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Comparative Analysis of Geometric Forms and Numerical Calculation of Fractals in Khayyam's Mausoleum Based on the Islamic Architecture's Definitions and Box-Counting Method

Hengame Rezazade [a],*, Mahta Tari [a]

[a] *Department of Arts and Architecture ,Islamic Azad University, Yadegar-e- Imam Khomeini (RAH) Shar-e-Rey Branch, Tehran, Iran*

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Abstract

The interaction and balance between fractal components, which maintain to function in nature in perfect harmony with the highest efficiency, has been the source of inspiration for architects in different eras. Today, contemporary Iranian architecture needs a comprehensive study and familiarity with world issues, Hence Khayyam's mausoleum was chosen.the main purpose of this article is to analyze the geometry and numerical calculation of the fractal size of the building and its semantic knowledge in the contemporary building, which leads to finding the architect's goals of how to design these forms and common concepts with Islamic architecture. After achieving how to use Islamic elements in a contemporary monument, we understand the interrelationship between fractal geometry and architectural design.With the box-counting technique, the fractal dimension was calculated by checking the two-dimensional plates in 4 different scales from the largest to the smallest scale to prove its decimal dimension. Then, information was collected by the descriptive-analytic method after determining the common features between fractal forms with the analytical and comparative process, the obtained quantitative data resulted in extracting similar cases from the building. finally, the relationship between the characteristics of fractal geometry and the architecture of the tomb was proved and the obtained patterns were matched with the concepts of self-similarity, iteration (proportion and symmetry), microscale and non-integer dimension. The elements used such as tiled ornaments, the Islamic star and Shamsa, domed-shaped roof, a regular numerical pattern of geometric shapes with symbolism, unity in diversity, infinity and dynamism are reminiscent of Islamic architectural style. Furthermore, fractal has been used in three methods: conceptual, mathematical and intuitive, and has been manifested in both structural and aesthetic aspects.

Keywords: Fractal geometry, Fractal dimension, Box-counting, Fractal's application, Khayyam's Mausoleum.

1. Introduction

Nature presents itself as a cascade of self-similar shapes and their relationship to natural shapes was first Introduced in the fractal geometry of nature by Benoit Mandelbrot in 1977 (Bovill, 1996: 3). Nowadays, with the expansion of urban areas and the development of technology, human need for nature is felt more than ever, and consequently the style of architecture has shifted to nature-inspired architecture. Fractal is an example of these issues that cause the stability of the building and with the maximum usage of space without any conflict with nature.
Fractal geometry with its characteristics of self-similarity, scaling and never-ending had a hand in architecture as a formative idea and form generator. Architects, when designing using fractal geometry, tend to use it aesthetically, creating complex decorative patterns which can be perceived by the public (Osama, Sherif & Ezzeldin, 2014: 39). By achieving the features of fractal shapes and studying investigations and previous findings, we decided to choose Khayyam's Mausoleum as our case of study due to its visual daintiness and profound impact on the viewer amongst the contemporary Iranian architectures.
It is worth noting that in the first Pahlavi architectural era, in addition to special attention of the government to ancient orientation in architecture, architects in this era, in addition to experiencing ancient orientation also paid attention to some post-Islamic architectural factors and occasionally tried to express national identity in Iranian architecture through applying national-religious identity by combining pre-and post-Islamic architectural elements and ornaments (Soheili, 2015:56). The mentioned trends are also followed in the second Pahlavi period.
The studying this building in recent period and synchronized with second Pahlavi is the entrance of educated architects like Houshang Seyhoun in architecture field on that time. Contrary to what some have described as a decline in contemporary Iranian architecture, we decided to demonstrate that this type of architecture also occurred in this period by adapting contemporary architectural features to world-renowned issues. And the main reason of choosing a tomb building as a case study in this paper is its

* Corresponding Author Email Address: rezazadehengame@gmail.com

distinctive examples available in Iranian's architectural history as Hillenbrand mentioned in his book "Undoubtedly, the tomb after the mosque is the most common public building in Iranian Islamic architecture. This type of building is rooted in the context and culture of Iranian society and is less of a city that does not have a quota of such buildings" (Hillenbrand, 2001: 273).

There are many relationships between architecture, arts and mathematics for example the symmetry, the platonic solids[1], the polyhedral[2], the golden ratio[3], the spirals[4], the Fibonacci's sequence[5], still it is difficult to find some interconnections between fractal and architecture (Sala, 2001: 266). Therefore, in this paper we will try to detect similar points between the case of study and fractal concepts by using different techniques and calculating its dimension. In terms of fractal dimension, as Li et al. have pointed out "Fractal dimension (FD) is a useful feature for texture segmentation, shape classification, and graphical analysis in many fields. The box-counting approach is one of the frequently used techniques to estimates the FD of an image (Li, Du & Sun, 2009: 2460). So, we will attempt to examine this aspect of our focused monument based on its 2D checkered images as well.

The following aims can be mentioned below: Study of the effect of fractal geometry on a building related to contemporary Iranian architecture, finding the architect's goal of using this geometry in tomb design, Achieving interrelationships between fractal geometry and contemporary architectural design and extracting fractal pattern, to understand its meaning in Iran's contemporary era.

In this paper, we will try to explain the basis of architect's decision in creating concepts of fractal by analyzing and extracting geometric forms, try to remove the common features of the forms between our case of study and prior era's architecture (Islamic architecture period) and answer the following questions: 1) Does the structural aspects and forms of contemporary mausoleum like khayyam's tomb have fractal feature and if so, how is it? 2) Is there a connection between the features of this building and the intellectual foundations of similar buildings such as mosques with Islamic architecture, and what are these similarities? 3) What could be the architect's purpose and method to apply fractal geometry in his design?

2. Theoretical Foundations

2.1. Fractals

Fractal theory has gained wide popularity since (1977) with the release of the book by B. Mandelbrot (1983) entitled "The Fractal Geometry of Nature" . The fractal objects have a parametric property; such that all their parts are bonded by a relation in which the part is affected by the whole (Abdelsalam & Ibrahim, 2019: 32). Fractals are generally self-similar on multiple scales. So, fractals have a built-in form of iteration or recursion. Sometimes the recursion is visible in how the fractal is constructed. For example, Koch snowflake, Contor set, and the Sierpinski triangle are all generated using simple recursive rules. Self-similarity is present in nature (Bovill, 1996: 14). the small-scale feature also includes the self-similarity feature, so that the smaller components are similar in size and scale to the whole set, which is larger in scale. Therefore, according to the proposed theories and table 1, we will have a basis for determining and extracting fractal geometry from the building. According to the table 1 and the definitions proposed, Fig. 1 is the basis for extracting fractal patterns from our case of study in which we come to the result that Self-similarity, Small scale, Iteration and Decimal dimension are all common amongst fractal's sets in the position of line, surface and volume.

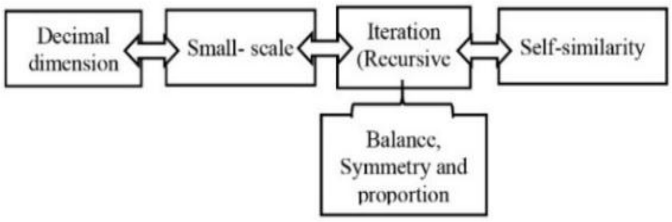


Fig. 1. Illustrate the main features of fractal geometry to extract similar geometric patterns.

In the following, we will observe that the fractal is represented in all three dimensions of line, plane and volume (according to the structure of forms and geometries extracted from our case of study.

2.2. Fractals in architecture

Fractal with its inherent complexity and rhythmic characteristics has also inspired many contemporary architectural design processes. The contemporary usage of fractals in architecture has resulted due to a range of varied concerns. One of the concerns is the organic metaphors of design as used by Peter Eisenman and Zvi Hecker (Alik & Ayyildiz, 2016: 286). For example Eisenman, A11 House's beginning point starts with two squares, then divides into four equal squares, and finally, by removing one square, takes the final L-shaped design (Fig. 2).

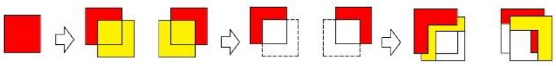


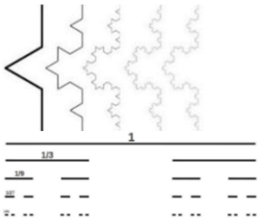
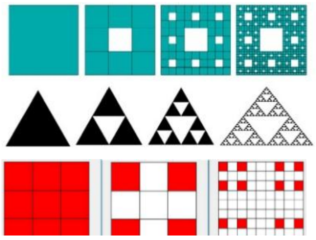
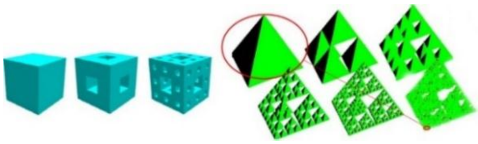
Fig. 2. The A11 house design process

- In European Architecture the structure of Eiffel Tower shows the existence of fractals in construction materials.
- In Gothic Architecture the existence of fractals is seen in the Catherdals in ornamentation .The application of fractals in architecture can be usually made in following different methods:

1. Conceptual methods: This uses fractal geometry and its concept as a guiding element to its theories. This method provides of theories. This method provides a theoretical solution that ultimately influence the final form.
2. Geometric-mathematical methods: Which uses scheme of counting squares to calculate the fractal dimension. This method is used to analyze the existing building also.
3. Geometric-intuitive method: This uses geometry as inspiration for creative expression.

Fractal analysis in Architecture can be done in two stages (Parashar & Bandyopadhyay, 2014: 94). As a result, according to Parashar and Bandyopadhyay's findings, fractals are present and used in architecture in different ways. This study will to examine how fractals can be used in monuments such as our mentioned building-Khayyam's mausoleum.

Table 1
Categorize fractal patterns and extract main features.

Fractal models	**Fractal characteristic**
	(In fractal lines): The curve is constructed from a line segment of unit length whose central third is extracted and replaced with two lines of length 1/3. This process is continued, with the protrusion of the replacement always on the same side of the curve, to get the Koch's Curve (Alik, 2015). The Cantor set demonstrates very clearly essential features that relate fractal structures to natural structures. The first feature is self-similarity from the large scale to the small-scale. The second important feature is the clustering of the points in the Cantor set (Bovill, 1996: 10).
	(In fractal plane): The Sierpinski Triangle is created by replacing an equilateral triangle of unit size, E_0, by three triangles of half its size, leaving the middle region empty, giving E_1 (Nurujjman, Hossain & Ahmed, 2017: 99). Growth with ½ size indicates its decimal dimension. According to the images, it is the development of small- scale and self-similar shapes from the central part (empty area) that illustrates the centrifugal nature of some fractal shapes. Fractal dimension of Contor cubes= Log8/log3= 1.890 (source: Khajvand Jafari, Asgari & Pishkoo, 2019: 2).
	(In fractal volume): An example of a fractal that occurs in volume is the Menger Sponge , which, as seen in the geometry of some fractals, is usually associated with filling and emptying a line, surface, or volume relative to a fraction. Smaller-scale components, similar to the overall shape of the whole volume, display fractals by creating positive and negative spaces.

2.3. Fractal dimension (based on mathematical definitions)

Fractal dimension is an proper indicator to describe the complexity of a certain geometry, and box-counting analysis is proved to be an effective and appropriate method for fractal dimension estimation which is widely used. However, traditional box-counting methods based on images may not consistently accurate (Wu et al, 2020:1). Hence, as we mentioned earlier, our goal is to prove and extract fractal patterns from both visual and arithmetic facets, using the same traditional counting method, we will achieve our goal of obtaining quantitative data of fractal patterns.
Mandelbrot and Falconer agree that we shall not give an exact definition but determine fractal by characteristics such as recursive, self-similarity, elaborate details, etc (Mandelbrot, 1985, 257). In fractal preferences, the fractal dimension is more important than scale. Due to the reason that they come in any size, the fractal dimension is maintained and it reflects the essence of fractals. This matter has fractal's application in science (Elgohari, 2019: 90). Therefore, calculating the fractal size of this building and analyzing its fractal patterns is fundamental because calculating the fractal dimension alone provides a lot of information about the type of geometry and their behavior.
Meanwhile, the box-counting technique has been used as a practical and profitable tool to achieve the desired fractal size in various cases such as many fields of engineering, software network, urban planning systems, and also architecture to estimate a fractal type building's size similar to our approach in this paper.
The Box-counting Dimension is utilized with Mandelbrot's formula for calculating fractal dimension, as researchers have previously done to determine the fractal dimension of prominent buildings such as the Robin House by Frank Lloyd Wright and other buildings.

2.4. Semantical and structural features of Fractal patterns in Iranian traditional architecture

According to previous researches and studies in the field of ornaments and architectural elements of the Islamic era, researchers have extracted and proved various geometric patterns and classified them into fractal forms. Also, not only formally but also semantically, the concept of fractal has been studied in architectural examples in the Islamic period that we can understand the nature of some of these concepts in the following definitions:
"Decorative elements such as geometric patterns in the most direct way depict unity in plurality and plurality in unity and show the manifestation of stability in change. The creation of spiritual space in this way has been considered as a reference to the world of monotheism" (Madadpour, 1995: 135). Obviously, as in any other prominent religion, culture and civilization, symbolism and symbolic representation have been common practices in the design of architectural spaces and their decoration and adornment (Soltanzadeh, 2015: 19).
Among the Islamic elements with the characteristics of fractal geometry in this tomb, we can mention: **The**

Islamic Star: a symbol of divinity, transcendence, eternity, immortality and hope. The high apexes of the star are a manifestation of light and spirituality (Cooper, 2007: 193) and **Shamsa (Decagram)**: which is a manifestation of the sun. In Persian literature, it is the source of radiance and the manifestation of zenith, beauty and height (Yahaqqi, 1996: 339). Just as Sarangi claims about Islamic architecture of Iran, this building's architecture follow a similar approach "In Persian architecture, it was geometry that provided diverse stylistic developments for constructions and designs; not only to serve a function, but also to evoke an emotional response by harmonization of the constructional elements, such as domes and columns and decorative elements" (Sarhangi, 1999: 96).

Through the use of numbers and geometry as a mathematical phrases, creation of shapes reminds eternal patterns of the example world. Due to this, mathematics are seen in the form of wisdom language, a tool for interpretation, which leads from the tangible world to the known world due to its nature. Mathematical abstraction is concerned with the sense, however, its nature is objective. These abstractions from the known world are used as an essential guide for eternal and objective essence in the divine order (Ardalan & Bakhtiari, 2005: 51). To conclude, the application of geometry and proportion in the design of the plan of Islamic era's buildings began with the design of the basic shape, before rotating and repeating the basic shape regularly.

According to the above definitions, in the following we will prove and demonstrate the similarities between the geometric and structural elements of Islamic architecture and our case of study with a comparative and deductive approach

2.5 Spatial and geometrical characteristics of Khayyam's mausoleum

Khayyam's mausoleum is located in Neishabour, and until 1956, when Houshang Seyhoun was asked to design a building in honor of Khayyam (philosopher, mathematician, astronomer, and poet of the quatrain).

The height of this building is 22 meters and it has 10 bases with a distance of 5 meters. From each of the bases, two diagonal blades move upwards, so that with the intersection of these blades, the total volume of the tower is made in space. The full and empty spaces, negative and positive spaces which are like the pattern of Menger Sponge that we have described it before in table 1, create fractal pattern in the volume and surface of Khayyam's Mausoleum, that result from the collision of the blades at the top of the building have created star-shaped patterns that symbolized the sky and ultimately, the stars become smaller towards the top of the dome, and ended to a five-feathered star which the sky of Neishabour appears thereby.

The entrance of Khayyam's Mausoleum is accessible from any direction, which on the other hand creates the highest interaction with the surrounding natural environment. In other words, designing a semi-open space to make the space eye-catching for visitors which is one of the most integral aspect of attracting visitors in building these monumental tombs. The symmetry, with its rhythmic repetition properties, proportions, and order, is reminiscent of fractal geometry. In the following, we will explore geometric patterns of Khayyam's Mausoleum as an example of tomb buildings to inspect the commonalities amongst them in terms of their self-similarity, iteration (symmetry and proportion), small-scale features. The perimeter of the tower is composed of triangular elements in perfect harmony with the decorations and geometric forms of the tower itself, which includes seven-feathered star that symbolized the seven heavens in Islamic believes. Additionally, The 12 triangles around the main building, the rhombuses 30 to 30, the five- feathered star at the top, create a numerical pattern (12×30+5=365) that symbolizes the number of days of the year and the astronomer characteristic of the person buried., the game with numbers, proportions and regular numerical pattern with the aim of embodying the character of Khayyam mathematician, with modern language and the use of fractal geometry has occurred symbolically.

He used to repeatedly focus in his interviews and lectures that copy and adaptation from western architecture is not the only remedy, but the points is to have a proper understanding of the Iranian architecture and western technology, try to shape the architecture of Iran and he considered the narrow-minded mimicry of western architecture as the plague of contemporary architecture (Ghaseminia & Soltanzadeh, 2016: 43). Specifically, we can summarize Seyhoun's architectural style as he himself described his design process " an artist must not only be sufficiently familiar with his own field's technique, but also with methods that have been conventional in the past and read and analyze them to know where art has moved in that particular field to achieve its current condition, and what is his mission in the continuation of the evolution. Therefore, to contemplate and act correctly, one must be especially aware of the history and records of that specific subject. Because an artist or architect never leaves the mother's womb, one must go and study, be educated, touch the works made and become acquainted with them, why and under what conditions those buildings were made in such a way. And now that I want to design, what are my conditions and confidently, they are different from the previous ones. My acquaintance is a historical acquaintance, persistent and from the past to the present (Mujdehi, 2016: 10).

3. Research Literature

Various researches have been done about fractals, most of which are specific to the field of traditional Iranian architecture in interior cases. Recent global and similar studies on the subject of fractal definitions and the techniques used can be found in these studies: "Fractals, architecture and sustainability" in which writer deal with how fractals relate to stability and calculate the fractal dimensions of Robie house and Kandariya Mahadev temple, (Parashar & Bandyopadhyay, 2014), Rumiez in "Fractal architecture" investigated about natural environment as an inspiration and represented the theory of organic in new architecture (Rumiez, 2013). In the field of box-counting dimension Wu et.al have examined different examples of fractals to verify box-counting methods' (based on images) accuracy based on mathematical definitions (Wu et al, 2020). Additionally, Amongst the researches which have been done in the field of urbanization we can mentione "Box-counting dimension of fractal urban form: stability issues and measurement design" in which authors examine box-counting dimension for urban forms and also declare that it can be used for projecting the size and growth of the municipal's areas as well (Jiang & Liu, 2012). Other researches in this field are: "Introduction of fractals and fractional dimensions" (Delkhosh, 2017), "Fractal structure of the urban objects" (Kholladi, 2004), "Typology of Iranian architectural tomb buildings" represent definitions and classified tomb's architectural style's during Iran's architectural periods (Abedi & Ebrahimi, 2015).

One of the main reasons for conducting this research is the lack of study resources in fractals in contemporary Iranian architecture. It can be said that no contemporary Iranian building has been studied in terms of having a pattern of fractal geometry with the techniques used in this article to extract the fractal dimension. So, writing balanced and understandable research on the concepts of fractal forms in terms of architecture and mathematics using the mentioned techniques in contemporary Iranian architecture is one of the innovative approaches of this paper.

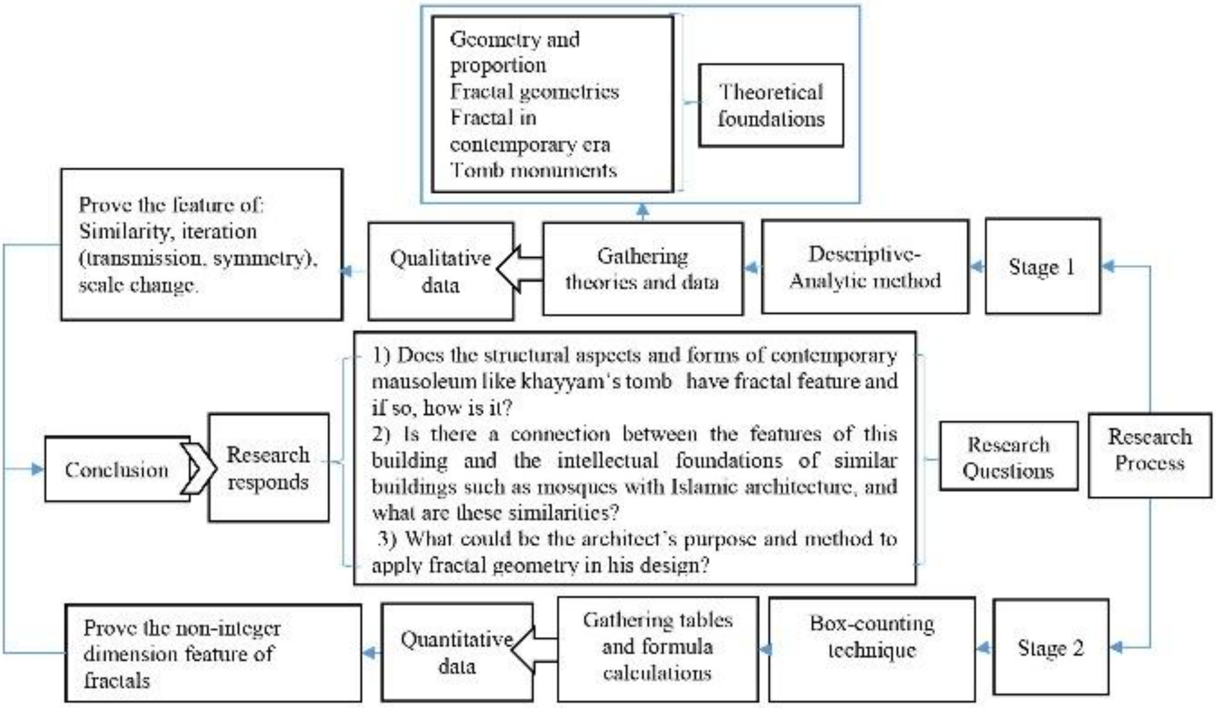


Fig. 3. The process of research to achieve this paper's results.

4. Research Methodology

The research method in this paper can be divided into two parts:

In the first part, with a practical purpose and a descriptive-analytic method, distinctive theories in this field have been collected by the method of library collection, from published articles and sources. Examines theories related to the geometry of fractal patterns from three aspects of line, surface and volume, and uses four common features between them as a basis for analyzes. Then, by matching and comparing during the analysis process, similar cases and in accordance with the proposed definitions were extracted from the monument. Meanwhile, computer software has been usedto access accurate drawings, all of which resulted in qualitative data that leads us to prove the hypothesis and answer research's questions.

In the computational part, we have calculated the fractal dimension with the help of box-counting technique. The elevation and plan, along with the site around it, are checkered into two-dimensional pages, and we have obtained data by counting the boxes containing the 2D images and placing the numbers in Mandelbrot's fractal dimension formula. Repeat this process by reducing the size of the squares, and then compare the numbers for each step with the smaller module. To sum up, this process is until the final conclusion is described in (Fig. 3):

In the final part of this section, using a comparative-deductive approach, similar patterns in Islamic architecture and the studied building are extracted and classified, further in the concluding part, it is used as a basis to achieve how to use fractal patterns in the recent era.

5. Research Findings

5.1. Box-counting dimension of Khayyam's mausoleum (based on mathematical formulas)

In order to reach our aims from mathematical points of view, we follow further procedure:

The connection between components of a fractal set and their scaling factor, can be presented as a formulas in mathematical language stated by Mandelbrot (Equation 1). Where a is the number of pieces and s is the reduction factor. For non-fractal the exponent D is an integer (Sala, 2001: 269).

$$a = \frac{1}{(S)^D}, D = \frac{\log\ (a)}{\log(\frac{1}{S})}$$

$$D_b = \frac{[\log(N(s_2\ ) - [\log(N(s_1\ )]}{[\log\left(N\left(\frac{1}{s_2}\right)\right) - \log\left(N\left(\frac{1}{s_1}\right)\right)]}$$

Equation 1: Formula for Box-counting dimension (Sala, 2001: 269).

First, we calculate Box-counting dimension of plan (Fig. 4). Quantitative data which we gain are located in table 2.

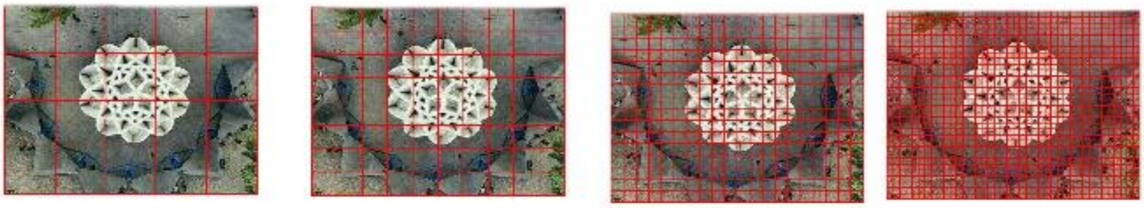


Fig. 4. Box-counting grids places over both plan and peripheral site.

Table 2

Box counting analyze of plan and its site.

step	Grid size (S)	Box count(N)	Log(s)	Log(N)
1	64	18	1,806	1,255
2	32	57	1,505	1,755
3	16	174	1,204	2,240
4	8	611	0,903	2,786

Now by increasing the size of the squares containing the building by increasing their scale, we compare the number of boxes to the smaller size in its following step, according to the formula:

$$D(64-\ 32) = \frac{\log(57) - \log(18)}{\log(64) - \log(32)} = \frac{0.5}{0.301} = 1.661$$

$$D(32-\ 16) = \frac{\log(174) - \log(57)}{\log(32) - \log(16)} = \frac{0.485}{0.301} = 1.611$$

$$D(6-\ 3) = \frac{\log(611) - \log(174)}{\log(16) - \log(8)} = \frac{0.546}{0.301} = 1.813$$

We repeat the same process for the elevation (Fig 5) and (Table 3):

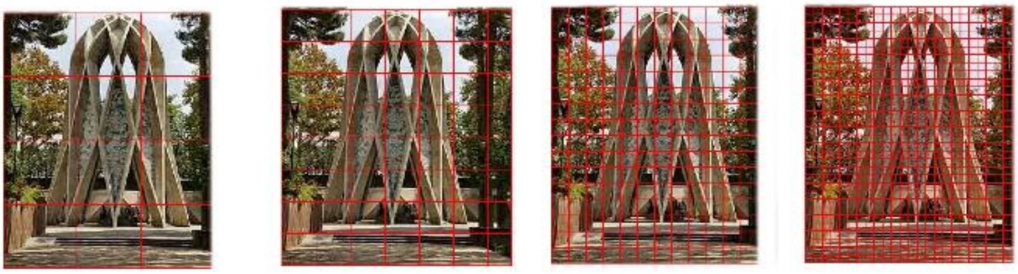


Fig. 5. Box-counting grids places over elevation of Khayyam's Mausoleum.

Table 3

Box counting analyze of elevation.

step	Grid size (S)	Box count(N)	Log(s)	Log(N)
1	64	12	1,806	1,079
2	32	30	1,505	1,477
3	16	98	1,204	1,991
4	8	326	0,903	2,517

$$D(64-\ 32) = \frac{\log(30) - \log(12)}{\log(64) - \log 32} = \frac{0.398}{0.301} = 1.322$$

$$D(32-\ 16) = \frac{\log(98) - \log(30)}{\log(12) - \log(6)} = \frac{0.514}{0.301} = 1.707$$

$$D(16-\ 8) = \frac{\log(329) - \log(98)}{\log(16) - \log(8)} = \frac{0.526}{0.301} = 1.747$$

Fractal has not been defined very strictly yet. But for most fractals, their fractal dimension is a non-integer value between 1 and 2, which is greater than the topological dimension (Wu et al, 2020, 3). The numbers that we have obtained, all of which are decimal, so we can concern it as a proof that the geometry of Khayyam's tomb is fractal.

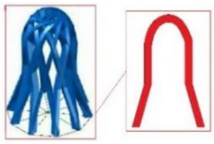
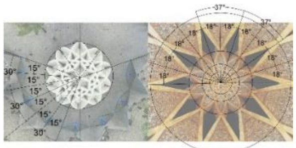
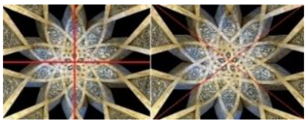
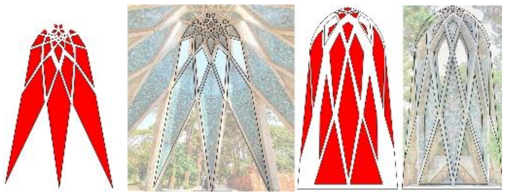
Also, according to our first table in which we has categorized both fractal's patterns and features, the Contor cubes' dimension showed very close figure (1.890) with our dimensions from checkered 2D images. So, with this detail, we come to the computational part of the answer to the first question of this research, which was previously raised.

5.2. Spatial and geometrical analysis of Khayyam's mausoleum (Visual analysis)

The purpose of this analysis is to find the properties of fractal forms by visual observation.

Visual examination of the building can be conducted by architects and viewers with great artistic sensitivity, who can reveal the main artistic characteristics of the building. This accompanies not only with incomparable aesthetic pleasure, but it can also encourage a fractal analysis research that leads and inspire rigorous mathematical calculation in fractal geometry (Shishin & Aldeen Ismail, 2016: 1088). As mentioned in the research method section, in the first stage, the theories were collected and then, by matching and analyzing the items in Khayyam's Mausoleum, we extracted fractal geometry patterns and proved them further (Table 4):

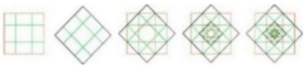
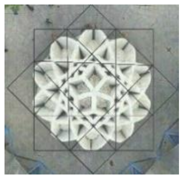
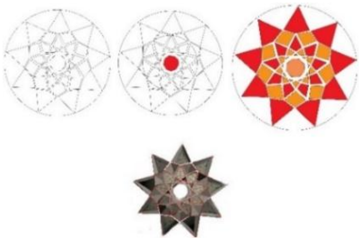
Table 4
Comparative-analytical analysis of theories and features of Khayyam's mausoleum.

Comparative results of matching extracted cases and theories	Extracted fractal items
- The general shape formed from a fixed module that by repeating itself has created the main form, which on the other hand has a structural role and provides the stability of the monument.	
- Patterns also exist in a scaling dimension, where similar forms occur at different magnification. When geometric self-similarity is defined on a hierarchy of scales, a self-similar fractal is created (Salingaros, 1999: 76).	
-"Similar" means that the relative proportions of the internal angles and shape's sides remain the same (Bovill, 1996: 15).	
- Dynamic architecture: The observer's eye is rhythmically drawn from the repeated patterns of rhombic-shaped structural and decorative elements to the center of the building (at the top of the dome). The features of self-similarity, change in scale, repetition and nonlinearity of rhombic elements according to the drawings are evident. Like the fractal model of the "Menger Sponge", which is formed by filling and emptying the original shape with similar shapes.	
- Self-similarity, Repetition and small-Scaling. The clockwise and counterclockwise spiral pattern, which starts from a center and continues in a spiral, originates from the fractal patterns in nature, such as the sunflower.	

In general, according to the words of the architect (Seyhoun), his complete familiarity with the concepts of Islamic architecture and fractals like these patterns is reveals to us, so that in this building we see a new expression of previous styles of past eras (for example application concrete in the body of building, which has previously been used in similar cases in traditional structures and ornaments). How to apply the elements of Islamic architecture with the inner concepts of unity in diversity, infinity, abstract representation of the universe and regular numerical patterns, all inspired by nature, is summarized below whith a comparative approach which we follow (Table 5):

Table 5
Features and similarities of geometric patterns of Khayyam's tomb with patterns of Islamic architectural decorations.

Feature of Islamic elements	**Features matching**	**Elements of Islamic architecture**	**Extracted fractal patterns from khayyam's mausoleum**
- Unity in plurality - Spiral growth from a single center (centrism) like spiral patterns of natural elements.	Self-similarity Proportion balance Centrism Iteration	 Ornaments of the Jameh Mosque of Isfahan	 Shamsa (seven-feathered star) under the dome.

The basis of Islamic architectural design: the growth of squares in harmony with the primary generating system (rotation and angle change).	Proportion Symmetry Iteration	 Islamic star formation by using the parameter of rotating the angle 45 degree (Broug, 2008).	
- Abstract representation of nature - Infinity (dome-shaped roof, tile decorations)	Self-similarity Iteration	 Dome of the sheikh lotfollah mosque	
- Unity in plurality: form growth under specified and regular angle and size - Infinity: Islamic star (symbol of eternity) shamsa - Inspired by nature - Dynamic architecture	Self-similarity Iteration Proportion and Symmetry dynamism	 Dome of the Jameh Mosque of Isfahan	

6. Conclusion

Fractal patterns were comprehensively studied in terms of the main features, computational techniques and their application in contemporary construction, and then we came to the traces of Islamic architecture, which is a manifestation of the artistic application of these elements in the tomb buildings. The process of conducting research and applying techniques in each stage was done with the aim of achieving answers to questions. 1) In order to answer the first question about the probability of a fractal pattern in this building, the present paper, while studying recent theories and assessing fractal models proposed to define fractal types, we based the four main features (Self-similarity, Iteration, small- scale and decimal dimension between them as the basis of our analysis. Prove the three main features first by visual analysis and finding common ground with the opinions expressed in the form of tables, and in the next step by Box-counting technique, calculate the fractal dimension of Khayyam's Mausoleum from two-dimensional plates, the fourth feature (non-integer dimension) is also proved. Because, as mentioned in the text, this aspect of fractals is their most important aspect, which provides various information about fractals (on average, quantitative data 1,695 and 1,592 were obtained for the plan and elevation at specific scales, which coordinated the fractal elements in plan and elevation that has led to the creation of balance and order in the building. In addition, the numbers calculated for different fractal models, including the two-dimensional model of Contor cubes sets (1.890), have a very close numerical correlation with the fractal dimension of the checkered 2D images in this study. Therefore, in general, both the decimal nature of the obtained dimension and the similarity of its numerical value with the calculated global numbers answered our first question in this paper.

2) In order to achieve the answer of second question, we came to the conclusion that the architect has adapted his work as in the past to the culture of its people by these items: precisely studying the architectural patterns of the Islamic era and identifying the current architecture, adapting his learnings to modern concepts, and decorating the form and geometry. Using elements such as: The use of elements such as the Islamic Star and Shamsa, which symbolizes the sky and stars in the architecture of Islamic mosques, and the preservation of the architectural style of tomb monuments in the form of 8 to 12 star-shaped structures that reach the unity point at the top like dome mosques with repetitive elements. All the efforts of the architect to create a concept have been in harmony with the intellectual foundations and architectural style of the Islamic period. Seyhoun's designs are not only in line with the architecture and modern sciences of the world, but also with the legacy of the architects of the past of his land. The extracted items that have common points with the definitions of fractal properties are described (see Fig. 6).
3) Regarding the architect's goals of applying fractal geometry, we reached the following conclusions: from an aesthetic point of view: the existence of regular proportions and mathematical relationships between the elements of this building with the aim of showing the personality of the buried person with an aesthetic approach in order to make tangible patterns understandable for visitors can be the designer's intentions to manifest his concept. It should be a channel for implementing the concept in the designer's mind because it has all the features proposed for this purpose. From the structural facets: the discussion of compatibility with nature and optimal use of space, which is one of the issues related to fractals, can be seen in the design of Khayyam's tomb. The columns, with their stable and branched form, are used both to distribute the structural forces in the best way and

as decorative elements for the construction of the tomb monument. In general, He has used fractal geometry in both static and structural aspects along with picturesque and eye-catching for the visitor. In other words, structures and decorations have become united and the most efficient space has been used, which today is one of the advantages of designing buildings with fractal forms in the world. Finally, according to the classification of Parashar, we demonstrate the method and manner of using fractals in Khayyam's Mausoleum (Fig. 7).

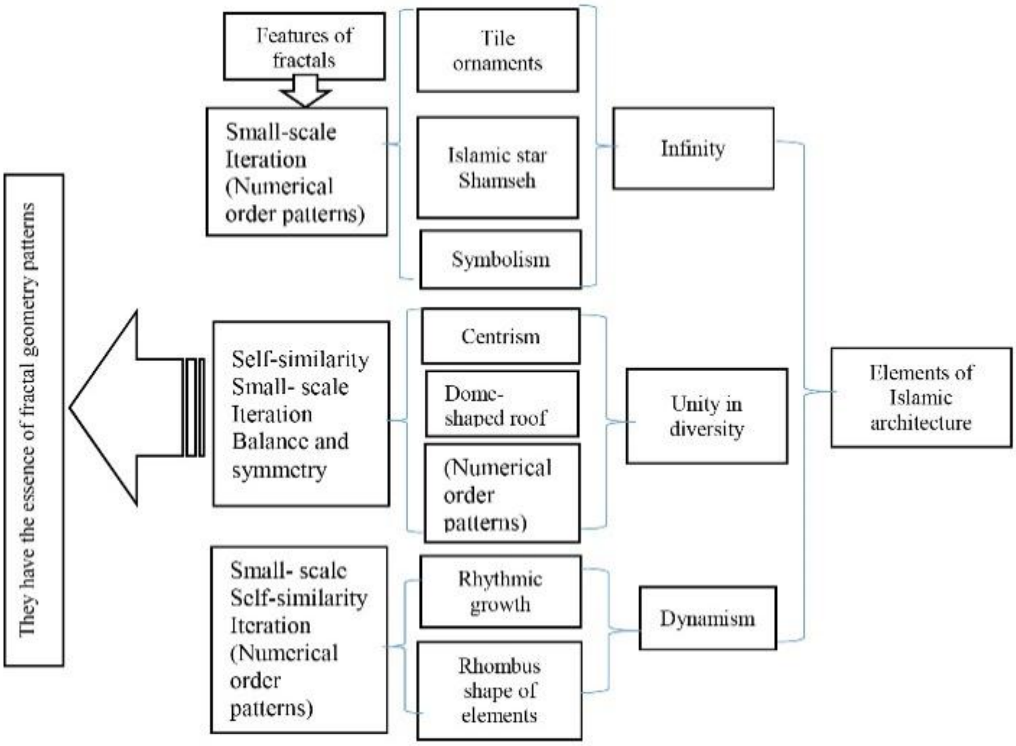


Fig. 6. Classification and study of architectural elements of the Islamic era in terms of fractal features.

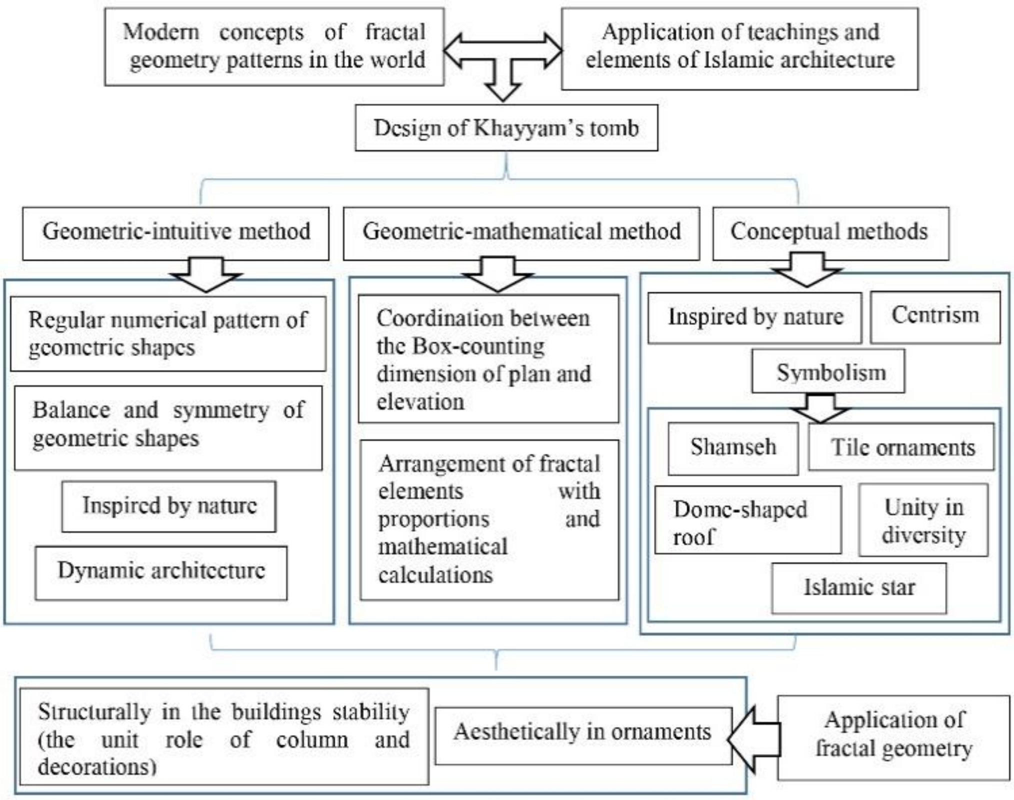


Fig. 7. An illustration of how fractal geometry is used in the Khayyam's Mausoleum

Endnotes

1. Regular polygons whose faces can consist of regular and identical polygons.
2. Three dimensional shapes with flat faces and edges that are polygon with a root in ancient Greek words.
3. Proportion which can be found in every natural elements in universe and from the quantitative point of view it is similar to 1,618.
4. A curve which move from a central point to the farther points by revolving around the point.
5. Series of numbers beginning with 0 and 1, where a number is the addition of the last two figures: 0, 1, 1, 2, 3, 5, 8, 13, … .

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