# Direct Exact Feedback Linearization based control of the of the Output Voltage in the Minimum phase DC-DC Choppers

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**Abstract**

*In this paper, a novel approach for control of the DC-DC buck converter in high-power and low-voltage applications is proposed. Designed method is developed according to state feedback linearization based controller ,which is able to stabilize output voltage in a wide range of operation. It is clear that in high-power applications, parasitic elements of the converter may become comparable with load value and hence, in this paper all of the converter parasitic elements are modeled during development of the controller and the state feedback coefficients, are optimized using the optimal control theory. In order to evaluate the accuracy and effectiveness of the proposed method, designed controller is simulated using MATLAB/Simulink toolbox. Presented simulation result proves that the developed controller has acceptable dynamic and steady-state responses. The controller is robust to the load variation and variety of parameters.*

**Keywords:** Nonlinear controller Buck converter; state feedback; parasitic elements; exact linearization; optimal control.

## 1- Introduction

The switched mode DC-DC converters are some of the simplest power electronic circuits which convert one level of electrical voltage into another level by switching action. DC-DC converters are used widely in different industrial applications like DC-motor speed control, maximum power point tracking of the photovoltaic systems and communication equipment, and also are used extensively in personal computers, computer peripherals, and adapters of consumer electronic devices to provide dc voltages. Buck, Boost and Buck-Boost converters are different topologies of the standard DC-DC choppers. From controller design viewpoint, these converters are nonlinear and time-variant [1]. Many control methods are used for control of switch mode DC-DC converters and the simple and low cost controller structure is always in demand for most industrial and high performance applications. Usually linear controllers are used to regulate output voltage of the converter, considering the linearized model [2].

Although the design and implementation of the linear regulators are completely simple, however its application may result in instability of the system in a wide range of operation. In fact, acceptable response may not be obtained using a linear controller in specific applications such as modern processors power supplies where converter load value is changed in a very wide range [3].Considering nonlinear characteristic of the DC-DC converters, it is better to use a nonlinear control technique in output voltage regulation. Main advantages of these controllers are their ability to react immediately to a transient condition. Main nonlinear controllers are: passivity-based control [4], sliding mode [5], adaptive back stepping [6] and feedback linearization.Sliding mode control is robust controller respect to uncertain parameters of the model and also, it is easy to implement [7].

However, variation of the switching frequency, the steady-state error and chattering are the main problems of sliding mode method in practical applications [8]. On the other hand, adaptive back stepping technique has been developed successfully in DC-DC converters [9].Reference [10] introduced the state feedback exact linearization method used in switching-mode power converter. The original nonlinear system was converted into linear system by proper nonlinear coordinate and state feedback transformation. Then, the control law was derived based on the linear optimal control method. But it wasn't included parasitic elements. In this study, the nonlinear model of the CCM Buck converter with attendance of parasitic elements is set up by state-space average method. Then, based on the state feedback exact linearization method of differential geometry theory, the controllability and the involute conditions are been testified. The original nonlinear system is converted into linear system by proper nonlinear coordinate and state feedback transformation. Then, the control law is derived based on the linear optimal control method. The dynamic system optimal problem is called the optimal control problem of the linear system and quadratic performance index if we select the quadratic function integral of the state variables and the control variables for a linear system. It also called a linear quadratic problem simply. Its optimal solution can expressed to a unified resolution expression, and a simple linear state feedback control law can be derived.

The optimal linear quadratic control has been successfully used in engineering practice widely.In section II a nonlinear model of the CCM Buck converter that includes parasitic elements and is suitable for the differential geometry theory is set up by state space average modelling method. In section III, a feedback control law is derived based on the state feedback linearization method. Then, a kind of quadratic performance index is proposed based on the passivity considerations. At last, the coefficients of feedback control law are optimized using the optimal quadratic control theory. The simulation results are presented in Section IV. Section V contains the conclusions.

## 2- The State-Space Averaging Model of CCM Buck Converter

The buck converter circuit converts a higher DC input voltage to lower DC output voltage. The basic buck DC-DC converter topology is shown in Fig.1. Considering inductor current ($x_1$) and output voltage

($x_2$) as state variables, the DC-DC buck converter can be modeled in the continuous conduction mode of operation using averaging technique.

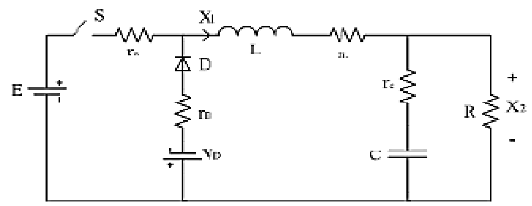


**Fig.1.** Main circuit of Buck converter

During ON-state of the power switch ( $0 < t < uT_s$ ), equivalent circuit of the converter is illustrated in Fig.2. In this condition, state-space model can be written as:

$$\begin{cases} \dot{x}_1 = \dfrac{1}{L}(-(r_s + r_L)x_1 - x_2 + E) \\ \dot{x}_2 = \dfrac{1}{C}x_1 - \dfrac{1}{RC}x_2 \end{cases} \quad (1)$$

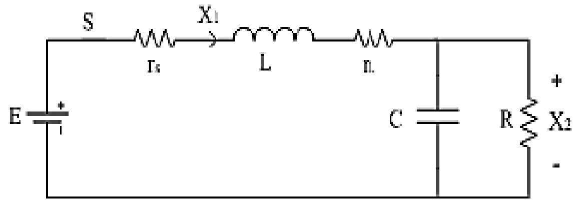


**Fig.2.** Main circuit of Buck converter when the switch is ON

Similarly as shown in Fig.1, during

( $uT_s < t < T_s$ ), power switch is OFF and converter model can be written as:

$$\begin{cases} \dot{x}_1 = -\dfrac{1}{L}\left((r_D + r_L)x_1 + x_2 + V_D\right) \\ \dot{x}_2 = \dfrac{1}{C}x_1 - \dfrac{1}{RC}x_2 \end{cases} \quad (2)$$

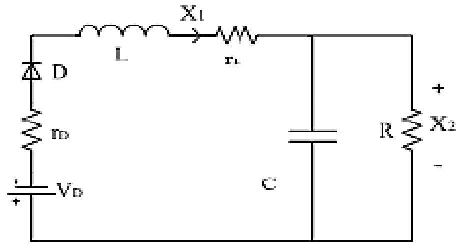


**Fig.3.** Main circuit of Buck converter when the switch is OFF

That using equations (3) averaged state-space model of the converter can be generally expressed as (4):

$$\begin{cases} A = UA_1 + (1 - U)A_2 \\ B = UB_1 + (1 - U)B_2 \end{cases} \quad (3)$$

$$\begin{cases} \dot{x}_1 = -\dfrac{1}{L}\left((r_S - r_D)u + r_D + r_L\right)x_1 \\ \qquad -\dfrac{1}{L}(x_2 - (\mathrm{E} + V_D)u + V_D) \\ \dot{x}_2 = \dfrac{1}{C}x_1 - \dfrac{1}{RC}x_2 \end{cases} \quad (4)$$

So using the technique of state-space averaging, the averaged state space model is given by:

$$\begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x) = x_2 - U_{ref} \end{cases} \quad (5)$$

$$f(x) = \begin{bmatrix} -\dfrac{1}{L}((r_D + r_L)x_1 + x_2 - V_D) \\ \dfrac{1}{C}x_1 - \dfrac{1}{RC}x_2 \end{bmatrix}$$

$$g(x) = \begin{bmatrix} -\dfrac{1}{L}((r_S - r_D)x_1 - E - V_D) \\ 0 \end{bmatrix}$$

$x = [x_1, x_2]$= $[i_L, v_o]$ is the state vector, $u$=$d$ where $d$ is the duty ratio of switch $S$, $U_{ref}$ is the expectation value of the output voltage.

## 3- System Design Based On State Feedback Exact Linearization Method

### 3-1-Principle of State Feedback Linearization Based on the Differential Geometry

**Definition (1):** For the SISO affine nonlinear system

$$\begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x) = x_2 \end{cases} \quad (6)$$

The necessary and sufficient conditions of state feedback linearization are :

**Condition (1.1):** For all $x$ in enclosure of $x^0$,the rank of matrix $[g(x) \quad ad_f g(x) \quad \cdots \quad ad_f^{n-1} g(x)]$ is equal to the dimension of the system (n).

**Condition (1.2):** The set of the vector fields $D = \{g(x) \quad ad_f g(x) \quad \cdots \quad ad_f^{n-2} g(x)\}$ is involutory in $x = x^0$ .

### 3-2-Controller Design for CCM Buck Converter

Based on the nonlinear model of CCM Buck converter showed as Fig.1 and the differential geometry theory, we can compute the Li bracket and Li derivatives as follow:

$$ad_f g(x) = \frac{\partial g(x)}{\partial x} f(X) - \frac{\partial f(x)}{\partial x} g(x) = \begin{bmatrix} (r_S - r_D)x_2 + (2r_D + r_L - r_S)V_D \\ +(r_D + r_L)E \\ \frac{1}{LC}\left((r_S - r_D)x_1 - E - V_D\right) \end{bmatrix} \quad (7)$$

The rank of matrix $[g(x) \quad ad_f g(x)]$ Equals to 2, and the system dimension is 2 as well, therefore the linearization condition (1.1) is satisfied.

As the system dimension n=2, the condition (1.2) is also satisfied. Hence, the CCM Buck converter can be linearized through a state feedback linearization.

Since

$$L_f h(x) = \frac{\partial h(x)}{\partial x} f(X) = \frac{1}{C} x_1 - \frac{1}{RC} x_2 \quad (8)$$

$$L_g h(x) = \frac{\partial h(x)}{\partial x} g(X) = 0 \quad (9)$$

$$L_g L_f h(x) = \frac{\partial L_f h(x)}{\partial x} g(X) = -\frac{1}{LC}\left((r_S - r_D)x_1 - E - V_D\right) \neq 0 \quad (10)$$

The system relative degree $r = 2 = n$. Based on Definition (1), the coordinate transformation of the CCM Buck converter is

$$\zeta = \begin{bmatrix} h(x) \\ L_f h(x) \end{bmatrix} = \begin{bmatrix} x_2 - U_{ref} \\ \frac{1}{C} x_1 - \frac{1}{RC} x_2 \end{bmatrix} \quad (11)$$

Therefore, the new linear system of ξ coordinate can be expressed as:

$$\dot{\zeta} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \zeta + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \nu \quad (12)$$

Where $\nu$ is the new input variable of the linear system (12). The feedback control law of original nonlinear system (5) is proposed as

$$u = \frac{-L^2{}_f h(x) + \nu}{L_g L_f h(x)} \quad (13)$$

Where the Li derivative is

$$L^2{}_f h(x) = \frac{\partial L_f h(x)}{\partial x} f(x) = -\frac{1}{LC}\left((r_D + r_L)x_1 + x_2 - V_D\right) + \frac{1}{RC^2} x_1 - \frac{1}{R^2 C^2} x_2 \quad (14)$$

### 3-3-Optimal Control and Selection of the Quadratic Performance Index Power Matrix Q and R

By nonlinear coordinate transformation (11) and the state feedback control law (13), the original nonlinear system (1) has been converted into a linear system. So, the synthesis question of the nonlinear system now can be transfer into the synthesis question of the linear system.

For a linear system

$$\begin{cases} \dot{\zeta} = A\zeta(\mathrm{t}) + \mathrm{B}\, \nu(t) \\ \zeta(t_0) = \zeta_0 \end{cases} \tag{15}$$

Defining a quadratic performance index

$$J = \frac{1}{2}\int_{t_0}^{\infty} \zeta^T Q\zeta + \nu^T R\nu \, dt \tag{16}$$

Where A and B are constant matrices, Q is a positive definite symmetric matrix or a positive semi-definite symmetric matrix, R is a positive definite symmetric matrix. The physical meanings of the performance index (16) are: the first item embody that the state error is close to zero during the control process and the terminal end, the second item could limit the amplitude of the control variable. Based on optimal quadratic control theory, the optimal control law is

$$\nu = -K\zeta \tag{17}$$

Where the gain matrix $K = B^T P$ and $P$ is the positive definite symmetric solution of the *Riccati* equation

$$PA + A^T P - PBR^{-1}B^T + Q = 0 \tag{18}$$

The basic ideas of passivity-based control method are based on the modification of the total energy function of the system and enhancing the dissipation structure of the system through suitable 'damping injections'. These procedures can force the total energy of the system to track the energy function we desired and make the whole closed-loop control system be passivity. Then, the state variables asymptotically stabilize to the desired equilibrium and the asymptotically stable closed-loop behavior can be achieved.

Based on passivity consideration, in this paper we construct the closed-loop system energy function:

$$H = \zeta^T Q\zeta = \frac{1}{2}L\Delta i_L^2 + \frac{1}{2}C\Delta v_O^2 \tag{19}$$

From equation (11), we get

$$\begin{cases} \zeta_1 = h(x) = x_2 - U_{ref} = \Delta v_O \\ \zeta_2 = L_f h(x) = \dfrac{1}{C}x_1 - \dfrac{1}{R_L C}x_2 \\ \quad = \dfrac{1}{C}\left(\Delta i_L - \dfrac{\Delta v_O}{R_L}\right) \\ \Delta i_L = x_1 - \dfrac{U_{ref}}{R_L} \end{cases} \tag{20}$$

A matrix form of (20) is given by

$$\zeta = \begin{bmatrix} 0 & 1 \\ \dfrac{1}{C} & -\dfrac{1}{R_L C} \end{bmatrix} \begin{bmatrix} \Delta i_L \\ \Delta v_O \end{bmatrix} \tag{21}$$

So corresponding to (19), Q is given by

$$Q = \begin{bmatrix} (\dfrac{L}{2R_L^2} + \dfrac{C}{2}) & \dfrac{LC}{2R_L} \\ \dfrac{LC}{2R_L} & \dfrac{LC^2}{2} \end{bmatrix} \tag{22}$$

And select another weighted matrix

$$R = (LC)^3 \tag{23}$$

From (12), we get

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{24}$$

Then, take A, B, Q and R into the Riccati equation, we get P and the feedback gain matrix K. Take the K into (17) and (20), we get the feedback control law u of the original nonlinear system (1).[1]

$$u = (-LC)/(((r\_S - r\_D)\, x\_1 - E - V\_D))[((RC(r\_D + r\_L) - L))/(RLC\hat{}2)\, x\_1 + (1/LC + 1/(R\hat{}2\, C\hat{}2))\, x\_2 - V\_D/LC + (-K\_1\, \zeta\_1 - K\_2\, \zeta\_2)] \tag{25}$$

In order to test the effectiveness of the control strategy and the feasibility of the selected power matrixes of the quadratic optimal performance index, simulations are performed for the closed-loop behavior of a CCM Buck circuit by MATLAB software.

## 4- SIMULATION RESULTS

The circuit parameters are as follows: input voltage $E$=32V, output voltage $v_O$=15V, load $R$L=10$\Omega$, $f_s$=100kHz, input inductor $L$=2mH, output capacitor $C$=10$\mu$F.Parasitic elements of circuit are: $r_L = 0.2$, $r_S = 0.1$, $r_D = 0.001$ and voltage drop of diode is $V_D = 0.8$ .Using MATLAB instruction *K_lqr(A,B,Q,R),* we get the feedback gain matrix $K = [k_1 \quad k_2]^T = [1.369 * 10^9 \quad 123445]^T$

### 4-1-First test

In the first test resistor of load is R=10$\Omega$ and input voltage (E) is constant in 32 volt. Fig.4 shows the dynamic behavior of inductor current and Fig.6 shows the dynamic behavior of output voltage, the setting time is extremely short about 0.5ms, there is no overshoot of the output voltage. Output ripple is rather small as show in Fig.5.

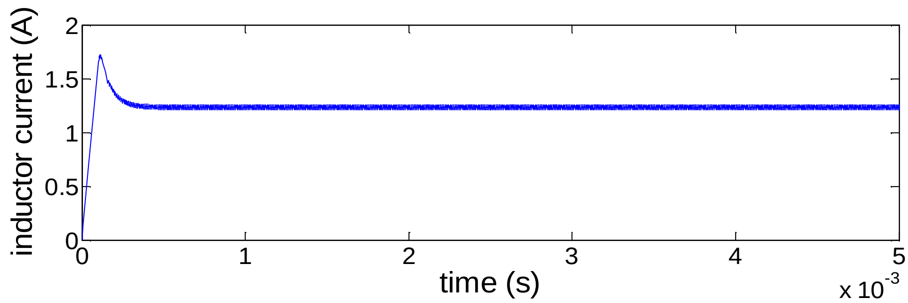


**Fig.4.** Dynamic response of Inductor current ($i_L$)

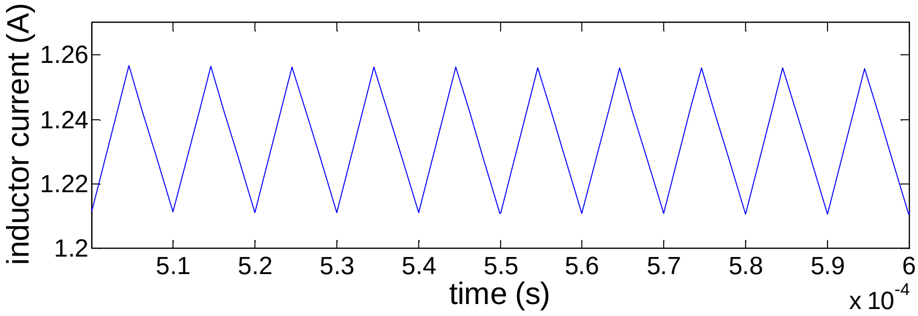


**Fig.5.** Zoom of Inductor current

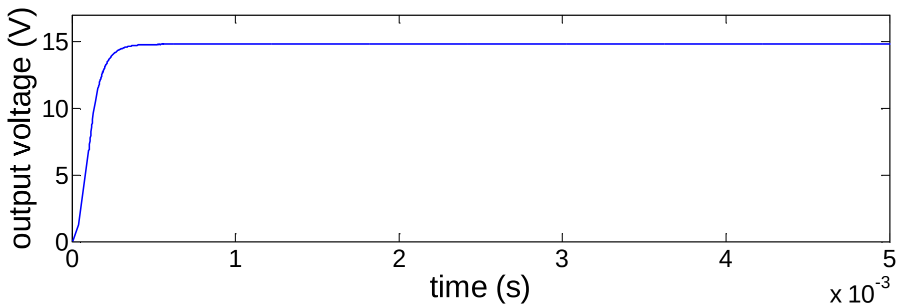


**Fig.6.** Dynamic response of output voltage $(v_O)$

4-2-Second test

In the second test, we want to test the dynamic response of system to load variation while load varies between 10$\Omega$ and 20$\Omega$ periodically. Fig.7 shows the Load resistance variation.Fig.8 shows the dynamic behavior of inductor current $(i_L)$, and Fig.9 shows the dynamic behavior of output voltage $(v_O)$ in the case of load variation.Inductor current $(i_L)$ and output voltage $(v_O)$ response fast about 0.5mS with small overshoots. So, it can be concluded that the closed-loop system based on the control scheme we have presented shows excellent stability and robustness to the load variation.

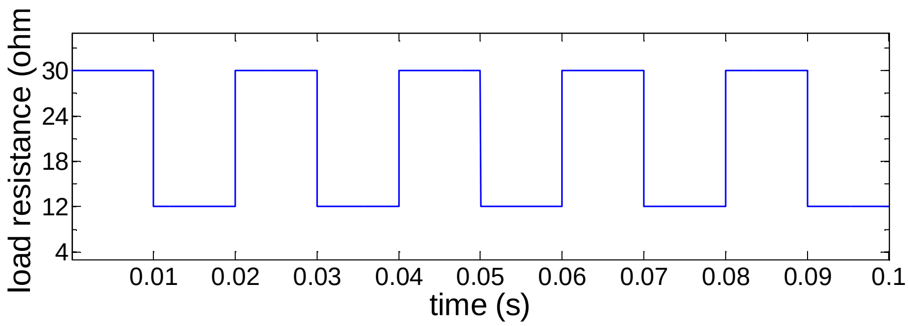


**Fig.7.** Load resistance variation

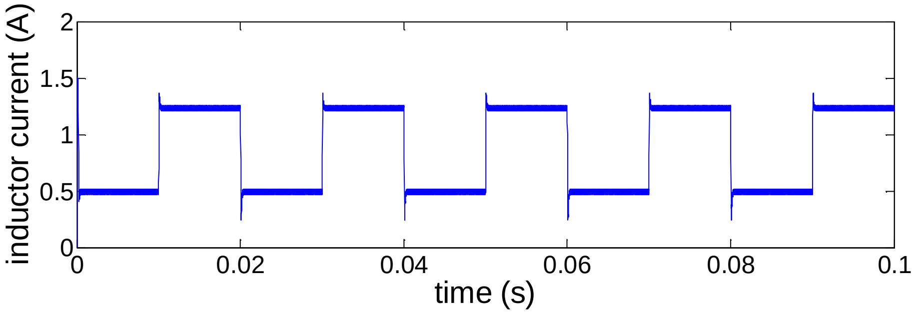


**Fig.8.** Dynamic response of Inductor current $(i_L)$ to load variation

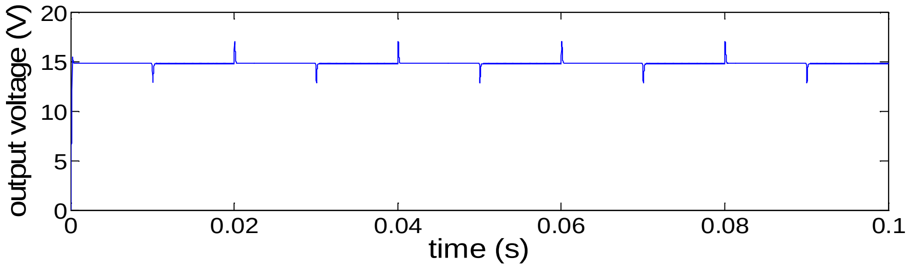


**Fig.9.** Dynamic response of output voltage ($v_O$) to load variation

4-3-Third test

In this test we examine the transient response to input voltage variation. Fig.10 shows the Input voltage variation that varies between 32V and 42V periodically. Fig.11 shows the dynamic behavior of inductor current ($i_L$) and Fig.12 shows the dynamic behavior of output voltage ($v_O$) in the case of input voltage variation. When input voltage varies between 32V and 42V periodically, the inductor current ($i_L$) and output voltage ($v_O$) are almost not be influenced. The closed-loop system shows strong robustness to the input voltage variation. This characteristic can be explained by the feedback control law (20) theoretically. The input voltage (E) acts as the denominator in equation (20), when it varies, the nonlinear feedback control law will automatically adjust the duty ratio of switch S direct to the variation and the output voltage can.

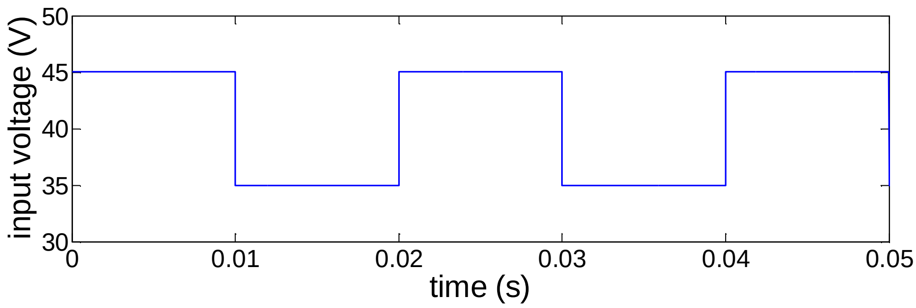


**Fig.10.** Input voltage variation

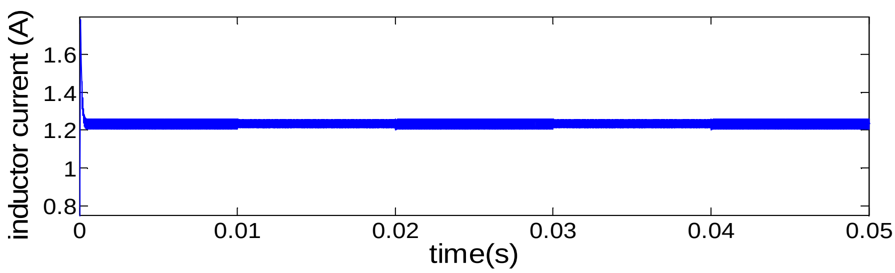


**Fig.11.** Dynamic response of Inductor current ($i_L$) to Input voltage variation

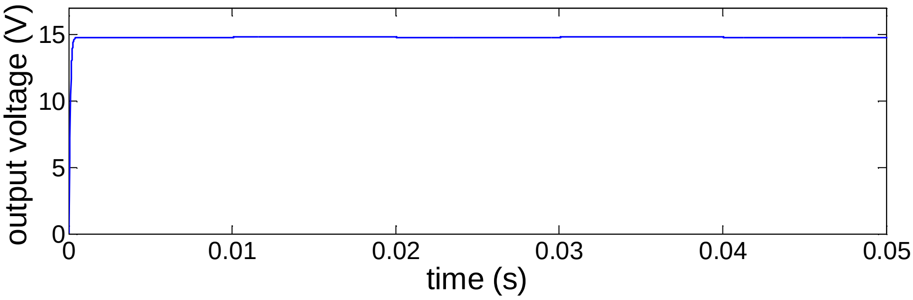


**Fig.12.** Dynamic response of output voltage $(v_O)$ to Input voltage variation

## 5- Conclusion

State feedback exact linearization control method has been applied in power electronics recently and proven to have overcome the shortcomings of the traditional linear control method used in switching-mode power converters in route. The nonlinear model for the CCM Buck converter is set up using state-space averaging theory and effect of the parasitic elements are modeled during state-space modeling of the converter. The nonlinear coordinate transformation and the state feedback control law are derived based on the state feedback linearization theory of differential geometry theory and the state feedback coefficients, are optimized using the optimal control technique. Further, based on passivity control theory, a quadratic performance index is proposed, and the weighted matrixes Q and R are presented also.

The closed-loop system shows strong robust to load variation and input voltage variation. Simulation results verify the correctness of the control strategy and the feasibility of the selected weighted matrixes. The proposed control law has the advantages of large signal stability, small overshoots of state variables, and short setting time.

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