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Joint Inspecting Interval Optimization and Redundancy Allocation Problem Optimization for Cold-Standby Systems with Non-Identical Components

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Abstract

In this paper, we study a redundancy allocation problem. The investigated problem has a system with *s* serially connected subsystems, which are under periodic inspection. Each subsystem has a 1-out-of-n cold-standby configuration with non-identical components. In each subsystem, component failures are diagnosed by a perfect switching system, and the first component on the standby queue starts working as a replacement for the failed component. This procedure will be continued until the last standby component gets in the service and fails during an inspection interval. The failures of the components are detected at inspection. The failed component(s) will be repaired during the next inspection interval and added to the standby queue. The subsystems can be in different states depending on their working component and the order of the components on the standby queue. We present an approach to calculate the subsystems-states transition probabilities. We minimize the subsystem's expected total cost by determining the optimal number of components and the optimal subsystem's inspection intervals. The expected total cost consists of downtime, repair, and inspection costs of the subsystems per unit time. Then, we determine the optimum allocated components to each subsystem under some constraints to find the optimal system inspection cost per unit time.

Keywords: Redundancy allocation problem, Periodic inspection, Inspection interval, Transition probabilities, Standby configuration, Markov theory.

1.Introduction

Redundancy allocation problem (RAP) and inspection interval optimization (IIO) are two important issues in production systems. Long inspection intervals will increase the System's failure cost, while short intervals will increase the System's inspection cost. Therefore, obtaining optimal inspection intervals is a cost-effective policy (Sharifi & Taghipour,2022). Determining the optimal configuration of the components inside a manufacturing system is another important issue that can improve system reliability. RAP is a well-known problem in reliability engineering (Sharifi & Sayyad, 2022)

In most cases, these two issues are investigated individually; however, they are interrelated. In this paper, we worked on a joint RAP and IIO problem and optimized both issues simultaneously. The under-studied System is a series-parallel system in which the subsystems are connected serially, and the components in each subsystem are connected in parallel. The components are considered non-identical to draw the problem near to real-case problems.

We first adopt the general formulas to calculate the subsystems' reliability. Next, we optimize the duration of the inspection interval and the System's configuration simultaneously, using a two-stage approach. In the first stage, we optimize the inspection interval of the components in each subsystem. In the second stage, considering the results of the first stage, we optimize the subsystem's reliability in terms of the redundant components in each subsystem.

In terms of inspection interval optimization, Taghipour et al., (2010) presented a model to find the optimal inspection intervals for complex repairable systems with two types of failure. A recursive approach was used to calculate the failure probability in every interval and expected downtimes. Nourelfath et al., (2012) considered a series-parallel system with imperfect preventive maintenance. A Markov process and a Universal Generating Function (UGF) algorithm were used to

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evaluate the System's availability and cost function. The entire solution space was initially partitioned into a set of subspaces, then Genetic Algorithms (GA) were used to select the best subspace and solution. In this study, the components' repair time was ignored in the proposed model to simplify it.

Mendes et al., (2014) developed a set of reliability formulas using conditional probabilities. The considered System has four different configurations, and the objective was to find the optimal inspection intervals. These configurations included an active redundancy system with non-repairable components, an active redundancy system with repairable components, a cold-standby redundant system with non-repairable components, and a cold-standby redundant System with repairable components. Conditional probabilities were also used to find the System's states probabilities. Due to the computational complexity of the developed models, systems with only two or three components were considered. Mendes et al., (2017) extended their previous work and used discrete-time Markov chains to define the transition probabilities of different system states. The optimal interval between inspections was obtained for multi-state redundant systems, considering the availability and costs of maintenance and production. Taghipour and Kassaei (2015) worked on a k-out-of-n load-sharing system with identical components. They assumed that the load of the failed component was distributed to the remaining components. In their model, the System is inspected periodically, and based on the number of failed components, they considered two different cases and developed a model to find the optimal inspection interval Their model aimed to minimize the total System expected cost. A simulation algorithm was presented to find the expected values in the objective function. Zhao and Nakagawa (2015) proposed three new inspection models. For each model, the total expected inspection and downtime costs were calculated, and the optimal policies that minimize the calculated costs were derived.

Zhao et al., (2016) compared periodic times and repair numbers for different policies in a replacement problem and presented a modified replacement model. Rezaei (2015) proposed a new reliability model by considering minimal and perfect repairs and optimizing the System's inspection interval. He considered a repairable system that consists of a rotor and filter with failure interactions. Huang et al., (2018) presented an advanced Bayesian analysis to determine appropriate non-periodic inspection intervals of fatigue-sensitive structures. He used the Bayesian approach to calculate the probability density function of the uncertain parameters and estimated the system reliability accurately, even with some uncertain parameters. The considered System contained a specific number of elements at only one fatigue-critical location. Yeh et al., (2019) modeled the failures and repairs as a continuous-time Markov chain to calculate the system reliability for the production systems with underlying serial structures. They optimized the redundancy and the frequency of inspection and maintenance tasks to maximize the System's profit.

Sharifi and Taghipour (2020) optimized a k-out-of-n System's inspection interval with non-identical components and a cold-standby configuration. They used a new method to optimize the System's inspection interval over a finite time horizon. Sharifi et al., (2021) optimized the inspection interval of a k-out-of-n system with load-sharing identical components whit a mixed redundancy strategy for deploying the components. Later, they extended their work by considering a condition-based approach to optimize the System's inspection interval (Sharifi et al., 2022)

In terms of RAP, different studies have been conducted by considering different assumptions (Teimouri & et al., 2016; Khorshidi & et al., 2016; Sharifi & et al., 2016; Gholinezhad & et al., 2017; Kim & Kim, 2016; Mellal & Salhi, 2021; Sharifi & et al., 2018; Kim, 2018; Sharifi & et al, 2019; Yeh, 2018; Huang & et al., 2019; Ouyang & et al., 2018; Sharifi & et al., 2019; Mousavi & et al.,2019; Hadipour & et al., 2019; Pourkarim & et al., 2018; Zaretalab & et al 2020; Wang & et al., 2020; Yeh, 2021; Chambari & et al., 2021; Zaretalab & et al., 2022; Reihaneh & et al., 2021)

This paper fills the gap in the current literature by considering repair action for systems with non-identical equipment/components. In this paper, we focus on a system with s serially connected subsystems. The subsystems act like one of the systems presented by Mendes et al., (2014) and we extend the general formula and calculate transition probabilities between different subsystems-states based on Markov theory. The subsystems are 1-out-of-n and contain non-identical components. We consider a periodic inspection problem for the subsystems. The failure of the operating components is only detected at inspection unless the last component fails and there is no standby component, and the subsystem shuts down. We find the optimal allocated components to each subsystem and optimal inspection interval for each subsystem to minimize total system inspection cost per unit time (ICPT).

We organize the contents of this paper as follows: an introduction to the problem along with a discussion of the related work is addressed in Section 1. In Section 2, we define the configuration and assumptions of the subsystem and discuss the proposed approach for calculating the transition probabilities. Section 3 deals with the calculation of cost matrixes. In this section, we also provide details about the calculation method of optimal inspection intervals for the subsystems. In Section 4, we present the redundancy allocation problem as well as some numerical examples to demonstrate the usefulness of the offered model. Lastly, Section 6 presents our conclusions and highlights some directions for further studies.

2.Systems Description and Transition Probabilities Formulation

For a system with the configuration described in *Section 1*, we first define the system states at the beginning of each inspection interval. Then we present a state-space diagram, as well as an exact formula for calculating the transition probabilities between all the states, $P_{i,j}(t)$, in which i is the system state at the beginning of the inspection interval and j is the system state at the end of the inspection interval with the related transition matrix. The formulas for the transition probabilities are easy to calculate for any number of components, *n*, in the System. The system assumptions are presented in *Section 2-1,* and the transition probabilities formulating procedure.

2.1.Assumptions

The system assumptions are as follows:

- The System is 1-out-of-n with *n* non-identical components,
- The failure of each component is independent of other components,
- The failure rate of the components is constant; therefore, the lifetime of the components follows an exponential distribution, and
- The failure rate of the i^{th} component is equal to λ_i.

According to the above-mentioned assumptions, the working probability of a component during the inspection interval by duration τ is equal to $e^{-\lambda_i \times t}$, and its probability of failure is equal to $\left(1 - e^{-\lambda_i \times t}\right)$.

2.2.System Configurations and Transition Probabilities

In this part, we first discuss the transition probabilities and transition matrix. Then, we present the transition probabilities formulas of the System.

2.2.1.Transition Probabilities and Rransition Matrix

In the presented problem, the working component and the order of the components on the standby queue at the beginning of an inspection interval are considered the system state. In the rest of the paper, when we use the term "inspection interval," we refer to the beginning of the inspection interval. After defining system states, the main objective is to calculate the transition probabilities between different system states during the inspection interval. The transition matrix in this problem is a matrix whose elements are the system transition probabilities.

2.2.2.Subsystems-States Transition Probabilities

In the investigated System (subsystems), one component works at the start of the inspection time interval by duration, and other components are in the standby queue. During the inspection interval, a standby component starts working using a perfect switch after the failure of a working component. The failed component at inspection intervals is observed, and the failed component is repaired during the next inspection interval. The repaired component is then added to the System at the end of the next inspection interval, and the new component is placed at the end of the standby components queue. We consider that component repair time is less than the inspection interval. After the System's failure (i.e., all its components fail), the system failure will be detected at the end of the inspection interval. In this case, all failed components will be repaired during the next inspection interval, and the System will start working at the end of that interval. Figure 1 is the schematic operation of System III with three components.

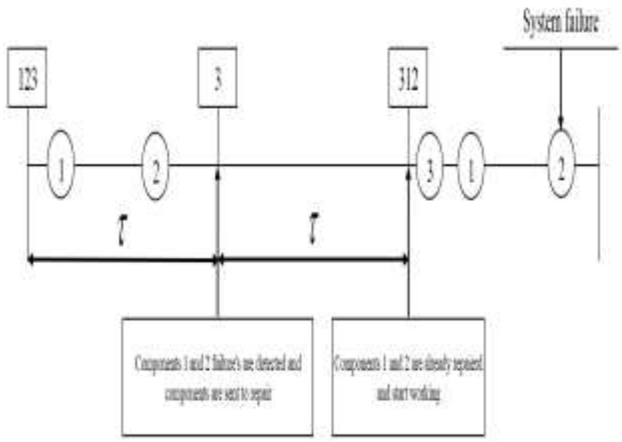

Fig. 1: Schematic operation of System III

In this model, each state is defined by an ordered pair such as (w, k) in which w is the working component and k is the total number of components that are not failed (working and standby components). The reachable states from the state (w, k) can be determined using Equations (1-3) as follows:

$$(w,k)\rightarrow\left(w+s-n\left[\frac{w+s-1}{n}\right],n-s\right)\quad;\quad s=0,1,\ldots,k\quad;\quad k\neq n \tag{1}$$

$$(w,n)\rightarrow\left(w+s-n\left[\frac{w+s-1}{n}\right],n-s\right)\quad;\quad s=0,1,\ldots,n-1 \tag{2}$$

$$(w,n)\rightarrow f\quad;\quad s=n \tag{3}$$

In equations (1) to (3), s is the number of failed components during an inspection interval, and f is the state where the System stops working. The transition matrix for this System has (n^2+1) rows and columns. Let's assume in this matrix first row and column belong to the state $(1,n)$, the second row and column belong to the state $(1,n-1)$, …, row and column n^2 belong to the state $(n,1)$, and the last row and column belong to the state that the System fails (f). So, in the transition matrix, the state (i,k) is addressed to row and column number $(i.n-k+1)$. We can calculate all elements of the transition matrix as follows:

$$p_{(w.n-k+1),(w.n-n+1)}(\tau)=e^{-\lambda_w.\tau}\quad;\quad\begin{cases}w=1,2,\ldots,n\\k=1,\ldots,n\\s=0\end{cases} \tag{4}$$

$$p_{(w.n),\left\{\left(w+1-n\left[\frac{w}{n}\right]\right).n-n+2\right\}}(\tau)=1-e^{-\lambda_w.\tau}\quad;\quad\begin{cases}w=1,\ldots,n\\k=1\\s=k=1\end{cases} \tag{5}$$

$$p_{(w.n-k+1),\left\{\left(w+s-n\left[\frac{w+s-1}{n}\right]\right).n-n+s+1\right\}}(\tau)=$$

$$\left\{\prod_{m=w}^{w+s-1}\lambda_{\left\{m-n\left[\frac{m-1}{n}\right]\right\}}\right\}.\sum_{m=w}^{w+s}\frac{e^{-\lambda_{\left\{m-n\left[\frac{m-1}{n}\right]\right\}}.\tau}}{\prod_{\substack{v=w\\k\neq m}}^{w+s}\left\{\lambda_{\left\{v-n\left[\frac{v-1}{n}\right]\right\}}-\lambda_{\left\{m-n\left[\frac{m-1}{n}\right]\right\}}\right\}}\quad;\quad\begin{cases}w=1,\ldots,n\\k=2,\ldots,n-1\\s=1,\ldots,k-1\end{cases} \tag{6}$$

$$p_{(w.n-k+1),\left\{\left(w+s-n\left[\frac{w+s-1}{n}\right]\right).n-n+s+1\right\}}(\tau)=1-\sum_{\substack{j=1\\j\neq\left(w+s-n\left[\frac{w+s-1}{n}\right]\right).n-n+s+1}}^{n^2+1}p_{i,j}(\tau)\quad;\quad\begin{cases}w=1,\ldots,n\\k=1,\ldots,n-1\\s=k\end{cases} \tag{7}$$

$$P_{(w.n-n+1),(n^2+1)}(\tau)=1-\sum_{j=1}^{2^n}p_{ij}(\tau)\quad;\quad i=1,\ldots,n \tag{8}$$

$$P_{(w.n-k+1),(n^2+1)}(\tau)=0\quad;\quad k=1,\ldots,n-1 \tag{9}$$

and for calculating the last row the transition matrix, we can use Equations (10) and (11) as follows:

$$p_{(2^n+1),1}(\tau)=1 \tag{10}$$

$$p_{(2^n+1),j}(\tau)=0 \quad ; \quad \forall j=2,\dots,n \tag{11}$$

3.Cost Functions and Inspection Interval Optimization

We consider four different costs based on the system states at the beginning and the end of each inspection interval, which can be defined as follows are provided in reference (Huang, 2018).

3.1.System Cost

The investigated System (subsystems) has a standby configuration with the component repair during the next inspection interval. The cost matrix has (n^2+1)rows and columns. The cost function for each inspection interval depends on the system state at the beginning of that interval and define as follows:

- *Rule 1*: If, at the beginning of an interval, the System is in state $\{(w,k);\ \ w,k=1,\dots,n;\}$, it means that $(n-k)$ components fail during the previous interval, and the failed components are repaired during the present interval.
  Rule 1-1: If during this interval less than k components fail, the System still is in working condition, and the system cost is equal to $\{C_{ins}+(n-k)\times C_r\}$.
  Rule 1-2: $(k=n)$ and if all components fail during this inspection interval, the System fails, and we have system downtime. In this condition and at the end of the inspection interval, we do not have any available (repaired) component, and the System moves to state f. The System's downtime is equal ρ, and the system cost in this condition is equal to $\{C_{ins}+\rho\times C_p\}$.
  Rule 1-3: If $(k<n)$ and all components fail during this inspection interval, the System fails, and we have system downtime. The System's downtime is equal $\rho_{(w.n-k+1)}$ and the system cost in this condition is equal to $\{C_{ins}+(n-k)\times C_r+\rho\times C_p\}$.
- *Rule 2*: If the System starts working (i.e., it's all components are working) and then fails during the previous inspection interval, at the current inspection interval, all failed components will be repaired, and the System starts working at the end of the present inspection interval. So, the system cost in this condition is equal to $\{C_s+n\times C_r+\tau\times C_p\}$.

Therefore, the system cost is calculated as presented in Equations (12-15).

$$C_{(w.n-k+1),\left\{\left(w+s-n\left[\frac{w+s-1}{n}\right]\right).n-n+s+1\right\}}=C_{ins}+(n-k).C_r \quad ; \begin{cases} w=1,\dots,n \\ k=1,\dots,n \\ s=0,\dots,k-1 \end{cases} \tag{12}$$

$$C_{(w.n-k+1),\left\{\left(w+k-n\left[\frac{w+k-1}{n}\right]\right).n-n+k+1\right\}}=C_{ins}+(n-k).C_r+C_p.\rho_{(i.n-k+1)} \quad ; \begin{cases} w=1,\dots,n \\ k=1,\dots,n-1 \\ s=k \end{cases} \tag{13}$$

$$C_{(w.n-n+1),(n^2+1)}=C_{ins}+C_p.\rho_{(i.n-n+1)} \quad ; \begin{cases} w=1,\dots,n \\ k=n \\ s=k \end{cases} \tag{14}$$

$$C_{(n^2+1),1} = nC_r + C_s + C_p.\tau \quad (15)$$

The cost matrix is a matrix in which its elements are system state cost.

3.2.Inspection Interval Calculation

Assume that the system mission horizon is equal to T_h. If we plan to inspect the System each τ time interval, the number of inspections during the System's mission horizon is calculated as presented in Equation (16).

$$m = \left\lfloor \frac{T_h}{\tau} \right\rfloor \quad ; \quad \tau = 1,2,\ldots,T_h \quad (16)$$

Assuming $\pi_{k,i}$ is a $1 \times (n^2+1)$ dimensional matrix in which $\{\pi_{k,i}(\tau); \quad i = 1,\ldots,n^2+1\}$ is the probability that the System is in state i at the beginning of the k^{rd} inspection interval when each inspection interval is equal to τ. So, we have:

$$\pi_{1,i}(\tau) = [1 \quad 0 \quad \cdots \quad 0] \quad ; \quad i = 1,\ldots,n^2+1 \quad (17)$$

$$\pi_{k,i}(\tau) = \pi_{1,i}(\tau) \times P^{k-1}(\tau) = \pi_{k-1,i}(\tau) \times P(\tau) \quad ; \quad k = 1,2,\ldots,m \ ; \ i = 1,\ldots,n^2+1 \quad (18)$$

Consider that:

$$C_i(\tau) = \sum_{j=1}^{n^2+1} p_{ij}(\tau).c_{ij}(\tau) \quad ; \quad i = 1,2,\ldots,n^2+1 \quad (19)$$

In which $p_{i,j}(\tau)$ and $c_{i,j}(\tau)$ are the (i,j) element of P (transition matrix) and C (cost matrix), respectively. $C_i(\tau)$ represents each inspection interval cost, if the System starts from state i at the beginning of the inspection interval. So, the total present worth of the system cost is equal to:

$$Total\ Present\ worth\ of\ cost = \sum_{k=1}^{m}\sum_{i=1}^{n^2+1}\left(P/F, \%r, k\right).\pi_{k,i}(\tau).C_i(\tau) \quad (20)$$

In equation (20), r is the daily interest rate and $(P/F, r\%, k)$ calculates the present worth of k^{rd} interval cost. The inspection cost per unit time (ICPT) is equal:

$$ICPT = \frac{(Total\ Present\ worth\ of\ cost) + \sum_{i=1}^{n} C_{pur}(i)}{T_h} \quad (21)$$

In equation (30), $C_{pur,i}$ is the purchasing price of component number i.

4.Redundancy Allocation Problem

The redundancy allocation problem is one of the most practical problems in reliability, as presented by Fyffe et al. [38] in 1968. This problem aims to maximize system reliability under system cost and weight constraints. In this section, we present a redundancy allocation problem with s parallel subsystems. The subsystems are under periodic inspection under the assumption presented in *Section 2*. The presented problem aims to minimize system inspection cost per unit time (ICPT). The decision variable in this problem is the number of allocated components to each subsystem among different available components. We consider that a limited budget is available to purchase the initial components at the beginning of the System's mission horizon. The mathematical problem is presented as follows:

$$Min\ Z = \sum_{i=1}^{s}\sum_{j=1}^{N_i} CPUT_{i,j}.x_{i,j} \quad (22)$$

$$\sum_{i=1}^{s}\sum_{j=1}^{N_i} w_{i,j}.x_{i,j} \le W \quad (23)$$

$$\sum_{i=1}^{s}\sum_{j=1}^{N_i} c_{i,j}.x_{i,j} \le C \quad (24)$$

$$\sum_{j=1}^{N_i} x_{i,j} = 1 \quad ; \quad \forall i \tag{25}$$

$$N_i = 2^{n_i} - 1 \tag{26}$$

$$1 \le n_i \le n_{Max,i} \quad ; \quad \forall i \tag{27}$$

$$x_{i,j} \in \{0, 1\} \quad ; \quad \forall i, j \tag{28}$$

For calculating the values of $ICPT_{i,j}$, for the subsystems, we consider all combinations and for each combination, the value of $ICPT_{i,j}$ can be calculated by using the procedure presented in section 4-1.

4.1.Numerical Example for Inspection Interval Optimization

To demonstrate the usefulness of the presented formulas in calculating the transition and cost matrices, we consider a system with 3 components and calculate the transition and cost matrixes using the related Equations. Next, we consider a system with different components and calculate the best inspection interval and system cost. In this System, the mission horizon is 3650 days (10 years) and $C_{ins} = 200$, $C_r = 200$, $C_p = 1000$, $C_s = 500$ and $C_{Pur,1} = 2000$. Also, the failure rate and the purchasing price of components are calculated as follows:

$$\lambda_i = 0.015 + 0.005(i-1) \quad ; \quad i = 1,2,\ldots,10 \tag{29}$$

$$C_{Pur}(i) = 2000 - 5(i-1) \quad ; \quad i = 1,2,\ldots,10 \tag{30}$$

We calculate the optimum system ICPT and the optimum inspection interval for two examples as follows:

1. Example 1: monthly interest rate $r = 0\%$, the component purchasing cost is not considered.
2. Example 2: monthly interest rate $r = 0.25\%$, the component purchasing cost is considered.

Table 1
 contains the optimal ICPT, inspection interval (days) and the number of inspections during the System's horizon (10 years) for the two examples. The ICPT of both examples is presented in figure 2.

Component number	Example 1			Example 2		
	Number of inspections	Inspection interval (Days)	ICPT	Number of inspections	Inspection interval (Days)	ICPT
1	1216	3	136.7990	1216	3	118.4917
2	456	8	43.9619	521	7	42.5546
3	260	14	23.9892	260	14	24.8250
4	173	21	16.9181	173	21	18.6641
5	125	29	13.7440	130	28	16.1279
6	110	33	12.0695	110	33	14.9851
7	86	42	11.1296	86	42	14.5566
8	82	44	10.5648	82	44	14.4552
9	68	53	10.2258	68	53	14.5757
10	62	58	10.0257	62	58	14.8094

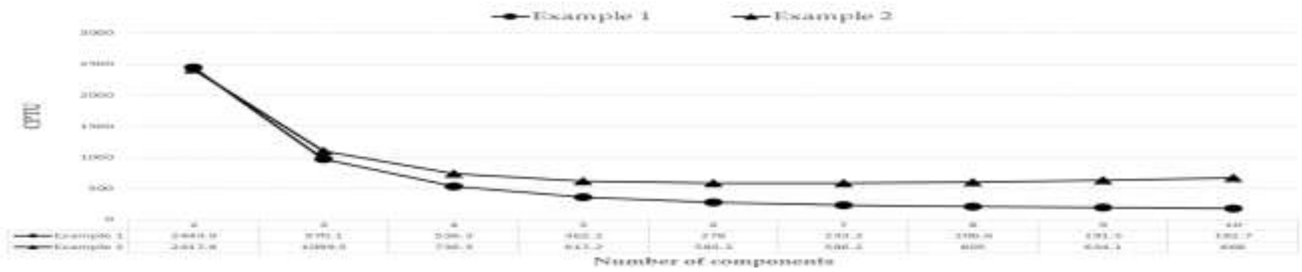

Fig. 2. Optimal ICPT for two examples.

As we expected, when we do not consider the component's purchasing price, by increasing the system components, the system ICPT decreases. The system reliability during each inspection interval depends on the number of components in the System, and when we do not spend any money on purchasing the components, the System applies more components (as it is available). But, if we consider the components purchasing price (as in example 2), the System with 8 components has a better ICPT compared to the System with more or fewer components.

4.2.A Numerical Example of the Redundancy Allocation Problem

We consider a system with 7 subsystems, which are under periodic inspection. We consider that for each subsystem, five different components type are available to allocate. So, the total combinations of components are equal to $\left(2^5 - 1 = 31\right)$. The other subsystem parameters are presented in Table 2.
For calculating the failure rate of each component in all subsystems, we used Equation (29), and for calculating the weight of each component in all subsystems, we used equation (31) as follow:

$$w_{i,j} = w_{1,j} - (i-1),\ i = 1,2,\ldots,5 \tag{31}$$

Table 2
Initial parameters for the presented example.

	$C_{Pur}(1)$	C_{ins}	C_r	C_P	C_s	$Lambda_1$	w_1
Subsystem-1	2000	200	200	1000	500	0.015	40
Subsystem-2	1250	200	150	750	350	0.025	25
Subsystem-3	1000	200	250	1250	550	0.015	40
Subsystem-4	1500	200	120	1150	450	0.035	20
Subsystem-5	2000	200	100	1500	650	0.015	45
Subsystem-6	1500	200	125	1100	450	0.030	25
Subsystem-7	1250	200	150	850	550	0.010	15

For each subsystem with the parameters presented in Table 2, we first calculate the subsystem ICPT using the procedure presented in sections 4-1. The results for ICPTs are presented in Table 3. Also, the weight and initial cost of each combination for each subsystem are calculated and presented in Tables 4 and 5 consequently. In these tables, the first column represents the subsystems combination index (j), and the second column defines the combinations of the components.

Table 3
Subsystems ICPT for all component combinations.

Combination index	Components combination	Subsystem						
		1	2	3	4	5	6	7
1	1	118.4917	128.2100	130.6647	177.2464	139.0017	161.8461	92.1866
2	2	133.8090	136.9224	146.4036	183.3865	157.2757	174.0635	110.1055
3	3	146.1139	145.3955	161.7002	189.4396	175.0361	181.6412	126.0866
4	4	158.0770	153.6388	176.5720	195.4075	184.3680	187.4915	137.4979
5	5	169.7119	161.6610	176.5583	201.2917	192.6977	193.2595	147.9211
6	12	42.5546	49.9217	45.6637	67.9935	46.5575	61.7714	31.6539
7	13	45.3651	52.2267	48.9351	70.2001	49.6016	64.5101	34.4636
8	14	48.0082	54.4186	51.5328	72.3323	52.5122	66.6076	36.9005
9	15	50.0473	56.4391	53.9960	74.3971	54.8076	68.4053	38.8176
10	23	49.7679	55.4614	53.6446	73.0416	54.3447	67.3416	39.2655
11	24	52.4662	57.6268	56.8730	75.4293	57.1191	69.4709	42.0474
12	25	55.0214	59.5025	59.2937	77.7402	59.7867	71.5268	44.2874
13	34	56.5525	60.2656	60.8009	78.4093	61.2642	72.2169	46.0283
14	35	58.9934	62.4010	63.7617	80.9558	64.3336	74.5197	48.6406
15	45	62.3576	65.1560	67.9637	83.2105	67.4766	77.3977	52.0807
16	123	24.8250	29.2493	26.1192	38.7629	25.2865	35.1090	18.0584
17	124	25.7650	30.1345	27.2324	39.7270	26.2836	36.1028	18.9258
18	125	26.6120	30.9595	28.1229	40.5705	27.0752	36.8579	19.6794
19	134	27.1676	31.2825	28.6846	40.7984	27.5576	37.1318	20.2543
20	135	28.0319	32.0592	29.5933	41.5667	28.5136	37.9552	21.0086
21	145	29.1577	33.0642	30.8823	42.5102	29.6763	38.9885	22.0485
22	234	29.1786	32.6987	30.9052	42.0010	29.5605	38.4949	22.4021
23	235	30.1073	33.5970	32.0603	42.8569	30.5508	39.4277	23.3009
24	245	31.4236	34.5668	33.4725	43.9068	31.8120	40.5965	24.5072
25	345	33.2519	35.8614	35.4694	45.2235	33.4703	41.7726	26.3927
26	1234	18.6641	21.4247	19.2837	27.4737	17.8454	24.9413	13.3284
27	1235	19.1104	21.8533	19.7319	27.9329	18.2602	25.3690	13.7041
28	1245	19.7216	22.3798	20.3446	28.4068	18.8157	25.9082	14.2190
29	1345	20.4808	23.0021	21.2654	28.9891	19.5763	26.6037	14.9371
30	2345	21.7667	23.8864	22.7435	29.7471	20.7284	27.3964	16.2940
31	12345	16.1279	17.7195	16.2957	22.1594	14.6947	20.2204	11.2684

Table 4
Subsystems purchasing cost for all component's combination.

Combination index	Components combination	Subsystem						
		1	2	3	4	5	6	7
1	1	2000	1250	1000	1500	2000	1500	1250
2	2	1950	1200	950	1450	1950	1450	1200
3	3	1900	1150	900	1400	1900	1400	1150
4	4	1850	1100	850	1350	1850	1350	1100
5	5	1800	1050	800	1300	1800	1300	1050
6	12	3950	2450	1950	2950	3950	2950	2450
7	13	3900	2400	1900	2900	3900	2900	2400
8	14	3850	2350	1850	2850	3850	2850	2350
9	15	3800	2300	1800	2800	3800	2800	2300
10	23	3850	2350	1850	2850	3850	2850	2350
11	24	3800	2300	1800	2800	3800	2800	2300
12	25	3750	2250	1750	2750	3750	2750	2250
13	34	3750	2250	1750	2750	3750	2750	2250
14	35	3700	2200	1700	2700	3700	2700	2200
15	45	3650	2150	1650	2650	3650	2650	2150
16	123	5850	3600	2850	4350	5850	4350	3600
17	124	5800	3550	2800	4300	5800	4300	3550
18	125	5750	3500	2750	4250	5750	4250	3500
19	134	5750	3500	2750	4250	5750	4250	3500
20	135	5700	3450	2700	4200	5700	4200	3450
21	145	5650	3400	2650	4150	5650	4150	3400
22	234	5700	3450	2700	4200	5700	4200	3450
23	235	5650	3400	2650	4150	5650	4150	3400
24	245	5600	3350	2600	4100	5600	4100	3350
25	345	5550	3300	2550	4050	5550	4050	3300
26	1234	7700	4700	3700	5700	7700	5700	4700
27	1235	7650	4650	3650	5650	7650	5650	4650
28	1245	7600	4600	3600	5600	7600	5600	4600
29	1345	7550	4550	3550	5550	7550	5550	4550
30	2345	7500	4500	3500	5500	7500	5500	4500
31	12345	9500	5750	4500	7000	9500	7000	5750

Table 5
subsystems weight for all component combinations.

Combination index	Components combination	Subsystem						
		1	2	3	4	5	6	7
1	1	40	25	40	20	45	25	15
2	2	39	24	39	19	44	24	14
3	3	38	23	38	18	43	23	13
4	4	37	22	37	17	42	22	12
5	5	36	21	36	16	41	21	11
6	12	79	49	79	39	89	49	29
7	13	78	48	78	38	88	48	28
8	14	77	47	77	37	87	47	27
9	15	76	46	76	36	86	46	26
10	23	77	47	77	37	87	47	27
11	24	76	46	76	36	86	46	26
12	25	75	45	75	35	85	45	25
13	34	75	45	75	35	85	45	25
14	35	74	44	74	34	84	44	24
15	45	73	43	73	33	83	43	23
16	123	117	72	117	57	132	72	42
17	124	116	71	116	56	131	71	41
18	125	115	70	115	55	130	70	40
19	134	115	70	115	55	130	70	40
20	135	114	69	114	54	129	69	39
21	145	113	68	113	53	128	68	38
22	234	114	69	114	54	129	69	39
23	235	113	68	113	53	128	68	38
24	245	112	67	112	52	127	67	37
25	345	111	66	111	51	126	66	36
26	1234	154	94	154	74	174	94	54
27	1235	153	93	153	73	173	93	53
28	1245	152	92	152	72	172	92	52
29	1345	151	91	151	71	171	91	51
30	2345	150	90	150	70	170	90	50
31	12345	190	115	190	90	215	115	65

We also consider that $W = 25000$ and $C = 500$. Then, we solve the presented example using Lingo 11.0 software.

The optimal value $x_{i,j}$ for this example using a full-enumeration technique is calculated as:

$$\left(x_{1,16}=1,\ x_{2,16}=1,\ x_{3,4}=1,\ x_{4,5}=1,\ x_{5,16}=1,\ x_{6,17}=1,\ x_{7,26}=1\right) \quad (32)$$

It means that the optimal subsystems' components combinations are:

$$\left\{(1,2,3),(1,2,3),(4),(5),(1,2,3),(1,2,3),(1,2,3,4)\right\} \quad (33)$$

Also, the System's weight is equal to 24728, the system components purchasing cost is equal to 500, and the optimal system ICPT is equal to 127.7982.

5.Conclusion and Further Studies

In this paper, we worked on a redundancy allocation problem. The subsystems have standby configuration with components repair. In these subsystems, the components were non-identical and had a constant failure rate. The subsystems were under periodic inspection, and the failure of the components was detected at inspection intervals. We calculated the optimal ICPT and inspection interval for the subsystems as well as the optimal number of subsystem components. Finally, we solved the mentioned redundancy allocation problem using Lingo 11.0 software.

Since the calculation of the transition probabilities is complicated, we considered a discrete time inspection policy. For future work, the problem can be drawn near real conditions by considering the continuous time inspection model. Moreover, time-dependent failure rates can be considered.

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