their approach is an adaptation of Wang and Lee's method [9]. Allahviranloo and Saneifard [13] utilized the concept of the center of gravity for the defuzzification of fuzzy numbers. Unfortunately, both Abbasbandy and Hajjari's method [12] and Allahviranloo and Saneifard's method [13] face difficulties in ranking symmetric fuzzy numbers relative to the y-axis.

We provide a comprehensive overview of various approaches to ranking fuzzy numbers, a critical aspect in decision-making under uncertainty. Overall, this literature review provides a comprehensive survey of the historical evolution, critiques, and refinements in the field of ranking fuzzy numbers, offering valuable insights for researchers and practitioners alike. To address the aforementioned limitations, this research presents a novel centroid point method that incorporates distance and area considerations for ranking fuzzy numbers.

3.Preliminaries

In this section, we briefly give some basic notions and important definitions that are related to our discussion.

Definition 1. A fuzzy subset u of the real line R with membership functions on $u(t): R \to [0,1]$ is called a fuzzy number if [14]:

(a) u is normal, i.e., there exists an element t_0 such that $u(t_0) = 1$,

(b) u is fuzzy convex, i.e., $u\{\lambda . t_1 + (1-\lambda). t_2\} \geq min\{u(t_1), u(t_2)\}$; $\forall t_1, t_2 \in R, \forall \lambda \in [0,1]$,

(c) $u\ (t)$ is upper semi-continuous,

(d) $supp\, u$ is bounded, where $supp\, u = cl\,\{t \in R: u(t) > 0\}$ and cl is a closure operator.

Definition 2. A fuzzy set $\tilde{A}$, defined on the universal set of real numbers R, is said to be a generalized fuzzy number if its membership function has the following characteristics [15]:

1.$\mu_{\tilde{A}}: R \to [0, w]$ is continuous,

2.$\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a\,] \cup [d, +\infty)$,

3. $\mu_{\tilde{A}}(x)$ strictly increasing on $[a,b]$ and strictly decreasing on $[c,d]$,

4.$\mu_{\tilde{A}}(x) = w$, for all $x \in [a,b]$, where $0 < w \leq 1$.

Definition 3. A generalized fuzzy number $\tilde{A} = (a,b,c,d;w)$ is said to be a generalized trapezoidal fuzzy number if its membership function is given by [16], as shown in Fig. 1:

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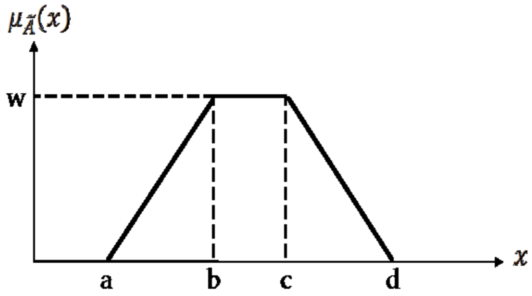

Fig 1. Generalized trapezoidal fuzzy number

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{w.(x-a)}{b-a} & a \le x \le b \\ w & b \le x \le c \\ \dfrac{w.(x-d)}{c-d} & c \le x \le d \end{cases}$$

Definition 4. A generalized fuzzy number $\tilde{A}$ where $\tilde{A} = (a, b, c; w)$ is called a generalized triangular fuzzy number if its membership function is given by [16], as shown in Fig. 2:$\mu_{\tilde{A}}(x)$

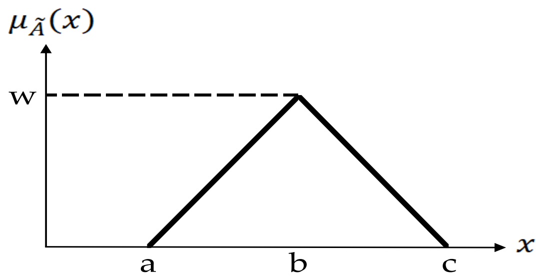

Fig 2. Generalized triangular fuzzy number

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{w.(x-a)}{b-a} & a \le x \le b \\ w & x = b \\ \dfrac{w.(x-d)}{c-d} & c \le x \le d \text{ where } 0 \le w \le 1 \end{cases}$$

Definition 5. A Fuzzy number $\tilde{A}$ is called non-negative if $\mu_{\tilde{A}}(x) = 0$, for all$x < 0$ [17] as shown in Fig3:

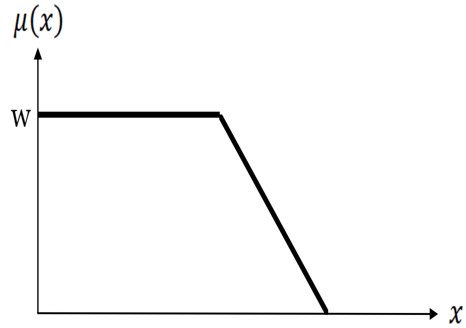

Fig 3. Nonnegative fuzzy number

Definition 6.A Fuzzy number $\tilde{A}$ is called non-positive, if$\mu_{\tilde{A}}(x) = 0$ for all$x > 0$,[17],as shown in Fig. 4:

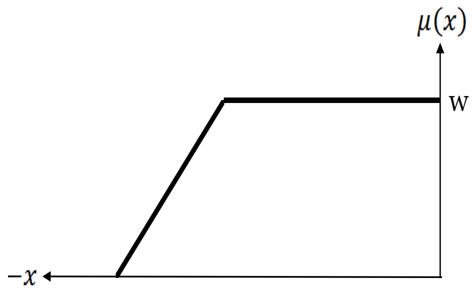

Fig 4.Non-positive fuzzy number

Definition 7. The area is the size or the magnitude of a two-dimensional shape. The entire surface or the whole floor of any geometric shape of a fuzzy number such as$\tilde{A} = (a_1, a_2, a_3, a_4; w_{\tilde{A}})$ can be calculated as follows [18]:

$$S(\tilde{A}) = \frac{1}{2} \times [(a_2 - a_1) + (a_4 - a_3)] \times w_{\tilde{A}} + (a_3 - a_2) \times w_{\tilde{A}} \qquad (1)$$

4.A Review of the Centroid Point Method

Definition 8. Assume there are n fuzzy numbers$\tilde{A}_1, \dots, \tilde{A}_n$ where$1 \le j \le n$.The centroid point of a fuzzy number $\tilde{A}_i$corresponded to a value$\bar{X}$ on the horizontal axis and a value $\bar{Y}$on the vertical axis. The centroid point$\{\bar{X}(\tilde{A}_i), \bar{X}(\tilde{A}_j)\}$ of a fuzzy number $\tilde{A}_i$ was defined as [7], [19]:

$$\bar{X}(\tilde{A}_i) = \frac{\int_a^b (x.\mu_{\tilde{A}_i}^L).dx + \int_b^c x.dx + \int_c^d (x.\mu_{\tilde{A}_i}^R).dx}{\int_a^b \mu_{\tilde{A}_i}^L.dx + \int_b^c dx + \int_c^d \mu_{\tilde{A}_i}^R.dx} \qquad (2)$$

$$\bar{X}(\tilde{A}_i) = \frac{\int_0^1 (y.g_{\tilde{A}_i}^L).dy + \int_0^1 (y.g_{\tilde{A}_i}^R).dy}{\int_0^1 g_{\tilde{A}_i}^L.dy + \int_0^1 g_{\tilde{A}_i}^R.dy} \qquad (3)$$

where $\mu_{\tilde{A}_i}^L$ and $\mu_{\tilde{A}_i}^R$ where the left and right membership functions of $\tilde{A}_i$ respectively, and $g_{\tilde{A}_i}^L$ and $g_{\tilde{A}_i}^R$ were inverse functionsof $\mu_{\tilde{A}_i}^L$ and $\mu_{\tilde{A}_i}^R$respectively. For a non-normal trapezoidal fuzzy number $\tilde{A} = (a, b, c, d; w)$ formulas lead to the following results, respectively [12]:

$$\bar{X}(\tilde{A}_i) = \frac{1}{3}\left\{a + b + c + d - \frac{c.d - a.b}{(c+d) - (a+b)}\right\} \qquad (4)$$

$$\bar{X}(\tilde{A}_i) = \frac{w}{3}\left\{1 + \frac{c - d}{(c+d) - (a+b)}\right\} \qquad (5)$$

Definition 9. Suppose$(\bar{X}, \bar{Y})$ is the center of gravity for the desired fuzzy number$\tilde{A}$. In this case, the distance of the center of gravity from the origin (original point) is obtained as follows [7]:

$$R(\tilde{A}) = \sqrt{(\bar{X})^2 + (\bar{Y})^2} \qquad (6)$$

By using $R(\tilde{A})$ given in (6), Cheng [7] can rank fuzzy numbers. The considered the larger the value of $R(\tilde{A})$, the better the ranking of $\tilde{A}$.

Definition 10. The area between the centroid point$\{\bar{X}(\tilde{A}), \bar{Y}(\tilde{A})\}$) and original point (0,0) of the fuzzy number $\tilde{A}$ was defined as [8]:

$$S(\tilde{A}) = \bar{X}(\tilde{A}).\bar{Y}(\tilde{A}) \qquad (7)$$

Chu and Tsao [8] ranked fuzzy numbers according to the area covered. They considered the larger the value of $S(\tilde{A})$, the better the ranking of $\tilde{A}$. Wang and Lee [9] proposed a revised method based on the Cho and Tsao's [8] method. They ranked the fuzzy numbers based on their $\bar{X}$ values if they are different. In instances where they exhibit equality, a further comparison is conducted based on their respective $\bar{Y}$'s values to establish their ranking.

Definition 11. Let E stand the set of non-normal fuzzy numbers, W be a constant provided that$\le w \le 1$ and $\gamma: E \to \{-w, 0, w\}$be a function that is defined as [12]:

$$\forall A \in E: \gamma(A) = Sign\left[\int_0^1 \{g_A^L(x) + g_A^R(x)\}.dx\right] \text{i.e.,}$$

$$\gamma(\tilde{A}) = \begin{cases} 1 & \inf(\text{supp}\,(\tilde{A})) \ge 0 \\ 0 & \sup\left(\text{supp}\,(\tilde{A})\right) < 0 \\ -1 & \tilde{A} = (a, b, d; w)\ and\ b + c = 0 \end{cases}$$

$$IR(\tilde{A}) = \gamma(\tilde{A}).\ \sqrt{\bar{X}(\tilde{A})^2 + \bar{Y}(\tilde{A})^2} \qquad (8)$$

Abbasbandy and Hajjari[12] used $IR(\tilde{A})$ to rank fuzzy numbers. They considered the larger the value of $IR(\tilde{A})$, the better the ranking of $\tilde{A}$.

Definition 12. Assume there are n fuzzy numbers$\tilde{A}_1, \dots, \tilde{A}_n$. The maximum crisp value$\tau_{max}$is defined as [13]:

$$\tau_{max} = max\{x | x \in Domain\ (\tilde{A}_1, \dots, \tilde{A}_n)\} \qquad (9)$$

Definition 13. The value$Dist(\tilde{A}_i)$ of the fuzzy numbers $\tilde{A}_i$, where $1 \le i \le n$, is defined as[13]:

$$Dist(\tilde{A}_i) = \sqrt{\left(\bar{x}_{\tilde{A}_i} - \tau_{max}\right)^2} + \sqrt{\bar{Y}_{\tilde{A}_i}^2} \qquad (10)$$

Now, by using $Dist(\tilde{A}_i)$ given in (9), for any two fuzzy numbers $\tilde{A}_i$ and $\tilde{A}_j$, Allahviranloo and Saneifard [12] order are determined based on the following rules:

- $Dist(\tilde{A}_1) < Dist(\tilde{A}_2)$ if, and only if$\tilde{A}_1 \succ \tilde{A}_2$,
- $Dist(\tilde{A}_1) > Dist(\tilde{A}_2)$ if, and only if $\tilde{A}_1 \prec \tilde{A}_2$,
- $Dist(\tilde{A}_1) = Dist(\tilde{A}_2)$ if, and only if $\tilde{A}_1 \sim \tilde{A}_2$.

Despite many efforts, most of the approaches with centroid points still have shortcomings. Now, the deficiency of the centroid point method is analyzed and discussed through the following examples.

Example 1. Consider fuzzy numbers$\tilde{A} = (2,3,4)$, $\tilde{B} = (6,7,8)$, as shown in Fig. 5.
According to Cheng's method[7], we have:

$$\left.\begin{array}{l}\bar{X}(\tilde{A}) = 3, \bar{Y}(\tilde{A}) = 0.33 \to R(\tilde{A}) = 3.02 \\ \bar{X}(\tilde{B}) = 7, \bar{Y}(\tilde{B}) = 0.33 \to R(\tilde{B}) = 7.01\end{array}\right\} \to \tilde{A} \prec \tilde{B}$$

$$\left.\begin{array}{l}\bar{X}(-\tilde{A}) = -3, \bar{Y}(-\tilde{A}) = 0.33 \to R(-\tilde{A}) = 3.02 \\ \bar{X}(-\tilde{B}) = -7, \bar{Y}(-\tilde{B}) = 0.33 \to R(-\tilde{B}) = 7.01\end{array}\right\} \to -\tilde{A} \prec -\tilde{B}$$

This method cannot give a reasonable ranking for fuzzy numbers and their images.

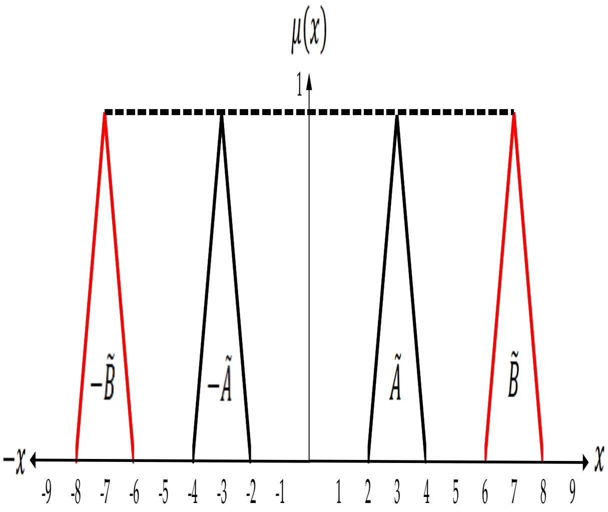

Fig 5. Fuzzy numbers$\tilde{A}$, $\tilde{B}$, $-\tilde{A}$, and $-\tilde{B}$in Example 1

Example 2. Consider fuzzy numbers$\tilde{A} = (1,2,3; 1)$, $\tilde{B} = (9,10,11; 0.1)$ adopted from[9], as shown in Fig. 6. Capously, $\tilde{A}$ is smaller than $\tilde{B}$. However, the ranking outcome by Chu and Tsao's [8] method is contrary to one's intuitions [9].

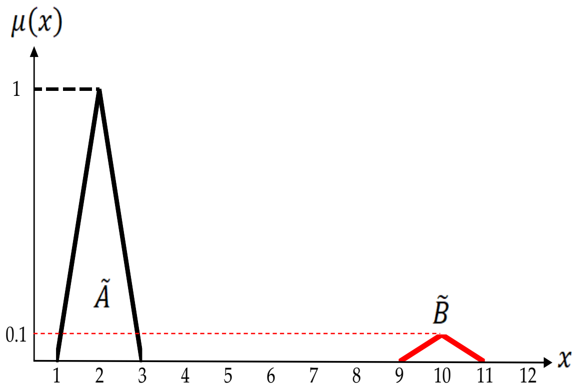

Fig 6.Fuzzy numbers $\tilde{A}$ and$\tilde{B}$ in Example 2

Example 3. Consider fuzzy numbers $\tilde{A} = (0.2,0.5,0.8)$, $\tilde{B} = (0.4,0.5,0.6)$ adopted from[10], as shown in Fig. 7. According to Wang and Lee's [9]method, we have:

$$\bar{X}(\tilde{A}) = \bar{X}(\tilde{B}) \text{ and } \bar{Y}(\tilde{A}) = \bar{Y}(\tilde{B}) \to \tilde{A} \sim \tilde{B}$$

This approach cannot differentiate two fuzzy numbers with the same centroid point [10].

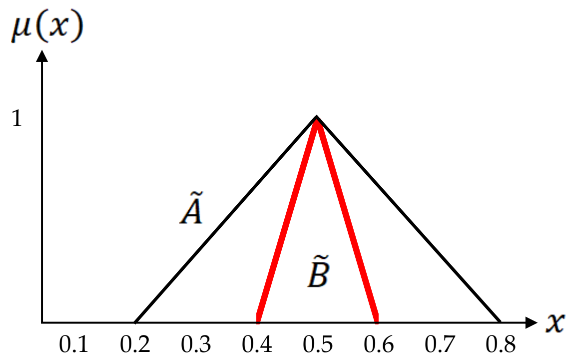

Fig 7. Fuzzy numbers $\tilde{A}$ and $\tilde{B}$ in Example 3

Example 4. Consider fuzzy numbers$\tilde{A} = (0.2,0.5,0.8)$, $\tilde{B} = (0.4,0.5,0.6)$ adopted from[9], as shown in Fig. 7. The ranking orders reported by Wang et al. [10] and NejadandMashinchi[6] are$\tilde{A} \prec \tilde{B}$ and $-\tilde{A} \prec -\tilde{B}$, which is illogical[11].
Wang et al.'s [10] and Nejad and Mashinchi's methods [6] cannot produce the correct ranking order under symmetric fuzzy number circumstances, and their methods lead to the same ranking orders.

Example 5. Let$\tilde{A} = (-1,0,1)$, $\tilde{B} = (-3,0,3)$be two triangular fuzzy numbers, as shown in Fig.8. The ranking orders reported by Abbasbandy and Hajjari [12] and Allahviranloo and Saneifard [13] are $\tilde{A} \sim \tilde{B}$. Thus, these methods cannot rank symmetric fuzzy numbers relative to the y-axis.

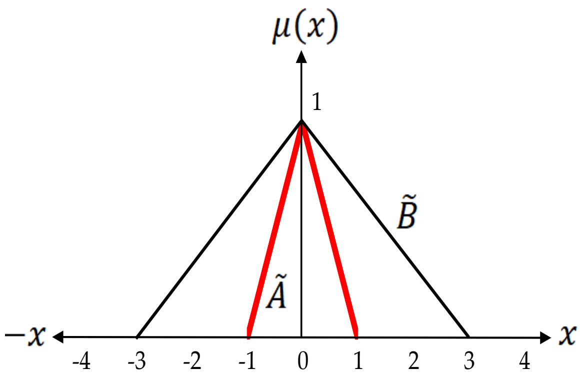

Fig 8.Fuzzy numbers $\tilde{A}$ and $\tilde{B}$ in Example 5

5.Ranking Fuzzy Numbers Based on the Center of the Area

This section introduces a new method for ranking generalized fuzzy numbers. By this method, we will resolve the shortcomings discussed in the previous section by proposing the following definition.

Definition 14. If an area lies in the$x - y$ plane and is bounded by the curve$y = f(x)$, as shown in Fig.9, then its centroid will be in this plane, and we obtain formulas located in the center of an area, namely, [20]:

$$\bar{X} = \frac{\int \tilde{x}.dA}{\int dA}, \quad \bar{Y} = \frac{\int \tilde{y}.dA}{\int dA} \tag{11}$$

These integrals can be evaluated by performing a single integration if we use a rectangular strip for the differential area element. For example, if we consider a horizontal strip, Fig.10, then$dA = x.dy$, and its centroid is located at$\tilde{x} = x/2$and$\tilde{y} = y$.

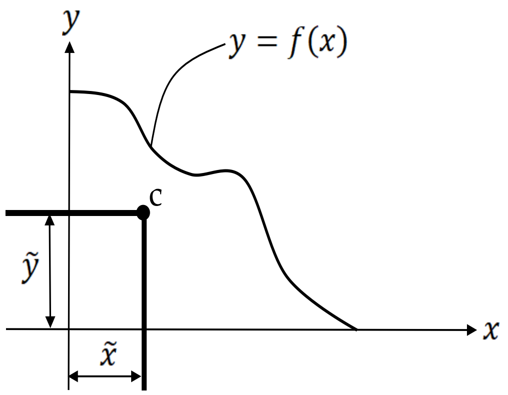

Fig 9.An area bounded by a curve in definition 8., [20]

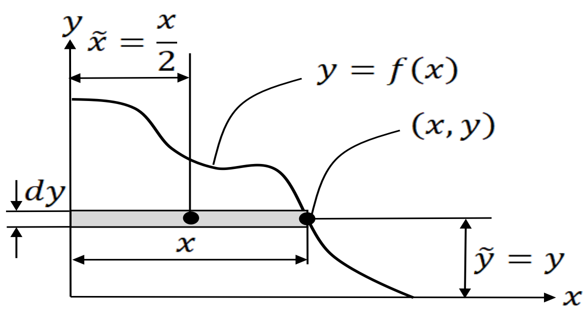

Fig 10.A horizontal strip in definition 8, [20]

Definition 15. A composite structure comprises interconnected elementary bodies, typically of rectilinear or right triangular configuration. Such a structure can frequently be decomposed into its constituent parts, with known weights and respective centroids. This information obviates the necessity for integration in the computation of the overall center of gravity of the composite body. [20].

Therefore, the center of gravity for the entire composite shape can be calculated as follows:

$$\bar{X} = \frac{\sum \tilde{x}.w}{\sum w}, \quad \bar{Y} = \frac{\sum \tilde{y}.w}{\sum w} \tag{12}$$

Note: the above formulas can also be used to get the area center of the compound shapes [20].

Theorem 1. Suppose b is the length, and h is the height of a given rectangle (Fig. 11). The center of the rectangle area is equal to the intersection of two diameters, and its coordinates are as follows:

$$\bar{X} = \frac{b}{2}, \quad \bar{Y} = \frac{h}{2} \tag{13}$$

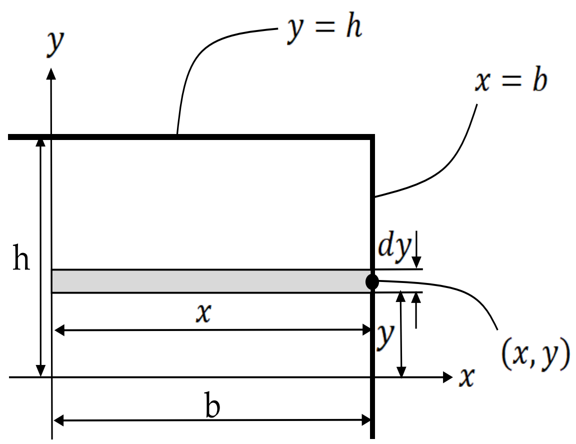

Fig11. Rectangular center area in Theorem 1

It is known that $dA = x.dy = b.dy$, $\tilde{y} = y$, and $\tilde{x} = {}^{b}/_{2}$. Then:

Proof:

$$\bar{X} = \frac{\int \tilde{x}.dA}{\int dA} = \frac{\int_0^h \frac{b}{2}.b.dy}{\int_0^h b.dy} = \frac{b^2.h/_2}{h.b} = \frac{b}{2}$$

$$\bar{Y} = \frac{\int \tilde{y}.dA}{\int dA} = \frac{\int_0^h y.b.dy}{\int_0^h b.dy} = \frac{b.h^2/_2}{h.b} = \frac{h}{2}$$

Theorem 2. Suppose h is the height of the right triangle, and b is its base (Fig. 12). In this case, the center coordinates are as follows:

$$\bar{X} = \frac{b}{3}, \quad \bar{Y} = \frac{h}{3} \tag{14}$$

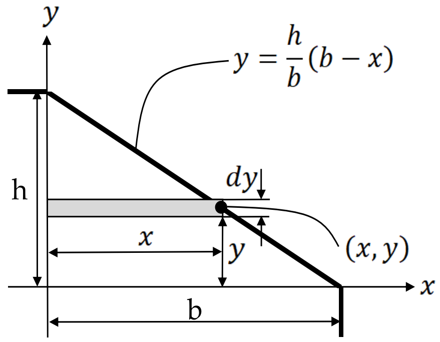

Fig 12: Right triangle area center in Theorem 2

Proof: It is known that $dA = x.dy = (b - y.b/h).dy$, $\tilde{y} = y$, and $\tilde{x} = x/2$. Then:

$$\bar{X} = \frac{\int \tilde{x}.dA}{\int dA} = \frac{\int_0^h \frac{x^2}{2}.dy}{\int_0^h x.dy} = \frac{\frac{1}{2}\int_0^h \left(b - \frac{y.b}{h}\right)^2.dy}{\int_0^h \left(b - \frac{y.b}{h}\right).dx} = \frac{h.b^2/6}{h.b/2} = \frac{b}{3}$$

$$\bar{Y} = \frac{\int \tilde{y}.dA}{\int dA} = \frac{\int_0^h y.\left(b - \frac{y.b}{h}\right)^2.dy}{\int_0^h \left(b - \frac{y.b}{h}\right).dx} = \frac{b.h^2/6}{h.b/2} = \frac{h}{3}$$

Note: $\bar{Y} = \frac{h}{3}$ is valid for any shape of the triangle [20].

Theorem 3: Now, assume that the right triangle (Fig. 13) is as follows:

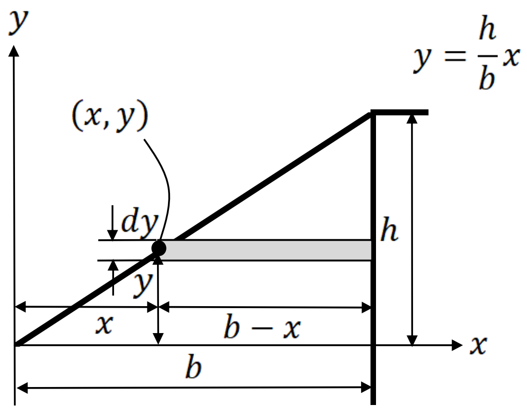

Fig. 13: Right triangle area center in Theorem 3.

It is known that $dA = (b - x).dy = (b - y.b/h).dy$, $\tilde{y} = y$, and $\tilde{x} = x + (b - x)/2 = (b + x)/2 = (b + y.b/h)/_2.dy$. Then:

$$\bar{X} = \frac{\int \tilde{x}.dA}{\int dA} = \frac{\int_0^h \left(b^2 - \frac{b^2.y^2}{h^2}\right)/_2.dy}{\int_0^h \left(b - \frac{y.b}{h}\right).dy} = \frac{h.b^2/3}{h.b/2} = \frac{2b}{3} \qquad (15)$$

Now, we propose the algorithm for ranking the fuzzy numbers,$\tilde{A}_1, \dots, \tilde{A}_n$ in$E$, based on the center of the area.

Algorithm:

(14)

Step 1: The first step is to calculate the area center of each generalized fuzzy number. Depending on whether the fuzzy number is nonnegative or non-positive, the following is applied:

$$\begin{cases} \bar{X} = X + \acute{X} \leftrightarrow if\ \tilde{A}\ is\ a\ non-negative\ fuzzy\ number\ (Figure\ 14) \\ \bar{X} = -X - \acute{X} \leftrightarrow if\ \tilde{A}\ is\ a\ non-positive\ fuzzy\ number\ (Figure\ 15) \end{cases} \qquad (16)$$
$$\bar{Y} = Y$$

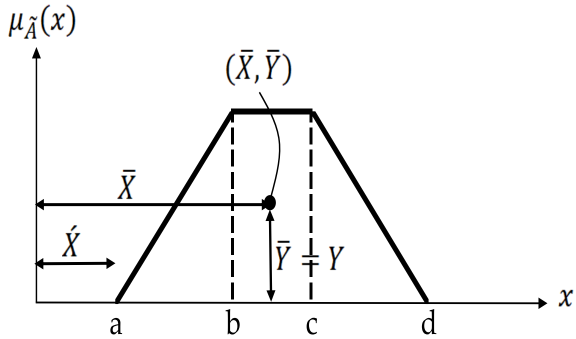

Fig. 14: Nonnegative fuzzy number

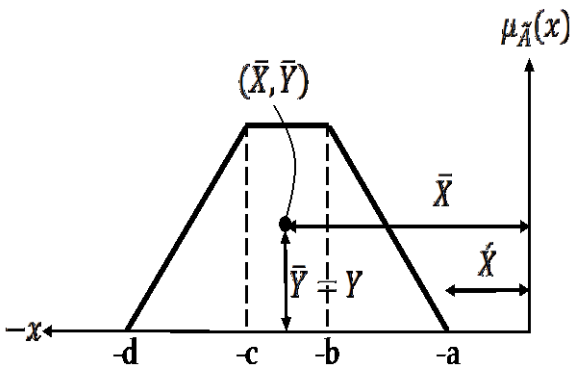

Fig 15. Non-positive fuzzy number.

Where (X, Y) is the area center of the fuzzy number that can be calculated by using the formulas mentioned in equation (11), and $\acute{X}$ is the least distance between the shapes of the fuzzy number from the origin of the coordinates. If a generalized fuzzy number is nonnegative then$\acute{X} = a$ (Figure 14) and if a generalized fuzzy number is non-positive then$\acute{X} = |d|$ (Figure 15).
Consider the scenario where the contour of the fuzzy number intersects the y-axis, implying that the fuzzy number is not strictly nonnegative or non-positive. Under such circumstances, the computation of the fuzzy number's area center proceeds as follows:

1) Divide the geometric contour of the fuzzy number into two distinct segments: the positive section and the negative section

2) Calculate the area of the positive section(S^+) and the negative section(S^-).

3) Calculate the area center of the positive$(\bar{X}^+, \bar{Y}^+)$and negative$(\bar{X}, \bar{Y})$ sections. For this purpose, divide the corresponding shape into simple shapes (right triangle and rectangles), calculate the area center of each shape, and finally obtain the area center of the positive and negative sections by using Equation 12.

4) The area center of the entire shape$(\bar{X}^T, \bar{Y}^T)$ is obtained by using the formulas in Definition 9 as follows:

$$\bar{X}^T = \frac{(S^+ \times \bar{X}^+) - (S^- \times \bar{X}^-)}{S^+ + S^-}$$
$$\bar{Y}^T = \frac{(S^+ \times \bar{Y}^+) + (S^- \times \bar{Y}^-)}{S^+ + S^-} \quad (17)$$

Note: Given the extensive application of general triangular and trapezoidal fuzzy numbers, the determination of their area centers is achieved through the following procedure: Partition the configuration of the generalized trapezoidal fuzzy number into two right triangles and a rectangle. Subsequently, compute the respective area centers of the rectangle and right triangles using the formulations provided in Theorems 1, 2, and 3. Finally, ascertain the trapezoidal area center employing Equation 12. Analogously, the area center of the generalized triangular fuzzy number is determined by segmenting the shape into two right triangles.

Step 2: Calculate the value of K according to the value of $\bar{X}$, as follows:

$$K = \begin{cases} -1 \ \ if \ \ \bar{X} < 0 \\ +1 \ \ if \ \ \bar{X} > 0 \end{cases} \quad (18)$$

Step 3: In this step, we calculate the R-value of the desired fuzzy number Ã by using the following formula:

$$R(\tilde{A}) = K_{\tilde{A}} \times \sqrt{\bar{X}^2 + \bar{Y}^2} \quad (19)$$

Step 4: Arrange the generalized fuzzy numbers in ascending order based on the value of R, where a higher R signifies a larger generalized fuzzy number. If two generalized fuzzy numbers possess identical R values, prioritize them by their respective areas. For example, suppose that $\tilde{A}$ and $\tilde{B}$ are two given generalized fuzzy numbers. Then:

$$\begin{aligned} &if \ \ R(\tilde{A}) > R(\tilde{B}) \to \tilde{A} > \tilde{B} \\ &if \ \ R(\tilde{A}) < R(\tilde{B}) \to \tilde{A} < \tilde{B} \\ &if \ \ R(\tilde{A}) = R(\tilde{B}) \to \tilde{A} \sim \tilde{B} \\ &if \ \ K_{\tilde{A}} \times S(\tilde{A}) > K_{\tilde{B}} \times S(\tilde{B}) \to \tilde{A} > \tilde{B} \\ &if \ \ K_{\tilde{A}} \times S(\tilde{A}) < K_{\tilde{B}} \times S(\tilde{B}) \to \tilde{A} < \tilde{B} \\ &if \ \ K_{\tilde{A}} \times S(\tilde{A}) = K_{\tilde{B}} \times S(\tilde{B}) \to \tilde{A} \sim \tilde{B} \end{aligned} \quad (20)$$

Now, using the algorithm, we solve Examples in section 3 and some other examples.

Example 6: Consider fuzzy numbers$\tilde{A} = (2,3,4)$, $\tilde{B} = (6,7,8)$in Example 1., as shown in Fig. 1. By using the proposed method, we rank these two fuzzy numbers as below:
At first, the calculations of the fuzzy number $\tilde{A}$are carried out.

Step 1: $\bar{X}_{\tilde{A}} = 3, \bar{Y}_{\tilde{A}} = 0.33$**,**
Step 2: $K = 1$**,**
Step 3: $R(\tilde{A}) = 1 \times \sqrt{3^2 + 0.33^2} = 3.02$
Now, calculations of the fuzzy number $\tilde{B}$ need to be done.

Step 1: $\bar{X}_{\tilde{B}} = 7, \bar{Y}_{\tilde{B}} = 0.33$**,**
Step 2: $K = 1$**,**
Step 3: $R(\tilde{B}) = 1 \times \sqrt{7^2 + 0.33^2} = 7.01$
Step 4:, $R(\tilde{A}) < R(\tilde{B})$, thus $\tilde{A} < \tilde{B}$),

Now, we rank the images of these fuzzy numbers by the proposed method:
At first, the calculations of the fuzzy number$-\tilde{A}$ are carried out.

Step 1: $\bar{X}_{-\tilde{A}} = -3, \bar{Y}_{-\tilde{A}} = 0.33$**,**
Step 2: $K = -1$**,**
Step 3: $R(-\tilde{A}) = -1 \times \sqrt{(-3)^2 + 0.33^2} = -3.02$

Now, calculations of the fuzzy number $-\tilde{B}$ need to be done.

Step 1: $\bar{X}_{-\tilde{B}} = -7, \bar{Y}_{-\tilde{B}} = 0.33$**,**
Step 2: $K = -1$**,**
Step 3: $R(-\tilde{B}) = 1 \times \sqrt{(-7)^2 + 0.33^2} = -7.01$
Step 4: $R(-\tilde{A}) > R(-\tilde{B})$, thus $-\tilde{A} > -\tilde{B}$),

The proposed method can rank fuzzy numbers and their images logically. Thus, the proposed method can overcome the shortcomings of Cheng's [7]method.

Example 7: Consider fuzzy numbers$\tilde{A} = (1,2,3;1)$, $\tilde{B} = (9,10,11;0.1)$in Example 2, as shown in Fig. 2. By using the proposed method, we rank these two fuzzy numbers as below:

At first, the calculations of the fuzzy number$\tilde{A}$ are carried out.

Step 1: $\bar{X}_{\tilde{A}} = 2, \bar{Y}_{\tilde{A}} = 0.33$,
Step 2: $K = 1$,
Step 3: $R(\tilde{A}) = 1 \times \sqrt{2^2 + 0.33^2} = 2.03$

Now, calculations of the fuzzy number $\tilde{B}$need to be done.

Step 1: $\bar{X}_{\tilde{B}} = 10, \bar{Y}_{\tilde{B}} = 0.033$,
Step 2: $K = 1$,
Step 3: $R(\tilde{B}) = 1 \times \sqrt{10^2 + 0.033^2} = 10.01$
Step 4: $R(\tilde{A}) < R(\tilde{B})$, thus $\tilde{A} < \tilde{B}$),

The ranking outcome of the proposed method is the same as one's intuitions. Thus, the proposed method can overcome the shortcomings of Chu and Tsao's[8]method.

Example 8: Consider fuzzy numbers $\tilde{A} = (0.2,0.5,0.8)$, $\tilde{B} = (0.4,0.5,0.6)$ in example 3, as shown in Fig. 3. By using the proposed method, we rank these two fuzzy numbers as below. At first, the calculations of the fuzzy number $\tilde{A}$ are carried out.
Step 1: $\bar{X}_{\tilde{A}} = 0.5, \bar{Y}_{\tilde{A}} = 0.33$,
Step 2: $K = 1$,
Step 3: $R(\tilde{A}) = 1 \times \sqrt{0.5^2 + 0.33^2} = 0.6$
Now, calculations of the fuzzy number $\tilde{B}$ need to be done.
Step 1: $\bar{X}_{\tilde{B}} = 0.5, \bar{Y}_{\tilde{B}} = 0.33$,
Step 2: $K = 1$,
Step 3: $R(\tilde{B}) = 1 \times \sqrt{0.5^2 + 0.33^2} = 0.6$
Step 4: $R(\tilde{A}) = R(\tilde{B})$.So,in this case, we use the area to rank fuzzy numbers.

$K_{\tilde{A}} \times S(\tilde{A}) = 0.3 > K_{\tilde{B}} \times S(\tilde{B}) = 0.1$, thus $\rightarrow \tilde{A} > \tilde{B}$.

The proposed method can rank fuzzy numbers with the same centroid point. Thus, the proposed method can overcome the shortcoming of Wang and Lee's [9] method.
Now, we rank the images of these fuzzy numbers by the proposed approach:
At first, the calculations of the fuzzy number $-\tilde{A}$ are carried out.

Step 1: $\bar{X}_{-\tilde{A}} = -0.5, \bar{Y}_{-\tilde{A}} = 0.33$,
Step 2: $K = -1$,
Step 3: $R(-\tilde{A}) = -1 \times \sqrt{(-0.5)^2 + 0.33^2} = -0.6$

Now, calculations of the fuzzy number $-\tilde{B}$ need to be done.

Step 1: $\bar{X}_{-\tilde{B}} = -0.5, \bar{Y}_{-\tilde{B}} = 0.33$,
Step 2: $K = -1$,
Step 3: $R(-\tilde{B}) = 1 \times \sqrt{(-0.5)^2 + 0.33^2} = -0.6$
Step 4:, $R(-\tilde{A}) = R(-\tilde{B})$.
So, in this case, we use the area to rank fuzzy numbers.

$K_{\tilde{A}} \times S(-\tilde{A}) = -0.3 < K_{\tilde{B}} \times S(-\tilde{B}) = -0.1$, thus $\rightarrow -\tilde{A} < -\tilde{B}$.

The proposed method can rank symmetric fuzzy numbers and their images logically. Thus, the proposed method can overcome the shortcomings of Nejad and Mashinchi's[6] method.

Example 9: Let$\tilde{A} = (-1,0,1)$, $\tilde{B} = (-3,0,3)$ be two triangular fuzzy numbers, as shown in Example 4 and Fig. 8. By using the proposed method, we rank these two fuzzy numbers as below. At first, the calculations of the fuzzy number $\tilde{A}$ are carried out.

Step 1: $\bar{X}_{\tilde{A}} = 0, \bar{Y}_{\tilde{A}} = 0.33$,
Step 2: $K = 1$,
Step 3: $R(\tilde{A}) = 1 \times \sqrt{0^2 + 0.33^2} = 0.33$

Now, calculations of the fuzzy number $\tilde{B}$ need to be done.

Step 1: $\bar{X}_{\tilde{B}} = 0, \bar{Y}_{\tilde{B}} = 0.33$,
Step 2: $K = 1$,
Step 3: $R(\tilde{B}) = 1 \times \sqrt{0^2 + 0.33^2} = 0.33$
Step 4: $R(\tilde{A}) = R(\tilde{B})$, So, in this case, we use the area to rank fuzzy numbers.

$K_{\tilde{A}} \times S(\tilde{A}) = 1 < K_{\tilde{B}} \times S(\tilde{B}) = 3$, thus $\rightarrow \tilde{A} < \tilde{B}$.

The proposed method can rank symmetric fuzzy numbers relative to the y-axis. Thus, the proposed method can overcome the shortcomings of Allahviranloo and Saneifard's [13] and Abbasbandy and Hajjari's [12] methods.

Example 10: Consider the data used in [3], i.e., the three normal fuzzy numbers $\tilde{A} = (5,6,7)$, $\tilde{B} = (5.9,6,7)$, and $\tilde{C} = (6,6,7)$ as shown in Fig. 16.By using the proposed method, we have:
$R(\tilde{A}) = 6.009$, $R(\tilde{B}) = 6.303$, and $R(\tilde{C}) = 6.338$R. Thus$\tilde{A} < \tilde{B} < \tilde{C}$. This is consistent with the ranking obtained by other approaches ([8], [10],[21], [22], [23], [24]). Table 1 summarizes the results obtained by different methods. It is imperative to observe that the ordering $\tilde{A} > \tilde{B} > \tilde{C}$, determined by the coefficient of variation (CV) index proposed by Cheng [7], is considered to be illogical and incongruent with human intuition ([21], [3]).

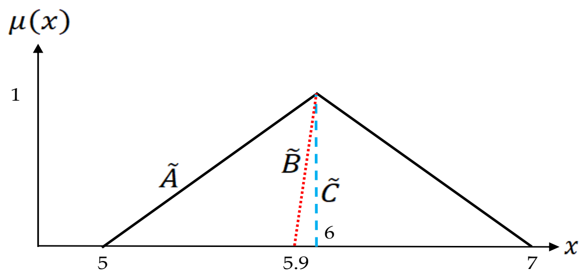

Fig. 16: Fuzzy numbers $\tilde{A}$, $\tilde{B}$, and $\tilde{C}$ in Example 10.

Table 1

The results of ranking fuzzy numbers in example 10

Ranking Approach	$\tilde{A}$	$\tilde{B}$	$\tilde{C}$	Ranking
Wang et al. [10]	0.2500	0.5339	0.5625	$\tilde{A} < \tilde{B} < \tilde{C}$
Wang and Lue [21]	0.5000	0.5710	0.5830	$\tilde{A} < \tilde{B} < \tilde{C}$
Asady [22]	0.6667	0. 8181	1.0000	$\tilde{A} < \tilde{B} < \tilde{C}$
Chen [23]	0.5000	0.5714	0.5833	$\tilde{A} < \tilde{B} < \tilde{C}$
Sign distance (p = 1) [24]	6.1200	12.4500	12.5000	$\tilde{A} < \tilde{B} < \tilde{C}$
Sign distance (p = 2) [24]	8.5200	8.8200	8.8500	$\tilde{A} < \tilde{B} < \tilde{C}$
Cheng [7]	6.0210	6.3490	6.7519	$\tilde{A} < \tilde{B} < \tilde{C}$
Abbasbandy and Hajjari[12]	6.0000	6.0750	6.0834	$\tilde{A} < \tilde{B} < \tilde{C}$
Our proposed method	6.0090	6.3030	6.3380	$\tilde{A} < \tilde{B} < \tilde{C}$

Example 11: Consider the following four sets of fuzzy numbers, proposed by Yao and Wu[25] (refer to [14]).
Set 1:$\tilde{u} = (0.4,0.5,1.0), \tilde{v} = (0.4,0.7,1.0), \tilde{z} = (0.4,0.9,1.0)$,
Set 2: $\tilde{u} = (0.3,0.4,0.7,0.9), \tilde{v} = (0.3,0.7,0.9), \tilde{z} = (0.5,0.7,0.9)$,
Set 3:$\tilde{u} = (0.3,0.5,0.7), \tilde{v} = (0.3,0.5,0.8,0.9), \tilde{z} = (0.3,0.5,0.9)$,
Set 4:$\tilde{u} = (0.0, 0.4,0.7,0.8), \tilde{v} = (0.2,0.5,0.9), \tilde{z} = (0.1,0.6,0.8)$,
The ranking results of Set 1, Set 2, Set 3, and Set 4 are given in Table 2. The details of our discussion for this example are given below.

In Set 1:$R(\tilde{u}) = 0.7159, R(\tilde{v}) = 0.7752, R(z) = 0.8353$. By Eq. (20) we have $R(\tilde{u}) < R(\tilde{v}) < R(\tilde{z})$. The conclusion is $\tilde{u} < \tilde{v} < \tilde{z}$. The ranking for our proposed method is the same as the other methods. But by the Cheng CV index[7], we have $\tilde{u} < \tilde{z} < \tilde{v}$ (see Figure 17).

In Set 2: $R(\tilde{u}) = 0.729, R(\tilde{v}) = 0.716, R(\tilde{z}) = 0.775$. By Eq. (20) we have $R(\tilde{v}) < R(\tilde{u}) < R(\tilde{z})$. The conclusion is $\tilde{v} < \tilde{u} < \tilde{z}$. Cheng CV Uniform Distribution method [7] and Cheng CV Proportional Distribution method [7] is $\tilde{z} < \tilde{v} < \tilde{u}$ which is shortcoming for this method [14] while the ranking order for Baldvin and Guild method[26] is $\tilde{u} \sim \tilde{v} < \tilde{z}$ and for the other methods including our proposed method and Rouhparvar and Panahi's [14] method (see Fig. 18).

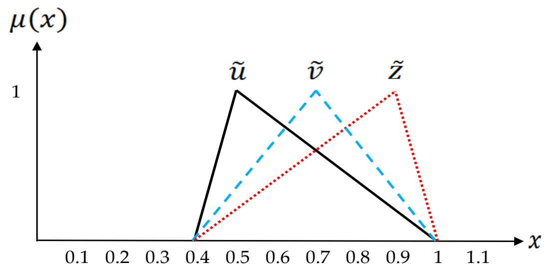

Fig. 17: Fuzzy numbers$\tilde{u}$, $\tilde{v}$, $\tilde{z}$ in Example 11, set 1

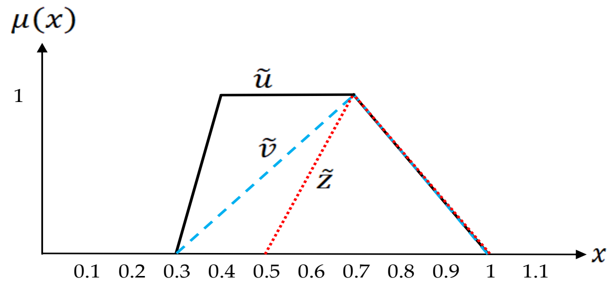

Fig. 18: Fuzzy numbers $\tilde{u}$, $\tilde{v}$, and $\tilde{z}$ in Example 11, set 2.

In Set 3:$R(\tilde{u}) = 0.601, R(\tilde{v}) = 0.764, R(\tilde{z}) = 0.715$. By Eq. (20) we have $R(\tilde{u}) < R(\tilde{z}) < R(\tilde{v})$. The conclusion is $\tilde{u} < \tilde{z} < \tilde{v}$. The result of Chen[23], Baldvin, and Guild[26]is $\tilde{u} < \tilde{v} < \tilde{z}$. The result of Chu and Tsao[8], Sign distance method[24], Length incentre point method, Cheng distance [7], Cheng CV uniform and Cheng CV proportional distribution method [7], are the same as our new method (see Fig. 19).

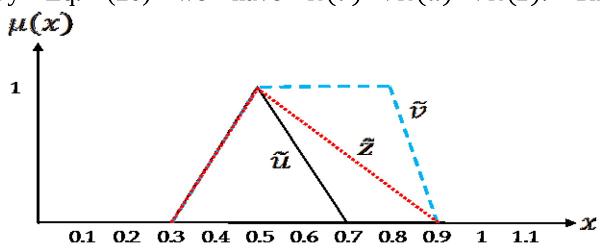

Fig. 19: Fuzzy numbers $\tilde{u}$, $\tilde{v}$, and $\tilde{z}$ in Example 11, set 3.

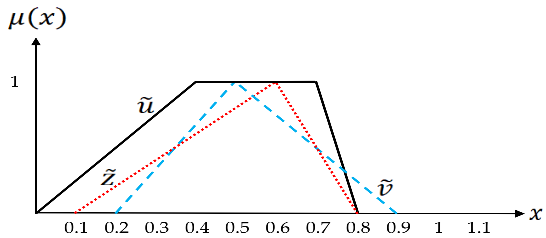

Fig. 20. Fuzzy numbers $\tilde{u}$, $\tilde{v}$, and $\tilde{z}$ in Example 11, set 4.

In Set 4: $R(\tilde{u}) = 0.62854, R(\tilde{v}) = 0.62847, R(\tilde{z}) = 0.6006$. By Eq. (20) we have $R(\tilde{u}) > R(\tilde{v}) > R(\tilde{z})$. The conclusion is $\tilde{z} < \tilde{v} < \tilde{u}$.The result of the Cheng CV uniform and Cheng CV proportional distribution method [7] is $\tilde{v} < \tilde{z} < \tilde{u}$It is easy to see that the ranking results for $\tilde{z}$ and $\tilde{v}$, obtained by these methods are unreasonable and are not consistent with human intuition (see Fig. 20).

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Vol 1, Issue 3, Summer 2024 , 35-47

Table 2
Comparative results of Example 11

Authors	**Fuzzy number**	**Set 1**	**Set2**	**Set 3**	**Set 4**
Choobineh and Li [27]	$\tilde{u}$	0.333	0.458	0.333	0.500
	$\tilde{v}$	0.500	0.583	0.417	0.583
Results	$\tilde{z}$	0.667	0.667	0.542	0.611
		$\tilde{u} < \tilde{v} < \tilde{z}$	$\tilde{u} < \tilde{v} < \tilde{z}$	$\tilde{u} < \tilde{v} < \tilde{z}$	$\tilde{u} < \tilde{v} < \tilde{z}$
	$\tilde{u}$	0.600	0.572	0.500	0.450
Yager [5]	$\tilde{v}$	0.700	0.650	0.550	0.525
	$\tilde{z}$	0.800	0.700	0.625	0.550
Results		$\tilde{u} < \tilde{v} < \tilde{z}$	$\tilde{u} < \tilde{v} < \tilde{z}$	$\tilde{u} < \tilde{v} < \tilde{z}$	$\tilde{u} < \tilde{v} < \tilde{z}$
	$\tilde{u}$	0.338	0.432	0.375	0.520
Chen [23]	$\tilde{v}$	0.500	0.563	0.425	0.570
	$\tilde{z}$	0.667	0.625	0.550	0.625
Results		$\tilde{u} < \tilde{v} < \tilde{z}$	$\tilde{u} < \tilde{v} < \tilde{z}$	$\tilde{u} < \tilde{v} < \tilde{z}$	$\tilde{u} < \tilde{v} < \tilde{z}$
	$\tilde{u}$	0.30	0.27	0.27	0.40
Baldvin and Guild[26]	$\tilde{v}$	0.33	0.27	0.37	0.42
	$\tilde{z}$	0.44	0.37	0.45	0.42
Results		$\tilde{u} < \tilde{v} < \tilde{z}$	$\tilde{u} \sim \tilde{v} < \tilde{z}$	$\tilde{u} < \tilde{v} < \tilde{z}$	$\tilde{u} < \tilde{v} \sim \tilde{z}$
	$\tilde{u}$	0.299	0.285	0.250	0.2440
Chu and Tsao[8]	$\tilde{v}$	0.350	0.325	0.315	0.2624
	$\tilde{z}$	0.399	0.350	0.275	0.2619
Results		$\tilde{u} < \tilde{v} < \tilde{z}$	$\tilde{u} < \tilde{v} < \tilde{z}$	$\tilde{u} < \tilde{z} < \tilde{v}$	$\tilde{u} < \tilde{z} < \tilde{v}$
	$\tilde{u}$	0.6	0.575	0.500	0.475
Yao and Wu [25]	$\tilde{v}$	0.7	0.650	0.625	0.525
	$\tilde{z}$	0.8	0.700	0.550	0.525
Results		$\tilde{u} < \tilde{v} < \tilde{z}$	$\tilde{u} < \tilde{v} < \tilde{z}$	$\tilde{u} < \tilde{z} < \tilde{v}$	$\tilde{u} < \tilde{v} \sim \tilde{z}$
	$\tilde{u}$	1.2	1.15	1,00	0.95
Sign distance method [24]	$\tilde{v}$	1.4	1.30	1.25	1.05
P = 1	$\tilde{z}$	1.6	1.40	1.10	1.05
Results		$\tilde{u} < \tilde{v} < \tilde{z}$	$\tilde{u} < \tilde{v} < \tilde{z}$	$\tilde{u} < \tilde{z} < \tilde{v}$	$\tilde{u} < \tilde{v} \sim \tilde{z}$
	$\tilde{u}$	0.8869	0.8756	0.7257	0.7853
Sign distance method [24]	$\tilde{v}$	1.0194	0.9522	0.9416	0.7958
P = 2	$\tilde{z}$	1.1605	1.0033	0.8165	0.8386
Results		$\tilde{u} < \tilde{v} < \tilde{z}$	$\tilde{u} < \tilde{v} < \tilde{z}$	$\tilde{u} < \tilde{z} < \tilde{v}$	$\tilde{u} < \tilde{v} < \tilde{z}$
	$\tilde{u}$	0.6435	0.5713	0.5000	0.4930
Length incentre point	$\tilde{v}$	0.7000	0.6286	0.6287	0.5335
method (I_X)[14]	$\tilde{z}$	0.7565	0.7000	0.5714	0.4991
Results		$\tilde{u} < \tilde{v} < \tilde{z}$	$\tilde{u} < \tilde{v} < \tilde{z}$	$\tilde{u} < \tilde{z} < \tilde{v}$	$\tilde{u} < \tilde{z} < \tilde{v}$
	$\tilde{u}$	0.7900	0.7577	0.7071	0.7106
Cheng distance [7]	$\tilde{v}$	0.8602	0.8149	0.8037	0.7256
	$\tilde{z}$	0.9268	0.8602	0.7458	0.7241
Results		$\tilde{u} < \tilde{v} < \tilde{z}$	$\tilde{u} < \tilde{v} < \tilde{z}$	$\tilde{u} < \tilde{z} < \tilde{v}$	$\tilde{u} < \tilde{z} < \tilde{v}$
	$\tilde{u}$	0.0272	0.0328	0.0133	0.0693
Cheng CV uniform	$\tilde{v}$	0.0214	0.0246	0.0304	0.0385
distribution [7]	$\tilde{z}$	0.0225	0.0095	0.0275	0.0433
Results		$\tilde{v} < \tilde{z} < \tilde{u}$	$\tilde{z} < \tilde{v} < \tilde{u}$	$\tilde{u} < \tilde{z} < \tilde{v}$	$\tilde{v} < \tilde{z} < \tilde{u}$
	$\tilde{u}$	0.0183	0.026	0.0080	0.0471
Cheng CV proportional	$\tilde{v}$	0.0128	0.146	0.0234	0.0236
distribution [7]	$\tilde{z}$	0.0137	0.057	0.0173	0.0255
Results		$\tilde{v} < \tilde{z} < \tilde{u}$	$\tilde{u} < \tilde{z} < \tilde{v}$	$\tilde{u} < \tilde{z} < \tilde{v}$	$\tilde{v} < \tilde{z} < \tilde{u}$
	$\tilde{u}$	0.7159	0.729	0.601	0.62854
Our propose method	$\tilde{v}$	0.7752	0.716	0.764	0.62847
	$\tilde{z}$	0.8353	0.775	0.715	0.60060
Results		$\tilde{u} < \tilde{v} < \tilde{z}$	$\tilde{v} < \tilde{u} < \tilde{z}$	$\tilde{u} < \tilde{z} < \tilde{v}$	$\tilde{z} < \tilde{u} \sim \tilde{v}$

Finally, a substantial body of contemporary scholarship has put forth a diverse array of methodologies for ranking Generalized Fuzzy Numbers [28,29]. And alternative ranking methods [30]. In recent years, there has been a notable surge in scholarly interest and attention towards the ranking of fuzzy sets [31-32]. In this manuscript, we have introduced a novel technique for prioritizing generalized fuzzy numbers, employing the centroid of the enclosed area as a key criterion. Our proposed method demonstrates a high level of effectiveness in systematically ranking a wide range of fuzzy numbers and their corresponding representations. Furthermore, it offers a straightforward means of ranking symmetric fuzzy numbers with the y-axis. The instances provided in this study serve to highlight the distinctive attributes of our proposed approach. When compared to established methodologies, it stands as a reasonable and highly efficient alternative. We posit that future research could explore the integration of the area centroid metric with other indices put forth in existing methods. This extension may facilitate the ranking of various forms of fuzzy numbers, such as intuitionistic fuzzy numbers, hesitant fuzzy numbers, and the like.

Conclusion

In this paper, we proposed a new method for ranking generalized fuzzy numbers based on the center of the area. The proposed method can effectively rank various fuzzy numbers and their images logically. Additionally, the proposed method can rank symmetric fuzzy numbers relative to the y-axis easily. The examples given in this paper illustrate that the proposed approach has distinct characteristics and compared with the existing approaches, it is reasonable and efficient. Combining the area center index with the other indexes presented in other methods, using the area center in the ranking of other kinds of fuzzy numbers such as intuitionistic fuzzy numbers, hesitant fuzzy numbers, etc, can be considered as future research. Looking ahead, an exciting avenue for future research lies in the integration of the area center index with complementary metrics from other established methodologies. This collaborative approach holds the potential to enhance the precision and versatility of fuzzy number ranking.

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