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Presenting a Multi-Objective Mathematical Model for Designing a Logistics Network with Transfer Pricing and Transportation Cost Allocation: A Robust Optimization Approach

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Abstract

Today, to satisfy the needs of customers in the supply chain, there have been considered the design and optimization of the logistic networks. The transfer pricing is one of the most important and the most complex issues that multinational companies faced to it. This article provides a multi-objective mathematical model in order to design a logistic network by considering the transfer pricing and the transportation cost allocation. There has been used the mixed integer nonlinear programming to model the problem. This network has three levels: the supplier, distribution center and the retailer. To deal with the uncertainty in the parameters of the model, there has been used the robust optimization approach and eventually phased solution approach by TH method.

Keywords: Transfer pricing, Logistic Network Design, Robust Optimization, TH Method.

1.Introduction

One of the main issues raised in the integrated logistic system is the optimal network structure that can minimize network structure costs during physical distribution and effective techniques for solving the logistic network design (Zhang & Xu, 2014). The transfer pricing is one of the most controversial issues of the multinational companies. The transfer pricing is a price that the purchasing department pays to the sales department of that company in order to obtain a product (Perron et al., 2010).

In order to avoid the arbitrary manipulation of the transfer prices by the multinational companies, most governments have adopted the transfer pricing regulations based on the arm's length principle and there are different methods to exploit the arm's length principle. One of these methods is taking into account the acceptable lower and upper limits for the transfer prices from an origin to a destination within a specific period represented the compatible possible intervals with the arm's length principle and the exact amount of the transfer price is determined by the model (Hammami et al., 2009).

Vidal and Goestschalckx (1997) identified some gaps and opportunities in order to investigate the methodology of the strategic and tactical designs of the global logistical systems. A lot of relevant researches on the international factors such as the transportation mode selection, the transportation cost allocation among the subsets and the non-linear effect on the international taxation were ignored and many models of the global supply chain had assumed that the transfer price is fixed and is given. Vidal and Goestschalckx (2001) investigated the transfer price in their article. Due to their point of view, the transfer price is one of the main issues in the maximization of the profits of the multinational corporations. They noted that the transfer pricing methods can have the significant impact on the taxable income, charges and profit after tax of the multinational corporations. They delivered a model to optimize the global supply and maximize the profits after tax of the multinational corporations included the transfer prices and the transportation price allocation as the variables in explicit making decision. Due to the complexity of the model, there has been used the meta-heuristic method to solve the problem.

Goestschalckx et al. (2002) focused, in their article, on the integration and combination of the previous researches and identified the new ideas on the network design of the global supply chain to determine the distribution-production allocation and the transfer prices. Qin et al. (2009) have proposed two issues about making decisions on the logistic multi-product field: inventory control and facility location. These two issues have been combined and the logistic network design problem has been formed with regard to the retailer demand. In this problem, the demand has been considered the uncertainty and it follows the normal probability distribution. The model

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used in this article is the mixed integer nonlinear programming that minimizes the cost of the entire network due to the specific service level.

Fernandes et al. (2015) investigated some issue in a multinational company in order to maximize the profit after the tax using the goods flow determination, the transfer price and the transportation cost allocation between each subset. They considered the model of Vidal and Goestschalckx (2001) and again reformulated the model and proposed three other solutions. Shunko et al. (2013) considered a multinational company with three entities. They investigated the transfer pricing impact of the decisions of the production or purchasing. They, also, identified an important issue that the multinational companies have been faced when regulating their transfer prices- the contradiction between the role of incentives and the role of the transfer price taxation.

Li (2013) designed the logistic integrated network. In fact, there has been investigated, in this article, the optimal location determination of the supplier, the allocation of the suppliers to facilities, sending demands and the management strategies of the supplies in the uncertain environment. As a whole, the facilities in demands and ask the items from the suppliers. To solve this kind of modeling, there has been delivered Lagrangian relaxation method.

Lin (2009) investigated the design of the integrated logistic networks, including the suppliers and retailers by taking into account the amount of the order due to the uncertain demand of the customer. They determined, in this article, the optimal areas and the wholesale price and the transportation price of the products for the suppliers and the amount of the demand for the retailers.

Hammami and Ferin (2014a) examined, in their paper, the mathematical optimization model for the design of the global supply chain that the emphasis on the transfer pricing is tangible and intangible element. They used the transfer pricing method of the profit split, delivered by Organization for Economic Cooperation & Development (OECD) and may be accepted by financial writers. This model is a multi-period, multi- level and multi-product model. They investigated in the same year in another article the development of an optimal model in the large scale specifically the redesign of the supply chain. The assumed model can help the managers make different logistic decisions. The integrated transfer price, in this model, has been achieved by two methods: the acceptable bounds on the value of transfer prices and the method of profit split that has been delivered by Organization for Economic Cooperation & Development (OECD) (Hammami & Ferin, 2014b).

Optimization under uncertainty comes mainly from two aspects: stochastic programming and robust optimization. In stochastic programming, indeterminate parameters are adjusted by the probability distribution function and the model searches the way in order to minimize the expected cost of the objective function. But in the robust approach, the probability function is incommensurable and the random parameters will be estimated among discrete scenarios or continuous intervals. In the discrete mode, there have been defined feasibility studies and various scenarios due to previous tests for every uncertain parameter and in the continuous mode, each uncertain parameter is determined in the final period. The main goal of the robust optimization is to minimize the worst cost or regret (Habibzadeh Bokani et al., 2016).

Pishvaee et al. (2011) suggested a robust optimization model in order to evaluate the inherent uncertainty of the input data based on the closed-loop supply chain network. First, they developed a mixed- integer linear programming model with the certain integer to design a closed-loop supply chain network.

Then, the suggested mixed- integer linear programming model has been delivered by the use of the recent development of the robust optimization theory. Vahdani et al. (2012) delivered a new model to design a reliable network of closed-loop supply chain facilities under the uncertainty. For this purpose, there has been developed a two objective mathematical formulation in order to minimize the total costs and transportation costs expected after the damage of the facilities of a logistic network. To solve this model, there has been introduced a new hybrid solution methodology combined with the robust optimization approach, queuing theory and fuzzy multi-objective programming. Vahdani and Naderi (2014) delivered a new mathematical programming model for the recycling network design in the iron and steel industry. This recycling network is the multi-echelon, multi-facility, interval stochastic, multi- product and multi-supplier. In addition, the purpose of this paper is to introduce a range random robust optimization methodology to deal with different uncertainties in the offered model.

According to the performed studies, it is obvious that in the models delivered by researchers, the transfer pricing models with the robust optimization approach have not been considered and, due to the current study, the networks with the shortage, hub and the uncertain demands have not been attention on the transfer pricing issues. In addition, most studies have been performed with regard to an objective function. Therefore, in this study, there has been offered a multi-objective mathematical model to deliver the model under the uncertainty conditions and the design of a logistics network in order to maximize the profits after tax and the service level. Because, the issues without uncertainty are formulated by certain parameters, but, in the real world, many parameters are faced with uncertainty and cannot be accurately determined. So, in this study, there will be developed a solution method based on the robust optimization approach to delivering model. The reason for the selection of the robust optimization and the attention to the uncertainty is that in this method, the variance of responses by solving the model is less than other methods. Also, since this method gives the worst possible solution, it enables the decision makers to provide better planning in the organizations (Vahdani et al., 2013).

The rest of this article is following sections: section 2 provides the proposed model in two certain and uncertain models. Section 3 delivers Fuzzy solution approach of the proposed model. Section 4 shows the numerical experiments. Sections 5 and 6 examine the sensitivity analysis and conclusion, respectively.

2.Proposed Model

2.1.Defining Problem

In the current study, the purpose is to provide a multi objective mathematical model to design a logistic network with the transfer pricing and the transportation cost allocation. The proposed model in this article is an integrated logistics network including suppliers, distribution centers, retailers and final customers. In this network, the suppliers sell several products to the distribution centers, then the retailers buy these products from the distribution centers and finally the products are sold to the customers. In this case, due to economies of scale, it is observed that which distribution centers will be the hub. All products will be processed in distribution centers. While distribution center open as a hub, the transportation cost and the processing cost decrease. In retailer level, all products may not be sold or shortage can be occurred. As a result, the holding cost of unsold products and the shortage costs are considered in the model.

One of the other objectives is the transportation cost and transportation saving cost allocation. It means that the transportation cost between two parts A and B, such as the supplier and the distribution center, may be allocated to the part A or B or both of them.

2.2.Model Assumption

- Each distribution center can act as a hub to save cost
- Suppliers, distribution centers and retailers should pay taxes
- Since the customer demand may not be fully satisfied, shortage may be occurring
- The lower and upper bounds should be considered for the transfer price

2.3.Model Symbols

2.3.1.Parameters

I: set of suppliers, indexed by i
J: set of distribution centers, indexed by j
K: set of retailers, indexed by k
P: set of products, indexed by p
TC_{ijp}: Transportation cost of each product p from supplier i to distribution centre (hub) j
TC'_{ijp}: saving cost of transportation of each product p from supplier i to distribution centre j when j has been opened as a hub
TC_{jkp}: transportation cost of each product p from distribution centre (hub) j to retailer k
TC'_{jkp}: saving cost of transportation of each product p transported from distribution centre (hub) j to retailer k when j has been opened as a hub
F_j: fixed cost of opening a distribution centre j as a hub
A_j: unit processing cost of products in distribution centre j
B_j: Processing savings cost per unit of product p in distribution centre (hub) j, if j has been opened as a hub
T_i: supply capacity of supplier i
N_j: processing capacity of distribution centre (hub) j
TAX_i: tax rate of supplier i
TAX_j: tax rate of distribution center (hub) j
TAX_k: tax rate of retailer k
TP^l_{ip}: lower bounds of transfer price of product p transported from supplier i
TP^u_{ip}: upper bounds of transfer price of product p transported from supplier i
TP^l_{jp}: lower bounds of transfer price of product p transported from distribution center (hub) j
TP^u_{jp}: upper bounds of transfer price of product p transported from distribution center (hub) j
R_{kp}: customer demand of product p in retailer k, it is an uncertain variable
$F_k(r)$: probability density function of customer demand at retailer k
PR_{kp}: retail price of the product p in retailer k
H_{kp}: unit holding cost of unsold product p in retailer k U_{kp}: unit shortage cost of product p in retailer k

2.3.2.Decision Variables

x_{ijp}: amount of product p transported from supplier i to distribution centre (hub) j
x_{jkp}: amount of product p transported from distribution centre (hub) j to retailer k
tp_{ip}: transfer price of the product p transported from supplier i

tp_{jp}: transfer price of the product p transported from distribution center(hub) j

z_j: if distribution center j opens as a hub, $z_j=1$; otherwise $z_j=0$

$ibts_i^+$: profit before tax of supplier i

$ibts_i^-$: loss before tax of supplier i

$ibts_j^+$: profit before tax of distribution centre (hub) j

$ibts_j^-$: loss before tax of distribution centre (hub) j

$ibts_k^+$: profit before tax of retailer k

$ibts_k^-$: loss before tax of retailer k

ys_{ij}: transportation cost between supplier i and distribution centre (hub) j, allocated to supplier i

y_{ij}: transportation cost between supplier i and distribution centre (hub) j, allocated to distribution centre (hub) j

yd_{jk}: transportation cost between distribution centre (hub) j and retailer k, allocated to distribution centre (hub) j

yr_{jk}: transportation cost between distribution center (hub) j and retailer k, allocated to retailer k

scs_{ij}: saving cost of transportation between supplier i and distribution center j, if j opened as a hub, allocated to supplier i

sc_{ij}: saving cost of transportation between supplier i and distribution centre j, if j opened as a hub, allocated to distribution center j

scd_{jk}: saving cost of transportation between distribution centre j and retailer k, if j opened as a hub, allocated to distribution center j

scr_{jk}: saving cost of transportation between distribution centre j and retailer k, if j opened as a hub, allocated to the retailer k

q_{kp}: total order of the retailer k from the product p

3 .Deterministic Mathematical Model

In this section we provide the model firstly. Then, we explain the objective functions and constraints.

$$\max w_1 = \sum_i [(1-TAX_i) ibts_i^+ - ibts_i^-] + \sum_j [(1-TAX_j) ibts_j^+ - ibts_j^-] + \sum_k [(1-TAX_k) ibts_k^+ - ibts_k^-] \quad (1)$$

$$\max w_2 = p(\sum_p R_{kp} \le \sum_p q_{kp}) = \int_0^{\sum_p q_{kp}} f_k(r)dr \quad \forall k \in K \quad (2)$$

$$\max w_2 = p(\sum_p R_{kp} \le \sum_p q_{kp}) = \int_0^{\sum_p q_{kp}} f_k(r)dr \quad \forall k \in K \quad (3)$$

$$\sum_j \sum_k \sum_p x_{jkp} = \sum_k \sum_p q_{kp}$$

$$\sum_j x_{jkp} = q_{kp} \quad \forall k \in K\ \forall p \in P \quad (4)$$

$$\sum_j \sum_k \sum_p x_{jkp} = \sum_i \sum_j \sum_p x_{ijp} \quad (5)$$

$$\sum_k x_{jkp} = \sum_i x_{ijp} \quad \forall j \in J\ \forall p \in P \quad (6)$$

$$\sum_j \sum_p x_{ijp} \le T_i \quad \forall i \in I \quad (7)$$

$$\sum_k \sum_p x_{jkp} \le N_j \quad \forall j \in J \quad (8)$$

$$1 \le \sum_j z_j \le M \quad (9)$$

$$-\sum_j \sum_p ys_{ij} x_{ijp} + \sum_j \sum_p sc_{ij} z_j x_{ijp} + \sum_j \sum_p tp_{ip} x_{ijp} = ibts_i^+ - ibts_i^- \quad \forall i \in I \quad (10)$$

$$-\sum_{i}\sum_{p} y_{ij}x_{ijp} - F_j z_j - \sum_{i}\sum_{p} A_j x_{ijp} + \sum_{i}\sum_{p} z_j B_j x_{ijp} + \sum_{i}\sum_{p} sc_{ij} x_{ijp} z_j - \sum_{k}\sum_{p} yd_{jk} x_{jkp} + \sum_{k}\sum_{p} scd_{jk} x_{jkp} z_j + \sum_{k}\sum_{p} tp_{jp} x_{jkp} = ibts_j^+ - ibts_j^- \quad \forall j \in J \tag{11}$$

$$\sum_{p} PR_{kp} \left(\int_0^{q_{kp}} rF_k(r)dr + \int_{q_{kp}}^{+\infty} q_{kp} F_k(r)dr \right) - \sum_{p} H_{kp} \left(\int_0^{q_{kp}} (q_{kp} - r) F_k(r)dr \right) - \sum_{p} U_{kp} \cdot \int_{q_{kp}}^{+\infty} (r - q_{kp}) F_k(r)dr - \sum_{j}\sum_{p} yr_{jk} x_{jkp} + \sum_{j}\sum_{p} scr_{jk} x_{jkp} z_j = ibts_k^+ - ibts_k^- \quad \forall k \in K \tag{12}$$

$$ys_{ij} + y_{ij} = \sum_{p} TC_{ijp} x_{ijp} \quad \forall i \in I, \forall j \in J \tag{13}$$

$$yd_{jk} + yr_{jk} = \sum_{p} TC_{jkp} x_{jkp} \quad \forall j \in J, \forall k \in K \tag{14}$$

$$scs_{ij} + sc_{ij} = \sum_{p} TC'_{ijp} x_{ijp} z_j \quad \forall i \in I, \forall j \in J \tag{15}$$

$$scd_{jk} + scr_{jk} = \sum_{p} TC'_{jkp} x_{jkp} z_j \quad \forall j \in J, \forall k \in K \tag{16}$$

$$ys_{ij} \geq scs_{ij} \quad \forall i \in I, \forall j \in J \tag{17}$$

$$y_{ij} \geq sc_{ij} \quad \forall i \in I, \forall j \in J \tag{18}$$

$$yd_{jk} \geq scd_{jk} \quad \forall j \in J, \forall k \in K \tag{19}$$

$$yr_{jk} \geq scr_{jk} \quad \forall j \in J, \forall k \in K \tag{20}$$

$$TP_{ip}^l \leq tp_{ip} \leq TP_{ip}^U \quad \forall i \in I, \forall p \in P \tag{21}$$

$$TP_{jp}^l \leq tp_{jp} \leq TP_{jp}^U \quad \forall j \in J, \forall p \in P \tag{22}$$

$$x_{ijp}, x_{jkp}, ys_{ij}, y_{ij}, yd_{jk}, yr_{jk}, ibts_i^+, ibts_i^-, ibts_j^+, ibts_j^-, ibts_k^+, ibts_k^- \geq 0 \tag{23}$$

$$z_j \in \{0,1\} \tag{24}$$

The proposed model has two objective function. The first objective function (1) maximizes profit after tax. On the other word, it can maximize the total profit of the logistic network. In the second objective function shown in the equation (2), the service level has been defined as satisfied demand probability or lack of shortage probability and it has been maximized. Constraints (3) represents that the total value of the output goods from all distribution centers should be equal to the total value of the orders from all retailers. Constraints (4) represent that the order level for each retailer k and the product p should be equal to the total amount of output from all distribution centers. Constraints (5) show that the total amount of products that are going to all retailers from all distribution centers should be equal to the total amount of all products that are going to the distribution centers from the suppliers. Constraints (6) show that for each distribution center j and the product p, the total output of all suppliers should be equal to total input all retailers. Constraints (7) and (8), respectively, represent the supplier's capacity and the distribution center's capacity. Constraints (9) represent that total open hubs should be less than or equal to a number of the potential hubs and they should be larger than or equal 1.

Constraints (10) show, the net income before tax of the suppliers, including transportation costs, ransportation saving cost if distribution center opened as a hub and the transfer pricing. It should be noted that, these costs are between supplier and distribution center and both are allocated to the supplier. Constraints (11) represents net income before tax of the distribution centers including the transportation costs between the supplier and the distribution center allocated to the distribution center, fixed cost of opening distribution center as a hub, the cost of processing products, saving cost of processing products if there is opened hub, the transportation cost between the distribution center and the retailer allocated to the distribution center, the transportation saving cost between the distribution center and the retailer allocated to the distribution center if there is opened hub and the transfer pricing. Constraints (12) represents the net income before tax of the retailers including the proceeds from the sale of the products, the holding cost of unsold products, shortage cost, transportation cost between the distribution center and the retailer allocated to the retailer and the transportation saving cost between the distribution center and the retailer allocated to the retailer if there is opened the hub. The income from the sale of the product; the holding cost of unsold products and the shortage cost are calculated as follows:

- income from the sale

$$\sum_p PR_{kp} \min(r_{kp}, q_{kp}) \;\; \forall k \in K \tag{25}$$

- Maintenance cost of unsold products

$$\sum_p H_{kp} \left[q_{kp} - r_{kp}\right]^+ \;\; \forall k \in K \tag{26}$$

Where $(x\text{-}y)^+$ denotes max [(x-y), 0]

- Shortage cost

$$\sum_p U_{kp} \left[r_{kp} - q_{kp}\right]^+ \;\; \forall k \in K \tag{27}$$

The total amount of these costs and incomes is as follows:

$$\sum_p PR_{kp} \min(r_{kp}, q_{kp}) - \sum_p H_{kp} \left[q_{kp} - r_{kp}\right]^+ - \sum_p U_{kp} \left[r_{kp} - q_{kp}\right]^+ - \sum_j \sum_p yr_{jk} x_{jkp} + \sum_j \sum_p scr_{jk} x_{jkp} z_j \tag{28}$$

Since the customer demand is uncertain, we can only obtain the expected income before tax as follows:

$$E\left[\sum_p PR_{kp} \min(r_{kp}, q_{kp}) - \sum_j \sum_p yr_{jk} x_{jkp} + \sum_j \sum_p scr_{jk} x_{jkp} z_j\right] - \sum_p E\left[\sum_p H_{kp} \left[q_{kp} - r_{kp}\right]^+ + \sum_p U_{kp} \left[r_{kp} - q_{kp}\right]^+\right] = V_1 - V_2 \tag{29}$$

$$\begin{aligned} V_1 &= E\left[\sum_p PR_{kp} \min(r_{kp}, q_{kp}) - \sum_j \sum_p yr_{jk} x_{jkp} + \sum_j \sum_p scr_{jk} x_{jkp} z_j\right] = \\ &\sum_p PR_{kp} \left[\int_0^{q_{kp}} rF_k(r)dr + \int_{q_{kp}}^{+\infty} q_{kp} F_k(r)dr\right] - \sum_j \sum_p yr_{jk} x_{jkp} + \sum_j \sum_p scr_{jk} x_{jkp} z_j \end{aligned} \tag{30}$$

$$\begin{aligned} V_2 &= \sum_p E\left[\sum_p H_{kp} \left[q_{kp} - r_{kp}\right]^+ + \sum_p U_{kp} \left[r_{kp} - q_{kp}\right]^+\right] = \\ &\sum_p \left[H_{kp} \left(\int_0^{q_{kp}} (q_{kp} - r_{kp}) F_k(r)dr\right) - U_{kp} \left(\int_{q_{kp}}^{+\infty} (r_{kp} - q_{kp}) F_k(r)dr\right)\right] \end{aligned} \tag{31}$$

The first statement in the equation (31), $H_{kp}(\int_{0}^{q_{kp}}(q_{kp}-r_{kp})F_k(r)dr)$ represents the total unsold products holding cost and the second statement, $U_{kp}(\int_{q_{kp}}^{+\infty}(r_{kp}-q_{kp})F_k(r)dr)$ represents the total shortage cost.

The constraints (13) express the transportation cost allocation between the supplier and the distribution center. The constraints (14) show the transportation cost allocation between the distribution center and the retailer. The constraints (15) represent the transportation saving cost allocation between the supplier and the distribution center if there is opened as a hub. The constraints (16) represent the transportation saving cost allocation between the distribution center and the retailer if there is opened as a hub. The constraints (17), (18), (19) and (20) show that if the transportation cost of every section have gotten positive value, the transportation saving cost of that section can have some level or not if there is opened as a hub. But if there is no amount of the cost, the saving cost amount of that section should not have any amount of cost.

Constraints (21) and (22) bound the transfer price of the suppliers and the distribution centers for the product p. the constraints (23) and (24) show bound of variables.

3.1.Robust Mathematical Model

In this section, the robust model has been delivered using the robust programming approach for the studied problem, with the assumption of the existence of the uncertainty in some studied parameters such as the fixed cost of an opened distribution center as a hub (F_i), unit processing cost of products in a distribution center j (A_j), the supply capacity of a supplier (T_i), processing capacity of the distribution center(hub) (N_j), tax rate of the supplier (TAX_i), tax rate of the distribution center (TAX_j), tax rate of the retailer (TAX_k) and the saving cost of the processing of the products in the distribution centers if there is opened as a hub (B_j). In the proposed model, it is assumed that each uncertain parameter changes in Closed Bounded Box (Ben-Tal & Nemirovski, 2000; Ben-Tal et al., 2005). The general representation of the uncertain box is as follows:

$$u_{Box}=\{\xi\in\Re^n:\left|\xi_t-\bar{\xi}_t\right|\le\rho G_t,\quad t=1,2,...,n\} \tag{32}$$

Where is the normal value of the as tth parameters of vector, Gt is the positive amount and represents the scale of the uncertainty and is the uncertainty level. One specific case is related to the case that relevant deviation from nominal data is of size up to. For more information, the readers can refer to the sources of Ben-Tal and Nemirovski (1998), and Ben-Tal et al. (2009).

The robust state of the proposed model has been shown using the relation 33-66.

$$\max\ w_1 \tag{33}$$

$$\max w_2=p(\sum_p R_{kp}\le\sum_p q_{kp})=\int_{0}^{\sum_p q_{kp}} f_k(r)dr\quad \forall k\in K \tag{34}$$

$$\sum_i\left[\left((1-\overline{TAX}_i)ibts_i^+ +\eta_i^{TAX}\right)-ibts_i^-\right]+\sum_j\left[\left((1-\overline{TAX}_j)ibts_j^+ +\eta_j^{TAX}\right)-ibts_j^-\right]+ \sum_k\left[\left((1-\overline{TAX}_k)ibts_k^+ +\eta_k^{TAX}\right)-ibts_k^-\right]\ge w_1 \tag{35}$$

$$\rho_{TAX}\,\mathcal{G}_i^{TAX}\,ibts_i^+\le\eta_i^{TAX}\qquad\forall i\in I \tag{36}$$

$$\rho_{TAX}\,\mathcal{G}_i^{TAX}\,ibts_i^+\ge-\eta_i^{TAX}\qquad\forall i\in I \tag{37}$$

$$\rho_{TAX}\,\mathcal{G}_j^{TAX}\,ibts_j^+\le\eta_j^{TAX}\qquad\forall j\in J \tag{38}$$

$$\rho_{TAX}\,\mathcal{G}_j^{TAX}\,ibts_j^+\ge-\eta_j^{TAX}\qquad\forall j\in J \tag{39}$$

$$\rho_{TAX}\,\mathcal{G}_k^{TAX}\,ibts_k^+\le\eta_k^{TAX}\qquad\forall k\in K \tag{40}$$

$$\rho_{TAX}\, \vartheta_k^{TAX}\, ibts_k^+ \geq -\eta_k^{TAX} \quad \forall k \in K \tag{41}$$

$$\sum_j \sum_k \sum_p x_{jkp} = \sum_k \sum_p q_{kp} \tag{42}$$

$$\sum_j x_{jkp} = \mathrm{q}_{kp} \quad \forall \mathrm{k} \in \mathrm{K}\ \forall \mathrm{p} \in \mathrm{P} \tag{43}$$

$$\sum_j \sum_k \sum_p x_{jkp} = \sum_i \sum_j \sum_p x_{ijp} \tag{44}$$

$$\sum_k x_{jkp} = \sum_i x_{ijp} \quad \forall \mathrm{j} \in \mathrm{J}\ \forall \mathrm{p} \in \mathrm{P} \tag{45}$$

$$\sum_j \sum_p x_{ijp} \leq \bar{T}_i - \rho_T\, \vartheta_i^T \qquad \forall \mathrm{i} \in \mathrm{I} \tag{46}$$

$$\sum_k \sum_p x_{jkp} \leq \bar{N}_j - \rho_N\, \vartheta_j^N \quad \forall j \in J \tag{47}$$

$$1 \leq \sum_j z_j \leq M \tag{48}$$

$$-\sum_j \sum_p ys_{ij}\, x_{ijp} + \sum_j \sum_p sc_{ij}\, z_j\, x_{ijp} + \sum_j \sum_p tp_{ip}\, x_{ijp} = ibts_i^+ - ibts_i^- \quad \forall \mathrm{i} \in \mathrm{I} \tag{49}$$

$$\begin{aligned} &-\sum_{\mathrm{i}} \sum_{\mathrm{p}} \mathrm{y}_{\mathrm{ij}} \mathrm{x}_{\mathrm{ijp}} - \bar{\mathrm{F}}_{\mathrm{j}} (1+\rho_F) \mathrm{z}_{\mathrm{j}} - \sum_{\mathrm{i}} \sum_{\mathrm{p}} \bar{\mathrm{A}}_{\mathrm{j}} (1+\rho_A) \mathrm{x}_{\mathrm{ijp}} + \sum_{\mathrm{i}} \sum_{\mathrm{p}} \mathrm{z}_{\mathrm{j}} \bar{\mathrm{B}}_{\mathrm{j}} (1+\rho_B) \mathrm{x}_{\mathrm{ijp}} + \\ &\sum_{\mathrm{i}} \sum_p \mathrm{sc}_{\mathrm{ij}} \mathrm{x}_{\mathrm{ijp}} \mathrm{z}_{\mathrm{j}} - \sum_k \sum_p \mathrm{yd}_{\mathrm{jk}} \mathrm{x}_{\mathrm{jkp}} + \sum_k \sum_p scd_{jk}\, x_{jkp}\, z_j + \sum_k \sum_p tp_{jp}\, x_{jkp} = ibts_j^+ - ibts_j^- \quad \forall \mathrm{j} \in \mathrm{J} \end{aligned} \tag{50}$$

$$\begin{aligned} &\sum_p PR_{kp} \Big(\int_0^{q_{kp}} rF_k(\mathrm{r})dr + \int_{q_{kp}}^{+\infty} q_{kp} F_k(\mathrm{r})dr \Big) - \sum_p H_{kp} \Big(\int_0^{q_{kp}} (q_{kp} - r) F_k(\mathrm{r})dr \Big) - \\ &\sum_p U_{kp} . \int_{q_{kp}}^{+\infty} (\mathrm{r} - q_{kp}) F_k(\mathrm{r})dr - \sum_j \sum_p yr_{jk}\, x_{jkp} + \sum_j \sum_p scr_{jk}\, x_{jkp}\, z_j = ibts_k^+ - ibts_k^- \quad \forall \mathrm{k} \in \mathrm{K} \end{aligned} \tag{51}$$

$$ys_{ij} + y_{ij} = \sum_p TC_{ijp}\, x_{ijp} \quad \forall i \in I, \forall j \in J \tag{52}$$

$$yd_{jk} + yr_{jk} = \sum_p TC_{jkp}\, x_{jkp} \quad \forall j \in J, \forall k \in K \tag{53}$$

$$scs_{ij} + sc_{ij} = \sum_p TC'_{ijp}\, x_{ijp}\, \mathrm{z}_j \quad \forall i \in I, \forall j \in J \tag{54}$$

$$scd_{jk} + scr_{jk} = \sum_p TC'_{jkp}\, x_{jkp}\, \mathrm{z}_j \quad \forall j \in J, \forall k \in K \tag{55}$$

$$ys_{ij} \geq scs_{ij} \quad \forall i \in I, \forall j \in J \tag{56}$$

$$y_{ij} \geq sc_{ij} \quad \forall i \in I, \forall j \in J \tag{57}$$

$$yd_{jk} \geq scd_{jk} \quad \forall j \in J, \forall k \in K \tag{58}$$

$$yr_{jk} \geq scr_{jk} \quad \forall j \in J, \forall k \in K \tag{59}$$

$$TP_{ip}^{l} \leq tp_{ip} \leq TP_{ip}^{U} \quad \forall i \in I, \forall p \in P \tag{60}$$

$$TP_{jp}^{l} \leq tp_{jp} \leq TP_{jp}^{U} \quad \forall j \in J, \forall p \in P \tag{61}$$

$$x_{ijp}, x_{jkp}, ys_{ij}, y_{ij}, yd_{jk}, yr_{jk}, ibts_i^{+}, ibts_i^{-}, ibts_j^{+}, ibts_j^{-}, ibts_k^{+}, ibts_k^{-} \geq 0 \tag{62}$$

$$z_j \in \{0,1\} \quad \forall \mathrm{j} \in \mathrm{J} \tag{63}$$

$$\eta_i^{TAX} \geq 0 \quad \forall \mathrm{i} \in \mathrm{I} \tag{64}$$

$$\eta_j^{TAX} \geq 0 \quad \forall j \in J \tag{65}$$

$$\eta_k^{TAX} \geq 0 \quad \forall k \in K \tag{66}$$

4.Fuzzy Solution Approach

In previous researchers, there are several procedures to face the multi-objective problems. Among these procedures, the fuzzy approach has been more attracted. One of the main reasons that caused widely usage of these approaches is an ability of taking to account of satisfaction level of each objective function. In this paper, the fuzzy solution approach that is developed by Liu and et al. (2003) is used to solve the proposed model. The steps of this approach are as follows:

First step: Determine the positive ideal solution and the negative ideal solution for any objective function. In order to calculate the positive ideal solution and the negative ideal solution, namely$(\mathcal{W}_1^{PIS}, x_1^{PIS})$ and$(\mathcal{W}_2^{PIS}, x_2^{PIS})$ each certain model has been solved individually for each objective function, and then the positive ideal solution has been achieved, and also the negative ideal solution has been estimated as follows:

$$\mathcal{W}_1^{NIS} = \mathcal{W}_1(x_2^{PIS}), \mathcal{W}_2^{NIS} = \mathcal{W}_2(x_1^{PIS}) \tag{67}$$

Second step: Determine a linear membership function for any objective function as follows:

$$\mu_1(x) = \begin{cases} 1 & if \quad \mathcal{W}_1 < \mathcal{W}_1^{PIS} \\ \dfrac{\mathcal{W}_1^{NIS} - \mathcal{W}_1}{\mathcal{W}_1^{NIS} - \mathcal{W}_1^{PIS}} & if \quad \mathcal{W}_1^{PIS} \leq \mathcal{W}_1 \leq \mathcal{W}_1^{NIS} \\ 0 & if \quad \mathcal{W}_1 > \mathcal{W}_1^{NIS} \end{cases} \tag{68}$$

$$\mu_2(x) = \begin{cases} 1 & if \quad \mathcal{W}_2 > \mathcal{W}_2^{PIS} \\ \dfrac{\mathcal{W}_2 - \mathcal{W}_2^{NIS}}{\mathcal{W}_2^{PIS} - \mathcal{W}_2^{NIS}} & if \quad \mathcal{W}_2^{NIS} \le \mathcal{W}_2 \le \mathcal{W}_2^{PIS} \\ 0 & if \quad \mathcal{W}_2 < \mathcal{W}_2^{NIS} \end{cases} \tag{69}$$

In fact, $\mu_h(x)$ represents the satisfaction degree of h^{th} objective function. It should be mentioned that $\mu_1(x)$ has been used to minimize the objective function and $\mu_2(x)$ has been used to maximize the objective functions.

Third step: Transform the certain models of the mixed integer programming in a single- objective model of the mixed integer programming using the aggregation function calculated as follows:

$$\max \quad \lambda(x, y) = \psi\lambda_0 + (1-\psi)\sum_h \theta_h \mu_h(x, y) \tag{70}$$

$$\lambda_0 \le \mu_h(x, y), h = 1, 2 \tag{71}$$

$$x, y \in F(x, y), \lambda_0 and\ \lambda \in \{0,1\} \tag{72}$$

Fourth step: Determine the values of parametersθ_h، ρ, ψ and solve the single-objective models created in the previous step. If the solutions resulted for the decision makers are satisfactory, they have been stopped; otherwise, in order to achieve the new solution, we change the values of the parametersρ, if needed.

5.Numerical Experiments

In this section, there had been considered five experimental problems with different parameters, and their size of problems had been summarized in Table 1. These problems have been solved in certain and uncertain mode. The uncertain mode was solved through three uncertainties levels 0.2, 0.5 and 0.7. The uncertainty level in each problem is the same for all parameters. The importance of the objective functions in every problem is different, and the penalty coefficient has been considered 0.5 for all problems. All test problems are solved by GAMS Software. The results of the robust model solution on the levels of uncertainty 0.2, 0.5 and 0.7 and TH method are shown in Table 2, and the certain model and TH method are shown in Table 3. According to the tables 2 and 3, the uncertain model solution time is less than the certain model solution time and by increasing the uncertainty level, the objective functions are decreased, and they give us worse solution. In figure 1 and 2, there were shown the flows of goods and the transportation costs allocation and transportation saving cost allocation related to the example 1 about the numerical experiments.

Table 1
Size of test problem

Problem no.	No. of suppliers (i)	No. of distribution centers(j)	No. of retailers (k)	No. of products (p)
1	10	2	2	3
2	3	2	2	8
3	1	5	2	1
4	5	2	2	3
5	3	1	3	2

Table 2
Summary of test result from optimization robust model

Problem no.	Uncertainty level(ρ)	Robust model					TH method under uncertainty	
		W_1	W_2	W_3	W_4	Solution time	λ	Solution time
1	.2	4869289000	0	0.816	-	6:17.615	0.158	13:38.606
	.5	3162039000	0	0.510	-	8:18.473	0.155	03:05.099
	.7	1635342000	0	0.306	-	6:53.931	0.197	09:52.233
2	.2	4711817000	0.00004600336	1	-	7:03.614	0.158	01:57.760
	.5	3071109000	0	1	-	3:33.483	0.149	03:06.618
	.7	1035524000	0	0.975	-	6:56.958	0.15	06:06.836
3	.2	2845663000	0.000000000772161	0.448	-	01:11.642	0.145	00:57.437
	.5	2420512000	0.000000000721909	0.28	-	00:26.805	0.059	00:46.896
	.7	2083372000	0	0.168	-	00:46.908	0.061	00:30.981
4	.2	4764897000	0	0.816	-	02:54.891	0.226	02:23.082
	.5	2897237000	0	0.510	-	02:27.116	0.151	01:41.467
	.7	1500152000	0	0.306	-	03:02.276	0.061	01:52.431
5	.2	5587304000	0.0000000000431119	0.0000000000431117	1	00:38.691	0.125	00:01.376
	.5	3296278000	0.0000000000411497	0.0000000000411495	0.625	00:15.339	0.125	00:03.195
	.7	2942534000	0	0	0.375	00:21.519	0.125	00:13.866

Table 3
Summary of test result from deterministic model

Problem no.	Deterministic model					TH method under certainty	
	W_1	W_2	W_3	W_4	Solution time	λ	Solution time
1	6021592000	0.03	1	-	25:17.328	0.734	09:12.673
2	7669722000	0.009	1	-	30:58.370	0.464	03:11.361
3	6255677000	0.019	0.552	-	18:56.970	0.157	00:09.494
4	5617391000	0	1	-	18:52.165	0.752	02:00.350
5	6117815000	0.003	0.003	1	17:08.472	0.448	07:02.516

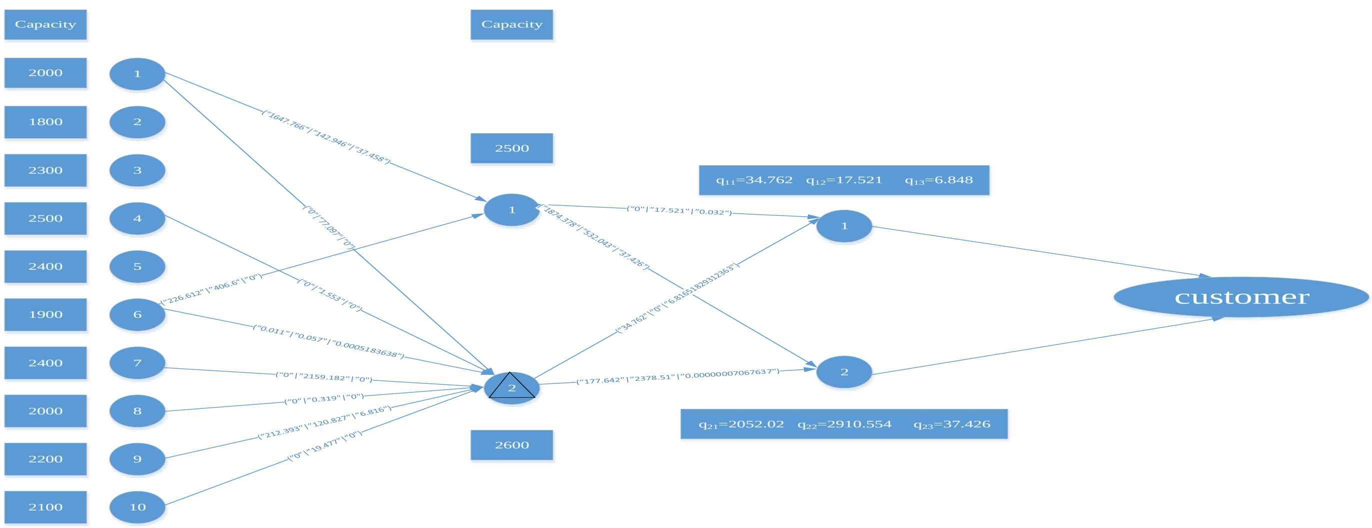

Fig. 1. Flow of the goods

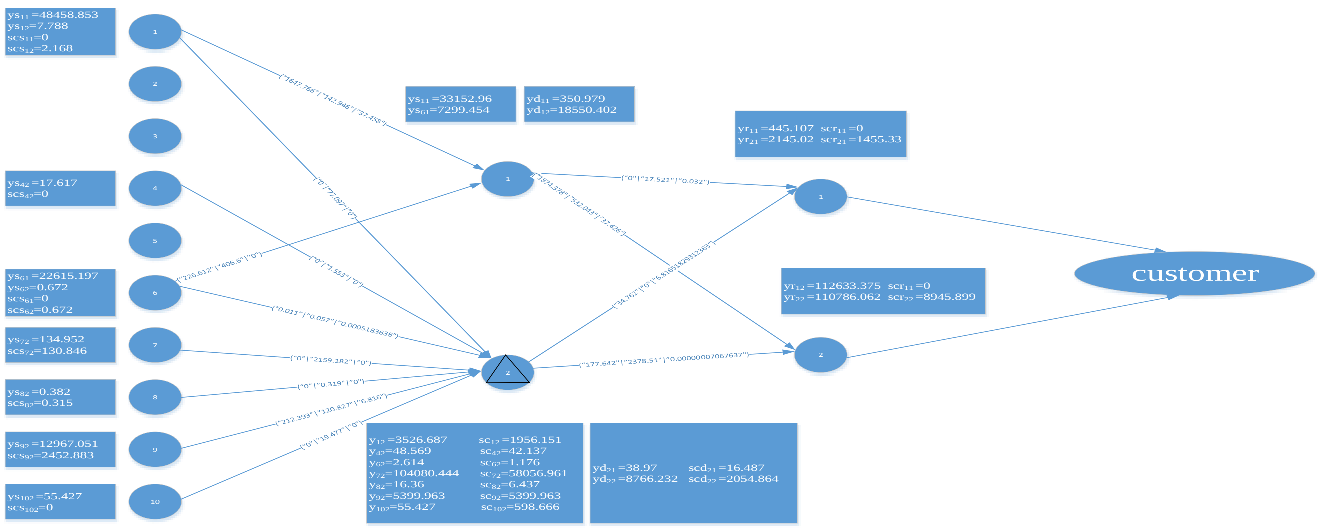

Fig. 2. Transportation cost allocation and transportation saving cost allocation

6.Sensitivity Analysis

6.1.The effect of uncertainty on the objective function

There have been solved eight trial problems at different uncertainty levels in range of 0.1 to 0.8 in order to investigate the uncertainty effect. Furthermore, in each problem, the uncertainty level is considered the same for all parameters. In figures 3 and 4, there has been shown the uncertainty sensitivity analysis. As it is obvious, by increasing the uncertainty level, the profit after tax is decreased, and we gain worse solution. By increasing the uncertainty level at the second and third objective function, the service level amount is decreased, and it gives us worse solution.

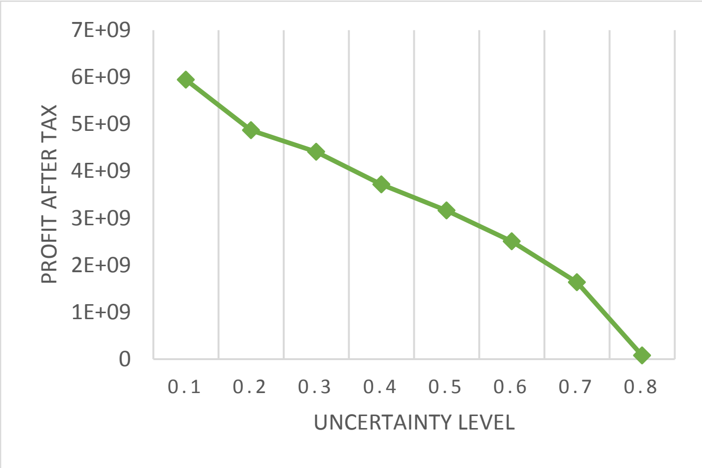

Fig. 3. Uncertainty level and first objective function

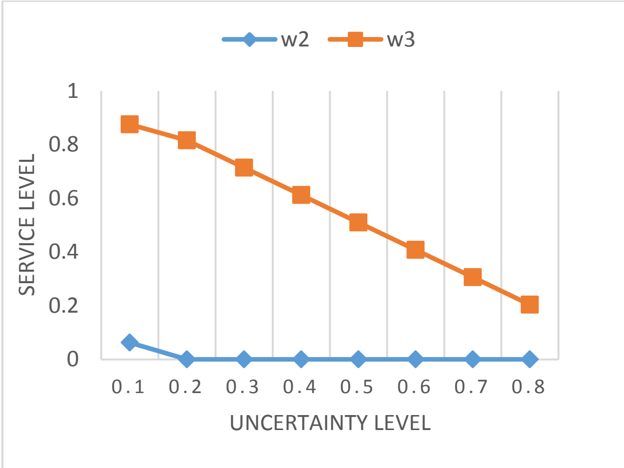

Fig. 4. Uncertainty level and second and third objective functions

6.1.The Effect of Penalty Coefficient on the Satisfaction Degree of Objective Function

In this section, there has been investigated the impact of the penalty coefficient on the satisfaction degree of the objective function. Furthermore, the uncertainty level is considered 0.2. Moreover, nine trial problems have been performed by different penalty coefficients in range of 0.1 to 0.9, and the sensitivity analysis is done. The results are shown in figures 4 and 5. By increasing different amounts of the penalty coefficient, the satisfactory degrees of the objective functions are not necessarily decreased or increased.

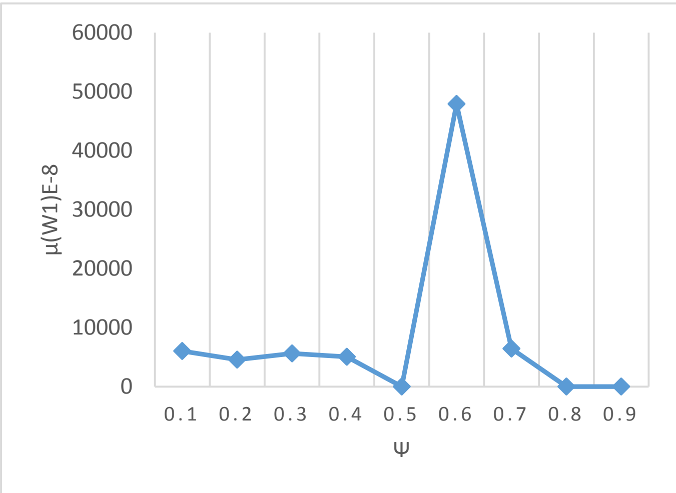

Fig. 5. Penalty coefficient and satisfaction degree of the first objective

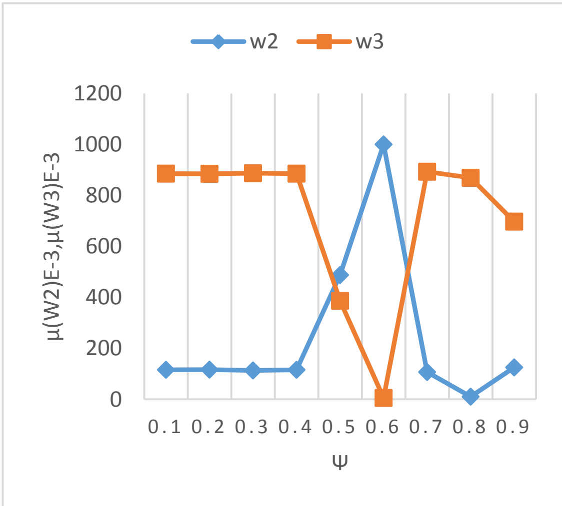

Fig. 6. Penalty coefficient and satisfaction degree of second and third function

7.Conclusion

In this paper, there has been a multi-objective mathematical model in order to design a logistic network by considering the transfer pricing and the transportation cost allocation and there has been used the robust optimization approach. This model has been developed using the robust optimization approach in the uncertain mode and then has been solved using a fuzzy approach to solve the multi-objective problems named TH. The proposed model has been solved in GAMS Optimization Software. Finally, there has been used the sensitivity analysis on the uncertainty level and the penalty coefficient. The robust model objective function values are worse than exact model and by increasing the uncertainty level, the objective functions generate worse value.It is recommended for the future researchers to use other methods, including the profit-split and the price resale method in order to determine the transfer pricing. Since the Presented issue is considered as NP-Hard roblems, it is suggested to use the heuristic and meta-heuristic algorithms.

References

Ben-Tal, A., & Nemirovski, A. (1998). Robust convex optimization. *Mathematics of Operations Research*, *2*, 769-805.

Ben-Tal, A., & Nemirovski, A. (2000). Robust solutions of linear programming problems contaminated with uncertain data. *Mathematical Programming*, *88*, 411-424.

Ben-Tal, A., Golany, B., Nemirovski, A., & Vial, J. P. (2005). Retailer-supplier flexible commitments contracts: a robust optimization approach. *Manufacturing & Service Operations Management*, *7*, 248-271.

Ben-Tal, A., El-Ghaoui, L., & Nemirovski, A. (2009). *Robust Optimization*. Princeton University Press.

Fernandes, R. Z, Pinho, C., & Gouveia, B. (2015). Supply chain networks design and transfer-pricing. *The International Journal of Logistics Management*, *26*(1), 128-146.

Goetschalckx, M., Vidal, C. J., & Doganc, K. (2002). Modeling and design of global logistics systems: A review of integrated strategic and tactical models and design algorithms. *European Journal of Operational*, Research *143*, 1-18.

Habibzadeh Baukani, F., Farhang Moghaddam, B., & Pishvaee, M. S. (2016). Robust optimization approach to capacitated single and multiple allocation hub location problems. *Computational and Applied Mathematics*, *35*, 45-65.

Hammami, R., Frein, Y., & Hadj-Alouane, A. B. (2009). A strategic-tactical model for the supply chain design in the delocalization context: mathematical formulation and a case study. *International Journal of Production Economics*, *122*, 351-365.

Hammamia, R., & Frein, Y. (2014a). Integration of the profit-split transfer pricing method in the design of global supply chains with a focus on offshoring context. *Computers & Industrial Engineering*, *76*, 243-252.

Hammamia, R., & Frein, Y. (2014b). Redesign of global supply chains with integration of transfer pricing: Mathematical modeling and managerial insights. *International Journal of Production Economics*, *158*, 267-277.

Li, X. (2013). An integrated modeling framework for design of logistics networks with expedited shipment services. *Transportation Research Part E*, *56*, 46-63.

Lin, L., Gen, M., & Wang X. (2009). Integrated multistage logistics network design by using hybrid evolutionary algorithm. *Computers & Industrial Engineering*, *56*(3), 854-873.

Liu, L., Huang, G. H., Liu, Y., Fuller, G. A., & Zeng, G. M. (2003). A fuzzy-stochastic robust programming model for regional air quality management under uncertainty. *Engineering Optimization*, *35*(2), 177-199.

Perron, S., Hansen, P., Digabel S. L., & Mladenovic, N. (2010). Exact and heuristic solutions of the global supply chain problem with transfer pricing. *European Journal of Operational Research*, *202*, 864-879.

Pishvaee, M. S., Rabbani, M., & Torabi S. A. (2011). A robust optimization approach to closed-loop supply chain network design under uncertainty. *Applied Mathematical Modelling*, *35*, 637-649.

Qin, J., Shi, F., Miao, L., & Tan, G. j. (2009). Optimal Model and Algorithm for Multi-Commodity Logistics Network Design Considering Stochastic Demand and Inventory Control. *Systems Engineering - Theory & Practice*, *29*(4), 176-183.

Shunko, M., & Gavirneni, S. (2007). Role of Transfer price in global supply chain with random demand. Journal of Industrial and Management Optimization, 3(1), 99-177.

Shunko, M., Debo, L., & Gavirneni, S. (2013). Transfer pricing and sourcing strategies for multinational firms. *Production and Operations Management*, *23*, 2043-2057.

Vahdani, B., Tavakkoli-Moghaddam, R., Modarres M., & Baboli A. (2012). Reliable design of a forward/reverse logistics network under uncertainty:

A Robust-M/M/c queuing model. *Transportation Research Part E*, *48*, 1152-1168.

Vahdani B., Tavakkoli-Moghaddam, R., Zandieh, M., & Razmi, J. (2012). Vehicle routing scheduling using an enhanced hybrid optimization approach. *Journal of Intelligent Manufacturing*, *23*, 759-774.

Vahdani, B., Tavakkoli Moghaddam, R., & Jolai, F. (2013). Reliable design of a logistics network under uncertainty: A fuzzy possibilistic-queuing model. *Applied Mathematical Modelling*, *37*(5), 3254-3268.

Vahdani, B., & Naderi-Beni, M. (2014). A mathematical programming model for recycling network design under uncertainty: an interval-stochastic robust optimization model. *International Journal of Advanced Manufacturing Technology*, *73*, 1057-1071.

Vidal, C. J., & Goetschalckx, M. (1997). Strategic production-distribution models: A critical review with emphasis on global supply chain models. *European Journal of Operational Research*, *98*, 1-18.

Vidal, C. J., & Goetschalckx, M. (2001). A global supply chain model with transfer pricing and transportation cost allocation. *European Journal of Operational Research*, *129*(1), 134-158.

Zhang, W., & Xu, D. (2014). Integrating the logistics network design with order quantity determination under uncertain customer demands. *Expert Systems with Applications*, *41*(1), 168-175.